Higher and partial angular moments $B \to K^{(*)}ll$

CP³ Origins
Cosmology & Particle Physics

Centre for Theoretical Physics · Higgs

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worked based on J. Gratrex, M. Hopfer and RZ , arXiv:1506.03970
"Generalised helicity formalism, higher moments in $B \to K_{J_K}(\to K\pi)\bar{\ell}_1\ell_2$"

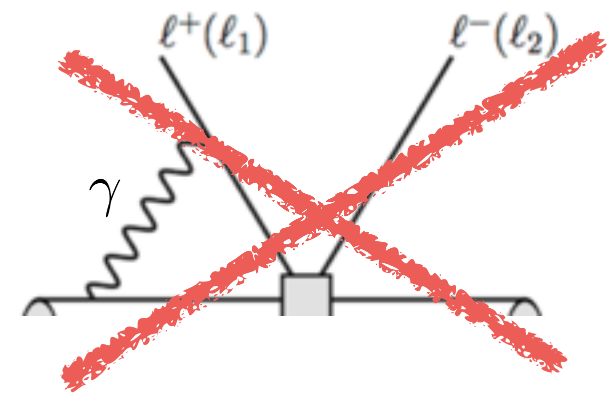
3 -5 Nov Implication of LHCb measurements (CERN)

Talk structure

Literature standard

[A] Angular distributions in LFA
(**lepton factorisation approximation**) using dim-6-H_{eff}

"lepton do not reinteract with process (broken QED)"



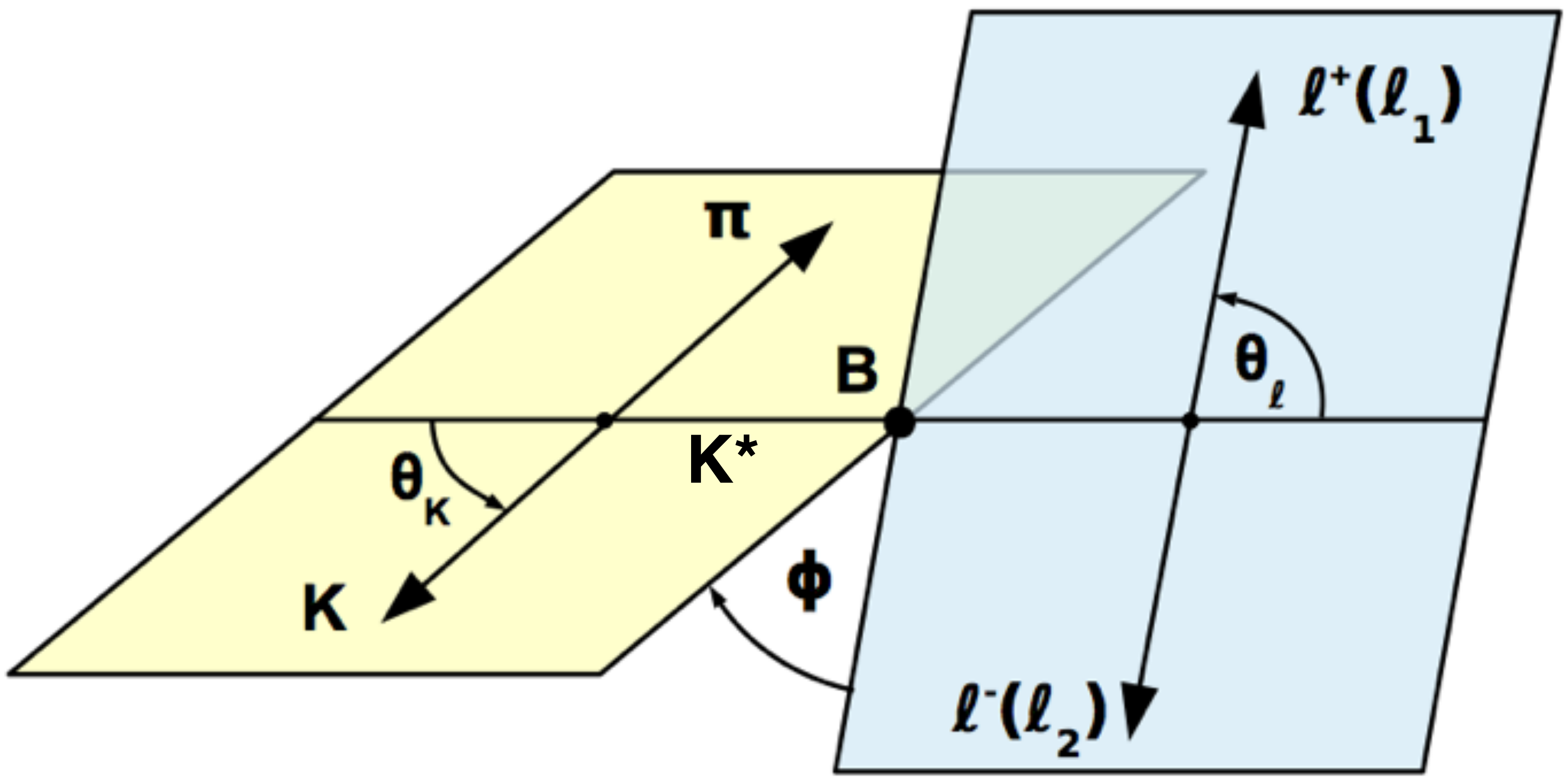
[B] **method of (partial) and higher angular moments**
"diagnosing some flavour-anomalies"

idea: LFA and QED-corrections differ in moments
⇒ assess size of QED through higher moments

Conclusions and summary

The decay topology B → V(→SS)l1 l2

For $\bar{B} \to \bar{K}^*(\to \bar{K}\pi)\ell^+\ell^-$ in particular



- **Kπ-pair** coming from K* is in **p-wave** (L=1) at **amplitude level**

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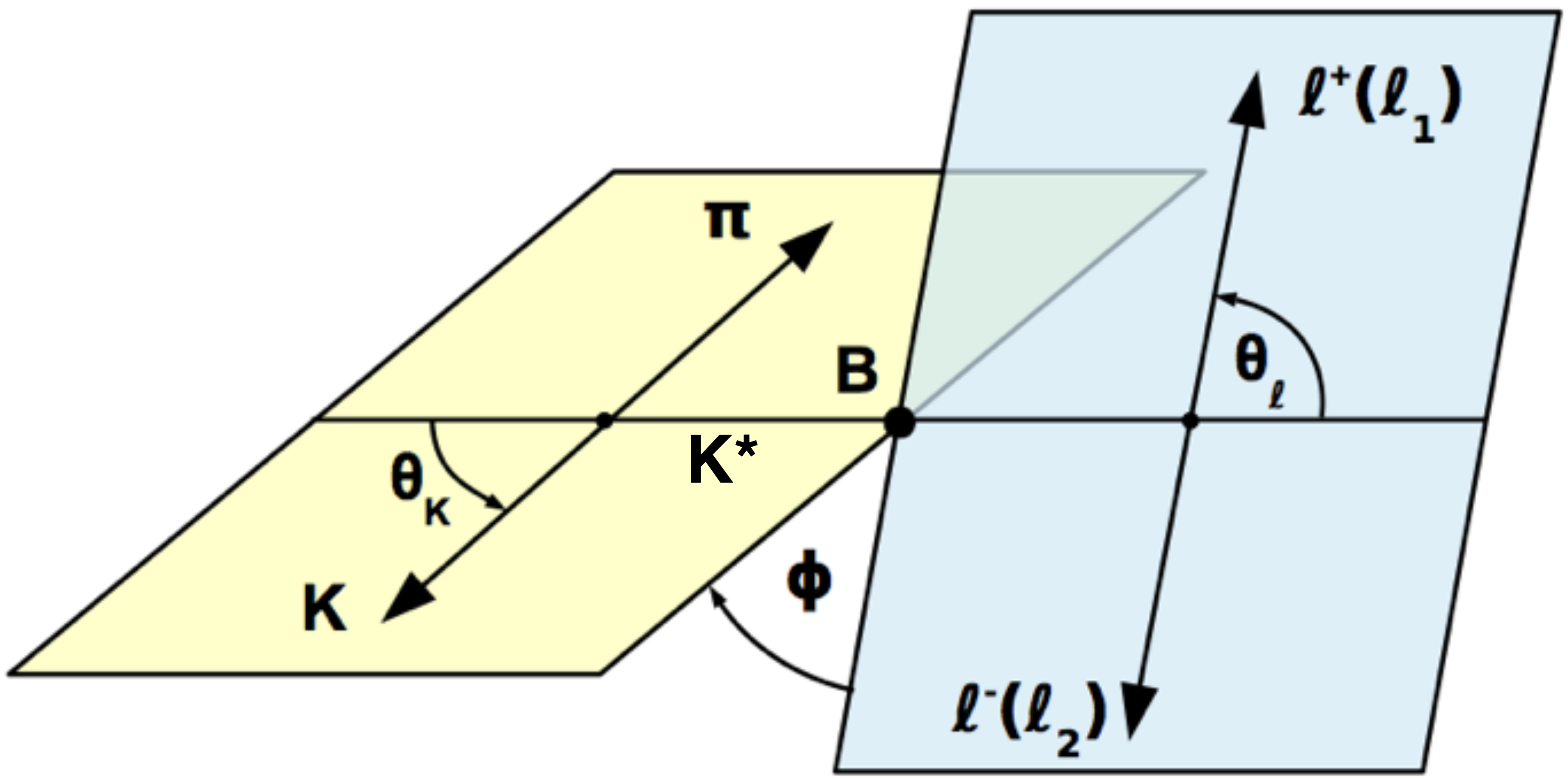

- **Kπ-pair** coming from K* is in **p-wave** (L=1) at **amplitude level**

what about lepton pair?

- principle no restriction - specifying approximation crucial

Lepton factorisation approximation (LFA)

- Heff of dim=6 with 10 operators

$$H^{\rm eff} = -\frac{4G_F}{\sqrt{2}}\frac{\alpha}{4\pi}V_{\rm ts}V^*_{\rm tb}\sum_{i=V,A,S,P,\mathcal{T}}(C_iO_i + C'_iO'_i)\;.$$

$$O_{S(P)} = \bar{s}_L b\,\bar{\ell}(\gamma_5)\ell\;, \qquad O_{V(A)} = \bar{s}_L\gamma^\mu b\,\bar{\ell}\gamma_\mu(\gamma_5)\ell\;,$$

$$O_{\mathcal{T}} = \bar{s}_L\sigma^{\mu\nu}b\,\bar{\ell}\sigma_{\mu\nu}\ell\;, \qquad O' = O|_{s_L\to s_R}$$

Literature standard & clearly dominant

SM: $C_V=C_9$ + long-distance; $C_A=C_{10}$ are relevant

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- lepton pair restricted to **S-** and **P-wave** at amplitude level in LFA

since decay rate square amplitude $\Longrightarrow$
$\sin(\theta_{K,l})^2$ $\cos(\theta_{K,l})^2$ - maximum-powers

[A] Angular distributions in lepton factorisation approximation

Differential decay rate

$$\frac{32\pi}{3}\frac{d^4\Gamma}{dq^2\ d\cos\theta_\ell\ d\cos\theta_K\ d\phi} = \mathrm{Re}\Big[G_0^{0,0}(q^2)\Omega_0^{0,0} + G_0^{0,1}(q^2)\Omega_0^{0,1} + G_0^{0,2}(q^2)\Omega_0^{0,2} + G_0^{2,0}(q^2)\Omega_0^{2,0} + G_0^{2,1}(q^2)\Omega_0^{2,1} + G_1^{2,1}(q^2)\Omega_1^{2,1} + G_0^{2,2}(q^2)\Omega_0^{2,2} + G_1^{2,2}(q^2)\Omega_1^{2,2} + G_2^{2,2}(q^2)\Omega_2^{2,2}\Big],$$

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$$\Omega_m^{l_K,l_\ell} \equiv D^{l_K}_{m,0}\left((\phi,\theta_K,-\phi)\right) D^{l_\ell}_{m,0}\left((0,\theta_\ell,0)\right)$$

$$D^{l}_{m,0}\left(\phi,\theta,-\phi\right) = \sqrt{\frac{(l-m)!}{(l+m)!}}P_{lm}(\cos\theta)e^{-im\phi}\ .$$

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$$G_2^{2,2} \sim \left(H_+^V\bar{H}_-^V + H_+^A\bar{H}_-^A - 2\left(H_+^T\bar{H}_-^T + 2H_+^{T_t}\bar{H}_-^{T_t}\right)\right)$$

Hadronic helicity amplitudes e.g. $H_\lambda^{V[A]} = \langle\bar{K}^*(\lambda)|\bar{s}\gamma^\mu[\gamma_5]b|\bar{B}\rangle\epsilon^*(\lambda)_\mu$

For completeness: connection standard literature-notation

- standard notation goes back at least to Treiman & Pais '68
  "pion phase shift information from Kl_4 decays"

$$\frac{8\pi}{3}\frac{d^4\Gamma}{dq^2\ d\cos\theta_\ell\ d\cos\theta_K\ d\phi} = (g_{1s} + g_{2s}\cos 2\theta_\ell + g_{6s}\cos\theta_\ell)\sin^2\theta_K +$$
$$(g_{1c} + g_{2c}\cos 2\theta_\ell + g_{6c}\cos\theta_\ell)\cos^2\theta_K +$$
$$(g_3\cos 2\phi + g_9\sin 2\phi)\sin^2\theta_K\sin^2\theta_\ell +$$
$$(g_4\cos\phi + g_8\sin\phi)\sin 2\theta_K\sin 2\theta_\ell +$$
$$(g_5\cos\phi + g_7\sin\phi)\sin 2\theta_K\sin\theta_\ell$$

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$$G_0^{0,0} = \frac{4}{9}\left(3\left(g_{1c}+2g_{1s}\right) - \left(g_{2c}+2g_{2s}\right)\right)\,,\quad G_0^{0,1} = \frac{4}{3}\left(g_{6c}+2g_{6s}\right)\,,\quad G_0^{0,2} = \frac{16}{9}\left(g_{2c}+2g_{2s}\right)\,,$$

$$G_0^{2,0} = \frac{4}{9}\left(6\left(g_{1c}-g_{1s}\right) - 2\left(g_{2c}-g_{2s}\right)\right)\,,\quad G_0^{2,1} = \frac{8}{3}\left(g_{6c}-g_{6s}\right)\,,\quad G_0^{2,2} = \frac{32}{9}\left(g_{2c}-g_{2s}\right)\,,$$

$$G_1^{2,1} = \frac{16}{\sqrt{3}}\underbrace{(g_5 + i g_7)}_{=\mathcal{G}_5}\,,\quad G_1^{2,2} = \frac{32}{3}\underbrace{(g_4 + i g_8)}_{=\mathcal{G}_4}\,,\quad G_2^{2,2} = \frac{32}{3}\underbrace{(g_3 + i g_9)}_{=\mathcal{G}_3}$$

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Convenience & illustration of $G_m^{lk,ll'}$'s

1. endpoint symmetries Hiller RZ'13

kinematic endpoint K* enhanced symmetry (threshold expansion in effective theory)

helicity amplitudes: $H_+^{V,A} = H_-^{V,A} = -H_0^{V,A}$

angular distribution two (one SM) parameters

$$G_0^{0,0} \neq 0\,, \quad G_0^{2,2} \to \mathrm{Re}[G_0^{2,2}]\,, \quad G_1^{2,2} \to -2\mathrm{Re}[G_0^{2,2}]\,, \quad G_2^{2,2} \to 2\mathrm{Re}[G_0^{2,2}]$$

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2. *examples of 12-angular observables in literature*

$$\langle P_2 \rangle_{\mathrm{bin}} = \frac{\left\langle 2G_0^{0,1} - G_0^{2,1} \right\rangle_{\mathrm{bin}}}{3\mathcal{N}_{\mathrm{bin}}}\,, \quad \langle P_4' \rangle_{\mathrm{bin}} = \frac{\left\langle \mathrm{Re}\left[G_1^{2,2}\right] \right\rangle_{\mathrm{bin}}}{4\mathcal{N}'_{\mathrm{bin}}}\,, \quad \langle P_5' \rangle_{\mathrm{bin}} = \frac{\left\langle \mathrm{Re}\left[G_1^{2,1}\right] \right\rangle_{\mathrm{bin}}}{2\sqrt{3}\mathcal{N}'_{\mathrm{bin}}}\,,$$

$$\left\langle \frac{d\Gamma}{dq^2} \right\rangle_{\mathrm{bin}} = \frac{3}{4} \left\langle G_0^{0,0} \right\rangle_{\mathrm{bin}}\,, \quad \langle P_6' \rangle_{\mathrm{bin}} = \frac{\left\langle \mathrm{Im}\left[G_1^{2,1}\right] \right\rangle_{\mathrm{bin}}}{2\sqrt{3}\mathcal{N}'_{\mathrm{bin}}}\,, \quad \langle A_{\mathrm{FB}} \rangle_{\mathrm{bin}} = \frac{1}{2} \frac{\left\langle G_0^{0,1} \right\rangle_{\mathrm{bin}}}{\left\langle G_0^{0,0} \right\rangle_{\mathrm{bin}}}\,,$$

forward backward type observables l_l=1 (odd in θ_l)

Some references on angular distributions

$O_{V,A}$ SM	$m_\ell = 0$	Krüger,Sehgal,Sinha,Sinha	'99
$O_{V,A}$ SM	$m_\ell \neq 0$	Faessler,Gutsche, Ivanov, Körner, Lyubivitskij	'02
idem		Krüger, Matias	'05
SM $B \to (K, K^*, K_2)$	$m_\ell = 0$	Lu, W.Wang	'11
add $O_{S,P}$	$m_\ell \neq 0$	Altmannshofer et al	'08
$B \to (K^{*,0}, K^*)$ 'S-wave	$m_\ell = 0$	Becirevic, Tayduganov	'12
add $O_{\mathcal{T}}$	$m_\ell \neq 0$	Gosh et al/Bobeth et al	'10'12
SM time-dependent	$m_\ell = 0$	Descotes-Genon, Virto	'15
all	$m_{\ell_1} \neq m_{\ell_2} \neq 0$*	our work	'15

in $G_m^{lk,ll}$ - basis expression relatively compact nevertheless provide mathematica notebook in arxiv-file results in *Mathematica notebook*

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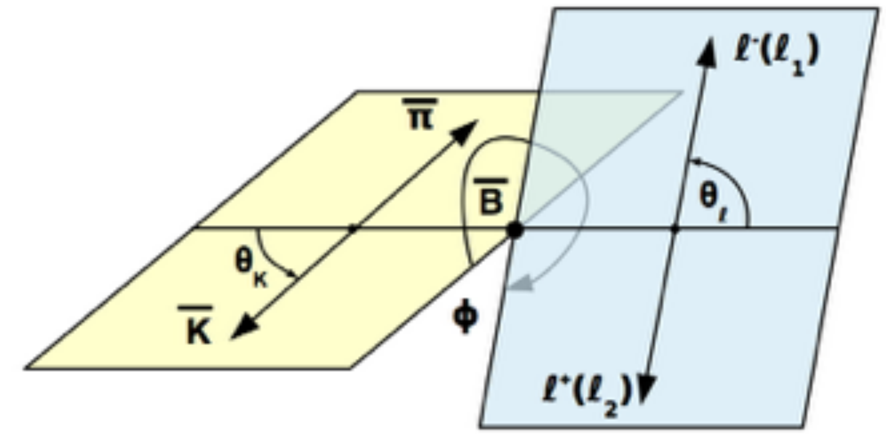
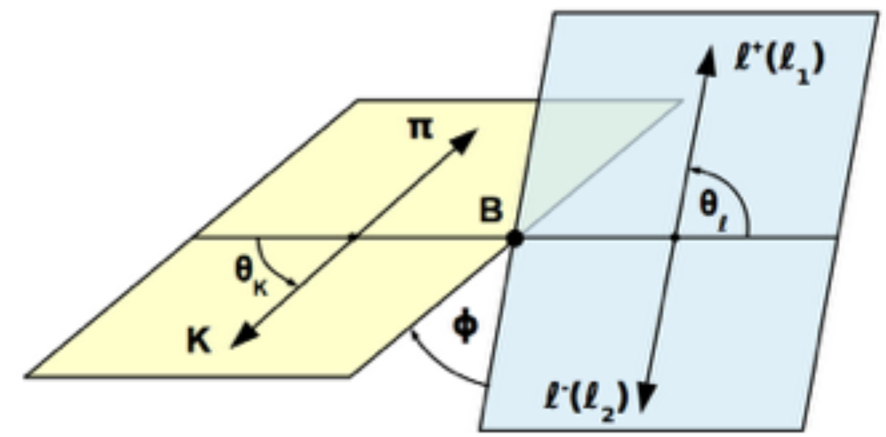
*$m_{l1} \neq m_{l2}$ useful for semileptonic & interesting for lepton flavour violation $B \to K\mu e$

Note on conventions

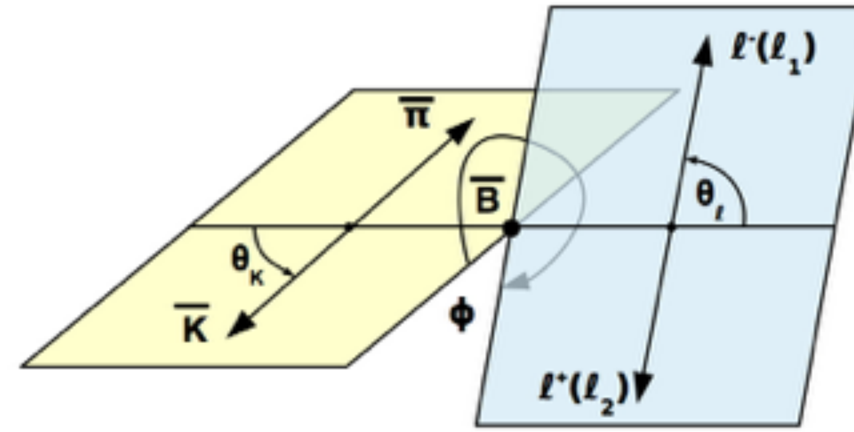
- need to define angles of B and anti-B decay
- we follow **LHCb conventions**:

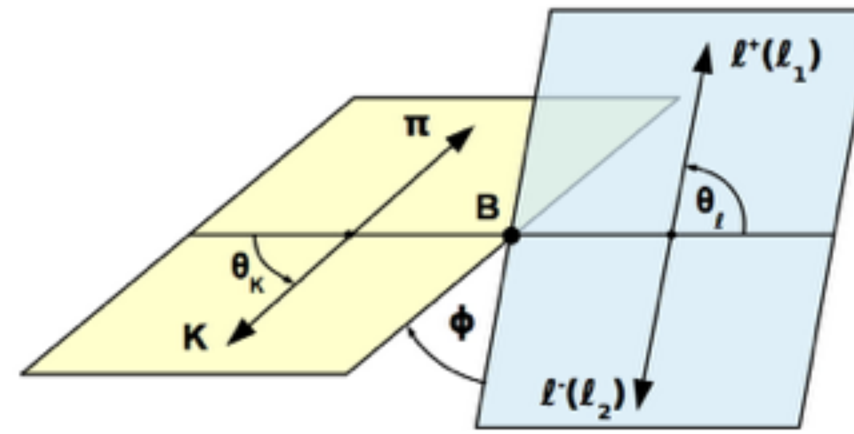




Note on conventions

- need to define angles of B and anti-B decay
- we follow **LHCb conventions**:
- **"theorist's conventions"** differ
  by matching our calculation we
  get the following diagram:





$$B \to K^*\ell\ell|_{\mathrm{LHCb}} \xrightarrow{g_{4,5,9} \to -J_{4,5,9}} B \to K^*\ell\ell|$$

$$B \to K^*\ell\ell|_{\mathrm{LHCb}} \xrightarrow{g_{7,8,9} \to -g_{7,8,9}} \bar{B} \to \bar{K}^*\ell\ell|_{\mathrm{LHCb}}$$

$$B \to K^*\ell\ell| \xrightarrow{J_{5,6,8,9} \to -J_{5,6,8,9}} \bar{B} \to \bar{K}^*\ell\ell|_{\mathrm{theory}}$$

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1) **differ** in translation in sign in **g(J)**$_{789}$: (i.e. $\phi \to -\phi$; affects P_6', P_8', S_9)
2) does not affect *current fits* but important when weak or strong phases included

How compute: 2 methods

- **Dirac-trace technology** (parameterisation of momenta - say in B-frame)

$$(\ell_2)^\mu = (f_\ell(E_2, q_0, -q_z), -|\vec{p}_\ell| \sin\theta_\ell \cos\phi, +|\vec{p}_\ell| \sin\theta_\ell \sin\phi, f_\ell(E_2, q_z, -q_0)) \ ,$$
$$(p_K)^\mu = (f_{K^*}(E_K, p_0, q_z), -|\vec{p}_K| \sin\theta_K, 0, -f_{K^*}(E_K, q_z, p_0)) \ ,$$

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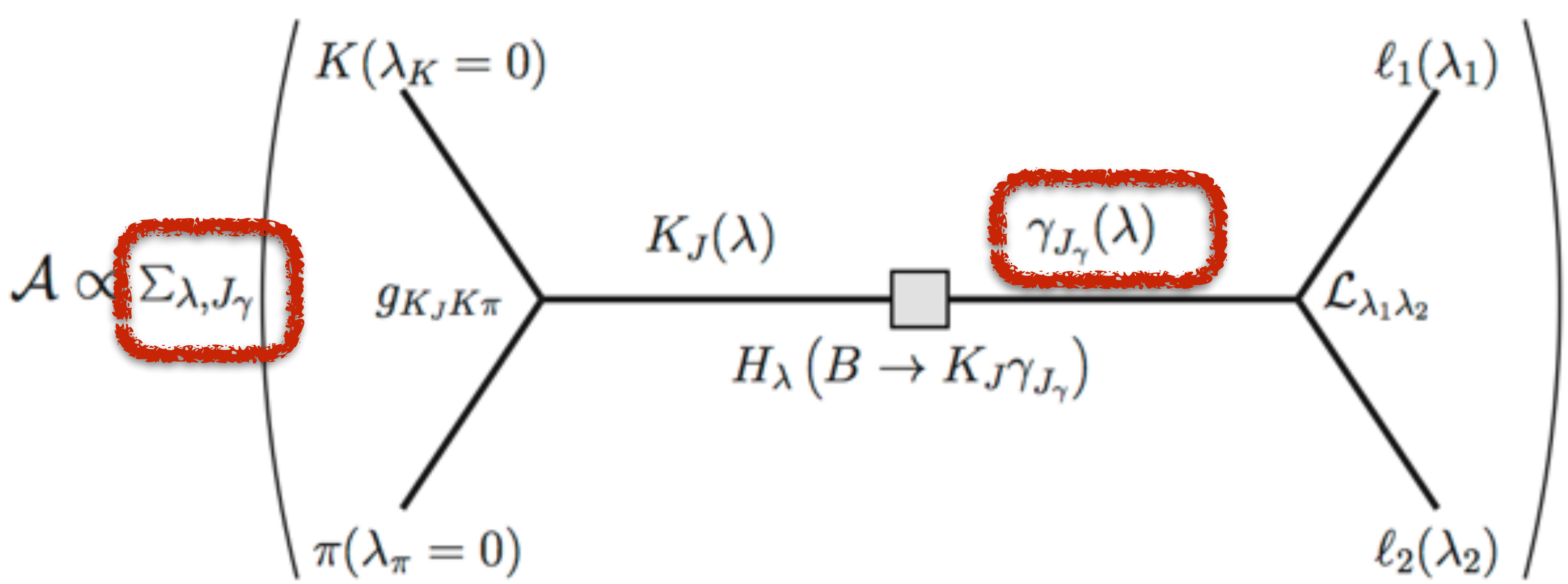
- **Jacob-Wick-technology** (use SO(3)/Wigner representation matrices)

generalised standard formalism: $B \to K_J(\to K\pi)\,\gamma^*(\to l_1 l_2)$ by decomposing SO(3,1) tensors into SO(3) irreps and summing J_γ (up to spin 2)

$$\underbrace{g_{\mu\nu}}_{SO(3,1)_{(j_1,j_2)=(\frac{1}{2},\frac{1}{2})}} = \underbrace{q_\mu q_\nu}_{\mathrm{SO(3)_{j=0}}} - \sum_{\lambda\in\{\pm,0\}} \underbrace{\omega_\mu(\lambda)\omega^*_\nu(\lambda')}_{\mathrm{SO(3)_{j=1}}}$$



[B] Method of (partial) and higher angular moments

"going beyond likelihood-fit which in standard form assumes LFA"

1. *focus on* ***B→Kll*** *(**1-angle**)*
2. *briefly discuss* ***B→K*(→Kπ)ll*** *(**3-angles**)*

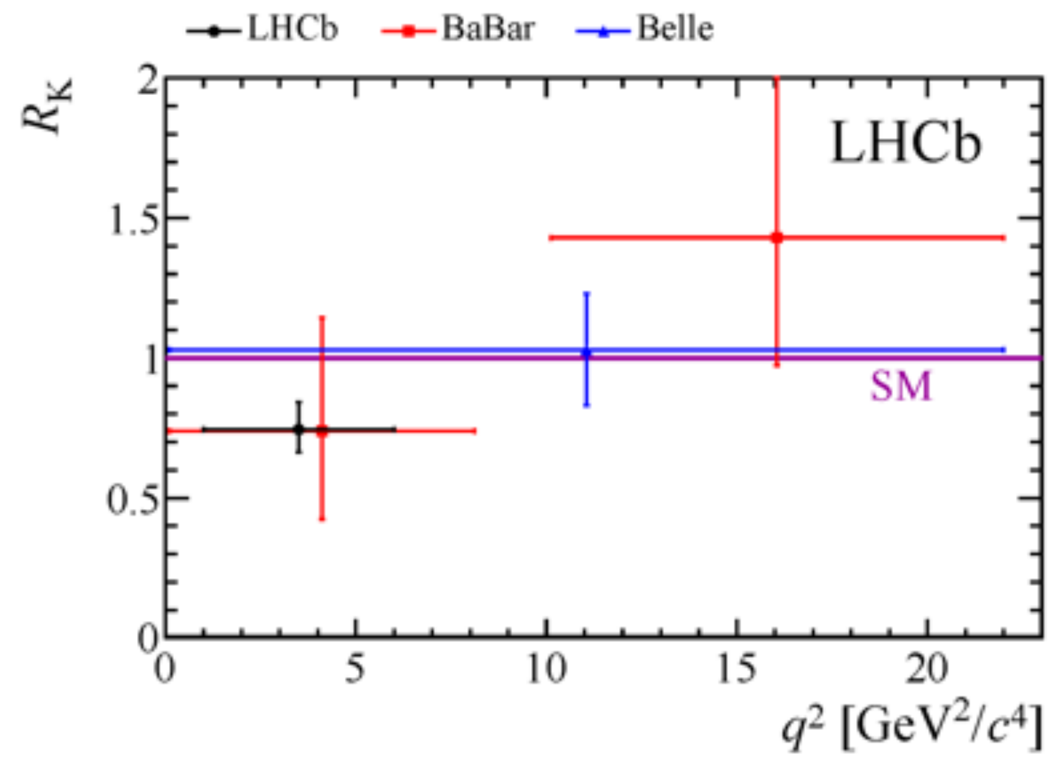
Current interest: R_K-anomaly

in combination with $H \to \mu\tau$ "anomaly" is rather interesting

$$R_K \equiv \frac{\mathcal{B}(B^+ \to K^+ \mu^+ \mu^-)}{\mathcal{B}(B^+ \to K^+ e^+ e^-)}$$

$$R_K|_{\rm SM} \simeq 1$$

Hiller Kruger'03



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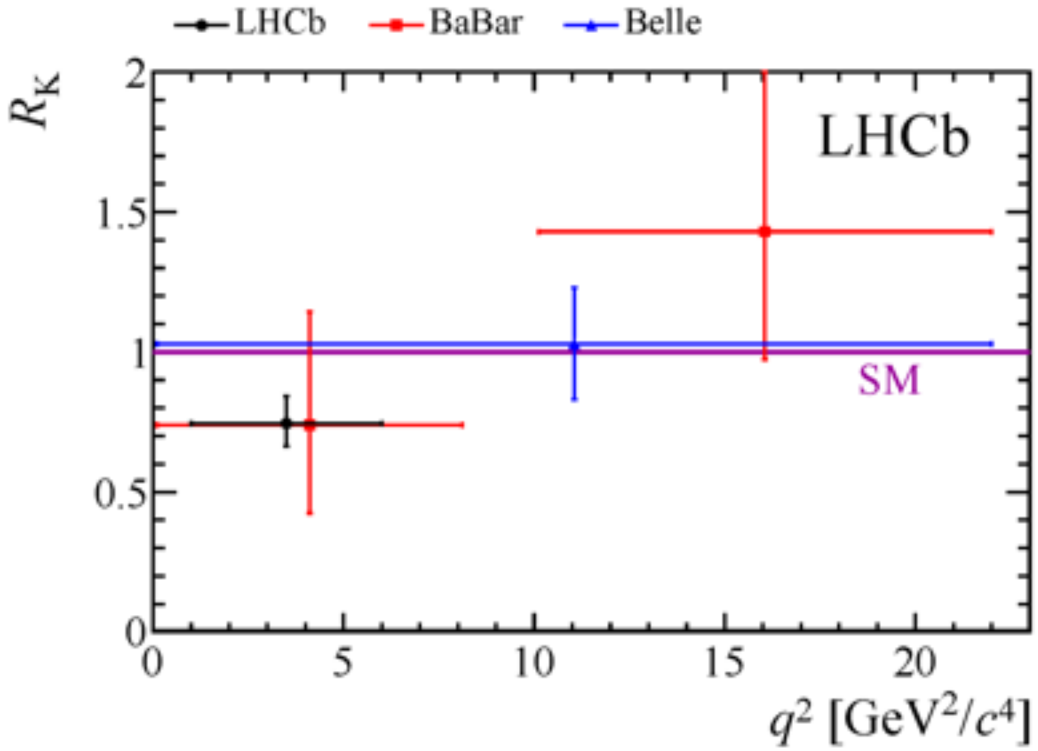

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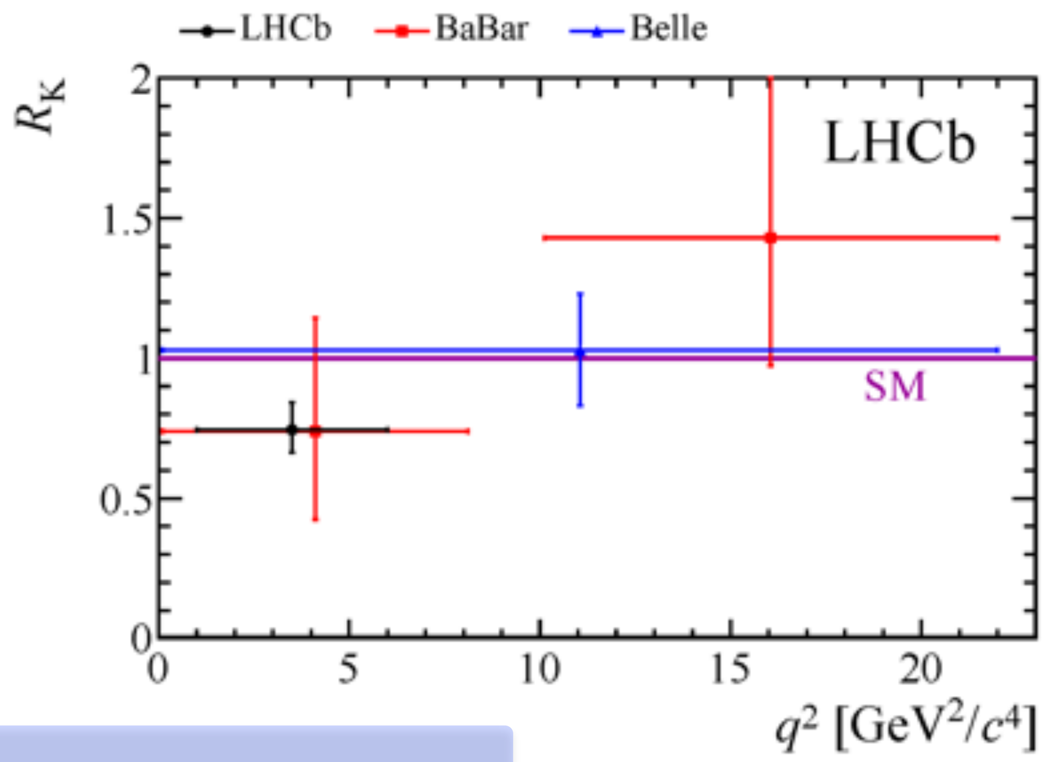

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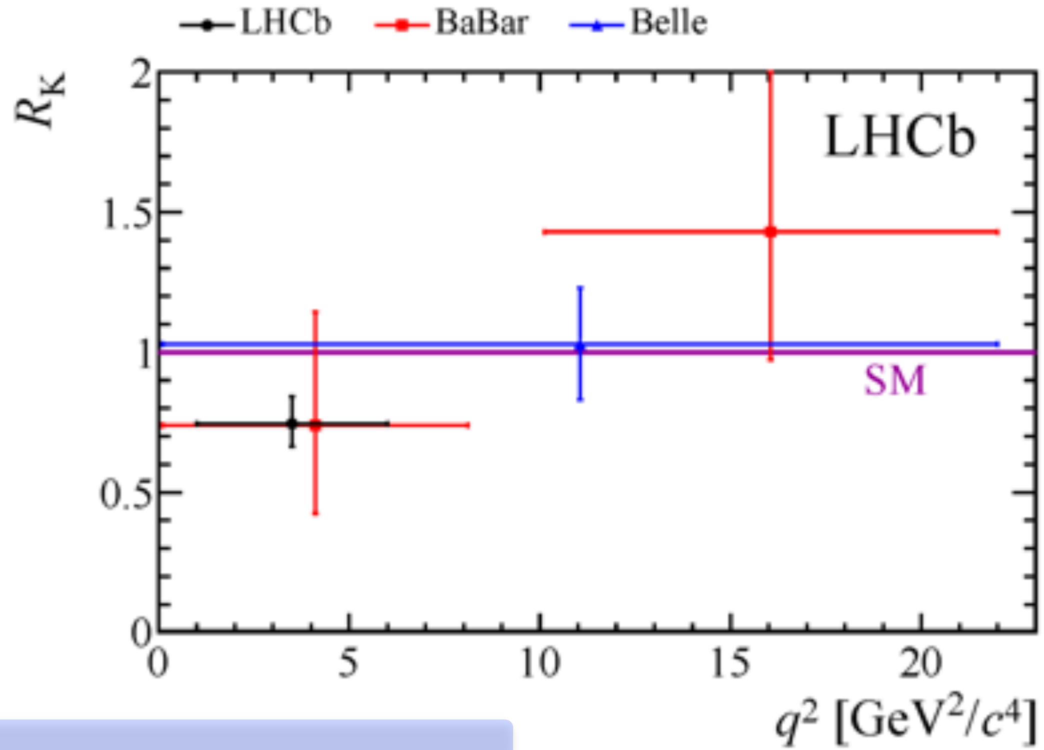

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$$R_\pi^{\rm sl} = \frac{\Gamma(\pi \to e^+\nu)}{\Gamma(\pi \to \mu^+\nu)} = \frac{m_e^2}{m_\mu^2}\frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2}(1+\delta_{\rm QED})\,, \quad |\delta_{\rm QED}| \simeq 4\%$$

an example

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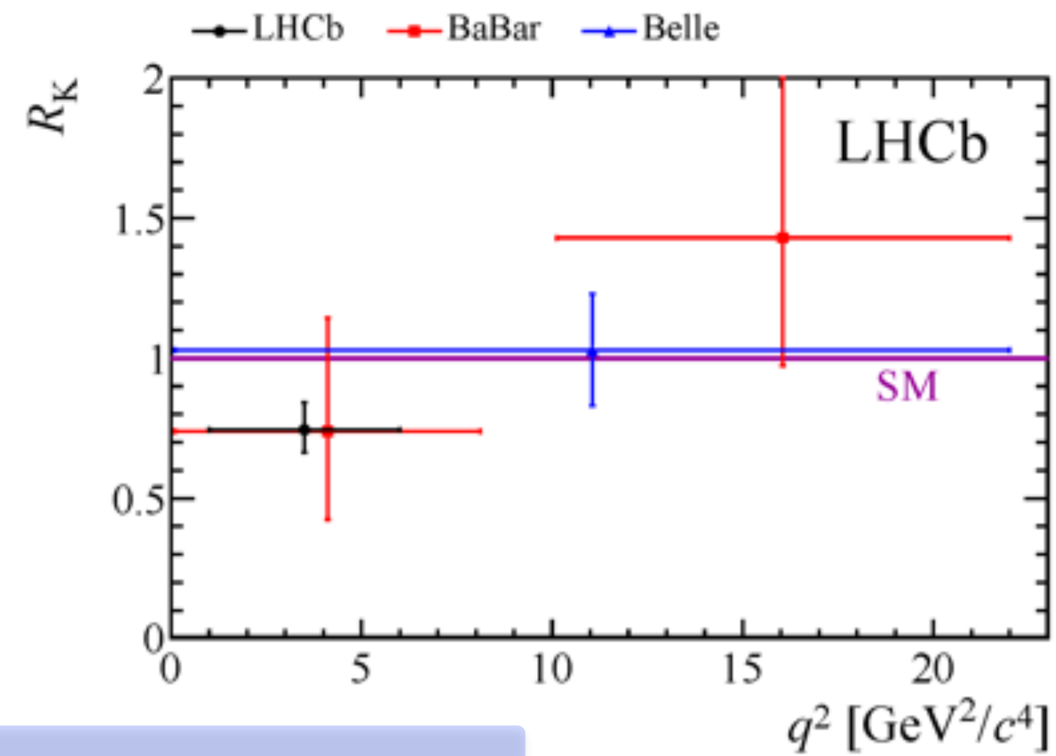

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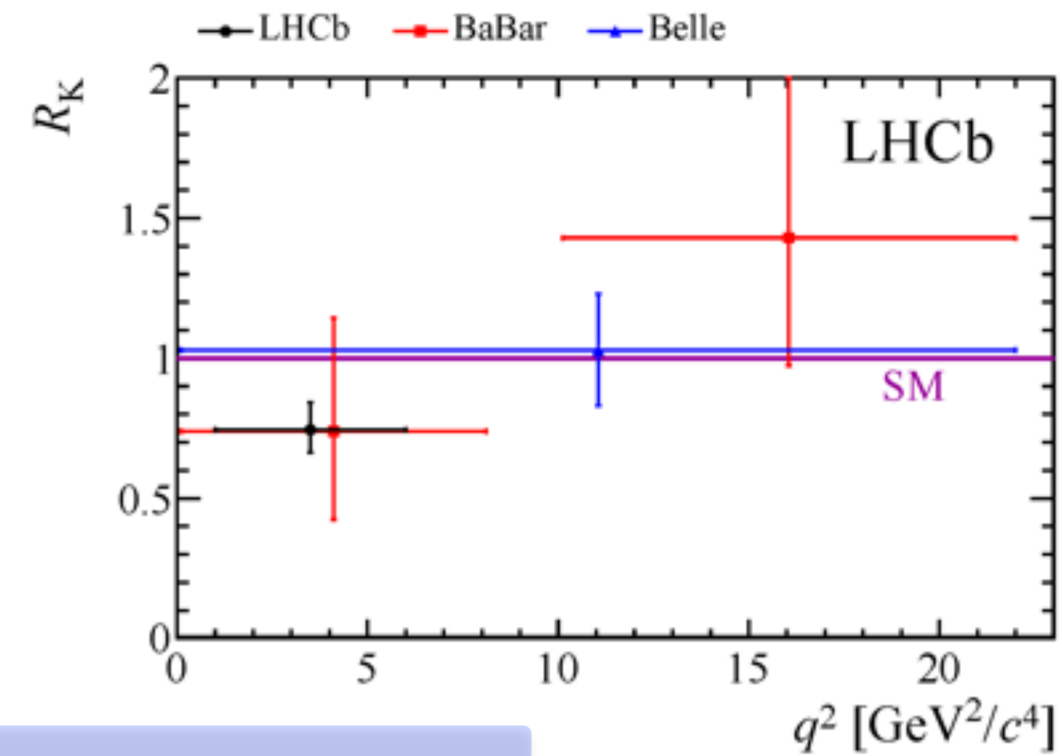

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an example

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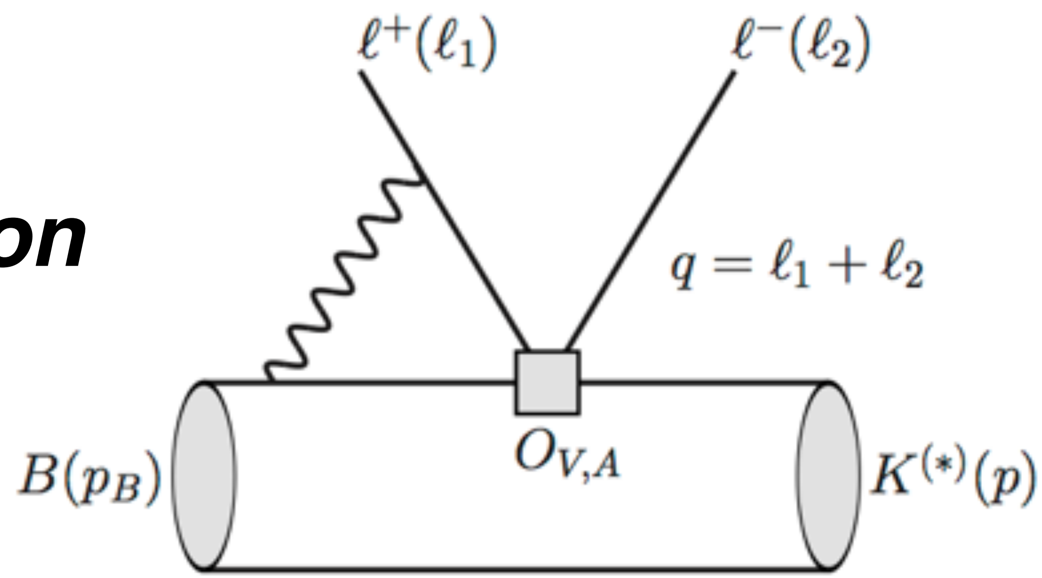
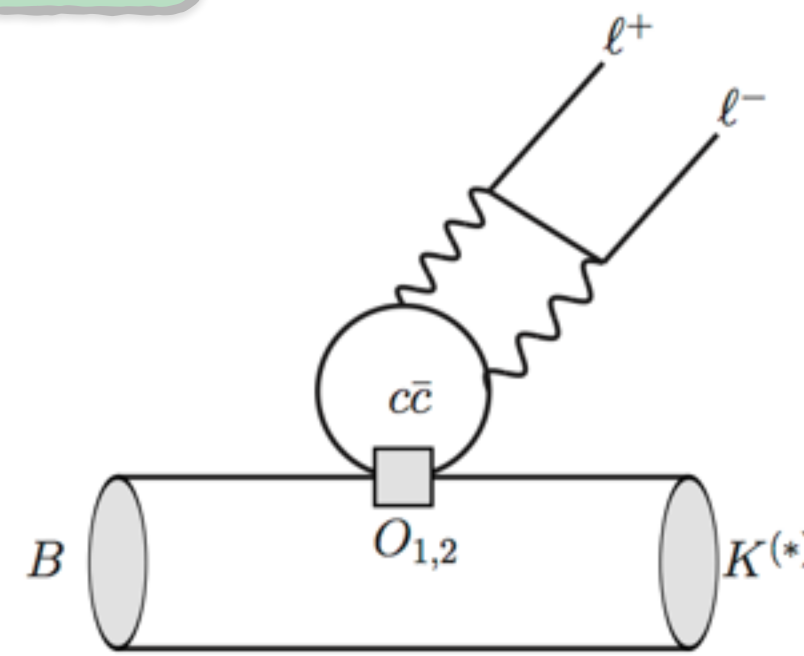
$\Longrightarrow$ suggest a way to diagnose/ bound effects in ang. distr.

non-factorisable QED corrections

effects:
A_{FB} without axial interaction

photon





- Becomes a proper 1→3 process and by crossing a 2→2 with Mandelstam variables

$$B(p_B) + \ell^-(-\ell_1) \to K(p) + \ell^-(\ell_2) \ ,$$

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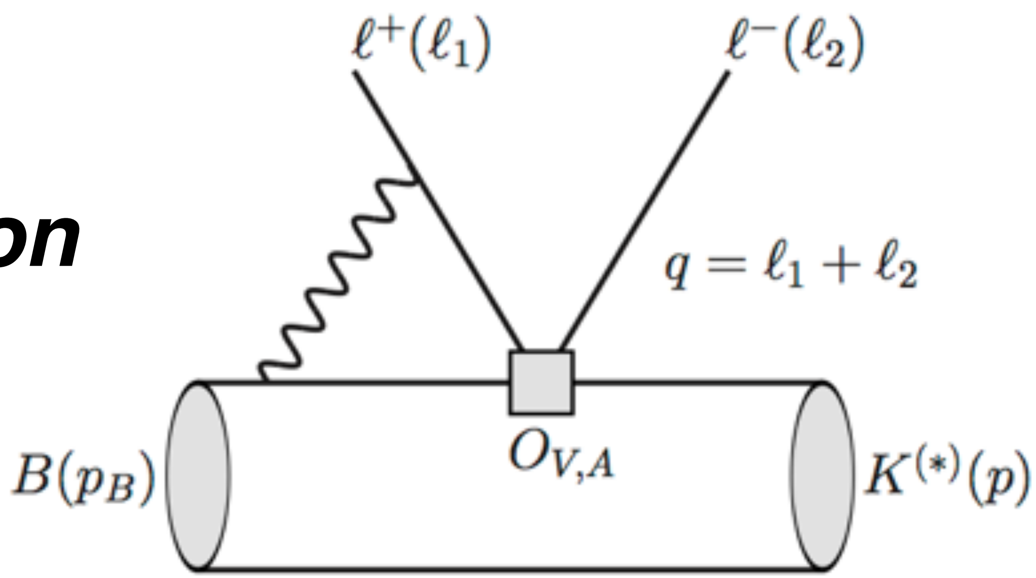
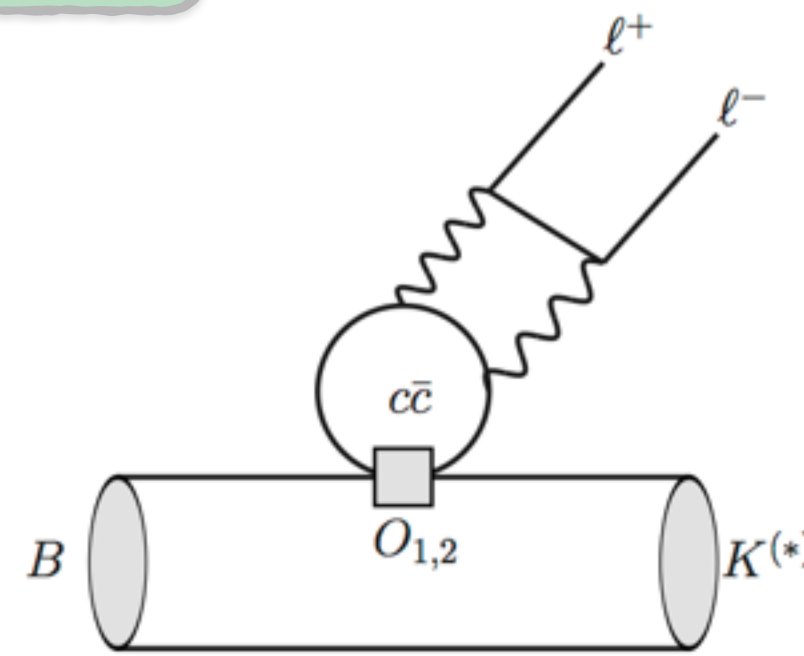
non-factorisable QED corrections

effects:
A_{FB} without axial interaction

photon





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- ⟹ s[u] enter logs ⟹ **no restriction sin(θ_l),cos(θ_l)-powers;**
Legendre polynomial [or $\Omega_m^{lk,ll}$] serves as a complete basis (non-vanishing higher moments)

$$\frac{d^2\Gamma(B \to K\ell^+\ell^-)}{dq^2 \ d\cos\theta_\ell} = \sum_{l_\ell \geq 0} G^{(l_\ell)} P_{l_\ell}(\cos\theta_\ell)$$

diagnosing QED effects $B \to K^{(*)} l^+ l^-$

- **$B \to K l^+ l^-$ moments:**

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1.

LFA	beyond LFA (eg. QED effects)
$M_{\bar{\ell}\ell}^{(l_\ell>2)} = 0$	$M_{\bar{\ell}\ell}^{(l_\ell>2)} \neq 0$

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2. likely QED-signature

$$M_{\bar{e}e}^{(l_\ell > 2)} \neq M_{\bar{\mu}\mu}^{(l_\ell > 2)}$$

$$|M_{\bar{e}e}^{(l_\ell > 2)}| > |M_{\bar{\mu}\mu}^{(l_\ell > 2)}|$$
$$\left[\alpha_{\rm QED}\, f(\ln\left(\frac{m_b}{m_\ell}\right))\text{-effects}\right]$$

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3. R_K is $M_{\bar{\ell}\ell}^{(l_\ell = 0)}$-moment - behaviour of moment in l_ℓ crucial
Rough computation suggests moderate fall-off
Amplitude: S-wave : D-wave = 1 : ~0.5(large uncertainty)

refinement: competitor signature

- **higher dimensional** operators (dimension 8,10….) $\delta H^{\text{eff}} = C^{(j)} O^{(j)} + ..$

$$O^{(j)} = \bar{s}_L \Gamma^{(j)}_{\mu_1 \ldots \mu_j} b \, \bar{\ell} \Gamma^{(j) \; \mu_1 \ldots \mu_j} \ell$$

with **higher SO(3)-spin** $\Gamma^{(j)}_{\mu_1 \ldots \mu_j} \equiv \gamma_{\{\mu_1} D^+_{\mu_2} \ldots D^+_{\mu_j\}}, \; D^+ \equiv \overleftarrow{D} + \overrightarrow{D}, \text{ with } \overrightarrow{D}$

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- QED versus higher dimensional operators

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QED wins even without logs for Wilson coefficients

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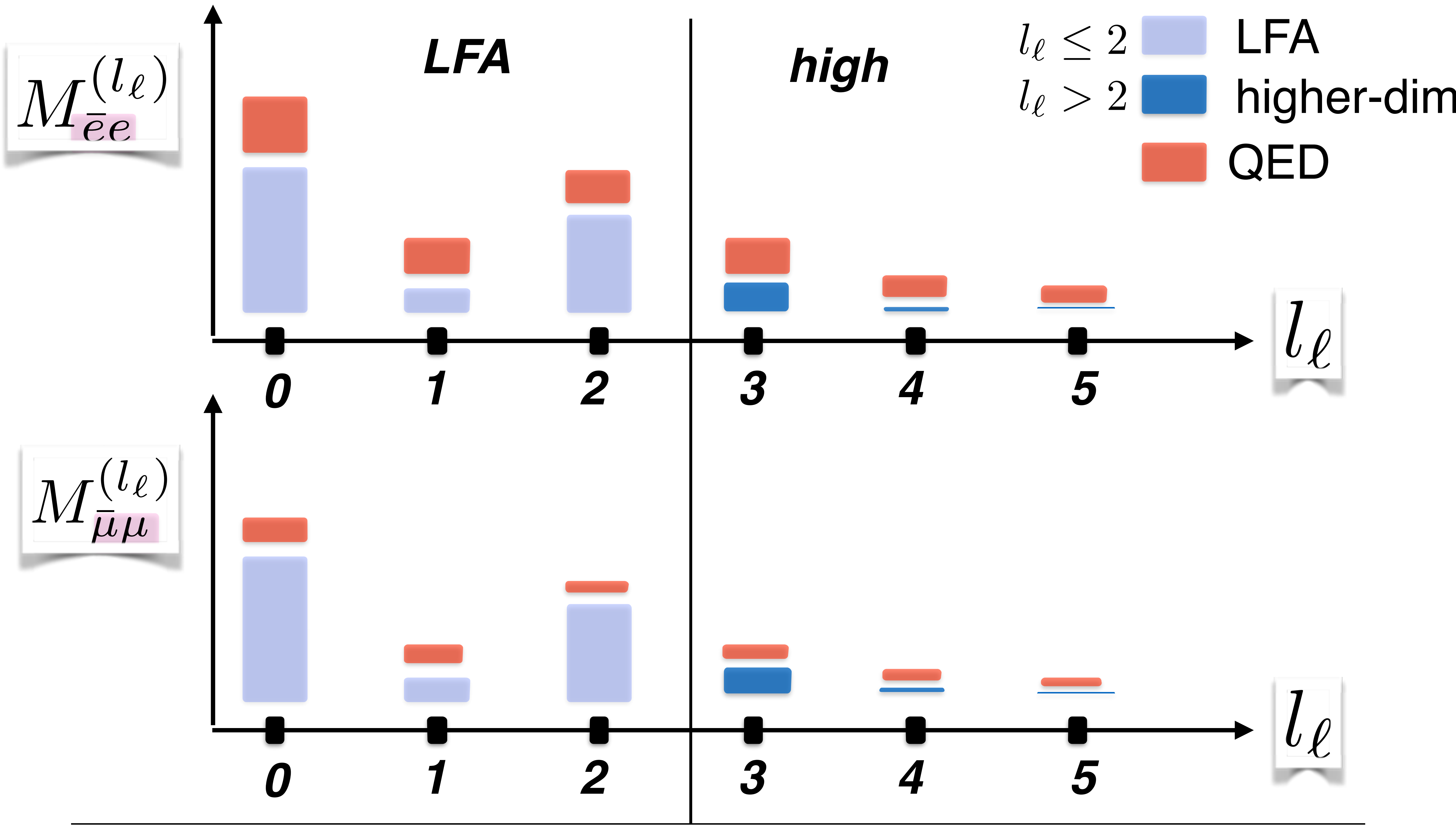
time for a graphical summary ….

qualitative overview of effects*



* emphasis on qualitative (size of effects are for illustration only)

[B.2] moments for $B \to K^*(\to K\pi)ll$
same idea but richer structure

Method of (partial) moments

- **method of moments** extendable to B->K*ll using orthogonality of Legendre P.

see also Beaujean, Chraszcz, Serra vanDyk ‘15

$$M_m^{l_K,l_\ell} \equiv \frac{1}{8\pi}\int_{-1}^{1} d\cos\theta_K \int_{-1}^{1} d\cos\theta_\ell \int_0^{2\pi} d\phi\, (\Omega_m^{l_K,l_\ell})^* \frac{d^4\Gamma}{d(\text{angles})} = \frac{(1+\delta_{m0})G_m^{l_K,l_\ell}}{2(2l_K+1)(2l_\ell+1)}$$

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- expect the full program of 1) and 2) to be equivalent in some statistical sense

discussion & conclusions

- **angler distribution** computed can be used for **any** $\boldsymbol{S \to V(\to S_1 S_2) l_1 l_2}$-decay
- **method of moments** can help to **clarify nature** of some $b \to s$-**anomalies**

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$\Longrightarrow$ *may result in* $\boldsymbol{B \to K^*(\to K \pi)\, \mu\mu}$ *signal window with downward shift in* q^2

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Now, LHCb-estimates this event to be negligible but general lesson is that checking higher moments could help to clarify matters

discussion & conclusions II

notation: $M_m^{l_K, l_\ell}$

- two examples of *characteristic moments*
  1) **S-wave** $K^{*,0}$-K^*-interference produces $M_m^{\mathrm{odd}, l_\ell} \neq 0$ not present K^*-only
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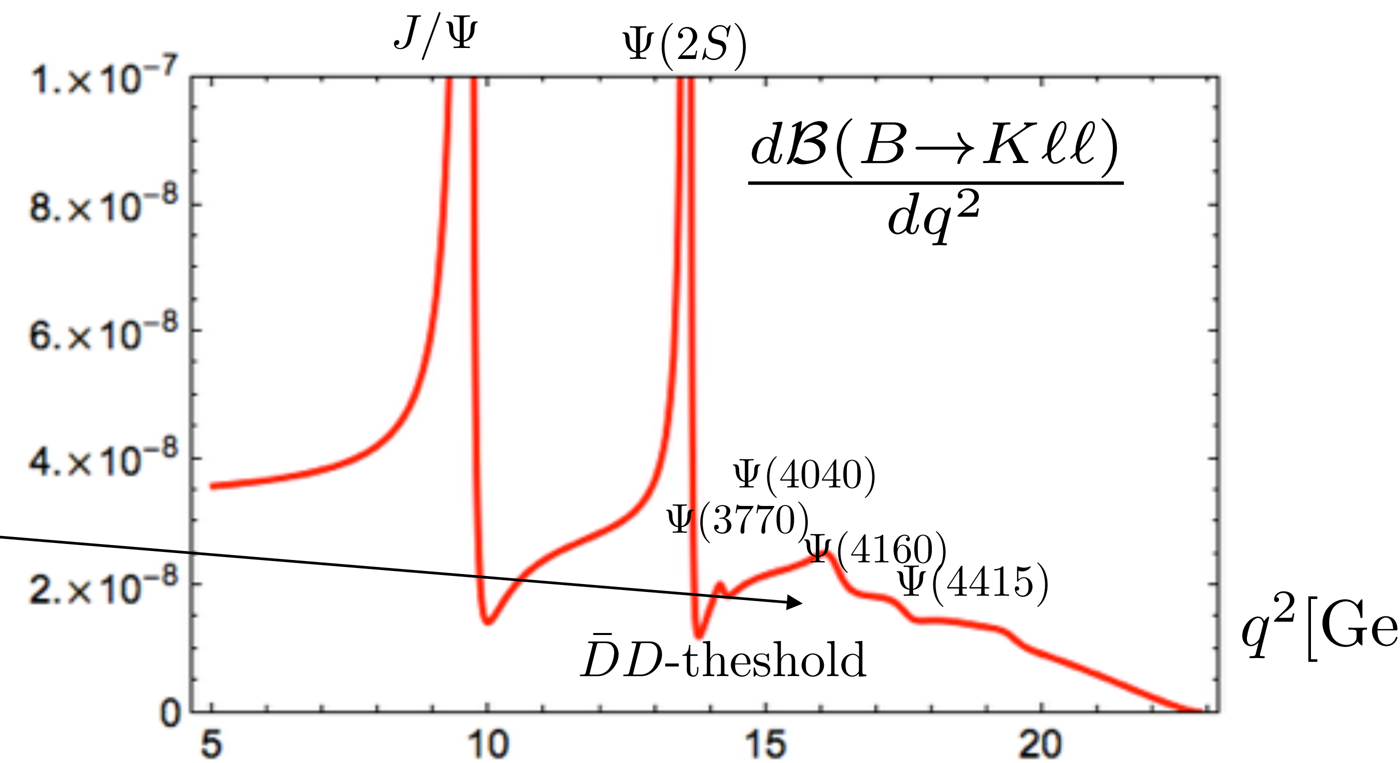
- **comment:** to be sure δC_9 is not charm: resonance-residues and -phases have to be *measured* (begun in $B \to K\mu\mu$ Lyon RZ'14 for broad resonances and is pursued by LHCb for narrow J/Ψ and Ψ(2S)-resonances)

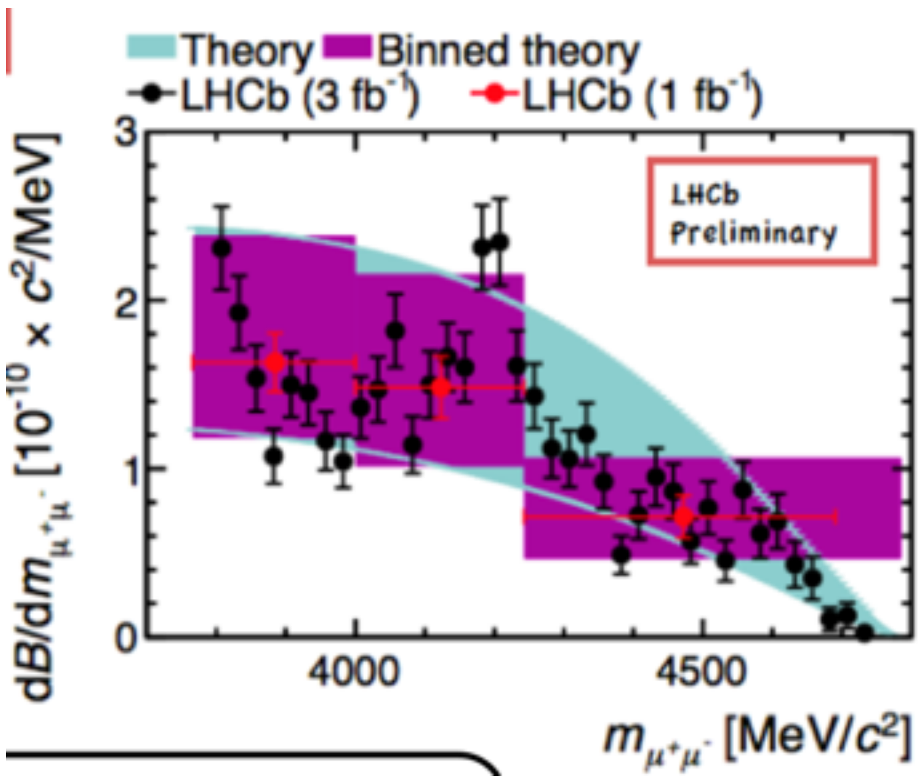


thanks for your attention

BACKUP

II.C comment charm resonances in B→K $^{(*)}$ll

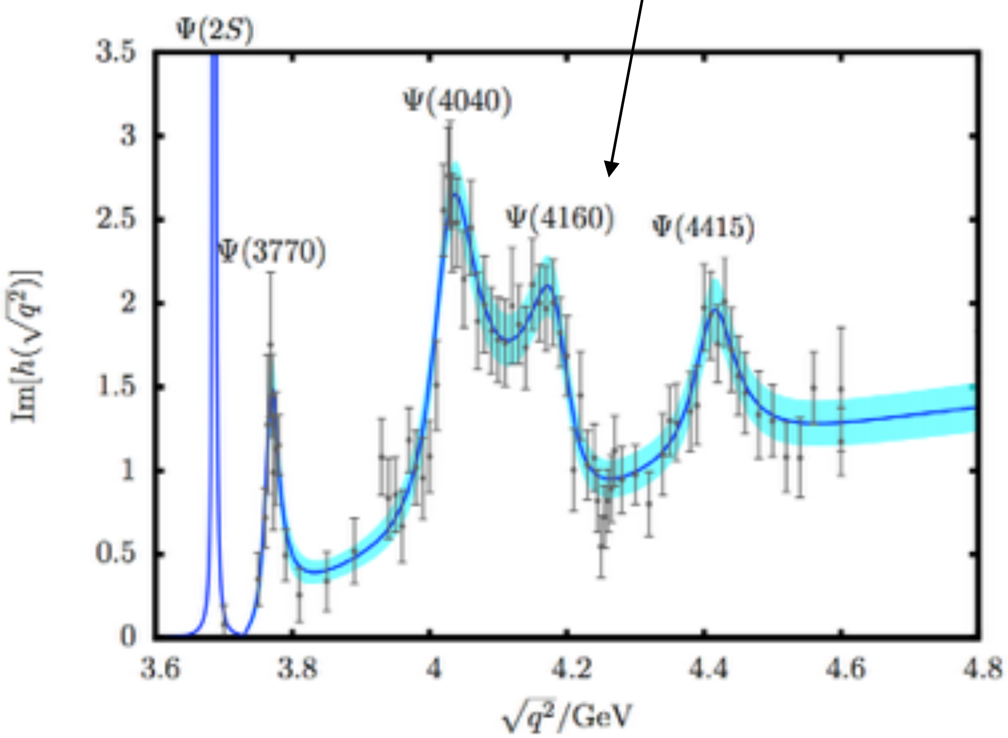
$$\mathrm{BF}(B \to K\ell\ell)$$

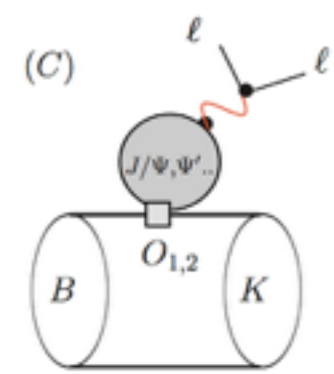


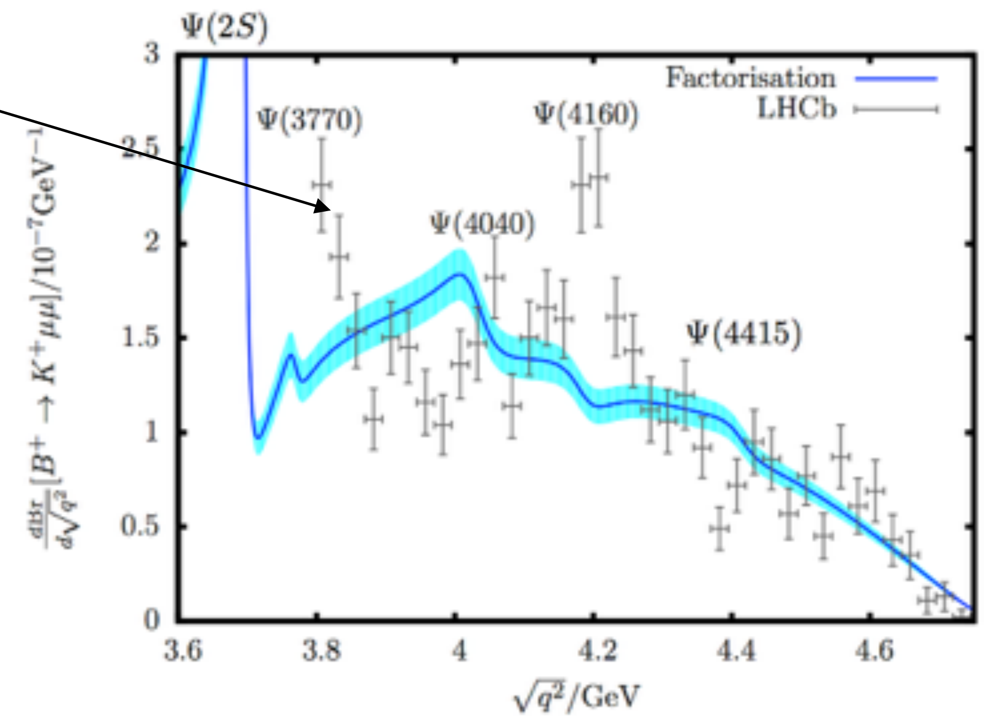
LHCb PRL 111 (2013)

pronounced J^{PC}= 1^{--} charm resonance structure

- Using a fit to BES-II data $e^+e^- \to$ hadrons able to check status of “naive” factorisation at high q^2 in $B \to Kll$







Lyon RZ 1406.0566

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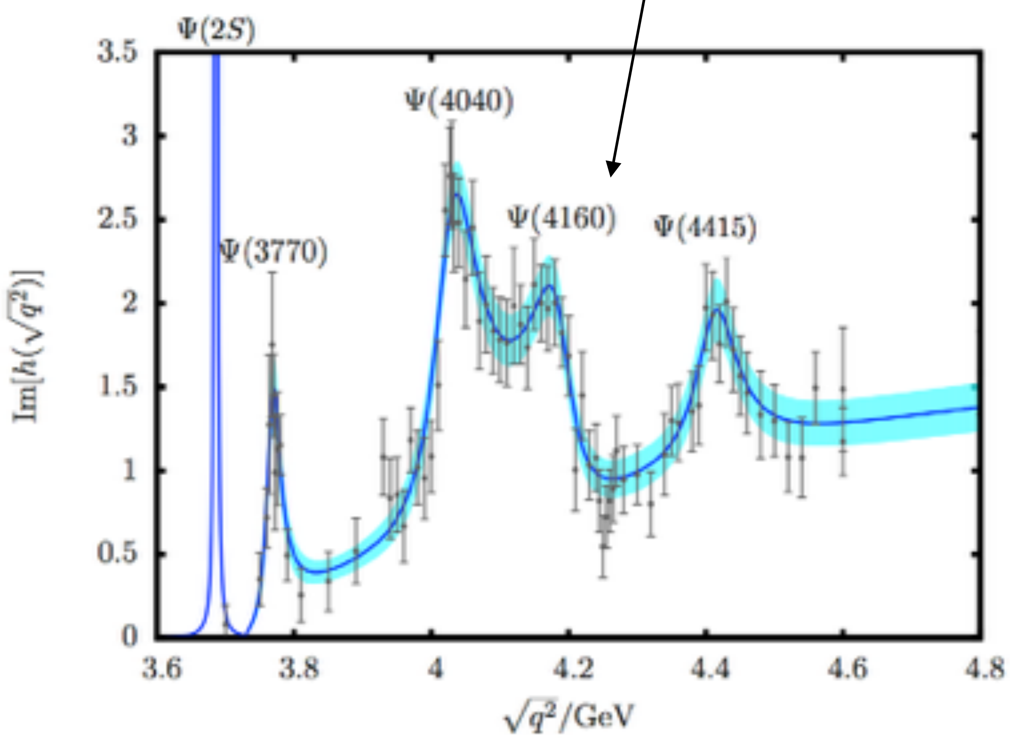
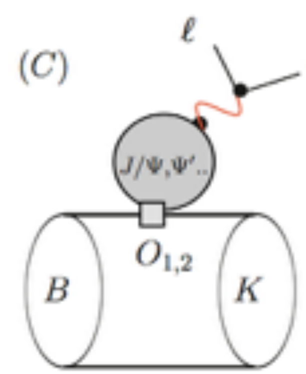
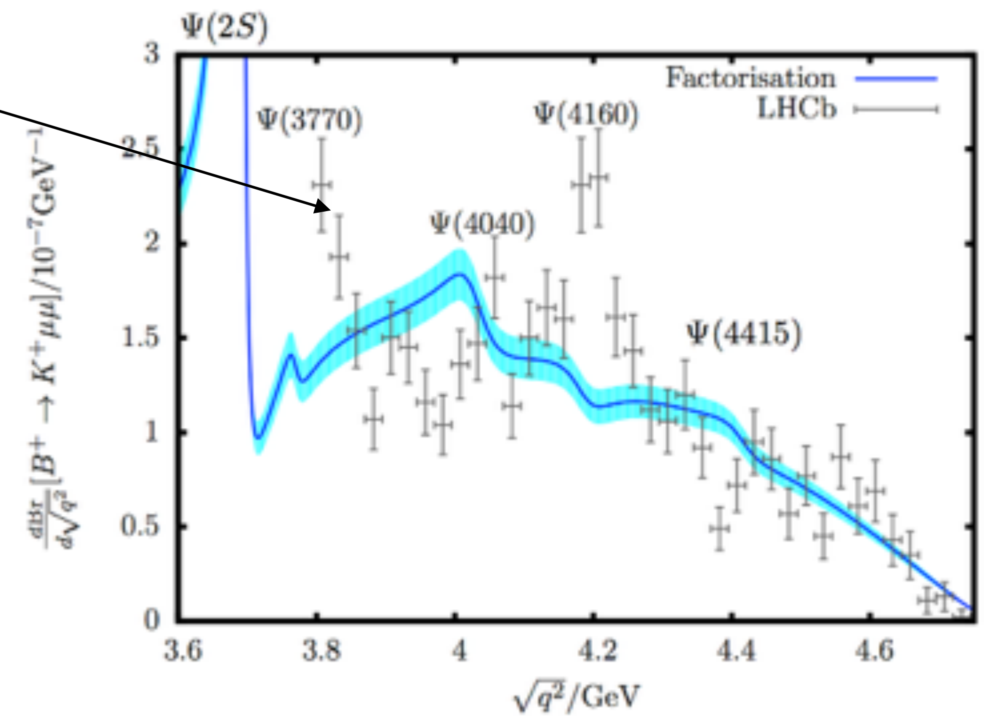
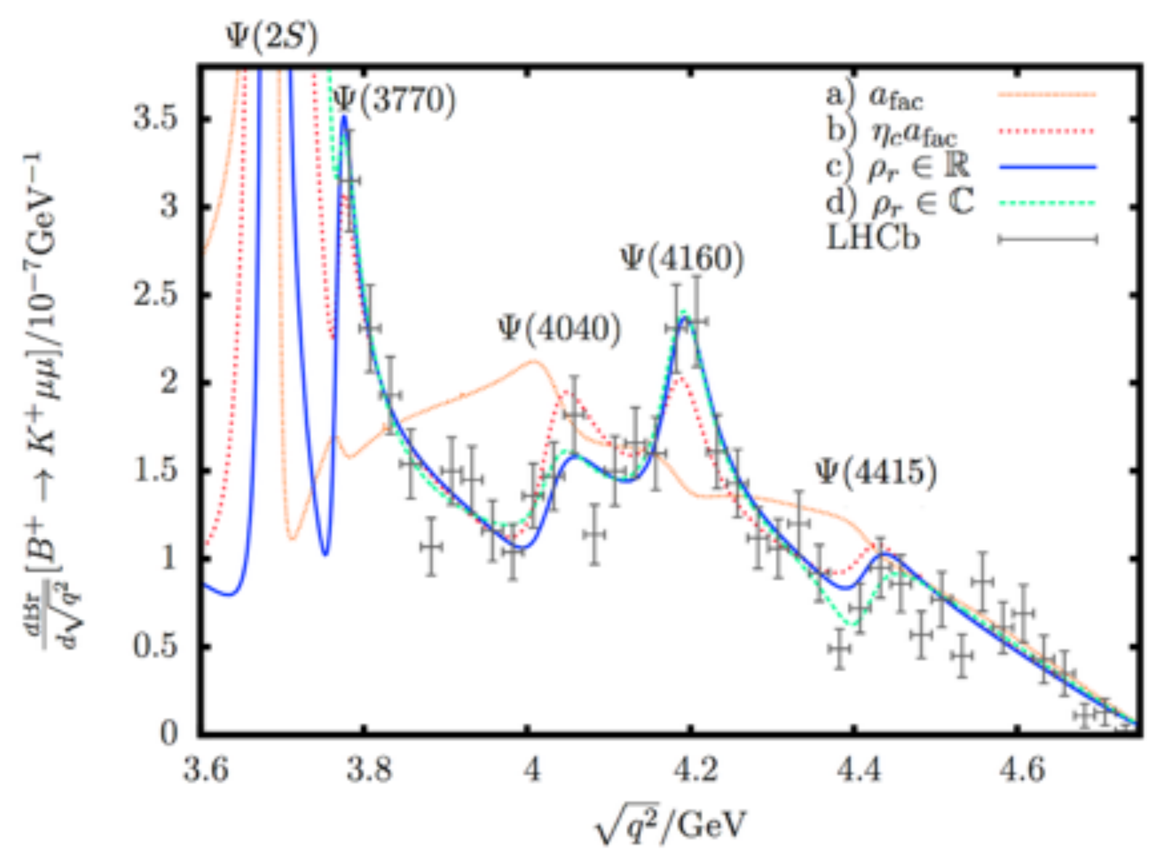




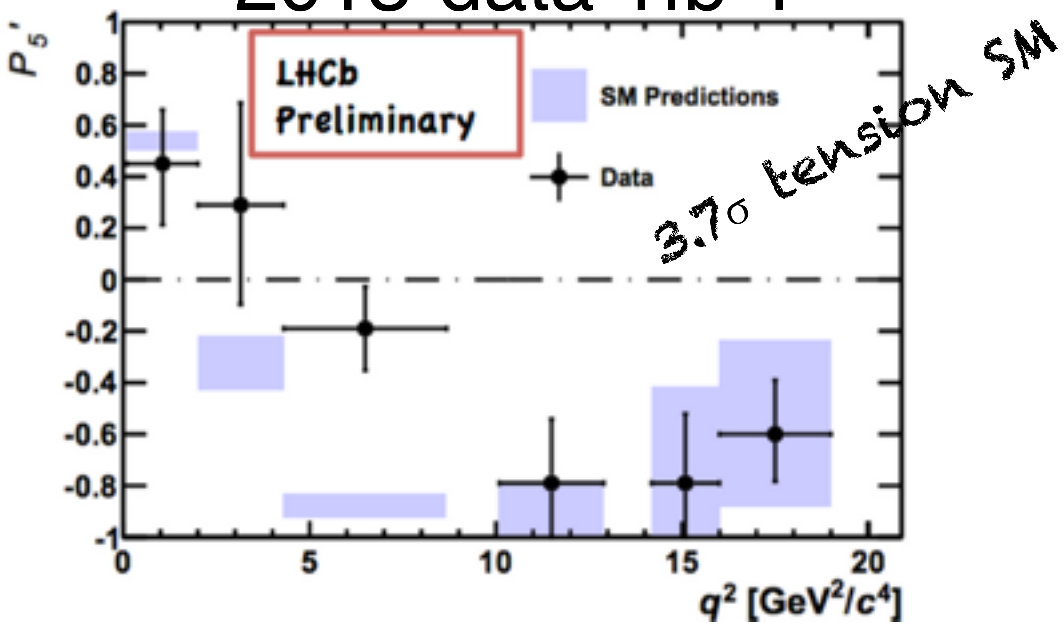


hight of resonances in naive fac. by factor ~(-2.5) fits the data well

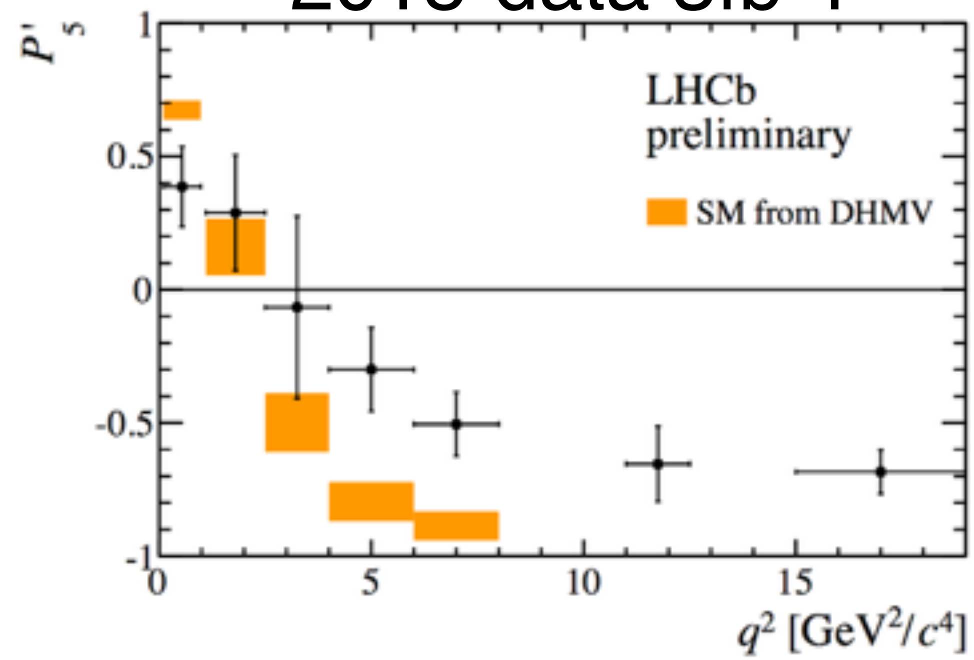


- Led us to speculate P_5'-anomaly in $B \to K^{(*)}ll$ might be related to charm (since charm pronounced)

2013-data 1fb-1



2015-data 3fb-1



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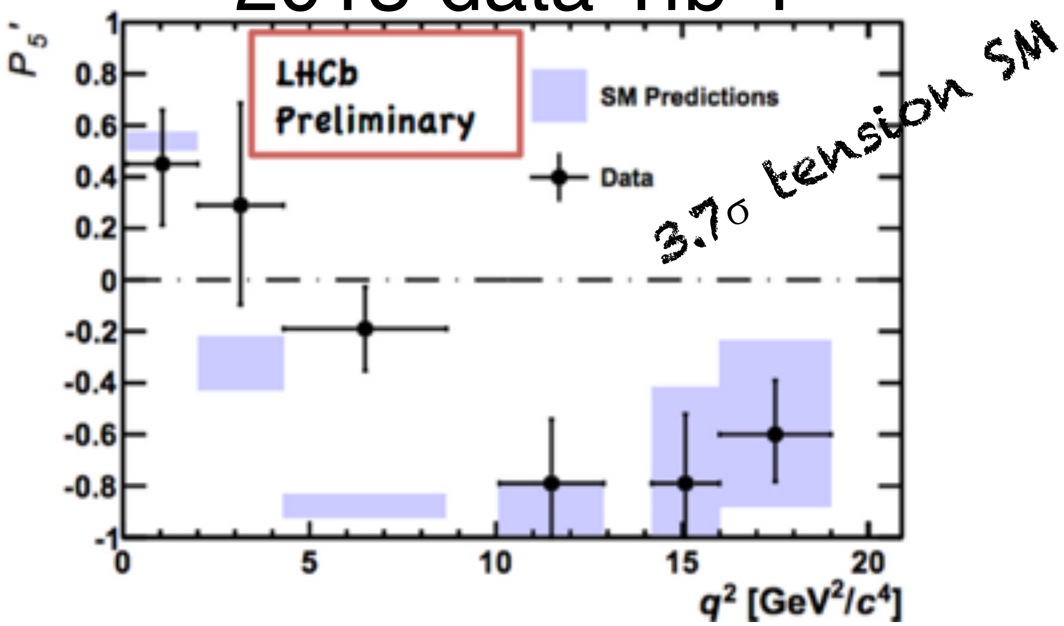

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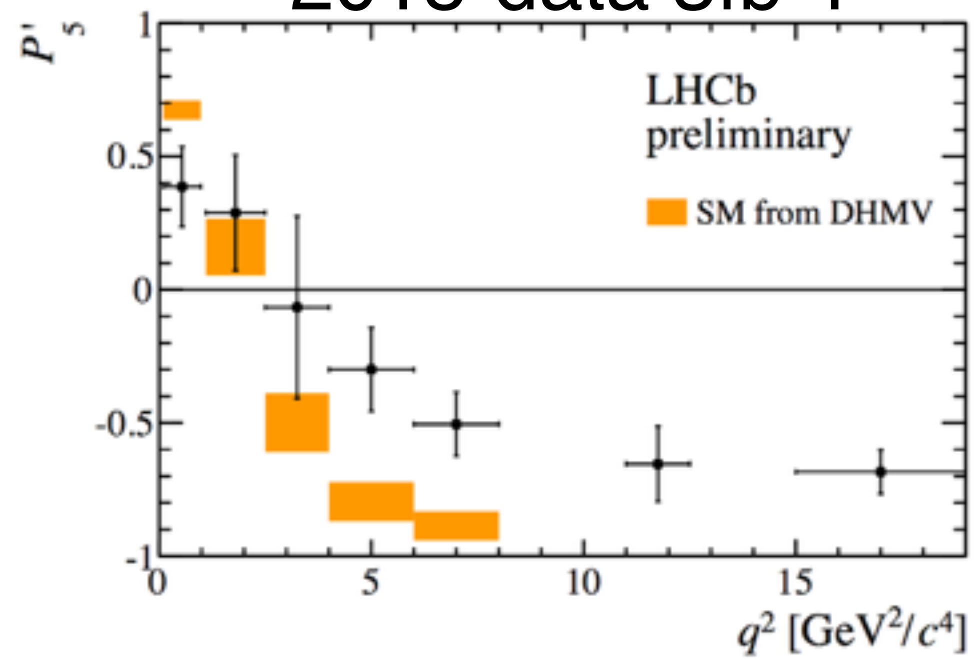

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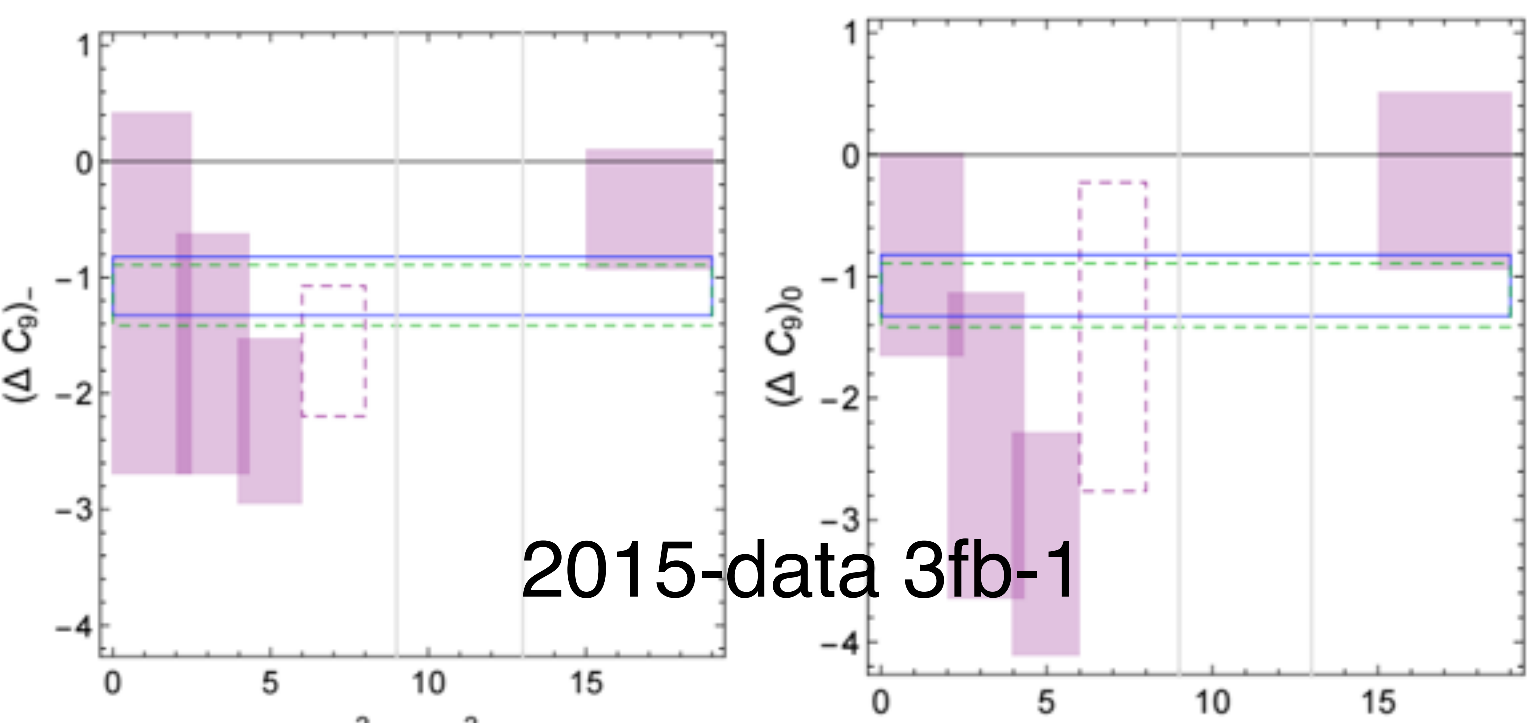
Straub's talk Moriond'15 (proceedings & Wolfgang's talk)



- effect same sign as in naive fac. in "-" versus "0" helicity
- my comment: that's what B→ J/Ψ K* experimental angular analysis predicts for J/Ψ,Ψ(2S)-contributions