Standard Model predictions for the mixing quantities ΔM_q, $\Delta\Gamma_q$ and a_{sl}^q

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Outline

- Motivation and Introduction
- Calculation of M_{12}^q
- Calculation of Γ_{12}^q
- How precise is the HQE?
- Conclusion

Motivation

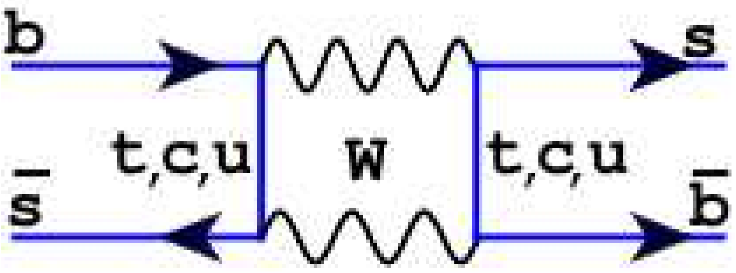
2015: High precision in experiment for many observables

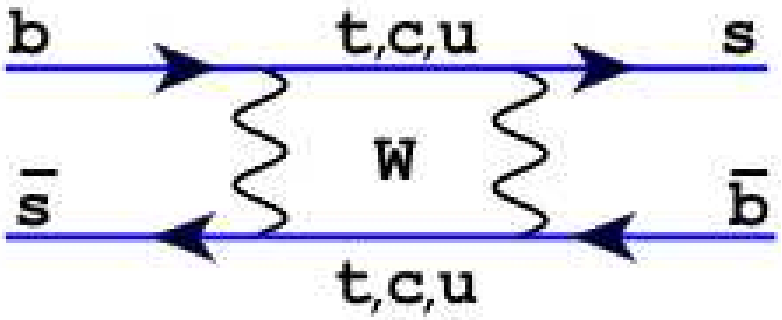
Theory has to cope with that!

- **Higher precision of SM predictions**
  - Higher orders in Perturbation Theory, e.g. **NNLO-QCD**
  - Precise non-perturbative parameters, e.g. **LATTICE, LCSR**
  - see e.g. talks by **Zwicky, Matias, Jäger, Meinel, van Dyk, Healy,...**

- **Challenge wide-spread assumptions that were justified some years ago**
  - Penguins can be neglected, see e.g. talk by **Nierste**
  - There is no NP acting in tree-level decays, see e.g. talk by **Tetlamatzi-Xolocotzi**

Introduction





$|M_{12}^q|$, $|\Gamma_{12}^q|$ and $\phi_{12}^q = \arg(-M_{12}^q/\Gamma_{12}^q)$ can be related to three observables:

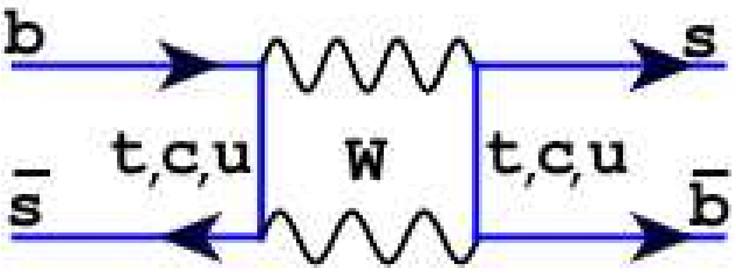
- **Mass difference:** $\Delta M_q := M_H - M_L \approx 2|M_{12}^q|$ **(off-shell)**
  $|M_{12}^q|$: heavy internal particles: t, SUSY, ...

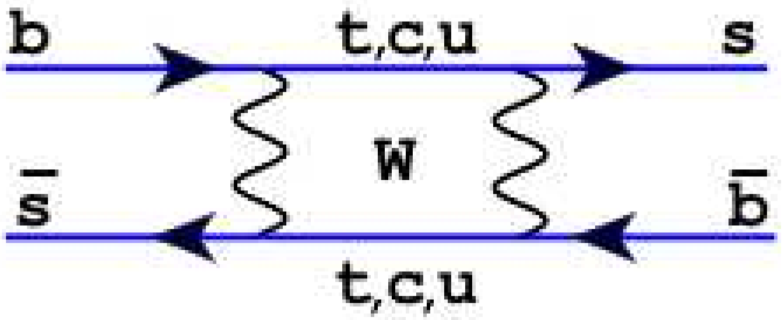
- **Decay rate difference:** $\Delta\Gamma_q := \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}^q| \cos\phi_{12}^q$ **(on-shell)**
  $|\Gamma_{12}^q|$: light internal particles: u, c, ... (almost) no NP!!!

- **Flavor specific/semi-leptonic CP asymmetries:** e.g. $B_q \to X l \nu$ (*semi-leptonic*)

$$a_{sl}^q \quad \equiv \quad a_{fs} = \frac{\Gamma(\overline{B}_q(t) \to f) - \Gamma(B_q(t) \to \overline{f})}{\Gamma(\overline{B}_q(t) \to f) + \Gamma(B_q(t) \to \overline{f})} = \left|\frac{\Gamma_{12}^q}{M_{12}^q}\right| \sin\phi_{12}^q = \Im\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right)$$

Mass difference ΔM_q





Calculating all the box diagrams

$$\begin{aligned} M_{12}^s &= \lambda_u^2 F(u,u) + \lambda_u \lambda_c F(u,c) + \lambda_u \lambda_t F(u,t) + \\ & \lambda_c \lambda_u F(c,u) + \lambda_c^2 F(c,c) + \lambda_c \lambda_t F(c,t) + \\ & \lambda_t \lambda_u F(t,u) + \lambda_t \lambda_c F(t,c) + \lambda_t^2 F(t,t) \end{aligned}$$

with the CKM structures $\lambda_q = V_{qs}^* V_{qb}$. Doing the loops one finds

$$F(p,q) = f_0 + f\left(\frac{m_q}{M_W}, \frac{m_p}{M_W}\right) ,$$

with a sizable constant value f_0 and a mass dependent term $f(m_q/M_W, m_p/M_W)$ that grows with the mass.

Mass difference ΔM_q

Using CKM unitarity $\lambda_u + \lambda_c + \lambda_t = 0$ one gets

$$\begin{aligned} M_{12}^s \quad = \quad & \lambda_c^2 \left[F(c,c) - 2F(u,c) + F(u,u) \right] \\ & + 2\lambda_c \lambda_t \left[F(c,t) - F(u,t) - F(u,c) + F(u,u) \right] \\ & + \lambda_t^2 \left[F(t,t) - 2F(u,t) + F(u,u) \right] \end{aligned}$$

- f_0 cancels in M_{12}^s due to GIM cancellation.
- If all internal masses would be equal (or zero), M_{12}^s would vanish.
- Looking at the CKM hierarchy we find

$$\lambda_c^2 \quad \propto \quad \lambda^4 \propto \lambda_c \lambda_t \propto \lambda_t^2$$

equal CKM factors in all terms, but first two terms are strongly GIM suppressed
To a good approximation we can write

$$M_{12}^s \quad \approx \quad \lambda_t^2 \left(F(t,t) - 2F(u,t) + F(u,u) \right) \propto \lambda_t^2 S(m_t) \, ,$$

with the Inami-Lim function $S(m_t)$

Mass difference ΔM_q

Formally integrating out the heavy particle (top and W = OPE) one gets

$$M_{12,q} = \frac{G_F^2}{12\pi^2} \lambda_t^2 M_W^2 S_0(x_t) B_{B_q} f_{B_q}^2 M_{B_q} \hat{\eta}_B$$

- Imaginary part only due to CKM structure
- 1 loop calculation $S_0(x_t = m_t^2/M_W^2)$ **Inami, Lim, '81**
- 2-loop perturbative QCD corrections $\hat{\eta}_B$ **Buras, Jamin, Weisz, '90**
- Hadronic matrix element: $\frac{8}{3} B_{B_q} f_{B_q}^2 M_{B_q} = \langle \bar{B}_q | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A} | B_q \rangle$

Theory **Artuso, Borissov, A.L., 2015 to appear** vs. Experiment : **HFAG 15**

$$\Delta M_d^{\rm SM} = 0.528 \pm 0.079 \text{ ps}^{-1} \qquad \Delta M_d^{\rm Exp} = 0.5055 \pm 0.0020 \text{ ps}^{-1}$$

$$\Delta M_s^{\rm SM} = 18.3 \pm 2.7 \text{ ps}^{-1} \qquad \Delta M_s^{\rm Exp} = 17.757 \pm 0.021 \text{ ps}^{-1}$$

- Perfect agreement, still room for NP
- Important bounds on the unitarity triangle and NP
- **Dominant uncertainty = Lattice**

Error budget for ΔM_s

ΔM_s^{SM}	ABL2015	LN2011	LN2006
Central Value	$18.3\,\mathrm{ps}^{-1}$	$17.3\,\mathrm{ps}^{-1}$	$19.3\,\mathrm{ps}^{-1}$
$\delta(f_{B_s}\sqrt{B})$	13.9%	13.5%	34.1%
$\delta(V_{cb})$	4.9%	3.4%	4.9%
$\delta(m_t)$	0.7%	1.1%	1.8%
$\delta(\alpha_s)$	0.1%	0.4%	2.0%
$\delta(\gamma)$	0.1%	0.3%	1.0%
$\delta(\lvert V_{ub}/V_{cb}\rvert)$	0.1%	0.2%	0.5%
$\delta(\overline{m}_b)$	$< 0.1\%$	0.1%	$- - -$
$\sum \delta$	14.8%	14.0%	34.6%

- Uncertainty dominated by non-perturbative parameters
- If lattice can achieve $\pm 5\%$ then overall $\pm 5\%$ might be possible (in 10 years?)

Numerical update **Artuso, Borissov, A.L., 2015 to appear**
of **A.L., Nierste 2011** and **A.L., Nierste 2006**

Lattice input for ΔM_s

We used the **FLAG** average

$$f_{B_s}\sqrt{B} = 216 \pm 15 \text{ MeV}$$

This is actually based on only one number in the literature from **HPQCD 2009**
There are currently four more (mostly preliminary) numbers on the market

$$f_{B_s}\sqrt{B} \approx 200 \text{ MeV} \Rightarrow \Delta M_s \approx 15.7 \text{ ps}^{-1}$$ **HPQCD 2014**

$$f_{B_s}\sqrt{B} \approx 211 \text{ MeV} \Rightarrow \Delta M_s \approx 17.4 \text{ ps}^{-1}$$ **ETMC 2014**

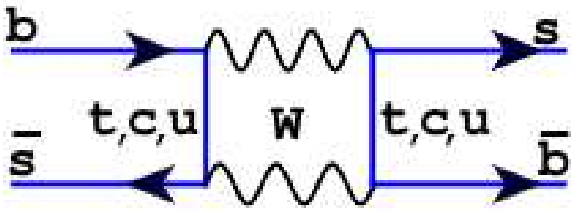
$$f_{B_s}\sqrt{B} \approx 227 \text{ MeV} \Rightarrow \Delta M_s \approx 20.2 \text{ ps}^{-1}$$ **Fermilab-MILC 2015**

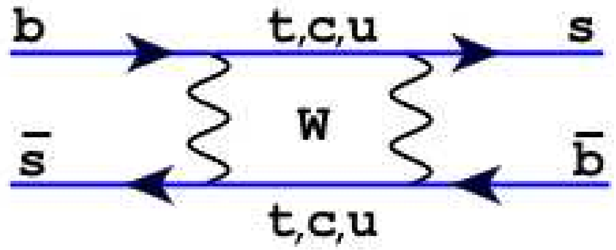
$$f_{B_s}\sqrt{B} \approx 262 \text{ MeV} \Rightarrow \Delta M_s \approx 27.2 \text{ ps}^{-1}$$ **RBC-UK 2015**

It will be great to see these numbers converge
and
to be included in the FLAG average

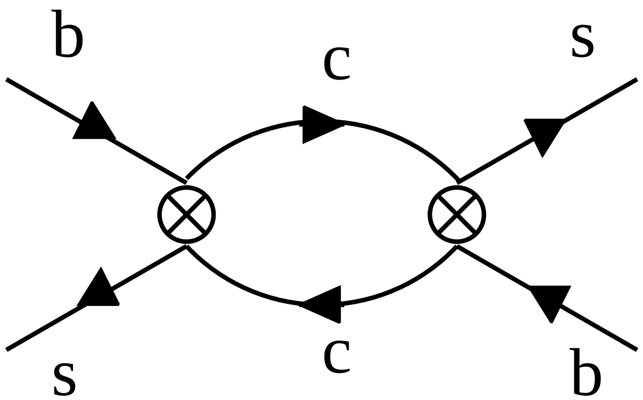
Calculation of Γ_{12}

Γ_{12} is given by the contribution of on-shell particles in the box-diagrams





up and charm cannot be integrated out thus we are left with



Second OPE = Heavy Quark Expansion (HQE)

’83,’85: Voloshin, Shifman; ’92: Bigi, Uraltsev;
’93: Blok, Shifman; ’92: Bigi, Uraltsev, Vainshtein

Calculation of Γ_{12}

$$\Gamma_{12} = \left(\frac{\Lambda}{m_b}\right)^3 \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + ...\right) + \left(\frac{\Lambda}{m_b}\right)^4 \left(\Gamma_4^{(0)} + ...\right) + \left(\frac{\Lambda}{m_b}\right)^5 \left(\Gamma_5^{(0)} + ...\right) + ...$$

'96: Beneke, Buchalla; '98: Beneke, Buchalla, Greub, A.L., Nierste;
'03: Beneke, Buchalla, A.L., Nierste; '03: Ciuchini, Franco, Lubicz, Mescia, Tarantino;
'06; '11: A.L., Nierste; '07 Badin, Gabianni,Petrov

HQE might be questionable - relies on quark hadron duality

Energy release is small $\Rightarrow$ naive dim. estimate: series might not converge

Historic Problems:

- Mid 90's: Missing Charm puzzle $n_c^{\text{Exp.}} < n_c^{\text{SM}}$, semi leptonic branching ratio
- Mid 90's: Λ_b lifetime is too short, i.e. $\tau(\Lambda_b) \ll \tau(B_d) = 1.520$ ps
- before 2003: $\tau_{B_s}/\tau_{B_d} \approx 0.94 \neq 1$
- 2010/11/13: dimuon asymmetry too large

The Heavy Quark Expansion

(Almost) all discrepancies disappeared:

- '12: $n_c^{\rm 2011PDG} = 1.20 \pm 0.06$ vs. $n_c^{\rm SM} = 1.23 \pm 0.08$ **Krinner, A.L., Rauh 1305.5390**
- HFAG '03 $\tau_{\Lambda_b} = 1.229 \pm 0.080$ ps^{-1} $\longrightarrow$ HFAG '15 $\tau_{\Lambda_b} = 1.466 \pm 0.010$ ps^{-1}

Shift by 3.0σ!

- **HFAG 2015:** $\tau_{B_s}/\tau_{B_d} = 0.990 \pm 0.004$
- 2010/11/13: dimuon asymmetry too large — Test Γ_{12} with $\Delta\Gamma_s$!

Theory arguments for HQE

$\Rightarrow$ calculate corrections in all possible "directions", to test convergence

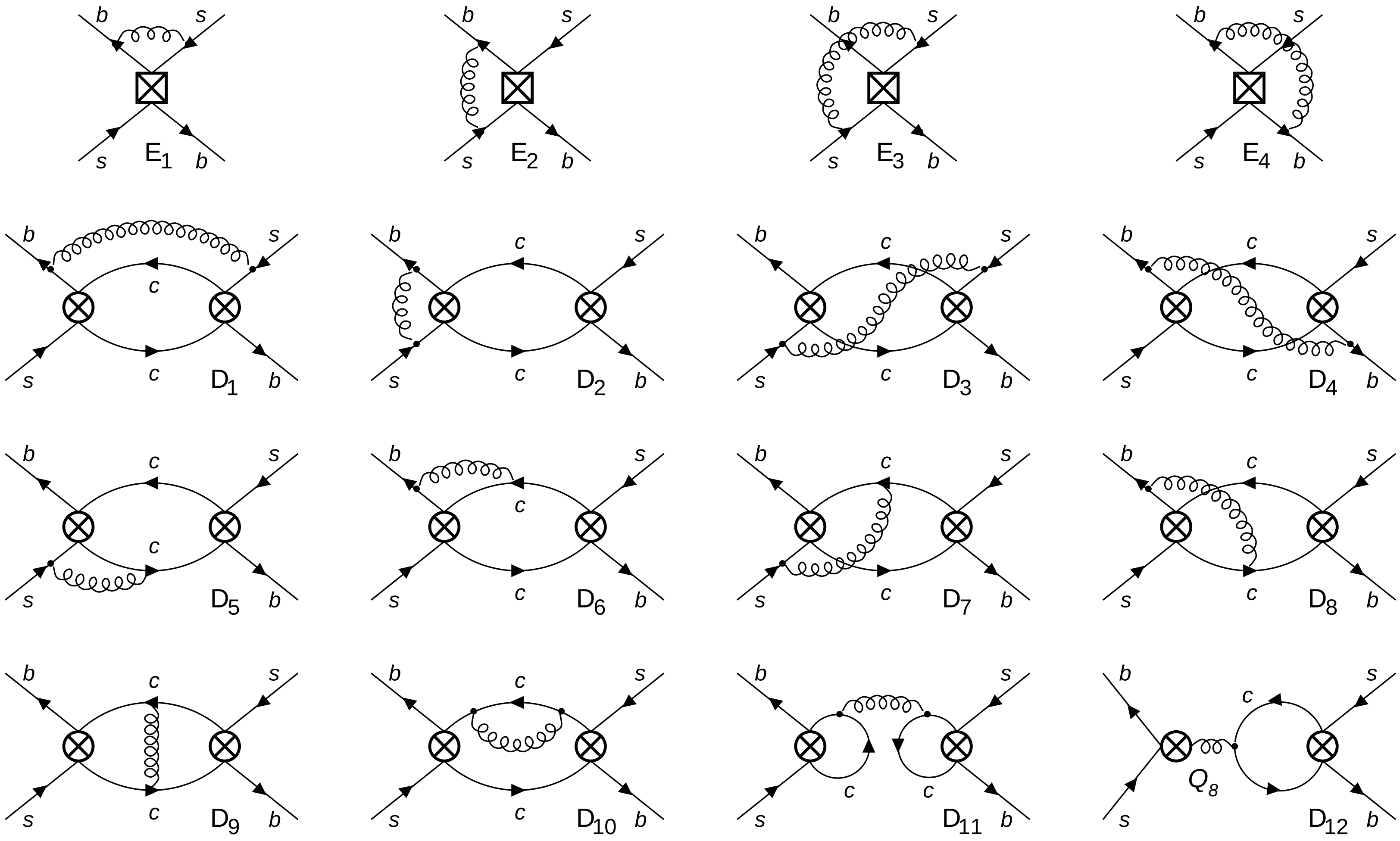
$$\begin{aligned} \Delta\Gamma_s &= \Delta\Gamma_s^0 \left(1 + \delta^{\rm Lattice} + \delta^{\rm QCD} + \delta^{\rm HQE}\right) \\ &= 0.144 \, \text{ps}^{-1} \left(1 - 0.14 - 0.06 - 0.19\right) \end{aligned}$$

$\Rightarrow$ looks ok!

No signs for any violation of quark-hadron duality

Calculation of Γ_{12}

Calculating LO (no gluons) plus the following diagrams



Calculation of Γ_{12}

$$\Gamma_{12} = \sum_{q=u,c} \sum_{p=u,c} \lambda_q \lambda_p \left(\Gamma_{qp}^{Q} \langle Q \rangle + \Gamma_{qp}^{Q_S} \langle Q_S \rangle + \Gamma_{qp}^{\tilde{Q}_S} \langle \tilde{Q}_S \rangle \right) + \left(\frac{\Lambda}{m_b} \right)^4 \Gamma_4 + \ldots.$$

with

$$\begin{aligned} Q &= (\bar{b}_i s_i)_{V-A} \cdot (\bar{b}_j s_j)_{V-A} \\ Q_S &= (\bar{b}_i s_i)_{S-P} \cdot (\bar{b}_i s_i)_{S-P} \\ \tilde{Q}_S &= (\bar{b}_i s_j)_{S-P} \cdot (\bar{b}_i s_j)_{S-P} \end{aligned}$$

and the matrix elements

$$\begin{aligned} \langle \bar{B}_s | Q | B_s \rangle &= \frac{8}{3} f_{B_S}^2 M_{B_S}^2 B \\ \langle \bar{B}_s | Q_S | B_s \rangle &= -\frac{5}{3} f_{B_S}^2 M_{B_S}^2 B_S' = -\frac{5}{3} f_{B_S}^2 M_{B_S} \frac{M_{B_S}^2}{(\bar{m}_b + \bar{m}_s)^2} B_S \\ \langle \bar{B}_s | \tilde{Q}_S | B_s \rangle &= \frac{1}{3} f_{B_S}^2 M_{B_S}^2 \tilde{B}_S' = \frac{1}{3} f_{B_S}^2 M_{B_S} \frac{M_{B_S}^2}{(\bar{m}_b + \bar{m}_s)^2} \tilde{B}_S \end{aligned}$$

f_{B_s}, B, B_S and $\tilde{B}_S$ have to be determined non-perturbatively!

B, B_S and $\tilde{B}_S$ are equal to one in vacuum insertion approximation

Calculation of Γ_{12}

Expanding in the small s momenta one gets contributions of dimension 7

$$
\begin{aligned}
R_0 &= Q_S + \tilde{Q}_S + \frac{1}{2} Q \\
R_1 &= \frac{m_s}{m_b} (\bar{b}_i s_i)_{S-P} (\bar{b}_j s_j)_{S+P} \\
R_2 &= \frac{1}{m_b^2} (\bar{b}_i \overleftarrow{D}_\rho \gamma^\mu (1-\gamma_5) D^\rho s_i)(\bar{b}_j \gamma_\mu (1-\gamma_5) s_j) \\
R_3 &= \frac{1}{m_b^2} (\bar{b}_i \overleftarrow{D}_\rho (1-\gamma_5) D^\rho s_i)(\bar{b}_j (1-\gamma_5) s_j)
\end{aligned}
$$

- Q, Q_S, $\tilde{Q}_S$, $m_b/m_s R_1$ and $m_b/m_s \tilde{R}_1$ form the so-called $\Delta B = 2$ **SUSY-basis**
  The corresponding matrix elements can be determined by current lattice technologies
- For R_2 and R_3 there exist no non-perturbative determinations
  A first estimate with QCD sum rules was made by **Mannel, Pecjak, Pivovarov**
  Current estimates rely on vacuum insertion approximation

Finally $\Delta\Gamma_s$ is measured!

Finally $\Delta\Gamma_s$ is measured! E.g. from $B_s \to J/\psi\phi$

LHCb Moriond 2012,...; ATLAS; CDF; DO; CMS

$$\Delta\Gamma_s^{\text{Exp}} = (0.081 \pm 0.006)\,\text{ps}^{-1}$$
$$\Delta\Gamma_s^{\text{SM}} = (0.088 \pm 0.020)\,\text{ps}^{-1}$$

HFAG 2015
ABL, to appear

- **HQE works also for $\Delta\Gamma_s$!!!**
- But there is a 2.4 σ discrepancy between CP-specific lifetimes and extractions from $B_s \to J/\psi K^+K^-, \pi^+\pi^-$!

$$\Delta\Gamma_d^{\text{Exp}} = (0.7 \pm 7) \cdot 10^{-3}\text{ps}^{-1}$$
$$\Delta\Gamma_d^{\text{SM}} = (2.61 \pm 0.59) \cdot 10^{-3}\text{ps}^{-1}$$

HFAG 2015
ABL, to appear

Theory Prediction for $\Delta\Gamma_s$

$\Delta\Gamma_s^{\mathrm{SM}}$	ABL, to appear	LN 2011	LN 2006
Central Value	0.088 ps^{-1}	0.087 ps^{-1}	0.096 ps^{-1}
$\delta(B_{\tilde{R}_2})$	14.8%	17.2%	15.7%
$\delta(f_{B_s}\sqrt{B})$	13.9%	13.5%	34.0%
$\delta(\mu)$	8.4%	7.8%	13.7%
$\delta(V_{cb})$	4.9%	3.4%	4.9%
$\delta(\widetilde{B}_S)$	2.1%	4.8%	3.1%
$\delta(B_{R_0})$	2.1%	3.4%	3.0%
$\delta(\bar{z})$	1.1%	1.5%	1.9%
$\delta(m_b)$	0.8%	0.1%	1.0%
$\delta(B_{\tilde{R}_1})$	0.7%	1.9%	$---$
$\delta(B_{\tilde{R}_3})$	0.6%	0.5%	$----$
$\delta(B_{R_1})$	0.5%	0.8%	$---$
$\delta(B_{R_3})$	0.2%	0.2%	$---$
$\delta(m_s)$	0.1%	1.0%	1.0%
$\delta(\gamma)$	0.1%	0.3%	1.0%
$\delta(\alpha_s)$	0.1%	0.4%	0.1%
$\delta(\lvert V_{ub}/V_{cb}\rvert)$	0.1%	0.2%	0.5%
$\delta(\bar{m}_t(\bar{m}_t))$	0.0%	0.0%	0.0%
$\sum\delta$	22.8%	24.5%	40.5%

- **Bag parameters of dimension 7 and 6**
- **Reduce μ-dependence: $\Gamma_3^{(2)}$ and $\Gamma_4^{(1)}$**
- **first step: Asatrian, Hovhannisyan, Yeghiazaryan, arXiv:1210.7939**

More precise prediction of $\Delta\Gamma_s$

Cancellation of non-perturbative uncertainties in the ratio Γ_{12}^q / M_{12}^q

$$-\frac{\Gamma_{12}^s}{M_{12}^s} = \frac{\Gamma_{12}^{s,cc}}{\tilde{M}_{12}^s} + 2\frac{\lambda_u}{\lambda_t}\frac{\Gamma_{12}^{s,cc} - \Gamma_{12}^{s,uc}}{\tilde{M}_{12}^s} + \left(\frac{\lambda_u}{\lambda_t}\right)^2 \frac{\Gamma_{12}^{s,cc} - 2\Gamma_{12}^{s,uc} + \Gamma_{12}^{s,uu}}{\tilde{M}_{12}^s}$$

with

$$\frac{\Gamma_{12}^{s,xy}}{\tilde{M}_{12}^s} = \frac{\pi m_b^2 \left[8G^{xy} 1 + 5G_S^{xy}\frac{B_S'}{B} + \mathcal{O}\left(\frac{1}{m_b}\right)\right]}{6M_W S_0(x_t)\hat{\eta}_B}$$

Relation to observables $\frac{\Delta\Gamma_q}{\Delta M_q} = -\Re\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right)$, $\quad a_{sl}^q = \Im\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right)$

More precise determination of $\Delta\Gamma_q$

$$\Delta\Gamma_s^{\mathrm{SM,2015\ II}} = \left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\mathrm{SM}} \Delta M_s^{\mathrm{Exp}} = 0.085 \pm 0.015\ \text{ps}^{-1}$$

Test of HQE

$$\left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\mathrm{Exp}} / \left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\mathrm{SM}} = 0.95 \pm 0.07 \pm 0.17$$

Error budget of $\Delta\Gamma_s / \Delta M_s$

$\Delta\Gamma_s^{\rm SM} / \Delta M_s^{\rm SM}$	ABL, to appear	LN 2011	LN 2006
Central Value	$48.1 \cdot 10^{-4}$	$50.4 \cdot 10^{-4}$	$49.7 \cdot 10^{-4}$
$\delta(B_{R_2})$	14.8%	17.2%	15.7%
$\delta(\mu)$	8.4%	7.8%	9.1%
$\delta(\widetilde{B}_S)$	2.1%	4.8%	3.1%
$\delta(B_{R_0})$	2.1%	3.4%	3.0%
$\delta(\bar{z})$	1.1%	1.5%	1.9%
$\delta(m_b)$	0.8%	1.4%	1.0%
$\delta(m_t)$	0.7%	1.1%	1.8%
$\delta(B_{\tilde{R}_1})$	0.7%	1.9%	$---$
$\delta(B_{\tilde{R}_3})$	0.6%	0.5%	$-----$
$\delta(B_{R_1})$	0.5%	0.8%	$---$
$\delta(B_{R_3})$	0.2%	0.2%	$---$
$\delta(\alpha_s)$	0.2%	0.8%	0.1%
$\delta(m_s)$	0.1%	1.0%	0.1%
$\delta(\gamma)$	0.0%	0.0%	0.1%
$\delta(\|V_{ub}/V_{cb}\|)$	0.0%	0.0%	0.1%
$\delta(V_{cb})$	0.0%	0.0%	0.0%
$\sum \delta$	17.3%	20.1%	18.9%

Semi-leptonic asymmetries

HQE works! SM predictions: **ABL 2015, update of A.L., U. Nierste, 1102.4274**

$$a_{fs}^{s} = (2.22 \pm 0.27) \cdot 10^{-5}$$
$$a_{fs}^{d} = -(4.7 \pm 0.6) \cdot 10^{-4}$$

Measurements from **BaBar, D0, LHCb**

$$a_{sl}^{s} = (-480 \pm 480) \cdot 10^{-5}$$ **HFAG 2015**

$$a_{sl}^{d} = (1 \pm 20) \cdot 10^{-4}$$ **HFAG 2015**

- Consistent with the SM
- Sizable space for NP
- But also consistent with the dimuon asymmetry **more data urgently needed**
  New interpretation of the dimuon asymmetry **Borissov, Hoeneisen 1303.0175**

$$A_{sl}^{b} = C_d a_{sl}^{d} + C_s a_{sl}^{s} + C_\Gamma \frac{\Delta\Gamma_d}{\Gamma_d}$$

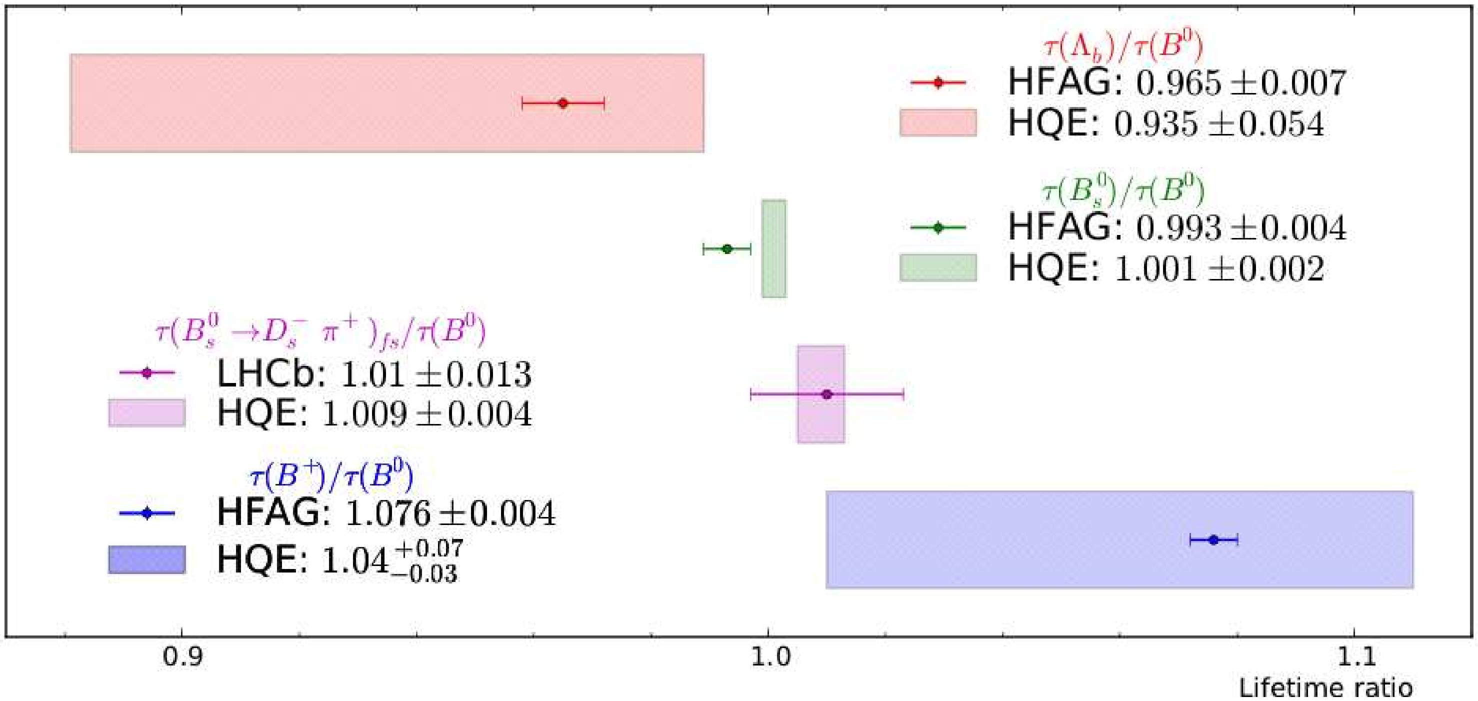
There is still sizable space for NP in $\Delta\Gamma_d$

Error budget for a_{sl}^s

$a_{fs}^{s,SM}$	ABL, to appear	LN 2011	LN 2006
Central Value	$2.22 \cdot 10^{-5}$	$1.90 \cdot 10^{-5}$	$2.06 \cdot 10^{-5}$
$\delta(\mu)$	9.5%	8.9%	12.7%
$\delta(\lvert V_{ub}/V_{cb}\rvert)$	5.0%	11.6%	19.5%
$\delta(\bar{z})$	4.6%	7.9%	9.3%
$\delta(B_{\tilde{R}_3})$	2.6%	2.8%	2.5%
$\delta(\gamma)$	1.3%	3.1%	11.3%
$\delta(B_{R_3})$	1.1%	1.2%	1.1%
$\delta(m_b)$	1.0%	2.0%	3.7%
$\delta(m_t)$	0.7%	1.1%	1.8%
$\delta(\alpha_s)$	0.5%	1.8%	0.7%
$\delta(B_{\tilde{R}_1})$	0.5%	0.2%	$---$
$\delta(\widetilde{B}_S)$	0.3%	0.6%	0.4%
$\delta(\mathcal{B}_{R_0})$	0.2%	0.3%	$---$
$\delta(\mathcal{B}_{R_2})$	0.1%	0.1%	$---$
$\delta(m_s)$	0.1%	0.1%	0.1%
$\delta(\mathcal{B}_{R_1})$	$< 0.1\%$	0.0%	$---$
$\delta(V_{cb})$	0.0%	0.0%	0.0%
$\sum \delta$	12.2%	17.3%	27.9%

How precise is the HQE?

Lifetimes can be used to test the HQE - **HFAG 2015** vs **A.L. 2014**



The theory prediction is strongly limited
by a **lack of up-to-date lattice values** for the matrix elements

- Matrix elements for B^+ and B_d stem from 2001
- Only an *exploratory* study for Λ_b from 1999
- No studies of charm - perturbative part looks promising **AL, Rauh 2013**

Outlook

Status 2015

1. SM and Experiment agree well for mixing quantities
2. HQE works very well
3. Still space for NP effects

Improvement of the SM precision

- Improvement in M_{12}: **maybe $\pm 5\%$ in 10 years?**
  - More precise lattice value of $f_B^2 B$
- Improvement in Γ_{12}: **maybe $\pm 5\%$ in 10 years?**
  - More precise lattice values of $\Delta B = 2$ dim 6 operators **(full SUSY basis)**
  - Non-perturbative determination of dim 7 operators
  - NNLO calculations: $\Gamma_3^{(2)}$ and $\Gamma_4^{(1)}$
- Precision of HQE
  - More precise lattice values of $\Delta B = 0$ dim 6 operators
  - First lattice values of $\Delta C = 0$ dim 6 operators