Multi-body B decays

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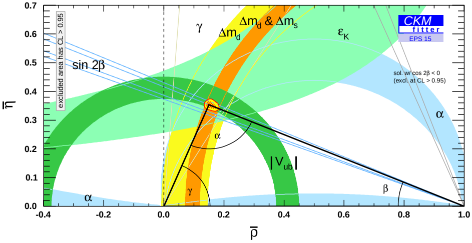
B decays physics goals

Plot from CKM Fitter collaboration



→ $\gamma(\phi_3)$: $B \to DK, \pi$
(GLW, ADS, GGSZ methods)
$b \to c\bar{u}s$ (tree level)

→ $\alpha(\phi_2)$: $B \to \pi\pi,\ \rho\pi,\ \rho\rho$
$b \to u\bar{u}d$ (loop sensitive)

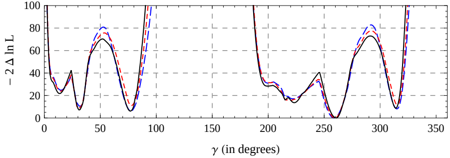
→ $\beta(\phi_1)$: $B \to K$ + charmonium
$b \to c\bar{c}d$ (loop sensitive)

- Amazing work by by the experimental community!
- Direct measurements (CKMFitter) : $\alpha(\phi_2) = 87.6^{+3.5}_{-3.3}$, $\beta(\phi_2) = 21.85^{+0.68}_{-0.67}$
- Direct γ measurement is statistics limited : $\gamma(\phi_3) = 73.2^{+6.3}_{-7.0}$

However, LHCb program to get $\delta\gamma$ to less than 1° (long run)

γ from three-body decays

- 3-body final state under SU(3) : $B \to K\pi\pi, K\overline{K}K$
  - $\rightarrow$ 6 final state symmetries : permutations of 3 particles
- Fully-symmetric state (Rey-Le Lorier, London, 1109.0881)
  - $\rightarrow$ More observables than unknowns $\Rightarrow$ γ can be extracted
  - $\rightarrow$ BB, Imbeault, London, 1303.0846



$\rightarrow$ SM-like : 77°

$\rightarrow$ 32°, 259°, 315°

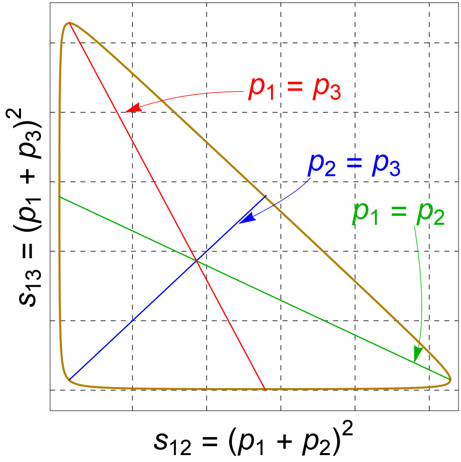
$K\pi\pi - K\overline{K}K$ puzzle ?

- Group theory analysis : I-spin, U-spin, SU(3) relations
  - $\rightarrow$ BB, Gronau, Imbeault, London, Rosner, 1402.2909

Three-body decays : Dalitz plots

- Three-body final state : $|P_1(p_1)P_2(p_2)P_3(p_3)\rangle$ $s_{ij} = (p_i + p_j)^2$

  $\rightarrow$ Momentum dependent. One relation $s_{12} + s_{23} + s_{13} = \text{constant}$



Features of a Dalitz plot:

- Independent measurements at different points may be possible
- Same SM weak phase (such as γ); Hadronic parameters are local
- Consistency checks : Flavor symmetries (SU(3), U-spin etc) provide amplitude relationships

Three-body decays : Dalitz plots

- General three-body amplitude : $\mathcal{A} = a + e^{i\phi} b$
  - $\rightarrow$ 2 parts separated by a relative weak phase ϕ (ignore overall phase)
  - $\rightarrow$ $a \equiv a(p_1, p_2, p_3)$, $b \equiv b(p_1, p_2, p_3)$: $|a|, |b|$, relative strong phase
  - $\rightarrow$ 4 unknowns at any general point on a Dalitz plot
- Only 2 (local/momentum dependent) observables
  - $\rightarrow$ CP averaged branching ratio $\propto |\mathcal{A}|^2 + |\overline{\mathcal{A}}|^2$
  - $\rightarrow$ direct-CP asymmetry $\propto |\mathcal{A}|^2 - |\overline{\mathcal{A}}|^2$
- The general three-body state is NOT a CP eigenstate!
  - $\rightarrow$ CP conjugate of $K_s(p_1)\pi^+(p_2)\pi^-(p_3)$ is $K_s(p_1)\pi^-(p_2)\pi^+(p_3)$
  - $\rightarrow$ Indirect/mixing-induced CP asymmetries not obvious

Bottom Line : Not enough observables to solve for γ

Can U-spin symmetry help?

- More observables in U-spin pairs : $B_s^0 \to K_S \pi^+ \pi^-, B_d^0 \to K_S K^+ K^-$
- Amplitudes have the same structure as before : $\mathcal{A} = a + e^{i\phi} b$

  $\to \mathcal{A}_d = V_{ub}^* V_{ud} T_d + V_{cb}^* V_{cd} P_d \qquad \mathcal{A}_s = V_{ub}^* V_{us} T_s + V_{cb}^* V_{cs} P_s$

  $\to \overline{\mathcal{A}}_d = V_{ub}^* V_{ud} \overline{T}_d + V_{cb}^* V_{cd} \overline{P}_d \qquad \overline{\mathcal{A}}_s = V_{ub}^* V_{us} \overline{T}_s + V_{cb}^* V_{cs} \overline{P}_s$
- T_d and $\overline{T}_d$ come from two regions of the same Dalitz plot

  $\to |p_1 p_2 p_3\rangle$ and $|p_1 p_3 p_2\rangle$ represent 2 regions of the same Dalitz plot
- U-spin relates hadronic parameters : $T_d = T_s$, ...

  $\to$ U-spin relationship : $|\mathcal{A}_d|^2 - |\overline{\mathcal{A}}_d|^2 = |\overline{\mathcal{A}}_s|^2 - |\mathcal{A}_s|^2$ (consistency check)
- Solve this problem using indirect-CP asymmetry : $\mathrm{Im}\left[(\mathrm{q/p})\mathrm{A}^*\overline{\mathrm{A}}\right]$

  $\to$ 2-body strategy : Fleischer, hep-ph/9903456

  $\to$ Independent measurement gives $B_d - \overline{B}_d$ mixing phase

A_{CP} in three-body decays

- Local (momentun dependent) branching fraction and CP asymmetry:

$$\Gamma(t) = \frac{\Gamma(B^0 \to f) + \Gamma(\overline{B}^0 \to f)}{2}$$

$$A_{CP}(t) = \frac{\Gamma(B^0 \to f) - \Gamma(\overline{B}^0 \to f)}{\Gamma(B^0 \to f) + \Gamma(\overline{B}^0 \to f)}$$

Note that $|f\rangle \neq |\overline{f}\rangle$

A_{CP} not a true "CP asymmetry"

Used for simplification

- Time-dependent Dalitz analysis : extract coefficients

$$\cos(\Delta m t) \propto \int_{\text{bin}} |\mathcal{A}|^2 - |\tilde{\mathcal{A}}|^2$$

$$\sin(\Delta m t) \propto \int_{\text{bin}} \text{Im}\left[(\text{q}/\text{p})\mathcal{A}^*\tilde{\mathcal{A}}\right]$$

$$\sinh(\Delta\Gamma t/2) \propto \int_{\text{bin}} \text{Re}\left[(\text{q}/\text{p})\mathcal{A}^*\tilde{\mathcal{A}}\right]$$

$$\cosh(\Delta\Gamma t/2) \propto \int_{\text{bin}} |\mathcal{A}|^2 + |\tilde{\mathcal{A}}|^2$$

$\rightarrow$ $\mathcal{A} = \langle f|B^0\rangle$, $\tilde{\mathcal{A}} = \langle \overline{f}|B^0\rangle$

$\rightarrow$ $\mathcal{A}$ has the form $a + e^{i\phi}b$

$\rightarrow$ Also for U-spin pair

Now we have enough observables to extract weak phase information

Further details in : BB, D. London, 1503.00737

Source of uncertainty

- In order to perform a fit we may use redefinitions:

$$\int_{\text{bin}} |\mathcal{A}|^2 \equiv |A'|^2 \,, \quad \int_{\text{bin}} |\tilde{\mathcal{A}}|^2 \equiv |\tilde{A}'|^2 \,, \quad \int_{\text{bin}} \mathrm{Im}\left[(\mathrm{q/p})\mathcal{A}^*\tilde{\mathcal{A}}\right] \equiv \ ?$$

- Use approximation : $\int_{\text{bin}} \mathrm{Im}\left[(\mathrm{q/p})\mathcal{A}^*\tilde{\mathcal{A}}\right] \approx \mathrm{Im}\left[(\mathrm{q/p})\mathcal{A}'^*\tilde{\mathcal{A}}'\right]$

  $\rightarrow$ Relation is exact for a point-sized bin

  $\rightarrow$ Approximation gets worse as bin size increases

- Given data set $\Rightarrow$ Larger statistical error for smaller bins
- Optimum bin size required to strike a balance

  $\rightarrow$ Vary bin size for fits to check its effect on physical parameters

Flavor-SU(3) symmetry

- Symmetry under interchange of u, d, s quarks
- 8 identical pseudoscalars under flavor SU(3) (8 generators of SU(3))

  $\rightarrow$ 3 Pions ($\pi^{\pm}, \pi^0$), 4 Kaons ($K^{\pm}, K^0, \overline{K}^0$) (+ a combination of η, η')

  $\rightarrow$ $|P_1 P_2 P_3\rangle$ has three identical particles under SU(3)

- The fully-symmetric state is simple : amplitude symmetrized in momentum

  $\rightarrow$ $\left|K^0(p_1)\pi^+(p_2)\pi^-(p_3)\right\rangle_{\rm FS} = \left|K^0(p_1)\pi^+(p_3)\pi^-(p_2)\right\rangle_{\rm FS}$

  $\rightarrow$ Mixing-induced CP asymmetry is simple : $\propto {\rm Im}\left[({\rm q/p})\mathcal{A}^*\overline{\mathcal{A}}\right]$

- Complication : amplitude analysis is required

  $\rightarrow$ Isobar method : $\mathcal{A}(s_{12}, s_{13}) \propto \sum_j c_j e^{i\phi_j} F^j_{\rm BW}(s_{12}, s_{13})$

  $\rightarrow$ Superposition of non-resonant and resonant (quasi-two-body) modes

Flavor-SU(3) relations

- 9 independent flavor-SU(3) matrix elements describe all $B \to 3P$ amplitudes

  $\to$ BB, Gronau, Imbeault, London, Rosner, 1402.2909

- 16 $b \to s$ decay channels and 16 $b \to d$ decay channels

  $\to$ Amplitudes of these decays depend on 9 matrix elements

  $\Rightarrow$ Not all amplitudes are independent : there are relations

- Isospin symmetry alone gives 6 relations in $b \to s$ (momentum dependent)

  $\to \ \mathcal{A}(B^+ \to K^0\pi^+\pi^0)_{\rm FS} \ = \ - \ \mathcal{A}(B_d^0 \to K^+\pi^0\pi^-)_{\rm FS}$

  $\to \ \sqrt{2}\mathcal{A}(B_s^0 \to 3\pi^0)_{\rm FS} \ = \ - \ \sqrt{3}\mathcal{A}(B_s^0 \to \pi^0\pi^+\pi^-)_{\rm FS}$ (4 more)

- Full SU(3) gives a 7th relation in $b \to s$ decays:

  $\to \ \sqrt{2}\mathcal{A}(B^+ \to K^+\pi^+\pi^-)_{\rm FS} \ = \ \mathcal{A}(B^+ \to K^+K^+K^-)_{\rm FS}$

  $\to$ U spin + FS version of : $\mathcal{A}(K_1^+\pi_2^+\pi^-) + \mathcal{A}(K_2^+\pi_1^+\pi^-) \ = \ \mathcal{A}(K^+K^+K^-)$

  Gronau, Rosner, hep-ph/0304178

Using flavor-SU(3) relations

- Relations can be checked using amplitude analysis
- Expect larger deviations from SU(3) in some regions

  $\rightarrow$ Near narrow resonances : resonance masses break flavor symmetries

  $\rightarrow$ Near kinematic boundaries since $m_K \neq m_\pi$
- No contributions from vector resonances to FS states

  $\rightarrow$ Smaller contribution to SU(3) breaking in FS states
- Integrate over the kinematically-allowed regions

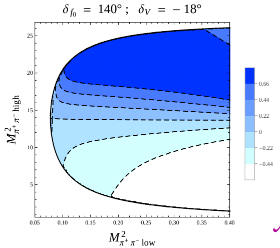
  $\rightarrow$ Expect smaller deviations from SU(3)

  $\rightarrow$ Possible cancellation of SU(3) breaking in the avergae
- Expect no (or tiny) deviations from Isospin relations

  $\rightarrow$ Deviations most likely show loopholes in amplitude analysis

Using Flavor SU(3) in B^+ decays

- Local asymmetries observed by LHCb in $B^+ \to \pi^+\pi^+\pi^-$
  - $\to$ Region : $s_{\pi^+\pi^- \mathrm{low}} < 0.4~\mathrm{GeV}^2$, $s_{\pi^+\pi^- \mathrm{high}} > 15~\mathrm{GeV}^2$
  - $\to$ Observed asymmetry : $A_{CP} \sim (60 \pm 10)\%$ Aaij et al.,1310.4740
- ρ (vector) & f^0 (scalar) channels interfere : BB, Gronau, Rosner, 1306.2625
  - $\to$ $\mathcal{A}_{B^+\to\pi^+\pi^+\pi^-}(s_{\mathrm{low}}, s_{\mathrm{high}}) = \mathcal{A}_\rho F_\rho(s_{\mathrm{low}}, s_{\mathrm{high}}) + \mathcal{A}_{f^0} F_{f^0}(s_{\mathrm{low}}, s_{\mathrm{high}})$
  - $\to$ $\mathcal{A}_{f^0}, \mathcal{A}_\rho$ from Flavor-SU(3) fits to $B \to PS, PV$ decays



$\to$ Large local CP asymmetries can be due to interference between different isobar channels

$\to$ Local effects can be smaller in FS state

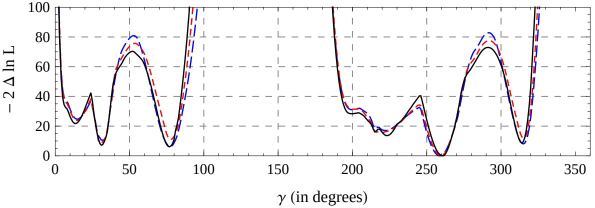
$\to$ Test for U-spin FS relation (local and integrated)

$$\mathcal{A}(B^+ \to \pi^+\pi^+\pi^-)_{\mathrm{FS}} = \sqrt{2}\mathcal{A}(B^+ \to \pi^+K^+K^-)_{\mathrm{FS}}$$

Three-body decay route to new physics

- γ extracted by applying SU(3) to fully symmetric state

  $\rightarrow$ Full Analysis : BB, Imbeault, London, 1303.0846



$\rightarrow$ SM-like : 77°

$\rightarrow$ Other solutions : 32°, 259°, 315°

$\rightarrow$ Discrete Ambiguity

- Key : γ extraction by applying SU(3) to other symmetry states

  $\rightarrow$ Break discrete ambiguity : more information from other states

- Also key : estimate systematic uncertainties in γ extraction

- Interesting situation : γ widely different from SM value

  $\rightarrow$ Significant SU(3) breaking ?

  $\rightarrow$ NP in three-body B decays; $K\pi\pi - KKK$ puzzle

Summary

- Three-body B decays may provide new information
- γ from three-body B decays is loop sensitive
- Flavor symmetry tests $\rightarrow$ U-spin, SU(3) :
  Compare flavor symmetry related decays
- U-spin related three-body decay pairs :
  Time-dependent Dalitz analysis for γ
- Amplitude analysis $\rightarrow$ SU(3) related final states
- Future theory studies of additional symmetry states

Three-body $b \to s$: more SU(3) relations

4 additional relations (isospin symmetry) :

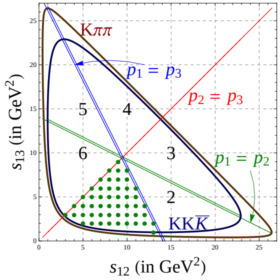
$$\sqrt{2}\mathcal{A}(B^+ \to K^0\pi^+\pi^0)_{\rm FS} = \mathcal{A}(B_d^0 \to K^0\pi^+\pi^-)_{\rm FS} + \sqrt{2}\mathcal{A}(B_d^0 \to K^0\pi^0\pi^0)_{\rm FS}$$

$$\sqrt{2}\mathcal{A}(B_d^0 \to K^+\pi^0\pi^-)_{\rm FS} = \mathcal{A}(B^+ \to K^+\pi^+\pi^-)_{\rm FS} + \sqrt{2}\mathcal{A}(B_d^0 \to K^0\pi^0\pi^0)_{\rm FS}$$

$$\mathcal{A}(B^+ \to K^+K^+K^-)_{\rm FS} + \sqrt{2}\mathcal{A}(B^+ \to K^+K^0\overline{K}^0)_{\rm FS}$$
$$= \sqrt{2}\mathcal{A}(B_d^0 \to K^0K^+K^-)_{\rm FS} + \mathcal{A}(B_d^0 \to K^0K^0\overline{K}^0)_{\rm FS}$$

$$\mathcal{A}(B_s^0 \to \pi^0K^+K^-)_{\rm FS} + \sqrt{2}\mathcal{A}(B_s^0 \to \pi^0K^0\overline{K}^0)_{\rm FS}$$
$$= -\sqrt{2}\mathcal{A}(B_s^0 \to \pi^-K^+\overline{K}^0)_{\rm FS} - \mathcal{A}(B_s^0 \to \pi^+K^-K^0)_{\rm FS}$$

γ from 3-body using flavor SU(3)



Fully-symmetric amplitudes :

$$2\mathcal{A}(B_d^0 \to K^+\pi^0\pi^-)_{\rm FS} = b\ e^{i\gamma} - \kappa\ c$$

$$\sqrt{2}\mathcal{A}(B_d^0 \to K^0\pi^+\pi^-)_{\rm FS} = -(d + \tilde{P}'_{uc})e^{i\gamma} - a + \kappa\ d$$

$$\sqrt{2}\mathcal{A}(B^+ \to K^+\pi^+\pi^-)_{\rm FS} = -(c + \tilde{P}'_{uc})e^{i\gamma} - a + \kappa\ b$$

$$\mathcal{A}(B_d^0 \to K^0K^0\overline{K}^0)_{\rm FS} = \alpha_{\rm SU(3)}(\tilde{P}'_{uc}\ e^{i\gamma} + a)$$

$$\mathcal{A}(B_d^0 \to K^+K^0K^-)_{\rm FS} = \alpha_{\rm SU(3)}\mathcal{A}(B^+ \to K^+\pi^+\pi^-)_{\rm FS}$$

- Hadronic parameters : $a, b, c, d, \alpha_{\rm SU(3)}, \tilde{P}'_{uc}$; Theory input : κ
- $\tilde{P}'_{uc} = 0$ for no CP-asymmetry in $3K_S$ channel
- Observables : $X = |\mathcal{A}|^2 + |\overline{\mathcal{A}}|^2$, $Y = |\mathcal{A}|^2 - |\overline{\mathcal{A}}|^2$, $Z = {\rm Im}[\mathcal{A}^*\overline{\mathcal{A}}]$
- Simplest fit : $\alpha_{\rm SU(3)} = 1$; 9 observables $\Rightarrow$ 7 hadronic unknowns + γ

  $\to$ Full Analysis : BB, Imbeault, London, 1303.0846

"$B \to \pi K$ puzzle" and γ

- Flavor SU(3) relates: $B^+ \to \pi^+ K^0,\ \pi^0 K^+\ ;\ \ B_d^0 \to \pi^- K^+,\ \pi^0 K^0$
- 9 measurements : 4 branching ratios + 4 direct CP asymmetries + 1 indirect CP asymmetry
- 9 unknowns : 4 magnitudes + 3 relative strong phases + β + γ
- SM can explain data but still room for NP
  - $\to$ Best fit value for $\gamma = (35.9 \pm 7.7)^\circ$: large deviation from $\gamma_{\rm tree}$
  - $\to$ Baek, Chiang, London, 0903.3086
- Problematic in (certain) NP models as well!
  - $\to$ Imbeault, Baek, London, 0802.1175
  - $\to$ Endo, Yoshinaga, 1206.0067