Multi-body B decays

Bhubanjyoti Bhattacharya

Université de Montréal

November 5, 2015

LHCb Implications Workshop,
3 - 5 Nov, 2015
CERN
$B$ decays physics goals

Plot from CKM Fitter collaboration

$\rightarrow \gamma(\phi_3) : B \to DK, \pi$
GLW, ADS, GGSZ methods
$b \to c\bar{u}s$ (tree level)

$\rightarrow \alpha(\phi_2) : B \to \pi\pi, \rho\pi, \rho\rho$
$b \to u\bar{u}d$ (loop sensitive)

$\rightarrow \beta(\phi_1) : B \to K +$ charmonium
$b \to c\bar{c}d$ (loop sensitive)

- Amazing work by the experimental community!

- Direct measurements (CKMFitter): $\alpha(\phi_2) = 87.6^{+3.5}_{-3.3}$, $\beta(\phi_2) = 21.85^{+0.68}_{-0.67}$

- Direct $\gamma$ measurement is statistics limited: $\gamma(\phi_3) = 73.2^{+6.3}_{-7.0}$

However, LHCb program to get $\delta\gamma$ to less than $1^\circ$ (long run)
A result

\[ \gamma \text{ from three-body decays} \]

- 3-body final state under SU(3) : \( B \to K\pi\pi, K\bar{K}K \)
  - 6 final state symmetries : permutations of 3 particles
- Fully-symmetric state \((\text{Rey-Le Lorier, London, 1109.0881})\)
  - More observables than unknowns \(\Rightarrow\) \(\gamma\) can be extracted
  - BB, Imbeault, London, 1303.0846

![Graph showing \( \gamma \) range]

- SM-like : 77°
  - 32°, 259°, 315°
- \( K\pi\pi - K\bar{K}K \) puzzle?

- Group theory analysis : I-spin, U-spin, SU(3) relations
  - BB, Gronau, Imbeault, London, Rosner, 1402.2909
Three-body final state: \[ |P_1(p_1)P_2(p_2)P_3(p_3)\rangle \quad s_{ij} = (p_i + p_j)^2 \]

\[ s_{12} + s_{23} + s_{13} = \text{constant} \]

Features of a Dalitz plot:

- Independent measurements at different points may be possible
- Same SM weak phase (such as $\gamma$);
- Hadronic parameters are local
- Consistency checks: Flavor symmetries (SU(3), U-spin etc) provide amplitude relationships
Three-body decays: Dalitz plots

- General three-body amplitude: \( A = a + e^{i \phi} b \)
  
  \( \rightarrow \) 2 parts separated by a relative weak phase \( \phi \) (ignore overall phase)

  \( \rightarrow a \equiv a(p_1, p_2, p_3) \), \( b \equiv b(p_1, p_2, p_3) \) : \( |a|, |b| \), relative strong phase

  \( \rightarrow \) 4 unknowns at any general point on a Dalitz plot

- Only 2 (local/momentum dependent) observables
  
  \( \rightarrow \) CP averaged branching ratio \( \propto |A|^2 + |\overline{A}|^2 \)

  \( \rightarrow \) direct-CP asymmetry \( \propto |A|^2 - |\overline{A}|^2 \)

- The general three-body state is **NOT** a CP eigenstate!
  
  \( \rightarrow \) CP conjugate of \( K_s(p_1)\pi^+(p_2)\pi^-(p_3) \) is \( K_s(p_1)\pi^-(p_2)\pi^+(p_3) \)

  \( \rightarrow \) Indirect/mixing-induced CP asymmetries not obvious

**Bottom Line**: Not enough observables to solve for \( \gamma \)
Can U-spin symmetry help?

- More observables in U-spin pairs: \( B^0_s \rightarrow K_S \pi^+ \pi^- \), \( B^0_d \rightarrow K_S K^+ K^- \)

- Amplitudes have the same structure as before: \( A = a + e^{i\phi} b \)

\[
A_d = V_{ub}^* V_{ud} T_d + V_{cb}^* V_{cd} P_d \\
A_s = V_{ub}^* V_{us} T_s + V_{cb}^* V_{cs} P_s
\]

\[
\overline{A}_d = V_{ub}^* V_{ud} \overline{T}_d + V_{cb}^* V_{cd} \overline{P}_d \\
\overline{A}_s = V_{ub}^* V_{us} \overline{T}_s + V_{cb}^* V_{cs} \overline{P}_s
\]

- \( T_d \) and \( \overline{T}_d \) come from two regions of the same Dalitz plot

\[
|p_1p_2p_3\rangle \text{ and } |p_1p_3p_2\rangle \text{ represent 2 regions of the same Dalitz plot}
\]

- U-spin relates hadronic parameters: \( T_d = T_s , \ldots \)

\[
\text{U-spin relationship: } |A_d|^2 - |\overline{A}_d|^2 = |\overline{A}_s|^2 - |A_s|^2 \text{ (consistency check)}
\]

- Solve this problem using indirect-CP asymmetry: \( \text{Im} \left[ \frac{q}{p} A^* \overline{A} \right] \)

\[
\text{2-body strategy: } \text{Fleischer, hep-ph/9903456}
\]

\[
\text{Independent measurement gives } B_d - \overline{B}_d \text{ mixing phase}
\]
$A_{CP}$ in three-body decays

- Local (momentum dependent) branching fraction and CP asymmetry:
  \[
  \Gamma(t) = \frac{\Gamma(B^0 \to f) + \Gamma(B^0 \to \bar{f})}{2} \\
  A_{CP}(t) = \frac{\Gamma(B^0 \to f) - \Gamma(B^0 \to \bar{f})}{\Gamma(B^0 \to f) + \Gamma(B^0 \to \bar{f})}
  \]
  Note that $|f\rangle \neq |\bar{f}\rangle$
  $A_{CP}$ not a true “CP asymmetry”
  Used for simplification

- Time-dependent Dalitz analysis: extract coefficients
  \[
  \cos(\Delta m t) \propto \int_{bin} |A|^2 - |\tilde{A}|^2 \\
  \sin(\Delta m t) \propto \int_{bin} \text{Im} \left[ (q/p)A^*\tilde{A} \right] \\
  \sinh(\Delta \Gamma t/2) \propto \int_{bin} \text{Re} \left[ (q/p)A^*\tilde{A} \right] \\
  \cosh(\Delta \Gamma t/2) \propto \int_{bin} |A|^2 + |\tilde{A}|^2
  \]
  \[
  \rightarrow A = \langle f|B^0\rangle, \quad \tilde{A} = \langle \bar{f}|B^0\rangle \\
  \rightarrow A \text{ has the form } a + e^{i\phi}b \\
  \rightarrow \text{Also for U-spin pair}
  \]
  Now we have enough observables to extract weak phase information

Further details in: BB, D. London, 1503.00737
In order to perform a fit we may use redefinitions:

\[ \int_{\text{bin}} |A|^2 \equiv |A'|^2, \quad \int_{\text{bin}} |\tilde{A}|^2 \equiv |\tilde{A}'|^2, \quad \int_{\text{bin}} \text{Im} \left[ \frac{q/p}{A^* \tilde{A}} \right] \equiv ? \]

Use approximation:

\[ \int_{\text{bin}} \text{Im} \left[ \frac{q/p}{A^* \tilde{A}} \right] \approx \text{Im} \left[ \frac{q/p}{A'^* \tilde{A}'} \right] \]

→ Relation is exact for a point-sized bin

→ Approximation gets worse as bin size increases

Given data set ⇒ Larger statistical error for smaller bins

Optimum bin size required to strike a balance

→ Vary bin size for fits to check its effect on physical parameters
Flavor-SU(3) symmetry

- Symmetry under interchange of $u, d, s$ quarks
- 8 identical pseudoscalars under flavor SU(3) (8 generators of SU(3))
  - $3$ Pions ($\pi^\pm, \pi^0$), $4$ Kaons ($K^\pm, K^0, \bar{K}^0$) (+ a combination of $\eta, \eta'$)
- $|P_1 P_2 P_3\rangle$ has three identical particles under SU(3)
- The fully-symmetric state is simple: amplitude symmetrized in momentum
  - $|K^0(p_1)^{(p_2)\pi^-(p_3)}\rangle_{FS} = |K^0(p_1)^{(p_3)\pi^-(p_2)}\rangle_{FS}$
  - Mixing-induced CP asymmetry is simple: $\propto \text{Im} [(q/p)A^*\bar{A}]$
- Complication: amplitude analysis is required
  - Isobar method: $A(s_{12}, s_{13}) \propto \sum_j c_j e^{i\phi_j} F_{BW}^j(s_{12}, s_{13})$
  - Superposition of non-resonant and resonant (quasi-two-body) modes
Flavor-SU(3) relations

- 9 independent flavor-SU(3) matrix elements describe all $B \rightarrow 3P$ amplitudes
  - $BB$, Gronau, Imbeault, London, Rosner, 1402.2909
- 16 $b \rightarrow s$ decay channels and 16 $b \rightarrow d$ decay channels
  - Amplitudes of these decays depend on 9 matrix elements
  - Not all amplitudes are independent: there are relations
- Isospin symmetry alone gives 6 relations in $b \rightarrow s$ (momentum dependent)
  - $A(B^+ \rightarrow K^0\pi^+\pi^0)_{FS} = - A(B^+_d \rightarrow K^+\pi^0\pi^-)_{FS}$
  - $\sqrt{2}A(B^0_s \rightarrow 3\pi^0)_{FS} = - \sqrt{3}A(B^0_s \rightarrow \pi^0\pi^+\pi^-)_{FS}$
    (4 more)
- Full SU(3) gives a 7th relation in $b \rightarrow s$ decays:
  - $\sqrt{2}A(B^+ \rightarrow K^+\pi^+\pi^-)_{FS} = A(B^+ \rightarrow K^+K^+K^-)_{FS}$
  - $U$ spin + FS version of: $A(K^+_1\pi^+_2\pi^-) + A(K^+_2\pi^+_1\pi^-) = A(K^+K^+K^-)$
    Gronau, Rosner, hep-ph/0304178
Using flavor-SU(3) relations

- Relations can be checked using amplitude analysis
- Expect larger deviations from SU(3) in some regions
  - Near narrow resonances: resonance masses break flavor symmetries
  - Near kinematic boundaries since $m_K \neq m_\pi$
- No contributions from vector resonances to FS states
  - Smaller contribution to SU(3) breaking in FS states
- Integrate over the kinematically-allowed regions
  - Expect smaller deviations from SU(3)
  - Possible cancellation of SU(3) breaking in the average
- Expect no (or tiny) deviations from Isospin relations
  - Deviations most likely show loopholes in amplitude analysis
Using Flavor SU(3) in $B^+$ decays

- Local asymmetries observed by LHCb in $B^+ \rightarrow \pi^+\pi^+\pi^-$
  - Region: $s_{\pi^+\pi^-\text{low}} < 0.4 \text{ GeV}^2$, $s_{\pi^+\pi^-\text{high}} > 15 \text{ GeV}^2$
  - Observed asymmetry: $A_{CP} \sim (60 \pm 10)\%$ Aaij et al., 1310.4740

- $\rho$ (vector) & $f^0$ (scalar) channels interfere: BB, Gronau, Rosner, 1306.2625
  - $A_{B^+\rightarrow \pi^+\pi^+\pi^-}(s_{\text{low}}, s_{\text{high}}) = A_{\rho} F_{\rho}(s_{\text{low}}, s_{\text{high}}) + A_{f^0} F_{f^0}(s_{\text{low}}, s_{\text{high}})$
  - $A_{f^0}, A_{\rho}$ from Flavor-SU(3) fits to $B \rightarrow PS, PV$ decays

  \[ \delta_{f^0} = 140^\circ; \quad \delta_{\rho} = -18^\circ \]

- Large local CP asymmetries can be due to interference between different isobar channels
- Local effects can be smaller in FS state
- Test for U-spin FS relation (local and integrated)
  \[ A(B^+ \rightarrow \pi^+\pi^+\pi^-)_{FS} = \sqrt{2} A(B^+ \rightarrow \pi^+K^+K^-)_{FS} \]
Three-body decay route to new physics

- $\gamma$ extracted by applying SU(3) to fully symmetric state
  → Full Analysis : BB, Imbeault, London, 1303.0846

- Key : $\gamma$ extraction by applying SU(3) to other symmetry states
  → Break discrete ambiguity : more information from other states

- Also key : estimate systematic uncertainties in $\gamma$ extraction

- Interesting situation : $\gamma$ widely different from SM value
  → Significant SU(3) breaking ?
  → NP in three-body $B$ decays; $K\pi\pi - KKK$ puzzle
Summary

- Three-body $B$ decays may provide new information
- $\gamma$ from three-body $B$ decays is loop sensitive
- Flavor symmetry tests $\rightarrow$ U-spin, SU(3) : Compare flavor symmetry related decays
- U-spin related three-body decay pairs : Time-dependent Dalitz analysis for $\gamma$
- Amplitude analysis $\rightarrow$ SU(3) related final states
- Future theory studies of additional symmetry states
Three-body $b \rightarrow s$: more SU(3) relations

4 additional relations (isospin symmetry):

\[
\sqrt{2} \mathcal{A}(B^+ \rightarrow K^0\pi^+\pi^0)_{FS} = \mathcal{A}(B^0_d \rightarrow K^0\pi^+\pi^-)_{FS} + \sqrt{2} \mathcal{A}(B^0_d \rightarrow K^0\pi^0\pi^0)_{FS}
\]

\[
\sqrt{2} \mathcal{A}(B^0_d \rightarrow K^+\pi^0\pi^-)_{FS} = \mathcal{A}(B^+ \rightarrow K^+\pi^+\pi^-)_{FS} + \sqrt{2} \mathcal{A}(B^0_d \rightarrow K^0\pi^0\pi^0)_{FS}
\]

\[
\mathcal{A}(B^+ \rightarrow K^+K^+K^-)_{FS} + \sqrt{2} \mathcal{A}(B^+ \rightarrow K^+K^0\bar{K}^0)_{FS}
\]

\[
= \sqrt{2} \mathcal{A}(B^0_d \rightarrow K^0K^+K^-)_{FS} + \mathcal{A}(B^0_d \rightarrow K^0K^0\bar{K}^0)_{FS}
\]

\[
\mathcal{A}(B^0_s \rightarrow \pi^0K^+K^-)_{FS} + \sqrt{2} \mathcal{A}(B^0_s \rightarrow \pi^0K^0\bar{K}^0)_{FS}
\]

\[
= - \sqrt{2} \mathcal{A}(B^0_s \rightarrow \pi^-K^+\bar{K}^0)_{FS} - \mathcal{A}(B^0_s \rightarrow \pi^+K^-\bar{K}^0)_{FS}
\]
\( \gamma \) from 3-body using flavor SU(3)

Fully-symmetric amplitudes:

\[
\begin{align*}
2\mathcal{A}(B^0_d \to K^+ \pi^0 \pi^-)_{FS} &= b \ e^{i\gamma} - \kappa \ c \\
\sqrt{2}\mathcal{A}(B^0_d \to K^0 \pi^+ \pi^-)_{FS} &= -(d + \tilde{P}^\prime_{uc})e^{i\gamma} - a + \kappa \ d \\
\sqrt{2}\mathcal{A}(B^+ \to K^+ \pi^+ \pi^-)_{FS} &= -(c + \tilde{P}^\prime_{uc})e^{i\gamma} - a + \kappa \ b \\
\mathcal{A}(B^0_d \to K^0 K^0 \bar{K}^0)_{FS} &= \alpha_{SU(3)}(\tilde{P}^\prime_{uc} e^{i\gamma} + a) \\
\mathcal{A}(B^0_d \to K^+ K^0 K^-)_{FS} &= \alpha_{SU(3)}\mathcal{A}(B^+ \to K^+ \pi^+ \pi^-)_{FS}
\end{align*}
\]

- Hadronic parameters: \( a, b, c, d, \alpha_{SU(3)}, \tilde{P}^\prime_{uc} \); Theory input: \( \kappa \)
- \( \tilde{P}^\prime_{uc} = 0 \) for no CP-asymmetry in 3\( K_S \) channel
- Observables: \( X = |\mathcal{A}|^2 + |\overline{\mathcal{A}}|^2 \), \( Y = |\mathcal{A}|^2 - |\overline{\mathcal{A}}|^2 \), \( Z = \text{Im}[\mathcal{A}^*\overline{\mathcal{A}}] \)
- Simplest fit: \( \alpha_{SU(3)} = 1 \); 9 observables \( \Rightarrow \) 7 hadronic unknowns + \( \gamma \)

\( \rightarrow \) Full Analysis: BB, Imbeault, London, 1303.0846
“$B \to \pi K$ puzzle” and $\gamma$

- Flavor SU(3) relates: $B^+ \to \pi^+ K^0, \pi^0 K^+; \quad B_d^0 \to \pi^- K^+, \pi^0 K^0$
- 9 measurements: 4 branching ratios + 4 direct CP asymmetries + 1 indirect CP asymmetry
- 9 unknowns: 4 magnitudes + 3 relative strong phases + $\beta + \gamma$
- SM can explain data but still room for NP
  → Best fit value for $\gamma = (35.9 \pm 7.7)\degree$: large deviation from $\gamma_{tree}$
  → Baek, Chiang, London, 0903.3086
- Problematic in (certain) NP models as well!
  → Imbeault, Baek, London, 0802.1175
  → Endo, Yoshinaga, 1206.0067