Probing CP violation systematically in differential distributions

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LHCb implications workshop

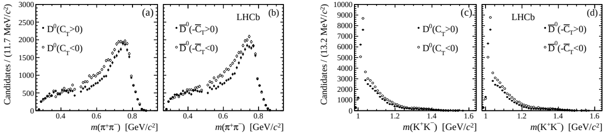
Multibody hadronic decays

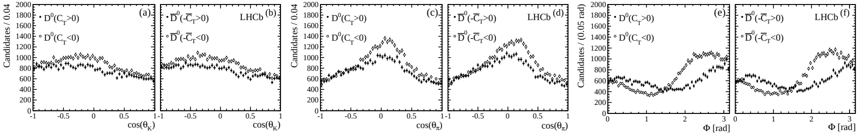
1. Large statistics

$B^0 \to K^+K^-K^\pm\pi^\mp$	1700 candidates	[1403.2888]
$B_s^0 \to K^+K^-K^+K^-$	4000	[1407.2222]
$\boldsymbol{D^0 \to K^+K^-\pi^+\pi^-}$	**170 000**	**[1408.1299]**
$B_s^0 \to K^+\pi^-K^-\pi^+$	700	[1503.05362]

...

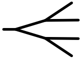
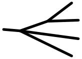
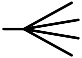
2. Multidimensional phase space





Multibody hadronic decays

3. Rich variety of interfering contributions

	Intermediate states in $D^0 \to K^+K^-\pi^+\pi^-$	Br $/10^{-4}$
	$(\phi\rho^0)_S, \quad \phi \to K^+K^-, \quad \rho^0 \to \pi^+\pi^-$	9.3 ± 1.2
	D	0.83 ± 0.23
	$(K^{*0}\overline{K}^{*0})_S, \quad K^{*0} \to K^\pm\pi^\mp$	1.48 ± 0.30
	$\phi(\pi^+\pi^-)_S, \quad \phi \to K^+K^-$	2.50 ± 0.33
	$(K^-\pi^+)_P(K^+\pi^-)_S$	2.6 ± 0.5
	$K_1^+K^-, \quad K_1^+ \to K^{*0}\pi^+$	1.8 ± 0.5
	$K_1^-K^+, \quad K_1^- \to \overline{K}^{*0}\pi^-$	0.22 ± 0.12
	$K_1^+K^-, \quad K_1^+ \to \rho^0 K^+$	1.14 ± 0.26
	$K_1^-K^+, \quad K_1^- \to \rho^0 K^-$	1.46 ± 0.25
	$K^*(1410)^+K^-, \ K^*(1410)^+ \to K^{*0}\pi^+$	1.02 ± 0.26
	$K^*(1410)^-K^+, \ K^*(1410)^- \to \overline{K}^{*0}\pi^-$	1.14 ± 0.25

[CLEO '12]

$\Longrightarrow$ Opportunities for CP violation

Probing CP violation systematically in differential distributions

CP-violating distributions

- With or without strong phases
- In untagged samples

Systematic analysis techniques

Differential CP violation

Compare the CP-conjugate amplitudes (squared)

$$\mathcal{M}(\{\vec{p}_i\}) \quad \text{and} \quad \bar{\mathcal{M}}(\{-\vec{p}_{\bar{\imath}}\})\Big|_{\vec{p}_{\bar{\imath}}=\vec{p}_i}$$

phase-space point by phase-space point (spinless case).

Contributions of definite · *strong* and *weak* phases
· $\hat{\mathrm{T}}$ transformation properties

$$\mathcal{M}(\{\vec{p}_i\}) = +a(\{\vec{p}_i\})\, e^{i(\delta_a+\varphi_a)} +b(\{\vec{p}_i\})\, e^{i(\delta_b+\varphi_b)} +c(\{\vec{p}_i\})\, e^{i(\delta_c+[\varphi_c+\pi/2])} + \dots$$

$$\bar{\mathcal{M}}(\{-\vec{p}_i\}) = +a(\{-\vec{p}_i\})\, e^{i(\delta_a-\varphi_a)} +b(\{-\vec{p}_i\})\, e^{i(\delta_b-\varphi_b)} +c(\{-\vec{p}_i\})\, e^{i(\delta_c-[\varphi_c+\pi/2])} + \dots$$

where $\hat{\mathrm{T}}$ is *motion reversal* (flips $\vec{p}$ and $\vec{\sigma}$).

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$$\bar{\mathcal{M}}(\{-\vec{p}_i\}) = \\ +a(\{+\vec{p}_i\})\, e^{i(\delta_a-\varphi_a)} \\ +b(\{+\vec{p}_i\})\, e^{i(\delta_b-\varphi_b)} \\ -c(\{+\vec{p}_i\})\, e^{i(\delta_c-[\varphi_c+\pi/2])} \\ +\dots$$

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where $\hat{\mathrm{T}}$ is *motion reversal* (flips $\vec{p}$ and $\vec{\sigma}$).

Triple products

$\hat{\mathrm{T}}$ oddity arises from

antisymmetric $\epsilon_{\mu\nu\rho\sigma} p^\mu q^\nu r^\rho s^\sigma$ contractions

of four independent momenta or spin vectors.

- in the Lagrangian: $i\tilde{F}^{\mu\nu} \equiv \frac{i}{2}\epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$
- in the presence of chiral fermions: $\gamma^5 \equiv \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$

In the p restframe,

$$\epsilon_{\mu\nu\rho\sigma} p^\mu q^\nu r^\rho s^\sigma = p^0 \; \vec{q}\cdot(\vec{r}\times\vec{s})$$

is a scalar *triple product*.

$\hat{\mathrm{T}}$-odd quantities are referred to as *triple products*.

CP violation and strong phases

Distributions of definite CP and $\hat{\text{T}}$ transformation properties

$$\left.\frac{d\Gamma}{d\{\vec{p}_i\}}\right|^{\hat{\text{T}}\text{-}^{\text{even}}_{\text{odd}}}_{\text{CP-}^{\text{even}}_{\text{odd}}} \equiv \frac{\mathbb{I} \pm \hat{\text{T}}}{2}\,\frac{\mathbb{I} \pm \text{CP}}{2}\,\frac{d\Gamma}{d\{\vec{p}_i\}}$$

$$\left.\frac{d\Gamma}{d\{\vec{p}_i\}}\right|^{\hat{\text{T}}\text{-even}}_{\text{CP-even}} \propto \; a\,a + b\,b + c\,c \\ +2\,a\,b\,\cos(\delta_a - \delta_b)\cos(\varphi_a - \varphi_b)$$

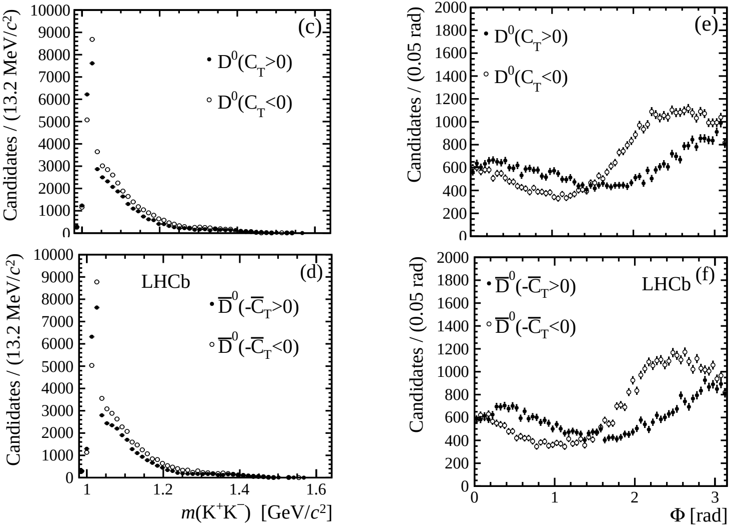
$$\left.\frac{d\Gamma}{d\{\vec{p}_i\}}\right|^{\hat{\text{T}}\text{-even}}_{\text{CP-odd}} \propto -2\,a\,b\,\sin(\delta_a - \delta_b)\,\sin(\varphi_a - \varphi_b)$$

$$\left.\frac{d\Gamma}{d\{\vec{p}_i\}}\right|^{\hat{\text{T}}\text{-odd}}_{\text{CP-odd}} \propto \; 2\,a\,c\,\cos(\delta_a - \delta_c)\sin(\varphi_a - \varphi_c) \\ +2\,b\,c\,\cos(\delta_b - \delta_c)\sin(\varphi_b - \varphi_c)$$

$$\left.\frac{d\Gamma}{d\{\vec{p}_i\}}\right|^{\hat{\text{T}}\text{-odd}}_{\text{CP-even}} \propto \; 2\,a\,c\,\sin(\delta_a - \delta_c)\,\cos(\varphi_a - \varphi_c) \\ +2\,b\,c\,\sin(\delta_b - \delta_c)\,\cos(\varphi_b - \varphi_c)$$

CP-violating distributions

$D^0 \to K^+K^-\pi^+\pi^-$ [1408.1299]



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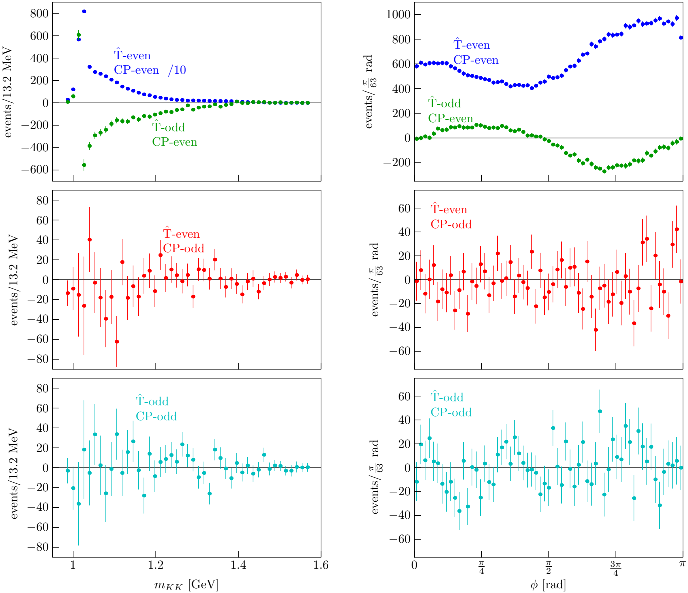

Probing CP violation systematically in differential distributions

CP-violating distributions

- With or without strong phases
- In untagged samples

Systematic analysis techniques

CP violation and untagged samples

– Tagging CP conjugate processes may cost efficiency.

– An untagged sample [as in 1503.05362]

e.g. $$\begin{cases} B_s^0 \to K^+(+\vec{p}_1) \quad \pi^-(+\vec{p}_2) \quad K^-(+\vec{p}_3) \quad \pi^+(+\vec{p}_4) \\ \bar{B}_s^0 \to K^-(-\vec{p}_1) \quad \pi^+(-\vec{p}_2) \quad K^+(-\vec{p}_3) \quad \pi^-(-\vec{p}_4) \end{cases}$$

$$\frac{\mathbb{I} + \mathrm{CP}}{2} \; \frac{\mathrm{d}\Gamma}{\mathrm{d}\{\vec{p}_i\}}$$

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$$\frac{\mathbb{I} + \mathrm{CP\hat{T}}}{2} \; \frac{\mathrm{d}\Gamma}{\mathrm{d}\{\vec{p}_i\}}$$

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$$\frac{\mathbb{I} + \mathrm{CP\hat{T}E}^*}{2} \, \frac{\mathrm{d}\Gamma}{\mathrm{d}\{\vec{p}_i\}}$$

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$$\frac{\mathbb{I} + \mathrm{CP}\hat{\mathrm{T}}\mathrm{E}^*}{2} \, \frac{d\Gamma}{d\{\vec{p}_i\}}$$

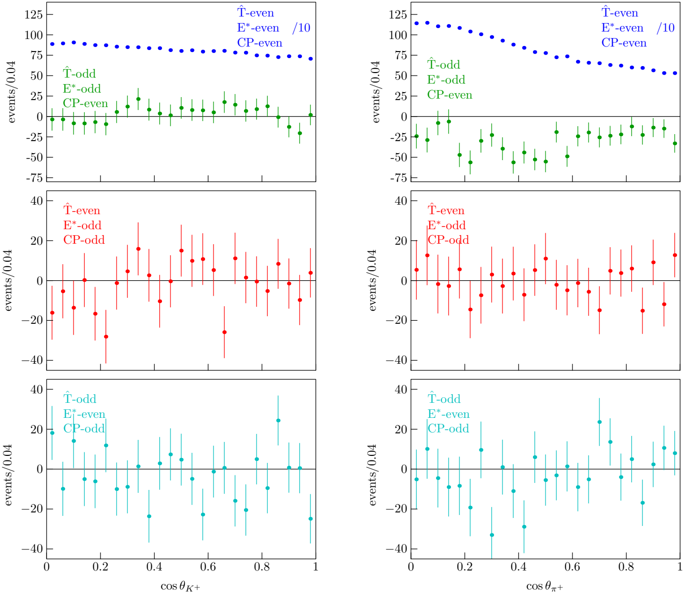
has two CP-odd distributions, of opposite $\hat{\mathrm{T}}$ and E^* parities

$$\frac{\mathbb{I} \pm \hat{\mathrm{T}}}{2} \, \frac{\mathbb{I} \mp \mathrm{E}^*}{2} \left(\frac{\mathbb{I} + \mathrm{CP}\hat{\mathrm{T}}\mathrm{E}^*}{2} \, \frac{d\Gamma}{d\{\vec{p}_i\}} \right) = \frac{\mathbb{I} \pm \hat{\mathrm{T}}}{2} \, \frac{\mathbb{I} \mp \mathrm{E}^*}{2} \, \frac{\mathbb{I} - \mathrm{CP}}{2} \, \frac{d\Gamma}{d\{\vec{p}_i\}}$$

CP violation and untagged samples

$D^0 \to K^+K^-\pi^+\pi^-$

[1408.1299]



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CP-violating distributions

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Systematic analysis techniques

Analysis techniques

Based on phenomenological parametrisations

- Full unbinned likelyhood fits
- Measurement of *expected* asymmetries

$\longrightarrow$ may miss *unexpected* manifestations of CP violation

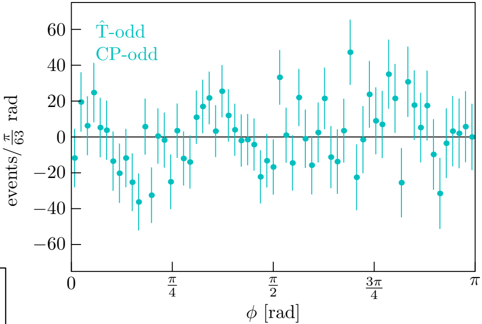
More systematic, relying some milder dynamical assumptions

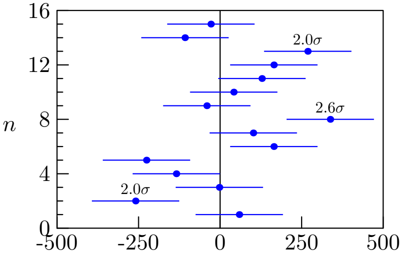
- Phase-space binnings [1408.1299]
- Decomposition in moments [Dighe et al '98, Beaujean et al '15, Gratrex et al '15]
- Series of asymmetries

i.e. integrated observables $\displaystyle\int d\{\vec{p}_i\}\ f(\{\vec{p}_i\})\ \frac{d\Gamma}{d\{\vec{p}_i\}}\Bigg|_{\text{CP-odd}}^{\hat{\text{T}}\text{-odd}}$

Systematic analysis techniques

$D^0 \to K^+ K^- \pi^+ \pi^-$

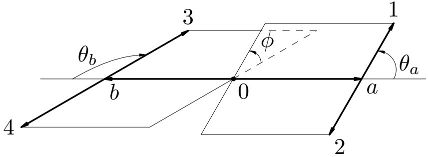




$$A_n \equiv \int \mathrm{d}\{\vec{p}_i\} \;\; \mathrm{sign}\{\sin n\phi\} \;\; \frac{\mathrm{d}\Gamma}{\mathrm{d}\{\vec{p}_i\}}\Bigg|_{\text{CP-odd}}^{\hat{\text{T}}\text{-odd}}$$

Generalized triple-product asymmetries

1. Fix a phase-space parametrisation



2. Angular asymmetries from partial-wave expansion

   For spinless external particles:

$$\mathcal{M} = 4\pi \sum_{j_a, j_b, \lambda} A_\lambda^{j_a, j_b}(m_a^2, m_b^2)\ Y_{j_a}^{\lambda}(\theta_a, \phi)\ Y_{j_b}^{\lambda}(\theta_b, 0)^*$$

3. The invariant masses can also induce change of signs

$$\Re\left\{\frac{1}{m_a^2 - M^2 + i\Gamma M}\right\} = \frac{m_a^2 - M^2}{(m_a^2 - M^2)^2 + \Gamma^2 M^2}$$

$$\longrightarrow \quad \int d\{\vec{p}_i\}\ \text{sign}\left\{ f_l(\,c\theta_a\,)\ f_m(\,c\theta_b\,)\ \sin n\phi \prod_i (m_a^2 - M_i^2) \prod_j (m_b^2 - M_j^2)\right\}\ \frac{d\Gamma}{d\{\vec{p}_i\}}\Bigg|_{\text{CP-odd}}^{\hat{\text{T}}\text{-odd}}$$

Probing CP violation systematically in differential distributions

High statistics allows for the accurate measurement of rich multidimensional differential distributions.

Symmetries characterize distributions measurable
· in the presence or absence of strong phases,
· in untagged samples.

Systematic procedures should be used to assess the departure from zero of CP-violating ones.

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