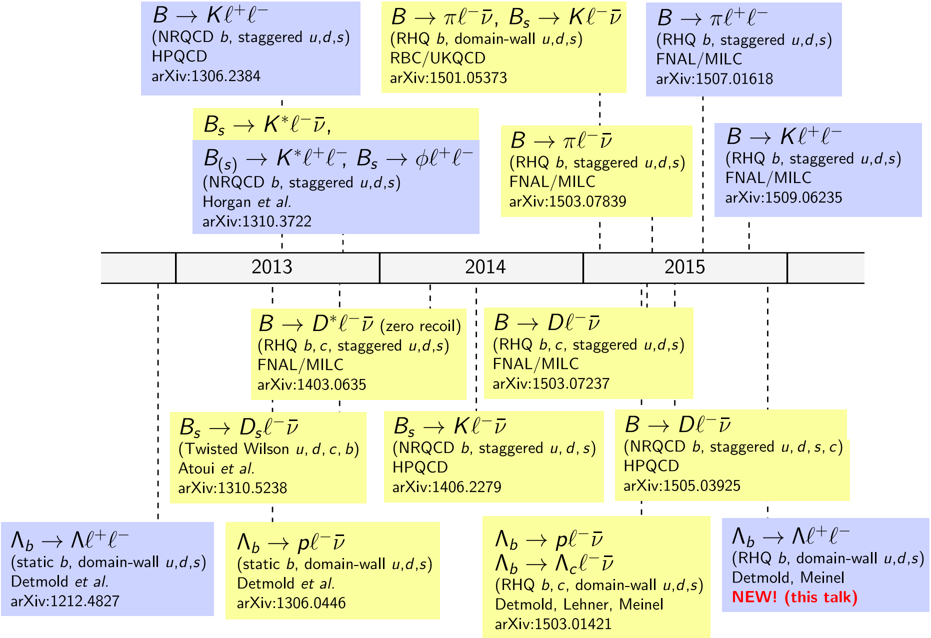
Lattice progress for semileptonic b decays

Stefan Meinel

THE UNIVERSITY OF ARIZONA

RIKEN BNL Research Center

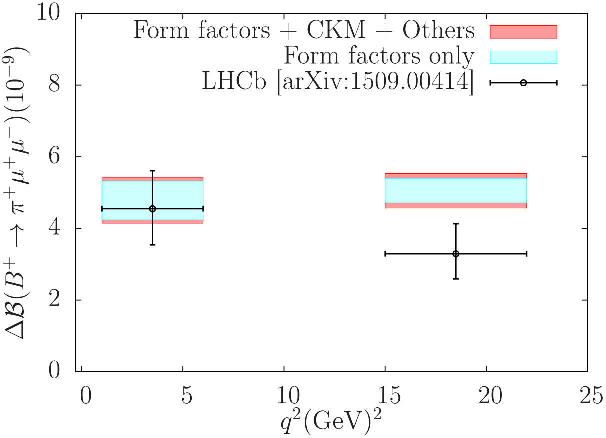
November 4, 2015



$B \to K\ell^+\ell^-$
(NRQCD b, staggered u,d,s)
HPQCD
arXiv:1306.2384
$B \to \pi\ell^-\bar{\nu}$, $B_s \to K\ell^-\bar{\nu}$
(RHQ b, domain-wall u,d,s)
RBC/UKQCD
arXiv:1501.05373
$B \to \pi\ell^+\ell^-$
(RHQ b, staggered u,d,s)
FNAL/MILC
arXiv:1507.01618
$B_s \to K^*\ell^-\bar{\nu}$,
$B_{(s)} \to K^*\ell^+\ell^-$, $B_s \to \phi\ell^+\ell^-$
(NRQCD b, staggered u,d,s)
Horgan *et al.*
arXiv:1310.3722
$B \to \pi\ell^-\bar{\nu}$
(RHQ b, staggered u,d,s)
FNAL/MILC
arXiv:1503.07839
$B \to K\ell^+\ell^-$
(RHQ b, staggered u,d,s)
FNAL/MILC
arXiv:1509.06235
2013
2014
2015
$B \to D^*\ell^-\bar{\nu}$ (zero recoil)
(RHQ b, c, staggered u,d,s)
FNAL/MILC
arXiv:1403.0635
$B \to D\ell^-\bar{\nu}$
(RHQ b, c, staggered u,d,s)
FNAL/MILC
arXiv:1503.07237
$B_s \to D_s\ell^-\bar{\nu}$
(Twisted Wilson u, d, c, b)
Atoui *et al.*
arXiv:1310.5238
$B_s \to K\ell^-\bar{\nu}$
(NRQCD b, staggered u, d, s)
HPQCD
arXiv:1406.2279
$B \to D\ell^-\bar{\nu}$
(NRQCD b, staggered u, d, s, c)
HPQCD
arXiv:1505.03925
$\Lambda_b \to \Lambda\ell^+\ell^-$
(static b, domain-wall u,d,s)
Detmold *et al.*
arXiv:1212.4827
$\Lambda_b \to p\ell^-\bar{\nu}$
(static b, domain-wall u,d,s)
Detmold *et al.*
arXiv:1306.0446
$\Lambda_b \to p\ell^-\bar{\nu}$
$\Lambda_b \to \Lambda_c\ell^-\bar{\nu}$
(RHQ b, c, domain-wall u,d,s)
Detmold, Lehner, Meinel
arXiv:1503.01421
$\Lambda_b \to \Lambda\ell^+\ell^-$
(RHQ b, domain-wall u,d,s)
Detmold, Meinel
NEW! (this talk)

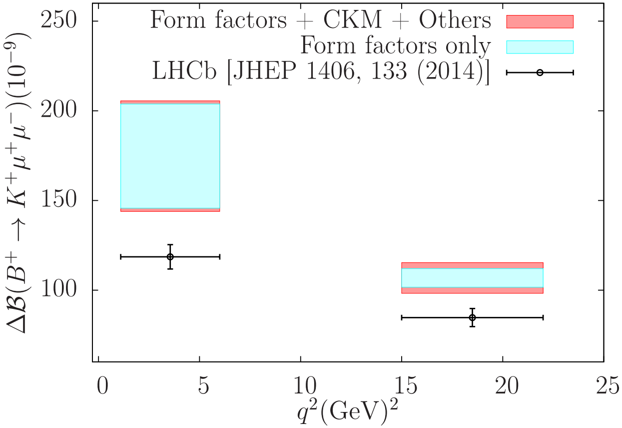
“Phenomenology of semileptonic B-meson decays with form factors from lattice QCD,”

[D. Du *et al.* (Fermilab Lattice and MILC Collaborations), arXiv:1510.02349]



“Phenomenology of semileptonic B-meson decays with form factors from lattice QCD,”

[D. Du *et al.* (Fermilab Lattice and MILC Collaborations), arXiv:1510.02349]



1. $\Lambda_b \to p\,\ell^-\bar{\nu}$ and $\Lambda_b \to \Lambda_c\ell^-\bar{\nu}$
2. $\Lambda_b \to \Lambda\ell^+\ell^-$
3. Outlook: multi-hadron and nonlocal matrix elements

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}(\Lambda_b \to p\mu\bar{\nu}) \;=\; |V_{ub}|^2 \times \text{function}\Big[\, \langle p \,|\, \bar{u}\gamma^\mu b \,|\Lambda_b\rangle,\;\; \langle p \,|\, \bar{u}\gamma^\mu\gamma_5 b \,|\Lambda_b\rangle \,\Big],$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}(\Lambda_b \to \Lambda_c\mu\bar{\nu}) \;=\; |V_{cb}|^2 \times \text{function}\Big[\, \langle \Lambda_c |\, \bar{c}\gamma^\mu b \,|\Lambda_b\rangle,\;\; \langle \Lambda_c |\, \bar{c}\gamma^\mu\gamma_5 b \,|\Lambda_b\rangle \,\Big]$$

Helicity form factors [T. Feldmann and M. Yip, PRD **85**, 014035 (2012)]:

$$\begin{aligned}
\langle F|\overline{q}\gamma^\mu b|\Lambda_b\rangle &= \overline{u}_F\Big[(m_{\Lambda_b}-m_F)\frac{q^\mu}{q^2}\,f_0 \\
&\quad+\frac{m_{\Lambda_b}+m_F}{s_+}\left(p^\mu+p'^\mu-(m_{\Lambda_b}^2-m_F^2)\frac{q^\mu}{q^2}\right)f_+ \\
&\quad+\left(\gamma^\mu-\frac{2m_F}{s_+}p^\mu-\frac{2m_{\Lambda_b}}{s_+}p'^\mu\right)f_\perp\Big]u_{\Lambda_b},
\end{aligned}$$

$$\begin{aligned}
\langle F|\overline{q}\gamma^\mu\gamma_5 b|\Lambda_b\rangle &= -\overline{u}_F\,\gamma_5\Big[(m_{\Lambda_b}+m_F)\frac{q^\mu}{q^2}\,g_0 \\
&\quad+\frac{m_{\Lambda_b}-m_F}{s_-}\left(p^\mu+p'^\mu-(m_{\Lambda_b}^2-m_F^2)\frac{q^\mu}{q^2}\right)g_+ \\
&\quad+\left(\gamma^\mu+\frac{2m_F}{s_-}p^\mu-\frac{2m_{\Lambda_b}}{s_-}p'^\mu\right)g_\perp\Big]u_{\Lambda_b}.
\end{aligned}$$

$$F=p,\Lambda_c,\quad \bar{q}=\bar{u},\bar{c},\quad s_\pm=(m_{\Lambda_b}\pm m_X)^2-q^2$$

“$\Lambda_b \to p\,\ell^-\,\bar{\nu}_\ell$ and $\Lambda_b \to \Lambda_c\,\ell^-\,\bar{\nu}_\ell$ form factors from lattice QCD with relativistic heavy quarks”

[W. Detmold, C. Lehner, S. Meinel, PRD **92**, 034503 (2015)]

Hopper/NERSC

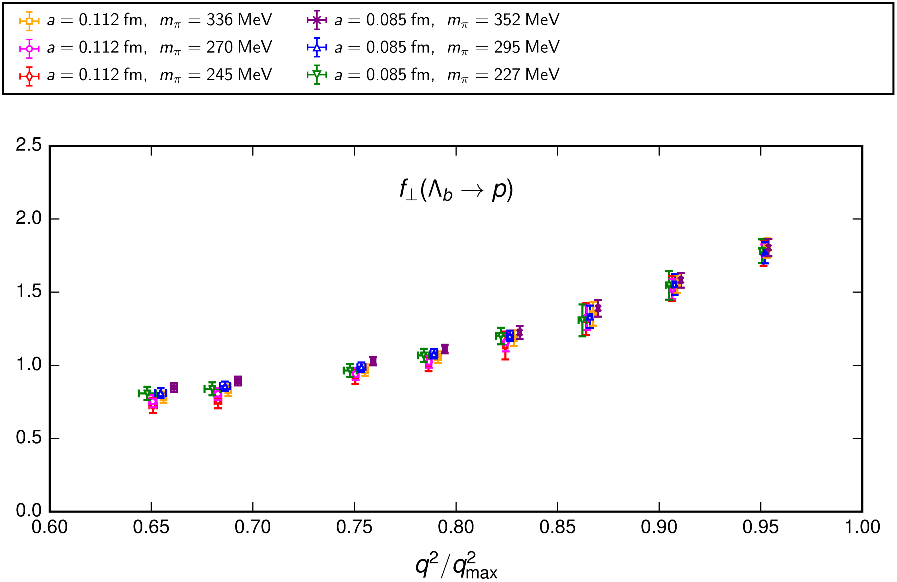

Stampede/TACC

- Gauge field configurations generated by the RBC and UKQCD collaborations
  [Y. Aoki *et al.*, PRD **83**, 074508 (2011)]

- u, d, s quarks: domain-wall action
  [D. Kaplan, PLB **288**, 342 (1992); V. Furman and Y. Shamir, NPB **439**, 54 (1995)]

- c, b quarks: "relativistic heavy-quark action"
  [A. El-Khadra, A. Kronfeld, P. Mackenzie, PRD **55**, 3933 (1997); Y. Aoki *et al.*, PRD **86**, 116003]

- "Mostly nonperturbative" renormalization
  [A. El-Khadra *et al.*, PRD **64**, 014502 (2001)]



$a = 0.112$ fm, $m_\pi = 336$ MeV
$a = 0.112$ fm, $m_\pi = 270$ MeV
$a = 0.112$ fm, $m_\pi = 245$ MeV
$a = 0.085$ fm, $m_\pi = 352$ MeV
$a = 0.085$ fm, $m_\pi = 295$ MeV
$a = 0.085$ fm, $m_\pi = 227$ MeV
$f_\perp(\Lambda_b \to p)$
0.0
0.5
1.0
1.5
2.0
2.5
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
$q^2/q^2_{\rm max}$

Combined chiral/continuum/kinematic extrapolation using modified z-expansion

[C. Bourrely, I. Caprini, L. Lellouch, PRD **79**, 013008 (2009)]

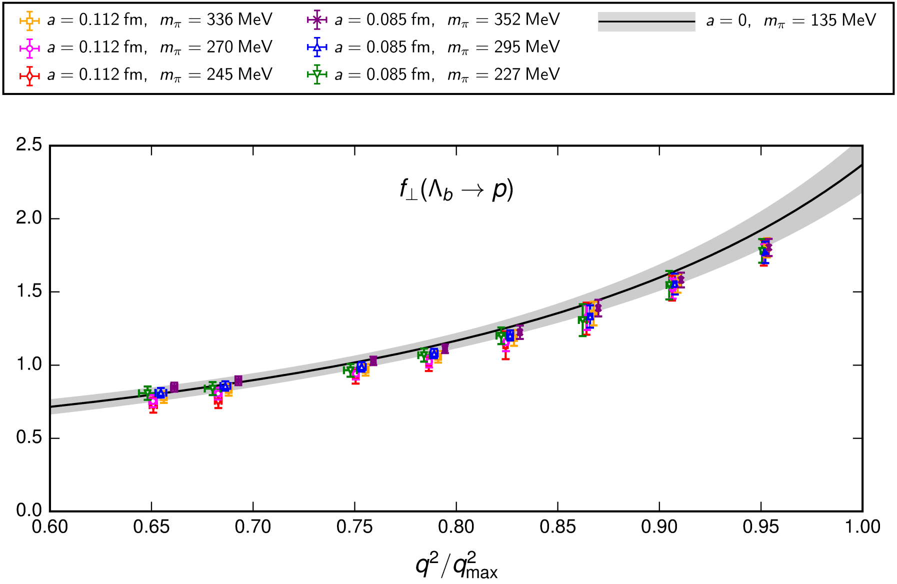
$$z^f(q^2) = \frac{\sqrt{t_+^f - q^2} - \sqrt{t_+^f - t_0}}{\sqrt{t_+^f - q^2} + \sqrt{t_+^f - t_0}},$$

"Nominal fit"

$$f(q^2) = \frac{1}{1 - q^2/(m_{\text{pole}}^f)^2} \left[a_0^f \left(1 + c_0^f \frac{m_\pi^2 - m_{\pi,\text{phys}}^2}{\Lambda_\chi^2} \right) + a_1^f \, z^f(q^2) \right] \times \left[1 + b^f \frac{|\mathbf{p}'|^2}{(\pi/a)^2} + d^f \frac{\Lambda_{\text{QCD}}^2}{(\pi/a)^2} \right],$$

“Nominal fit” in physical limit $a = 0$, $m_\pi = m_{\pi,\mathrm{phys}}$:

$$f(q^2) = \frac{1}{1 - q^2/(m^f_{\mathrm{pole}})^2} \left[a^f_0 + a^f_1 \, z^f(q^2) \right]$$



Gray band = statistical uncertainty.

"Higher-order fit":

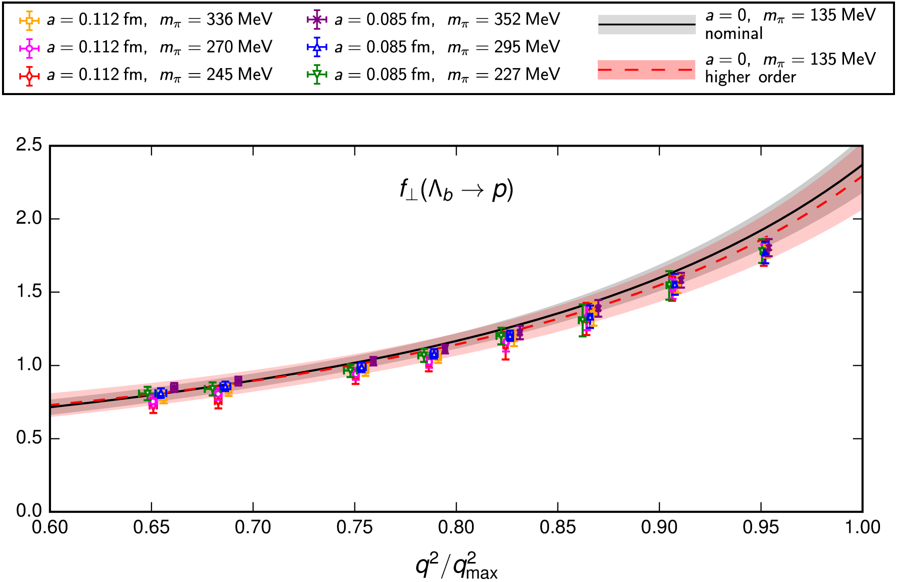
$$
\begin{aligned}
f_{\mathrm{HO}}(q^2) = \frac{1}{1-q^2/(m_{\mathrm{pole}}^f)^2}\Bigg[& a_0^f\left(1 + c_0^f\,\frac{m_\pi^2 - m_{\pi,\mathrm{phys}}^2}{\Lambda_\chi^2} + \tilde{c}_0^f\,\frac{m_\pi^3 - m_{\pi,\mathrm{phys}}^3}{\Lambda_\chi^3}\right) \\
& + a_1^f\left(1 + c_1^f\,\frac{m_\pi^2 - m_{\pi,\mathrm{phys}}^2}{\Lambda_\chi^2}\right) z^f(q^2) + a_2^f\,[z^f(q^2)]^2\Bigg] \\
\times\Bigg[& 1 + b^f\,\frac{|\mathbf{p}'|^2}{(\pi/a)^2} + d^f\,\frac{\Lambda_{\mathrm{QCD}}^2}{(\pi/a)^2} + \tilde{b}^f\,\frac{|\mathbf{p}'|^3}{(\pi/a)^3} + \tilde{d}^f\,\frac{\Lambda_{\mathrm{QCD}}^3}{(\pi/a)^3} \\
& + j^f\,\frac{|\mathbf{p}'|^2\Lambda_{\mathrm{QCD}}}{(\pi/a)^3} + k^f\,\frac{|\mathbf{p}'|\Lambda_{\mathrm{QCD}}^2}{(\pi/a)^3}\Bigg]
\end{aligned}
$$

Higher-order fit parameters constrained with Gaussian priors to be natural-sized.

Modified data correlation matrix to include other sources of uncertainty.

"Higher-order fit" in physical limit $a = 0$, $m_\pi = m_{\pi,\mathrm{phys}}$:

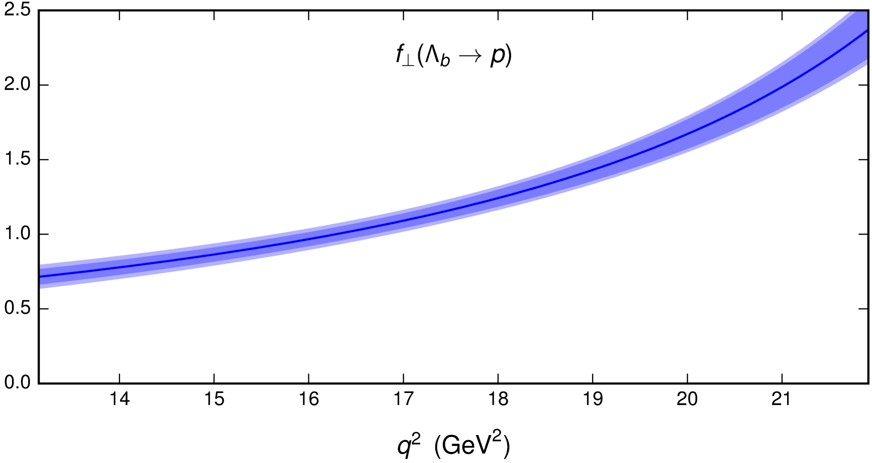
$$f_{\mathrm{HO}}(q^2) = \frac{1}{1 - q^2/(m^f_{\mathrm{pole}})^2} \left[a^f_0 + a^f_1 \, z^f(q^2) + a^f_2 \, [z^f(q^2)]^2 \right]$$



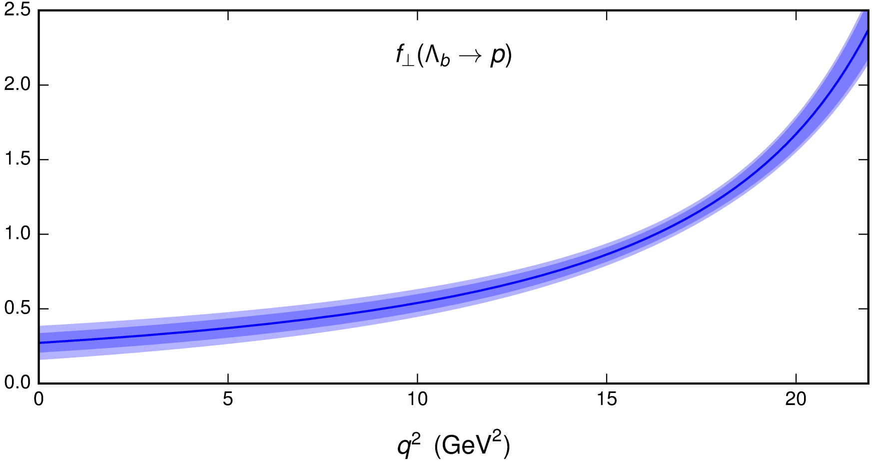
$a = 0.112$ fm, $m_\pi = 336$ MeV
$a = 0.112$ fm, $m_\pi = 270$ MeV
$a = 0.112$ fm, $m_\pi = 245$ MeV
$a = 0.085$ fm, $m_\pi = 352$ MeV
$a = 0.085$ fm, $m_\pi = 295$ MeV
$a = 0.085$ fm, $m_\pi = 227$ MeV
$a = 0$, $m_\pi = 135$ MeV nominal
$a = 0$, $m_\pi = 135$ MeV higher order
$f_\perp(\Lambda_b \to p)$
2.5
2.0
1.5
1.0
0.5
0.0
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
$q^2/q^2_{\rm max}$

Compute systematic uncertainty of any observable O using

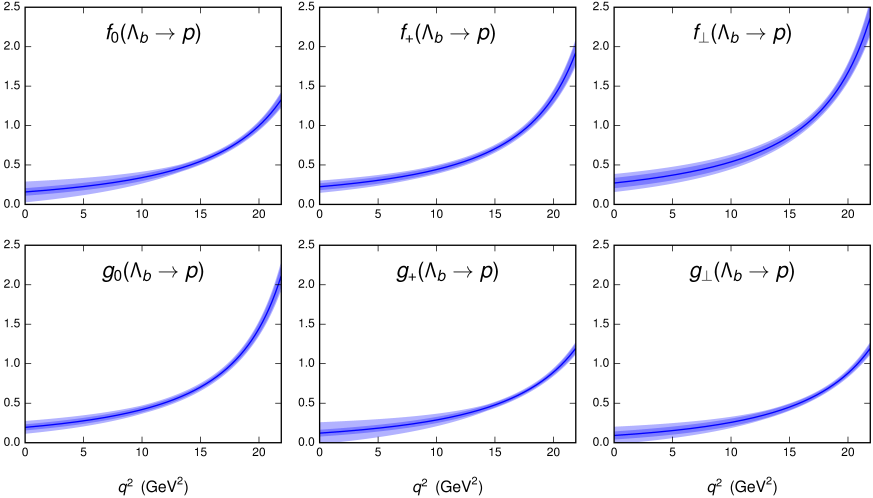
$$\sigma_{O,\text{syst.}} = \max\left(|O_{\text{HO}} - O|,\ \sqrt{|\sigma^2_{\text{HO}} - \sigma^2_O|} \right)$$



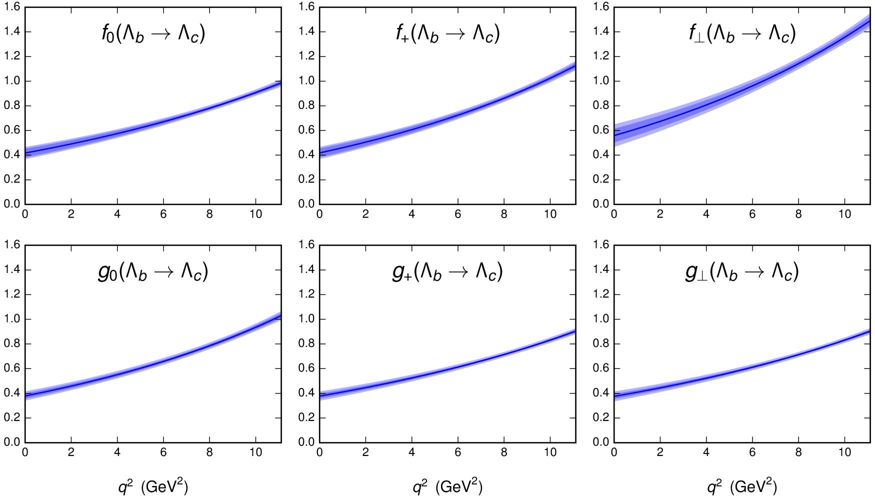
$f_\perp(\Lambda_b \to p)$
2.5
2.0
1.5
1.0
0.5
0.0
14
15
16
17
18
19
20
21
q^2 (GeV^2)



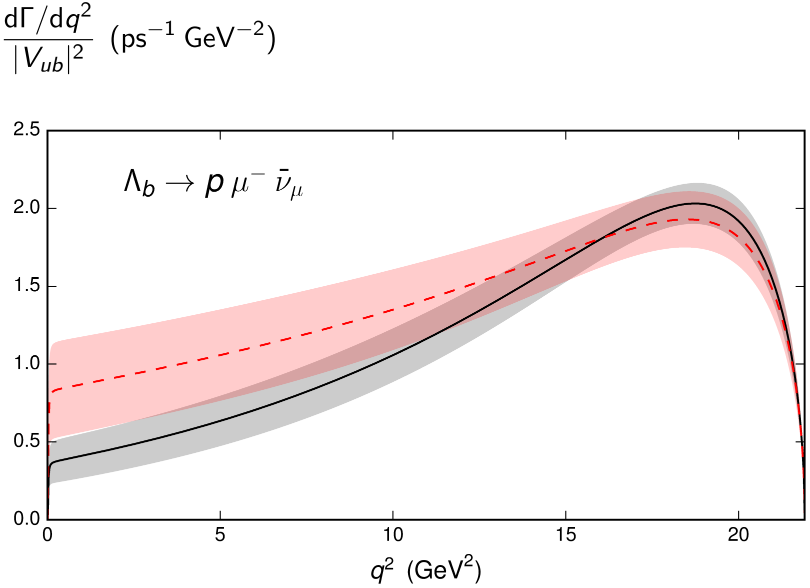
$f_\perp(\Lambda_b \to p)$
2.5
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
q^2 (GeV^2)



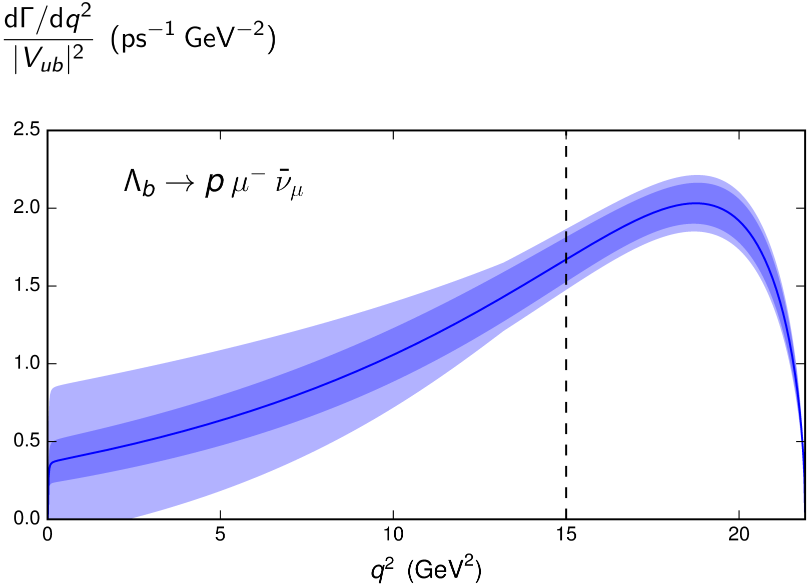
$f_0(\Lambda_b \to p)$
$f_+(\Lambda_b \to p)$
$f_\perp(\Lambda_b \to p)$
$g_0(\Lambda_b \to p)$
$g_+(\Lambda_b \to p)$
$g_\perp(\Lambda_b \to p)$
0.0
0.5
1.0
1.5
2.0
2.5
0
5
10
15
20
q^2 (GeV^2)



$f_0(\Lambda_b \to \Lambda_c)$
$f_+(\Lambda_b \to \Lambda_c)$
$f_\perp(\Lambda_b \to \Lambda_c)$
$g_0(\Lambda_b \to \Lambda_c)$
$g_+(\Lambda_b \to \Lambda_c)$
$g_\perp(\Lambda_b \to \Lambda_c)$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
0
2
4
6
8
10
q^2 (GeV^2)



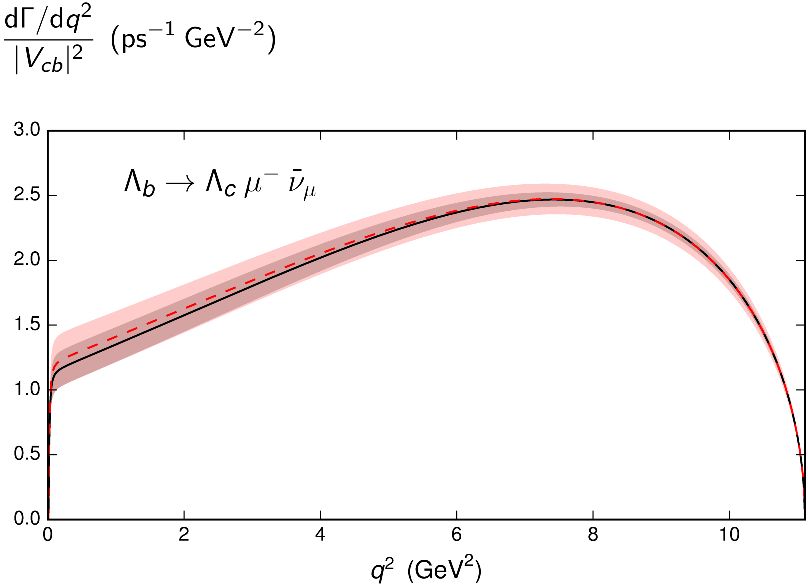
$\frac{d\Gamma/dq^2}{|V_{ub}|^2}$ (ps^{-1} GeV^{-2})
$\Lambda_b \to p\,\mu^-\,\bar{\nu}_\mu$
2.5
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
q^2 (GeV^2)



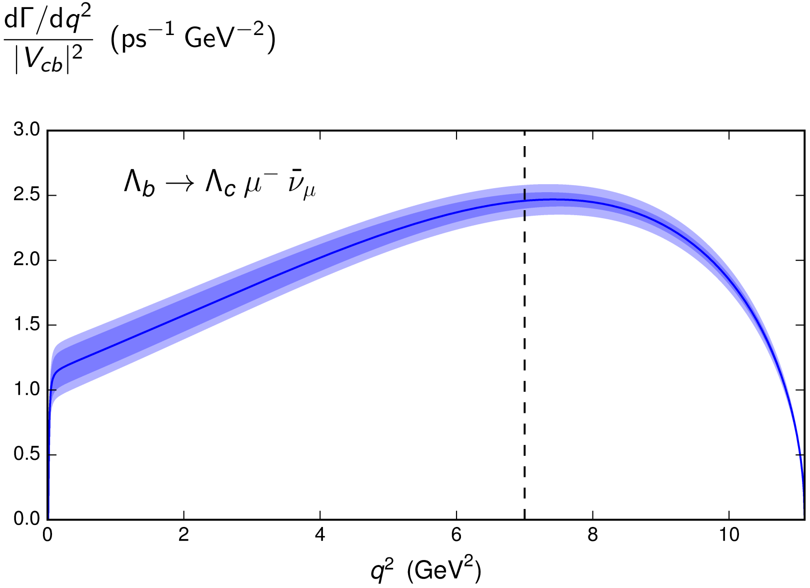
$\frac{d\Gamma/dq^2}{|V_{ub}|^2}$ (ps^{-1} GeV^{-2})
$\Lambda_b \to p\, \mu^- \bar{\nu}_\mu$
2.5
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
q^2 (GeV^2)

$$\frac{1}{|V_{ub}|^2} \int_{15\ \mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{\mathrm{d}\Gamma(\Lambda_b \to p\, \mu^- \bar{\nu}_\mu)}{\mathrm{d}q^2} \mathrm{d}q^2$$

$$= (12.31 \pm 0.76_{\,\mathrm{stat}} \pm 0.77_{\,\mathrm{syst}})\ \ \mathrm{ps}^{-1}$$



$\frac{d\Gamma/dq^2}{|V_{cb}|^2}$ (ps^{-1} GeV^{-2})
$\Lambda_b \to \Lambda_c \mu^- \bar{\nu}_\mu$
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
q^2 (GeV^2)



$\frac{d\Gamma/dq^2}{|V_{cb}|^2}$ (ps^{-1} GeV^{-2})
$\Lambda_b \to \Lambda_c\, \mu^- \, \bar{\nu}_\mu$
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
q^2 (GeV^2)

$$\frac{1}{|V_{cb}|^2} \int_{7\ \mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{\mathrm{d}\Gamma(\Lambda_b \to \Lambda_c\ \mu^- \bar{\nu}_\mu)}{\mathrm{d}q^2} \mathrm{d}q^2$$

$$= (8.37 \pm 0.16_{\,\mathrm{stat}} \pm 0.34_{\,\mathrm{syst}})\ \ \mathrm{ps}^{-1}$$

$$\frac{|V_{cb}|^2}{|V_{ub}|^2} \frac{\int_{15\ \mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{\mathrm{d}\Gamma(\Lambda_b \to p\, \mu^- \bar{\nu}_\mu)}{\mathrm{d}q^2} \mathrm{d}q^2}{\int_{7\ \mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{\mathrm{d}\Gamma(\Lambda_b \to \Lambda_c\, \mu^- \bar{\nu}_\mu)}{\mathrm{d}q^2} \mathrm{d}q^2}$$

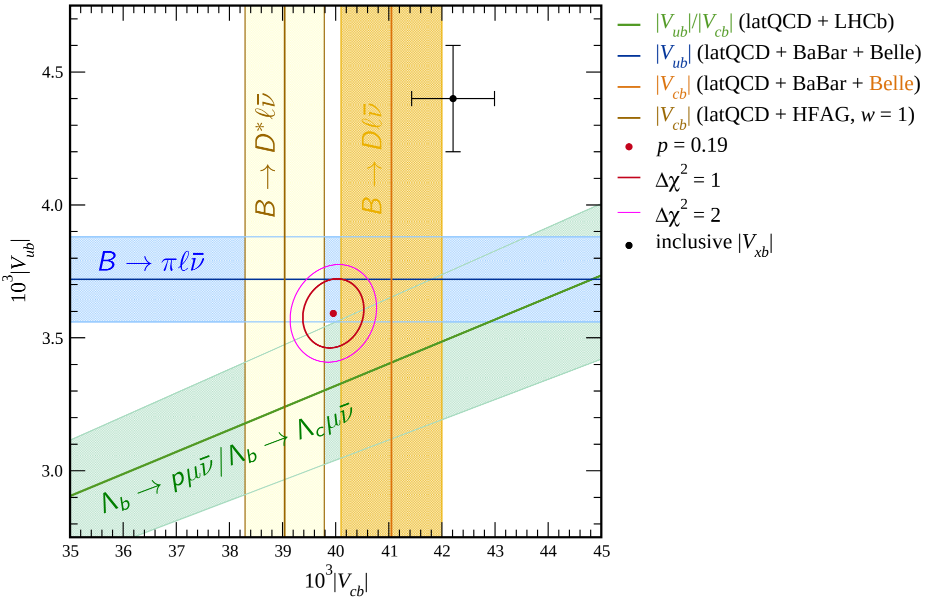
$$= 1.471 \pm 0.095_{\mathrm{stat.}} \pm 0.109_{\mathrm{syst.}}$$

Combine with LHCb measurement:

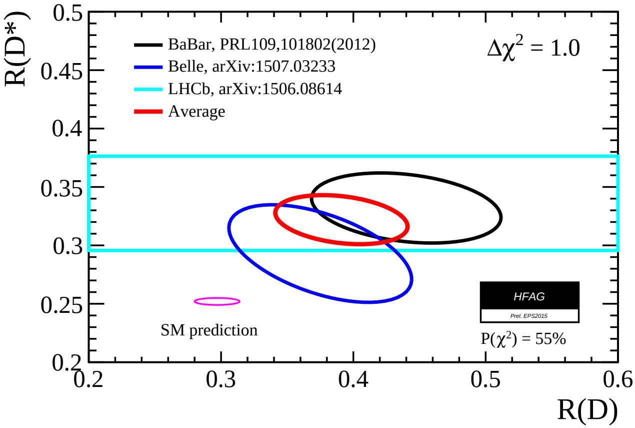
$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.004_{\,\mathrm{expt}} \pm 0.004_{\,\mathrm{lat}}$$

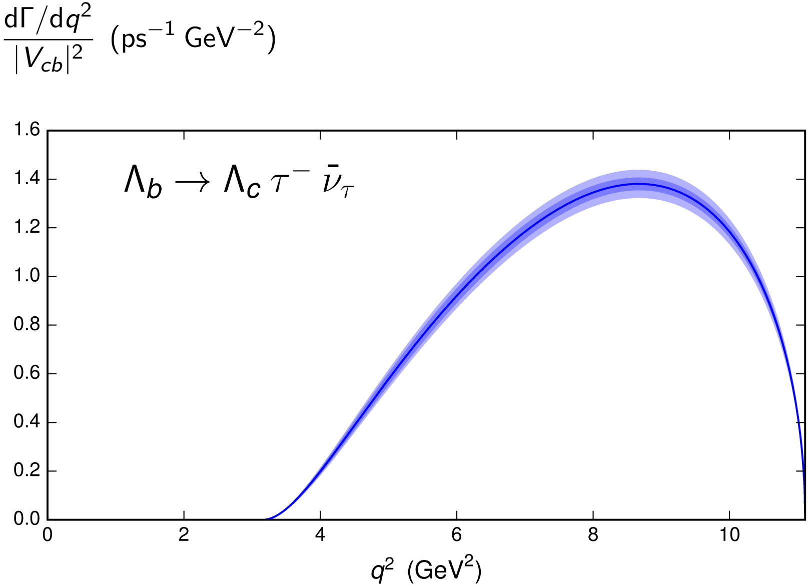
[LHCb Collaboration, Nature Physics **11**, 743-747 (2015)]

Plot by Andreas Kronfeld (private communication)



$$R[D^{(*)}] = \frac{\Gamma[\, B \to D^{(*)}\, \tau\, \bar{\nu}_\tau \,]}{\Gamma[\, B \to D^{(*)}\, \ell\, \bar{\nu}_\ell \,]_{\ell=e,\mu}}$$


R(D*)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
BaBar, PRL109,101802(2012)
Belle, arXiv:1507.03233
LHCb, arXiv:1506.08614
Average
$\Delta\chi^2 = 1.0$
SM prediction
HFAG
Prel. EPS2015
$P(\chi^2) = 55\%$
0.2
0.3
0.4
0.5
0.6
R(D)



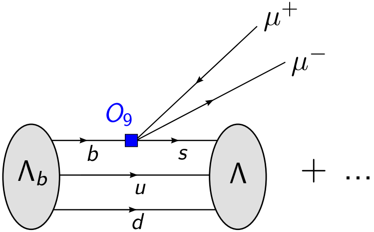
$\frac{d\Gamma/dq^2}{|V_{cb}|^2}$ (ps^{-1} GeV^{-2})
$\Lambda_b \to \Lambda_c \, \tau^- \, \bar{\nu}_\tau$
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
q^2 (GeV^2)

SM Prediction:

$$R[\Lambda_c] = \frac{\Gamma(\Lambda_b \to \Lambda_c\, \tau^- \bar{\nu}_\tau)}{\Gamma(\Lambda_b \to \Lambda_c\, \mu^- \bar{\nu}_\mu)} = 0.3328 \pm 0.0074 \pm 0.0070$$

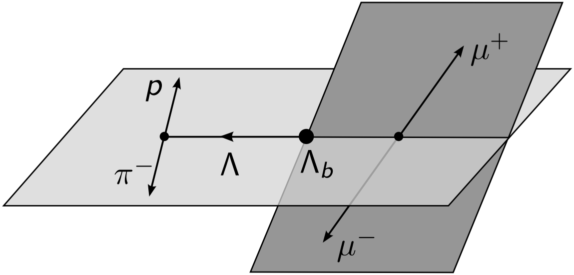
LHCb measurement?

1. $\Lambda_b \to p\,\ell^- \bar{\nu}$ and $\Lambda_b \to \Lambda_c \ell^- \bar{\nu}$
2. $\Lambda_b \to \Lambda \ell^+ \ell^-$
3. Outlook: multi-hadron and nonlocal matrix elements



Combines the best aspects of $B \to K^* \mu^+ \mu^-$ and $B \to K \mu^+ \mu^-$:

Λ has nonzero spin and is stable under strong interactions.



Angular observables studied in

T. Gutsche *et al.*, PRD 87, 074031 (2013),

P. Böer, T. Feldmann, D. van Dyk, JHEP 01, 155 (2015)

2012: $\Lambda_b \to \Lambda$ form factors from lattice QCD with static b quarks
[W. Detmold, C.-J. D. Lin, S. Meinel, M. Wingate, PRD **87**, 074502]

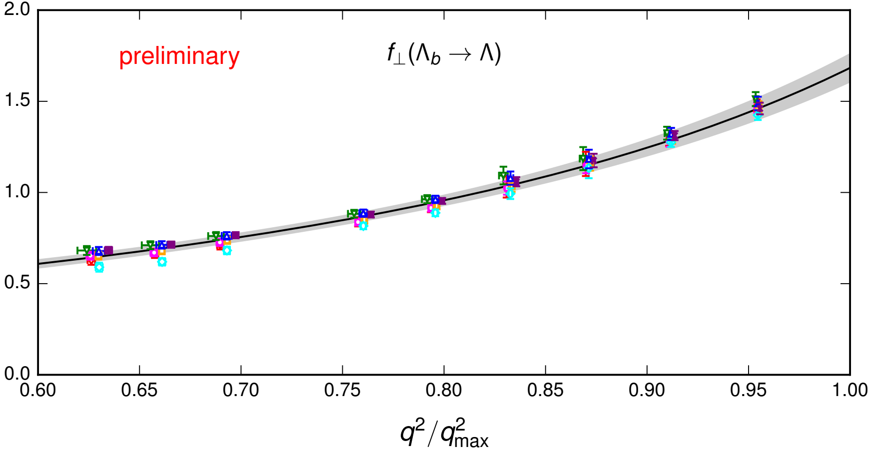
Introduces systematic uncertainty of order E/m_b where E is relevant hadronic energy scale

New! "$\Lambda_b \to \Lambda\, \ell^+ \ell^-$ form factors from lattice QCD with relativistic heavy quarks"

[W. Detmold and S. Meinel, in preparation]

Hopper/NERSC

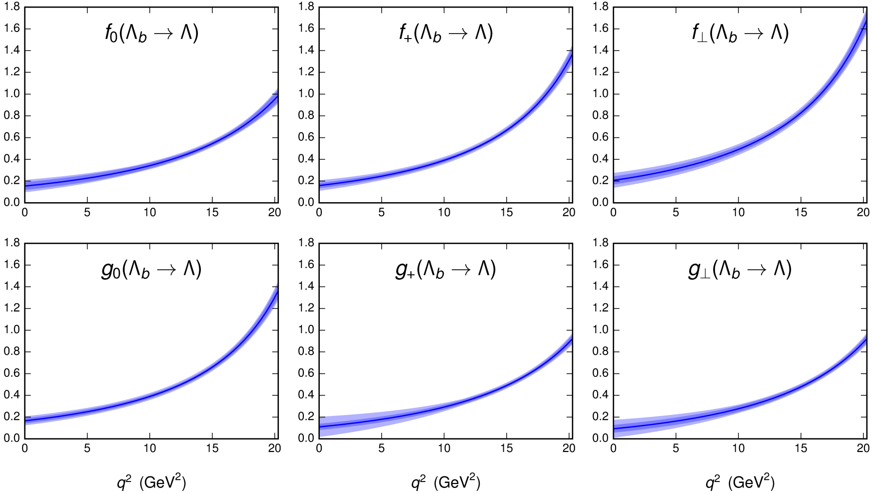

Stampede/TACC



Gray band = statistical uncertainty.

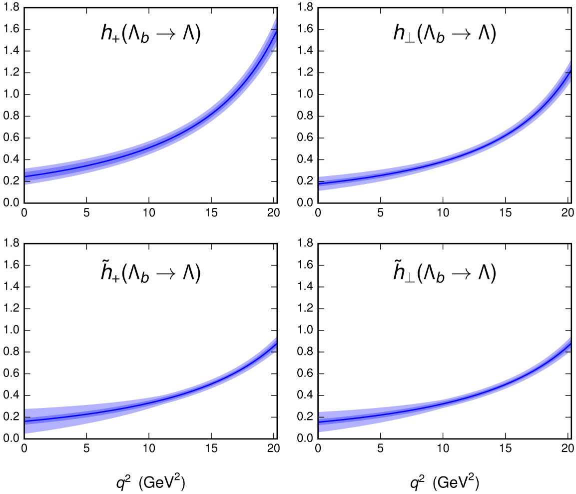
Vector and axial vector form factors

preliminary

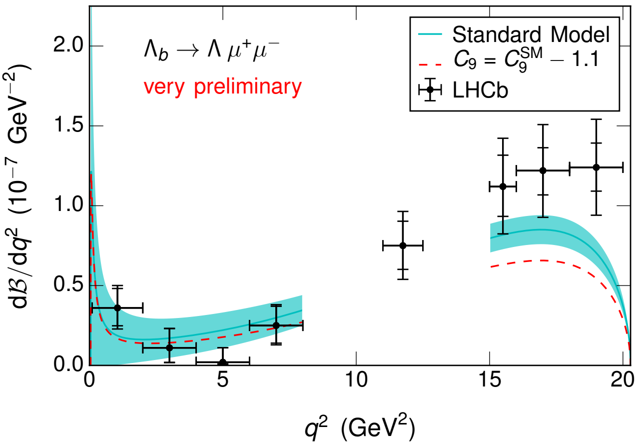


Tensor form factors

preliminary

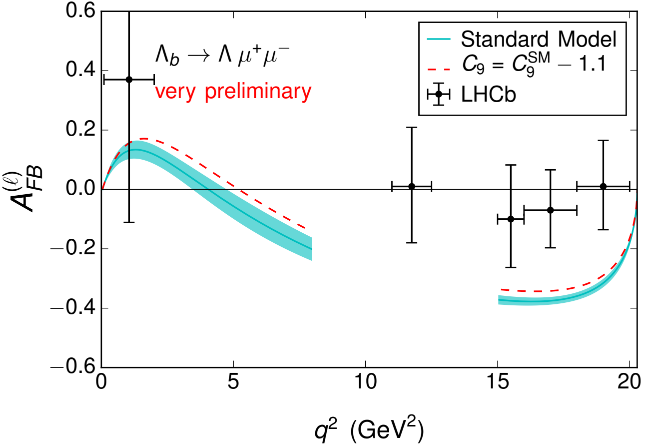


$\Lambda_b \to \Lambda\mu^+\mu^-$ differential branching fraction



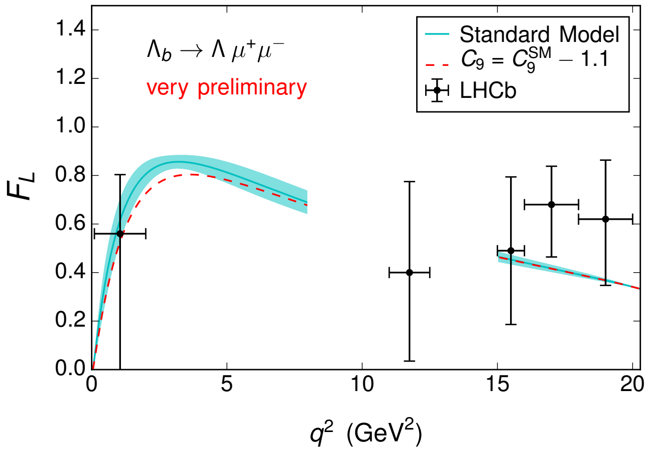
$\Lambda_b \to \Lambda\mu^+\mu^-$ lepton-side A_{FB}

$$\frac{d^2\Gamma}{dq^2\, d\cos\theta_\ell} = \frac{d\Gamma}{dq^2}\left[\frac{3}{8}\left(1+\cos^2\theta_\ell\right)(1-f_L) + A_{\mathrm{FB}}^{(\ell)}\cos\theta_\ell + \frac{3}{4} f_L \sin^2\theta_\ell\right]$$



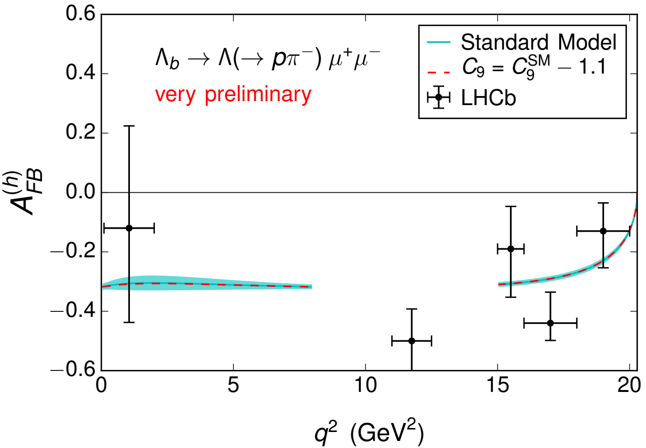
$\Lambda_b \to \Lambda\mu^+\mu^-$ lepton-side F_L

$$\frac{d^2\Gamma}{dq^2\, d\cos\theta_\ell} = \frac{d\Gamma}{dq^2}\left[\frac{3}{8}\left(1+\cos^2\theta_\ell\right)(1-F_L) + A_{FB}^{(\ell)}\cos\theta_\ell + \frac{3}{4}F_L\sin^2\theta_\ell\right]$$



$\Lambda_b \to \Lambda\mu^+\mu^-$ hadron-side A_{FB}

$$\frac{d^2\Gamma}{dq^2\, d\cos\theta_h} = \mathcal{B}(\Lambda \to p\pi^-)\frac{d\Gamma(\Lambda_b \to \Lambda\,\ell^+\ell^-)}{dq^2}\,\frac{1}{2}\left(1 + 2A_{\text{FB}}^{(h)}\cos\theta_h\right)$$

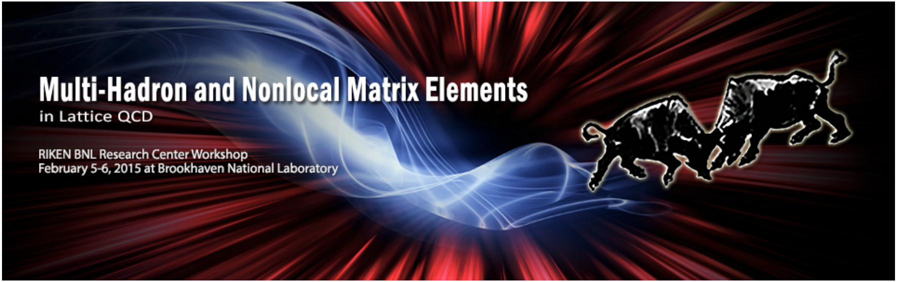


Fits of Wilson coefficients including $\Lambda_b \to \Lambda\mu^+\mu^-$ in progress

(Danny van Dyk's talk this afternoon).

1. $\Lambda_b \to p\,\ell^-\bar{\nu}$ and $\Lambda_b \to \Lambda_c\ell^-\bar{\nu}$
2. $\Lambda_b \to \Lambda\ell^+\ell^-$
3. Outlook: multi-hadron and nonlocal matrix elements

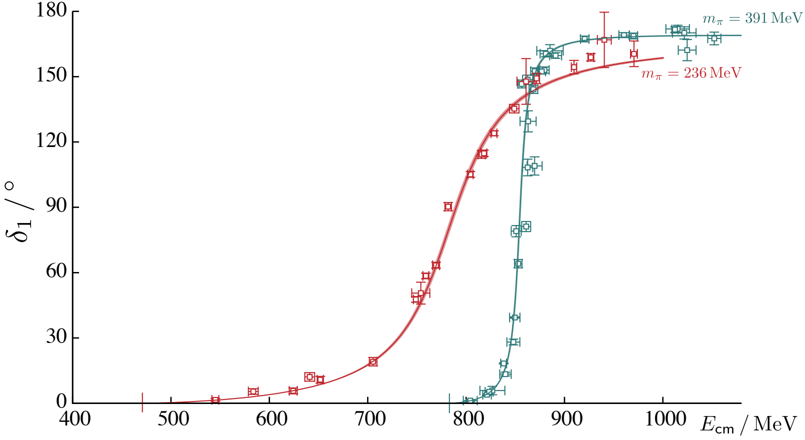
February 2015 workshop (slides and videos available online):



www.bnl.gov/mnme2015/

Coupled $\pi\pi$, $K\bar{K}$ scattering in P wave and the ρ resonance from latticeQCD

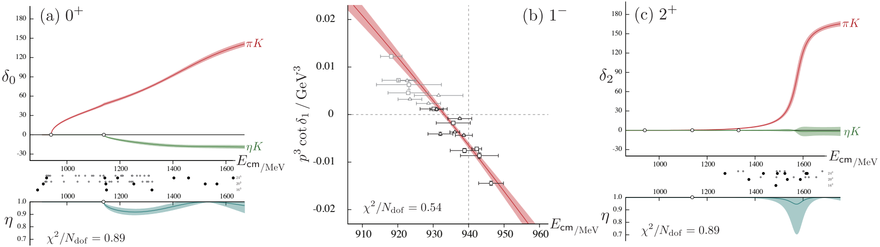
D. Wilson, R. Briceno, J. Dudek, R. Edwards, C. Thomas, arXiv:1507.02599



Resonances in coupled $K\pi$, $K\eta$ scattering from lattice QCD

J. Dudek, R. Edwards, C. Thomas, D. Wilson, PRL 113, 182001 (2014),

J. Dudek, R. Edwards, C. Thomas, D. Wilson, PRD 91, 054008 (2015)

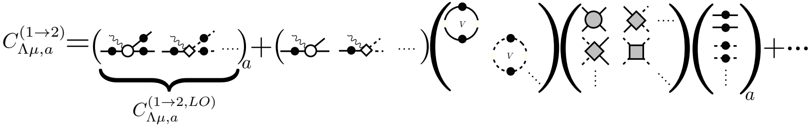


Rigorous method for computing $0 \to 2$, $1 \to 2$ and $2 \to 2$ hadronic matrix elements in lattice QCD

R. Briceno, M. Hansen, A. Walker-Loud, PRD 91, 034501 (2015),

R. Briceno, M. Hansen, arXiv:1502.04314,

R. Briceno, M. Hansen, arXiv:1509.08507



$$|\langle \mathbf{2}|\widetilde{\mathcal{J}}_A|\mathbf{1}\rangle| = \sqrt{\frac{1}{2E_1}}\sqrt{\mathcal{H}^{\rm in}_A\ \mathcal{R}\ \mathcal{H}^{\rm out}_A}$$

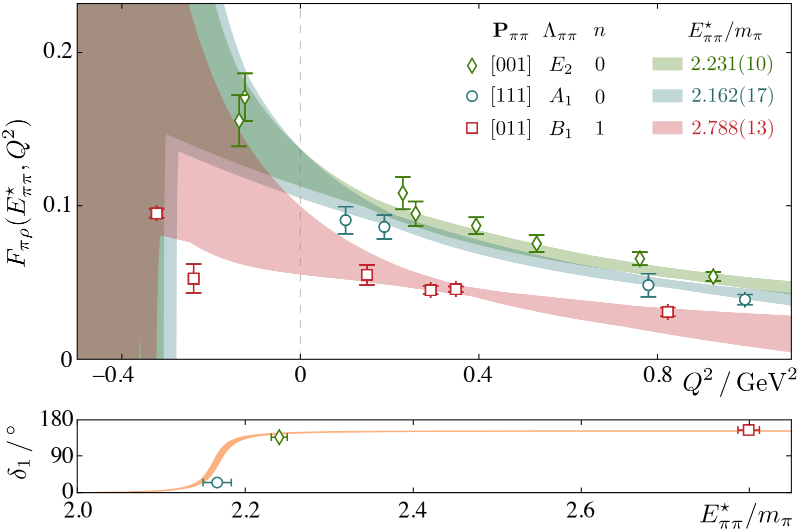
infinite volume transition amplitude

known finite volume function

finite volume matrix element

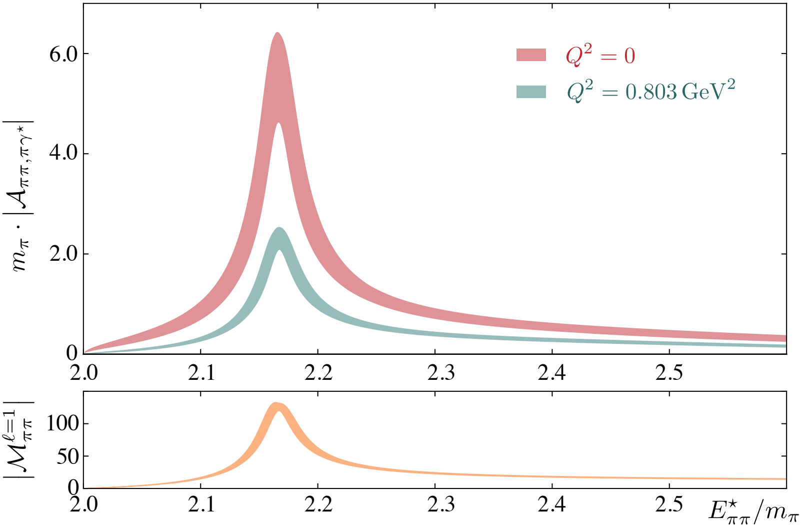
The resonant $\pi\gamma \to \pi\pi$ amplitude from lattice QCD

R. Briceno, J. Dudek, R. Edwards, C. Shultz, C. Thomas, D. Wilson, arXiv:1507.06622



The resonant $\pi\gamma \to \pi\pi$ amplitude from lattice QCD

R. Briceno, J. Dudek, R. Edwards, C. Shultz, C. Thomas, D. Wilson, arXiv:1507.06622

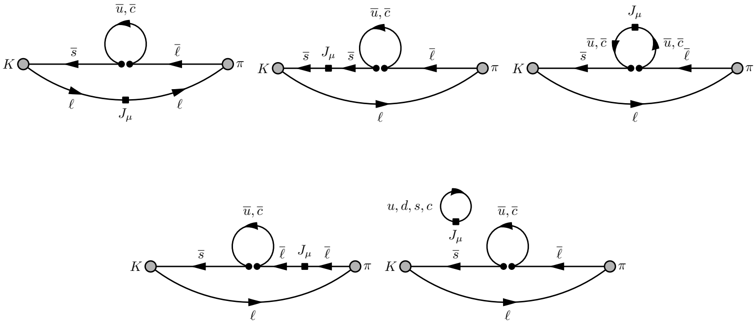


Long-distance contributions to rare kaon decays from lattice QCD

A. Portelli and X. Feng, talks at MNME 2015,

C. Sachrajda, arXiv:1503.01691,

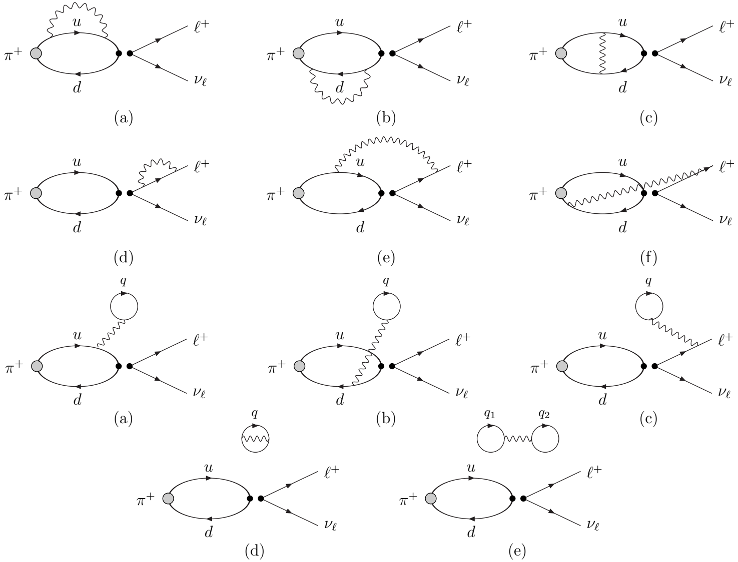
N. Christ, X. Feng, A. Portelli, C. Sachrajda, arXiv:1507.03094



$+ \ldots$

QED corrections to weak decays from lattice QCD

C. Sachrajda, talk at MNME 2015; N. Carrasco *et al.*, PRD 91, 074506 (2015)



Summary

- There has been substantial progress with lattice calculations of B and Λ_b semileptonic decay form factors.
- There are exciting new developments for multi-hadron and nonlocal matrix elements on the lattice.