$B \to \pi\ell^+\ell^-$ at large recoil: theory vs LHCb measurement

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Introduction

- $b \to d\ell^+\ell^-$ processes are CKM suppressed vs $b \to s\ell^+\ell^-$
- There is *CP*-asymmetry violation in SM
- $b \to d\ell^+\ell^-$ is a possible source of New physics
- LHCb has measured the branching fraction and *CP*-asymmetry of the $B^\pm \to \pi^\pm\mu^+\mu^-$ [LHCb-PAPER-2015-035 ArXiv:1509.00414]:

$$
\begin{aligned}
\mathrm{Br}(B^\pm \to \pi^\pm\mu^+\mu^-) &= (1.83 \pm 0.24 \pm 0.05) \times 10^{-8} \\
\mathcal{A}_{CP}(B^\pm \to \pi^\pm\mu^+\mu^-) &= -0.11 \pm 0.12 \pm 0.01
\end{aligned}
$$

- We calculate hadronic observables in $B \to \pi\ell^+\ell^-$ and predict *CP*-asymmetry at $0 < q^2 < 8\,\mathrm{GeV}^2$

Effective Weak Hamiltonian

$$H_{\text{eff}}^{b\to d} = \frac{4G_F}{\sqrt{2}}\left(\lambda_u \sum_{i=1}^{2} C_i\,\mathcal{O}_i^u + \lambda_c \sum_{i=1}^{2} C_i\,\mathcal{O}_i^c - \lambda_t \sum_{i=3}^{10} C_i\,\mathcal{O}_i\right) + h.c.$$

$C_i(\mu)$ — Wilson coefficients, O_i — dimension-six operators

$$\lambda_p = V_{pb}V_{pd}^* \quad (p = u, c, t)$$

$$\lambda_u \sim \lambda_c \sim \lambda_t \sim \lambda^3$$

Amplitude of the $B \to \pi \ell^+ \ell^-$ decay

$$A(B \to \pi \ell^+ \ell^-) = -\langle \pi \ell^+ \ell^- | H_{\rm eff}^{b \to d} | B(p+q) \rangle$$
$$= \frac{G_F}{\sqrt{2}} \frac{\alpha_{\rm em}}{\pi} \lambda_t f_{B\pi}^+(q^2) \Bigg[\left(\bar{\ell}\gamma^\mu \ell\right) p_\mu \Bigg(C_9 + \Delta C_9^{(B\pi)}(q^2)$$
$$+ \frac{2m_b}{m_B + m_\pi} C_7^{\rm eff} \frac{f_{B\pi}^T(q^2)}{f_{B\pi}^+(q^2)} \Bigg) + \left(\bar{\ell}\gamma^\mu \gamma_5 \ell\right) p_\mu C_{10} \Bigg],$$

where determined by nonlocal effects q^2-dependent addition to C_9

$$\Delta C_9^{(B\pi)}(q^2) = -16\pi^2 \frac{\lambda_u \mathcal{H}^{(u)}(q^2) + \lambda_c \mathcal{H}^{(c)}(q^2)}{\lambda_t f_{B\pi}^+(q^2)}$$

is the source of the direct CP-asymmetry

Hadronic input in $B \to \pi \ell^+ \ell^-$

$B \to \pi$ transition form-factors

$$\langle \pi(p) | \bar{d} \gamma^\mu b | B(p+q) \rangle \sim f^+_{B\pi}(q^2), \quad \langle \pi(p) | \bar{d} \sigma^{\mu\nu} q_\nu b | B(p+q) \rangle \sim f^T_{B\pi}(q^2)$$

We need the ratio $r_T(q^2) \equiv f^T_{B\pi}(q^2) / f^+_{B\pi}(q^2)$

Nonlocal effects

$$\mathcal{H}^{(u,c)}_\mu = i \int d^4x e^{iqx} \langle \pi(p) | \mathrm{T} \Big\{ j^{\mathrm{em}}_\mu(x), \Big[C_1 \mathcal{O}^{(u,c)}_1(0) + C_2 \mathcal{O}^{(u,c)}_2(0) + \sum_{k=3-6,8} C_k \mathcal{O}_k(0) \Big] \Big\} | B(p+q) \rangle = \left[(p \cdot q) q_\mu - q^2 p_\mu \right] \mathcal{H}^{(u,c)}(q^2)$$

$B \to \pi$ form factors

- We use LCSR results for $f^+_{B\pi}(q^2)$ fitted to BCL parametrization:
  [I.S. Imsong, A. Khodjamirian, Th. Mannel, D.V. Dyk (2014)]

$$
\begin{aligned}
f^+_{B\pi}(q^2) &= \frac{f^+_{B\pi}(0)}{1-q^2/m^2_{B*}}\left(1+b_1^+\left[z(q^2,t_0)-z(0,t_0)-\frac{1}{3}\left(z(q^2,t_0)^3-z(0,t_0)^3\right)\right]\right. \\
&+ \left. b_2^+\left[z(q^2,t_0)^2-z(q^2,t_0)^2+\frac{2}{3}\left(z(q^2,t_0)^3-z(0,t_0)^3\right)\right]\right)
\end{aligned}
$$

$$
\begin{aligned}
f_{B\pi}(0) &= 0.307 \pm 0.02 \\
b_1^+ &= -1.31 \pm 0.42 \\
b_2^+ &= -0.904 \pm 0.444
\end{aligned}
\qquad
\rho^{BCL} = \begin{pmatrix} 1.000 & 0.503 & -0.391 \\ 0.503 & 1.000 & -0.824 \\ -0.391 & -0.824 & 1.000 \end{pmatrix}
$$

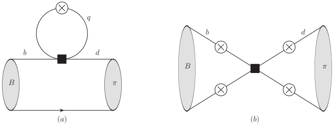
- In the large hadronic recoil for $f_T(q^2)$:

$$
f^T_{B\pi}(q^2) = r_T(q^2) f^+_{B\pi}(q^2) \simeq r_T(0) f^+_{B\pi}(q^2), \qquad r_T(0) = 0.98 \pm 0.02
$$

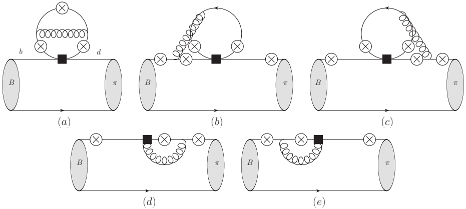
[G. Duplancic et al. (2008)]

Calculation of $H^{(u,c)}(q^2)$ at $q^2 < 0$

- LO, factorizable and weak annihilation (QCD factorization)
  [M. Beneke, Th. Feldmann, D. Seidel (2001)]

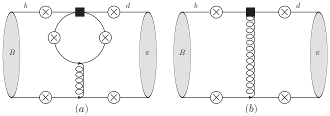


- NLO, factorizable
  [H.H.Asatryan, H.M. Asatrian, C. Greub, M. Walker (2002);
  H.M.Asatrian, K. Bieri, C. Greub, M. Walker (2004)]

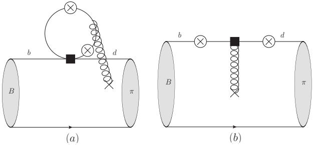


Calculation of $H^{(u,c)}(q^2)$ at $q^2 < 0$

- NLO, nonfactorizable (hard gluons)
  [M. Beneke, Th. Feldmann, D. Seidel (2001)]



- Soft gluons, nonfactorizable
  [A. Khodjamirian, Th. Mannel, A.A. Pivovarov, Y.-M. Wang (2010)]
  [A. Khodjamirian, Th. Mannel, Y.-M. Wang (2013)]



Dispersion Relations

Dispersion relations (analytic continuation of $\mathcal{H}^{(u,c)}(q^2)$ to $q^2 > 0$):

$$\mathcal{H}^{(u,c)}(q^2) = (q^2 - q_0^2)\Bigg[\sum_{V=\rho,\omega,J/\psi,\psi(2S)} \frac{k_V f_V A^{u,c}_{BV\pi}}{(m_V^2 - q_0^2)(m_V^2 - q^2 - i m_V \Gamma_V^{\rm tot})}$$

$$+ \int_{s_0^{u,c}}^{\infty} ds \frac{\rho^{(u,c)}(s)}{(s - q_0^2)(s - q^2 - i\epsilon)}\Bigg] + \mathcal{H}^{(u,c)}(q_0^2)$$

- $A^{u,c}_{BV\pi} = |A^{u,c}_{BV\pi}| e^{i\delta^{u,c}_{BV\pi}}$
- $|A^{u,c}_{BV\pi}|$ are extracted from nonleptonic $B \to V\pi$ decays
- $\delta^{u,c}_{BV\pi}$ are extracted from the fit of the dispersion relation to $\mathcal{H}^{(u,c)}(q^2)$ for $q^2 < 0$
- For $\rho^{(u,c)}(s)$ apply quark-hadron duality

Nonleptonic $B \to V\pi$ decays

The amplitude of the nonleptonic $B \to V\pi$ decay takes the form:

$$
\begin{aligned}
A(B \to V\pi) &= \langle V(q)\pi(p)|H_{\mathrm{eff(NL)}}^{b\to d}|B(p+q)\rangle \\
&= \frac{4G_F}{\sqrt{2}} m_V(\varepsilon_V^* \cdot p)\Big(\lambda_u A_{BV\pi}^u + \lambda_c A_{BV\pi}^c\Big)
\end{aligned}
$$

Ways of the extraction of $A_{BV\pi}^{u,c}$:

- Calculation in framework of QCD factorization approach [M. Beneke, M. Neubert (2003)]
- Experimental data on branching fraction and CP-asymmetry
- Isospin symmetry relations

The absolute values of the amplitudes in $B \to V\pi$

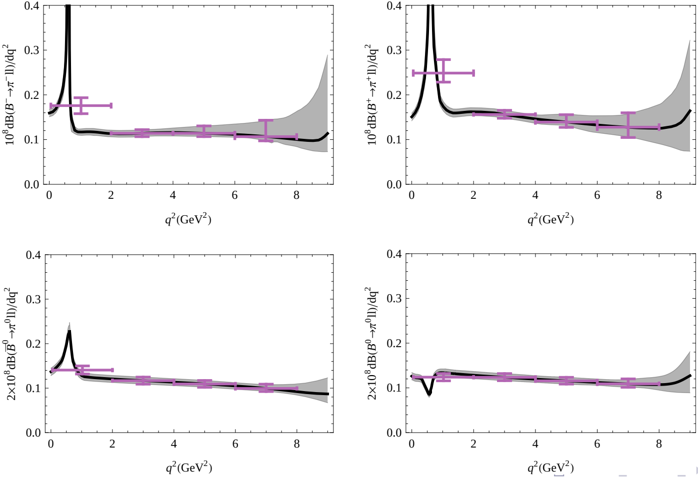
Mode	$\lvert A^u_{BV\pi}\rvert$	$\lvert A^c_{BV\pi}\rvert$
$B^\mp \to \rho^0\pi^\mp$	$20.8^{+2.7}_{-2.3}$ (QCDF)	$1.3^{+1.1}_{-0.4}$ (QCDF)
$B^\mp \to \omega\pi^\mp$	$19.1^{+2.7}_{-2.0}$ (QCDF)	$0.3^{+0.4}_{-0.1}$ (QCDF)
$B^\mp \to J/\psi\pi^\mp$	$0.5^{+9.7}_{-0.5}$ (Exp. data)	$29.2^{+1.4}_{-1.5}$ (Exp. data)
$B^\mp \to \psi(2S)\pi^\mp$	$3.5^{+6.7}_{-3.5}$ (Exp. data)	$32.3^{+2.0}_{-2.1}$ (Exp. data)

Mode	$\lvert A^u_{BV\pi}\rvert$	$\lvert A^c_{BV\pi}\rvert$
$B^0 \to \rho^0\pi^0$	$9.9^{+1.3}_{-1.4}$ (Exp. data)	0 (negligible)
$B^0 \to \omega\pi^0$	0 (negligible)	0 (negligible)
$B^0 \to J/\psi\pi^0$	$0.3^{+6.9}_{-0.3}$ (Isospin rel.)	$20.6^{+1.0}_{-1.1}$ (Isospin rel.)
$B^0 \to \psi(2S)\pi^0$	$2.4^{+4.7}_{-2.4}$ (Isospin rel.)	$22.8^{+1.4}_{-1.5}$ (Isospin rel.)

- Need more data on NL decays, e.g. $B \to \rho^{0\prime}\pi$, $B \to \omega'\pi$

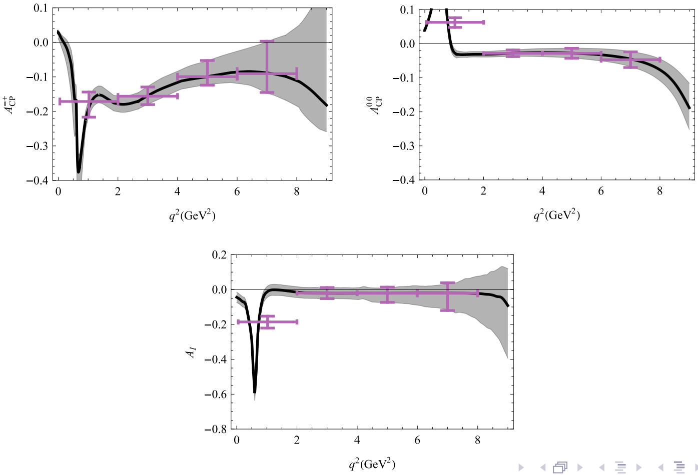
Observables in $B \to \pi \ell^+ \ell^-$

Dilepton invariant mass spectrum



Observables in $B \to \pi\ell^+\ell^-$

CP and isospin asymmetries



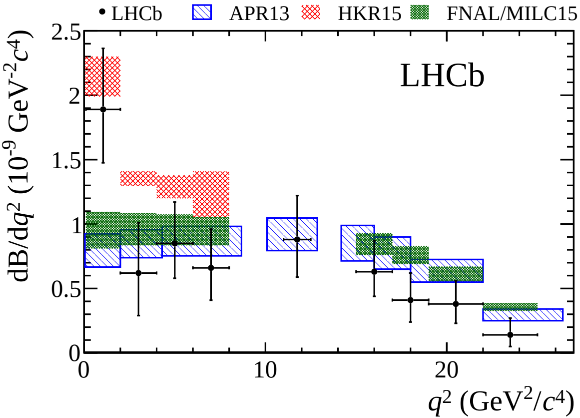
Observables in $B \to \pi \ell^+ \ell^-$

Binned values

Bin [GeV^2]	[0.05, 2.0]	[2.0, 4.0]	[4.0, 6.0]	[6.0, 8.0]	[1.0, 6.0]
$\mathcal{B}(B^-)$	$0.176^{+0.018}_{-0.018}$	$0.114^{+0.008}_{-0.007}$	$0.114^{+0.016}_{-0.007}$	$0.107^{+0.036}_{-0.009}$	$0.126^{+0.013}_{-0.010}$
$\mathcal{B}(B^+)$	$0.249^{+0.030}_{-0.020}$	$0.156^{+0.009}_{-0.008}$	$0.139^{+0.016}_{-0.011}$	$0.128^{+0.030}_{-0.023}$	$0.168^{+0.016}_{-0.012}$
$2 \times \mathcal{B}(\bar{B}^0)$	$0.140^{+0.009}_{-0.009}$	$0.117^{+0.008}_{-0.008}$	$0.109^{+0.008}_{-0.008}$	$0.099^{+0.010}_{-0.007}$	$0.119^{+0.008}_{-0.008}$
$2 \times \mathcal{B}(B^0)$	$0.124^{+0.008}_{-0.008}$	$0.124^{+0.008}_{-0.008}$	$0.116^{+0.008}_{-0.007}$	$0.109^{+0.011}_{-0.008}$	$0.121^{+0.008}_{-0.008}$
$\mathcal{A}_{CP}^{(-+)}$	$-0.171^{+0.027}_{-0.045}$	$-0.156^{+0.027}_{-0.024}$	$-0.099^{+0.047}_{-0.025}$	$-0.091^{+0.093}_{-0.053}$	$-0.143^{+0.035}_{-0.029}$
$\mathcal{A}_{CP}^{(\bar{0}0)}$	$0.063^{+0.014}_{-0.015}$	$-0.028^{+0.010}_{-0.010}$	$-0.028^{+0.015}_{-0.015}$	$-0.047^{+0.023}_{-0.023}$	$-0.008^{+0.013}_{-0.013}$
$\mathcal{A}_I$	$-0.195^{+0.033}_{-0.035}$	$-0.020^{+0.031}_{-0.032}$	$-0.021^{+0.035}_{-0.053}$	$-0.021^{+0.060}_{-0.100}$	$-0.063^{+0.033}_{-0.040}$

$$\mathcal{B}(B^- \to \pi^- \ell^+ \ell^- [q_1^2, q_2^2]) \equiv \frac{1}{q_2^2 - q_1^2} \int_{q_1^2}^{q_2^2} dq^2 \, \frac{dB(B^- \to \pi^- \ell^+ \ell^-)}{dq^2} \, .$$

Theory vs LHCb measurement



from LHCb-PAPER-2015-035 [ArXiv:1509.00414]

Comment concerning the ratio of branching fractions

$$R(q^2) \equiv \frac{d\Gamma(B \to \pi \ell^+ \ell^-)/dq^2}{d\Gamma(B \to K \ell^+ \ell^-)/dq^2}$$

$$R(q^2) = N(q^2) \left| \frac{V_{td}}{V_{ts}} \right|^2 \left| \frac{f^+_{B\pi}(q^2)}{f^+_{BK}(q^2)} \right|^2 \frac{\left| A^\pi(q^2) + B^\pi(q^2) + \alpha_u C^\pi(q^2) \right|^2 + |C_{10}|^2}{\left| A^K(q^2) + B^K(q^2) \right|^2 + |C_{10}|^2}$$

- $N(q^2)$ — kinematical factor
- $\alpha_u \equiv \frac{V_{ub} V^*_{ud}}{V_{tb} V^*_{td}} \sim 1$
- $A^{\pi,K}(q^2)$ account for short-distance effects
- $B^{\pi,K}(q^2)$, $C^\pi(q^2)$ account for nonlocal effects

Conclusion and outlook

- We analyse the nonlocal contributions in $B \to \pi \ell^+ \ell^-$ decays at large hadronic recoil
- We predict differential decay rate, direct CP-asymmetry and isospin asymmetry in $B \to \pi \ell^+ \ell^-$ for $0 < q^2 < 8$ GeV^2

Further improvements possible:

- More elaborated ansatz for hadronic dispersion relations (including radial excitations of light vector mesons)
- Precise measurements of the nonleptonic $B \to V\pi$ decays
- Measurement of the binned CP-asymmetry

Backup

Operator Basis

$$\mathcal{O}_9 = \frac{\alpha_{\rm em}}{4\pi}\left(\bar{d}_L\gamma^\mu b_L\right)\left(\bar{\ell}\gamma_\mu\ell\right), \quad \mathcal{O}_{10} = \frac{\alpha_{\rm em}}{4\pi}\left(\bar{d}_L\gamma^\mu b_L\right)\left(\bar{\ell}\gamma_\mu\gamma_5\ell\right),$$

$$\mathcal{O}_{7\gamma} = -\frac{e\, m_b}{16\pi^2}\left(\bar{d}_L\sigma^{\mu\nu}b_R\right)F_{\mu\nu},$$

$$\mathcal{O}_1^u = \left(\bar{d}_L\gamma_\mu u_L\right)\left(\bar{u}_L\gamma^\mu b_L\right), \quad \mathcal{O}_2^u = \left(\bar{d}_L^i\gamma_\mu u_L^j\right)\left(\bar{u}_L^j\gamma^\mu b_L^i\right),$$

$$\mathcal{O}_1^c = \left(\bar{d}_L\gamma_\mu c_L\right)\left(\bar{c}_L\gamma^\mu b_L\right), \quad \mathcal{O}_2^c = \left(\bar{d}_L^i\gamma_\mu c_L^j\right)\left(\bar{c}_L^j\gamma^\mu b_L^i\right),$$

$$\mathcal{O}_3 = \left(\bar{d}_L\gamma_\mu b_L\right)\sum_q\left(\bar{q}_L\gamma^\mu q_L\right), \quad \mathcal{O}_4 = \left(\bar{d}_L^i\gamma_\mu b_L^j\right)\sum_q\left(\bar{q}_L^j\gamma^\mu q_L^i\right),$$

$$\mathcal{O}_5 = \left(\bar{d}_L\gamma_\mu b_L\right)\sum_q\left(\bar{q}_R\gamma^\mu q_R\right), \quad \mathcal{O}_6 = \left(\bar{d}_L^i\gamma_\mu b_L^j\right)\sum_q\left(\bar{q}_R^j\gamma^\mu q_R^i\right),$$

$$\mathcal{O}_{8g} = -\frac{g_s m_b}{16\pi^2}\left(\bar{d}_L^i\sigma_{\mu\nu}(T^a)^{ij}b_R^j\right)G^{a\,\mu\nu}$$

Dispersion Relations, final form

$$\mathcal{H}^{(p)}(q^2) = (q^2 - q_0^2)\Bigg[\sum_{V=\rho,\omega,J/\psi,\psi(2S)} k_V f_V \frac{|A^p_{BV\pi}|\exp(i\delta^{(p)}_{BV\pi})}{(m_V^2 - q_0^2)(m_V^2 - q^2 - i m_V \Gamma_V^{\mathrm{tot}})}$$

$$+ \int\limits_{\tilde{s}_0(s_0)}^{4m_D^2} ds \frac{\rho^{(p)}_{LO}(s)}{(s - q_0^2)(s - q^2 - i\sqrt{s}\Gamma_{\mathrm{eff}}(s))}$$

$$+ |a_p|\exp(i\phi_a) + |b_p|\exp(i\phi_b)\frac{q^2}{4m_D^2}\Bigg] + \mathcal{H}^{(p)}(q_0^2), \quad p = u, c$$