# b->sll theory uncertainties


Sebastian Jäger

University of Sussex

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# Contents

Anomalies and their origin

Theory uncertainties in the B->K*ll angular distribution

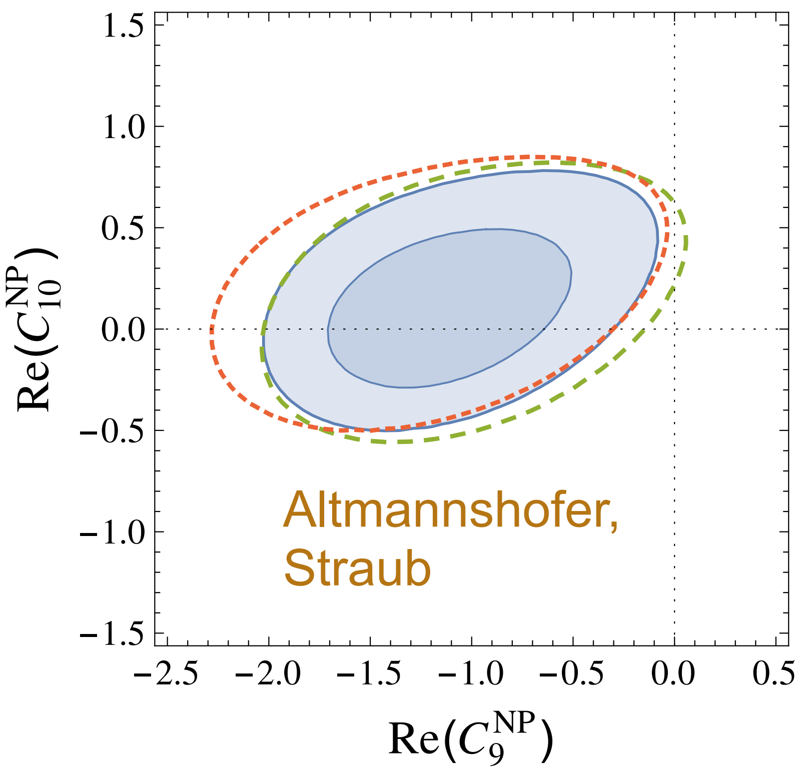
- form factors, power corrections, schemes, etc
- charm loops etc
- (a little) comparison with data

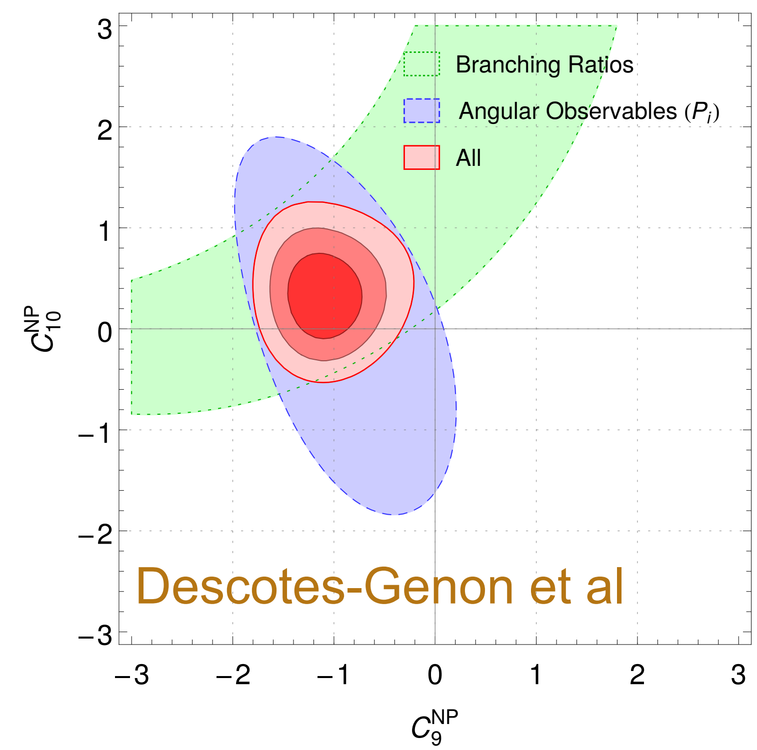
Conclusions

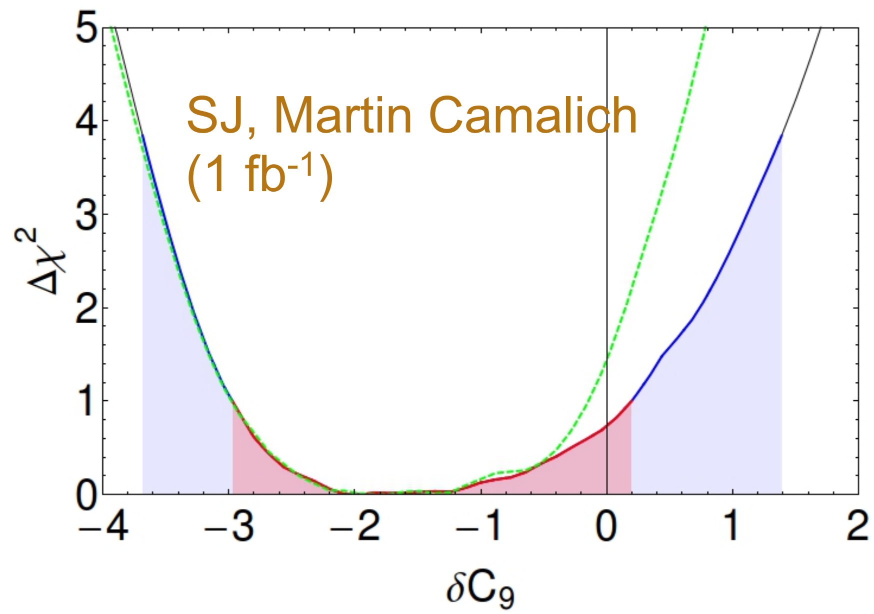
*Note - will focus on aspects relevant to $C_9$ determinations - at expense of (very interesting) right-handed current searches/electron physics - due to lack of time/space*

# Anomalies !

Fits of weak Hamiltonian to data on B->K(*)ll, Bs->mu mu, B->Xs gamma, B->phi ll, B->K*gamma prefer non-SM values.




also: Bobeth et al; Hurth-Mahmoudi; Silvestrini et al; Ghosh et al,...

Most (including speaker!) agree that best fit is for $C_9^{NP}$ ~ -1..-2 but differ on significance

BSM interpretations can be constructed, though no particularly compelling framework has emerged

# Prime suspects

$C_9$ : coupling of a particular four-fermion operator

$$Q_{9V} = \frac{\alpha_{\rm em}}{4\pi}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu l)$$

- easily obtained from Z' exchanges

Descotes-Genon et al; Altmannshofer et al; Crivellin et al; Gauld et al; ...

- vectorial (as opposed to chiral) coupling to leptons may be preferred by precision constraints and anomaly freedom (naturally predicts $R_K \neq 1$, too)

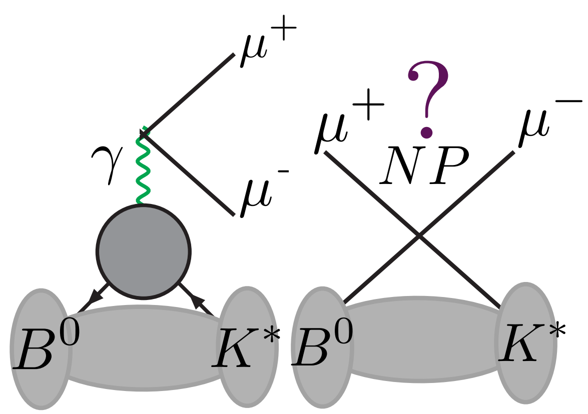
Altmannshofer-Gori-Pospelov-Yavin

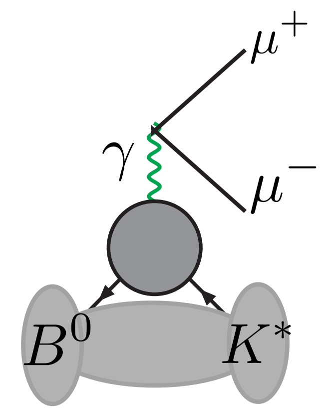
**Possible problem: BSM effects in $C_9$ can be mimicked by a range of SM effects - how well are they controlled?**

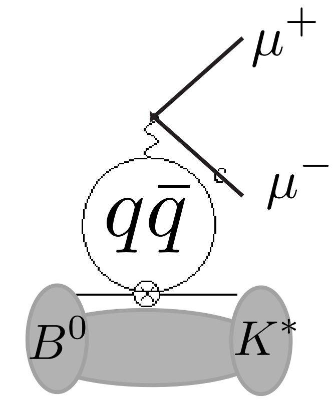
*note - these effects are all lepton-flavour-universal so no relevance for $R_K$ and other lepton universality tests*

# B->VII vector amplitudes

λ=+1/0/-1  helicity of vector meson




$$H_V(\lambda) \propto \tilde{V}_\lambda(q^2)C_9 - V_{-\lambda}(q^2)C_9' + \frac{2\,m_b m_B}{q^2}\left(\tilde{T}_\lambda(q^2)C_7 - \tilde{T}_{-\lambda}(q^2)C_7'\right) - \frac{16\,\pi^2 m_B^2}{q^2}\,h_\lambda(q^2)$$

no photon pole: vanishing relative contribution as $q^2$->0

photon pole at $q^2$=0

Only one form factor, drops out up to interference

photon pole at $q^2$=0

complicated nonlocal correction

Helicity +1 suppressed in heavy-quark limit (in SM) (basis for right-handed current tests)

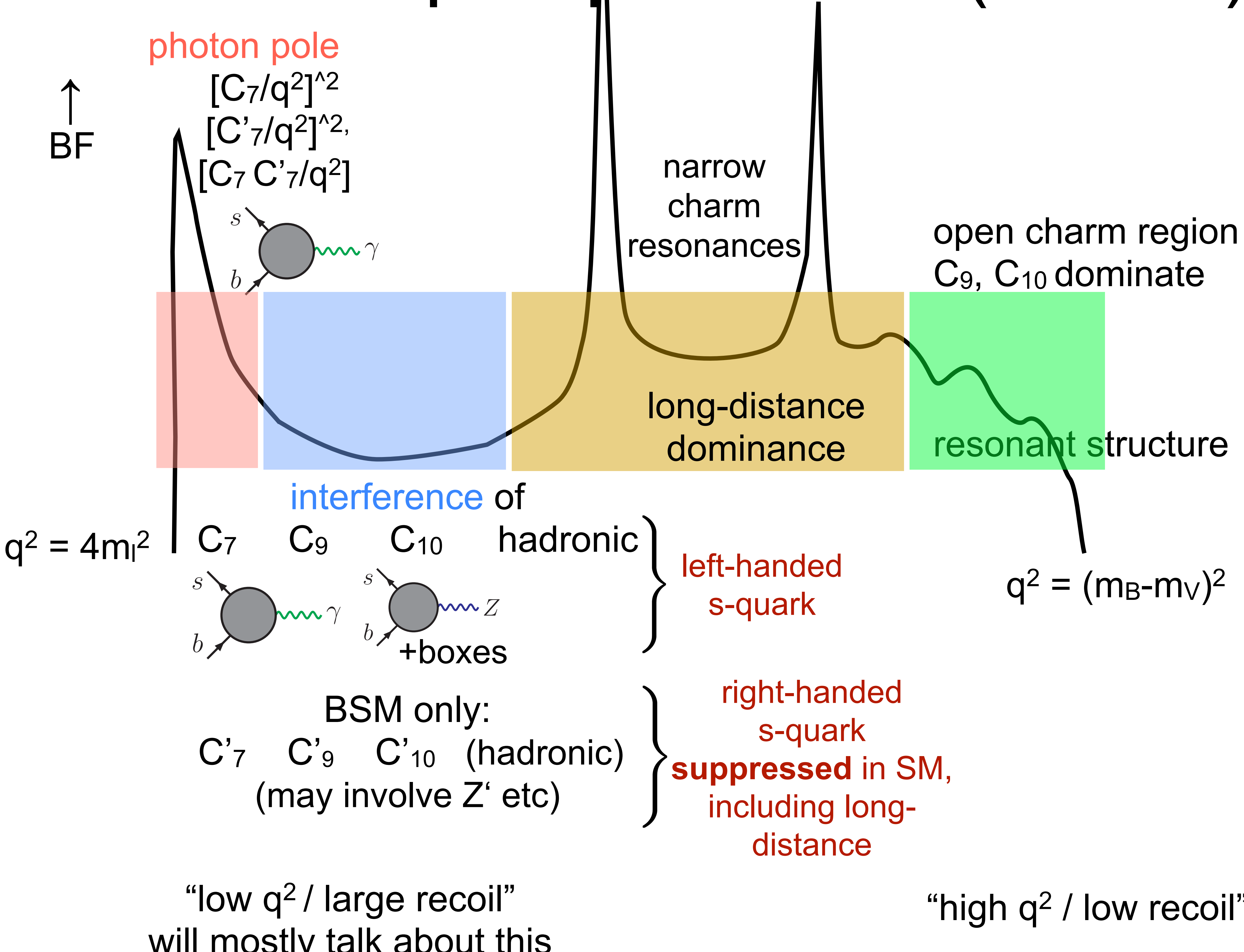
Burdman, Hiller 2000

Beneke, Feldmann, Seidel 2001

SJ, Martin Camalich 2012

λ=0 and λ=-1 amplitudes involve two nonperturbative form factors each, and nonlocal ("quark loop") contributions.
**Implies degeneracies between $C_9$ and nonperturbative physics.** (Eg, rescale $V_-$ and $C_9$ by opposite amount.)

# B->K*ll $q^2$ dependence (sketch)



# Form factors

Helicity amplitudes naturally involve helicity form factors

$$
\begin{aligned}
-im_B\tilde{V}_{L(R)\lambda}(q^2) &= \langle M(\lambda)|\bar{s}\not{\epsilon}^*(\lambda)P_{L(R)}b|\bar{B}\rangle, \\
m_B^2\tilde{T}_{L(R)\lambda}(q^2) &= \epsilon^{*\mu}(\lambda)q^\nu\langle M(\lambda)|\bar{s}\sigma_{\mu\nu}P_{R(L)}b|\bar{B}\rangle \\
im_B\tilde{S}_{L(R)}(q^2) &= \langle M(\lambda=0)|\bar{s}P_{R(L)}b|\bar{B}\rangle.
\end{aligned}
$$

~ Bharucha/Feldmann/Wick 2010

definitions here:
SJ, Martin Camalich 2012

- can express as linear combinations of traditional “transversity” FF brings in dependence on $q^2$ and meson masses - intransparent. (However S is essentially $A_0$ in the traditional nomenclature.)

- helicity form form factors **directly relevant** to B->V l l including the LHCb anomaly

in particular, **$V_-/T_-$ (co-)determines the zero crossing of both $A_{FB}$ and of $S_5/P_5'$, as far as form factors are concerned**

(Burdman; Beneke/Feldmann/Seidel)
SJ, Martin Camalich 2012,2014; ...

Form factors are a dominant source of theory uncertainty

At low $q^2$ (more or less) directly accessed by light-cone sum rules (LCSR), with associated systematics.
Reduce sensitivity by taking ratios; heavy-quark expansions; etc

# Form factor relations

The heavy-quark limit predicts simple relations between the (helicity) form factors, for instance:

Charles et al 1999
Beneke, Feldmann 2000
...
(SJ, Martin Camalich, WIP)

$$\frac{T_-(q^2)}{V_-(q^2)} = 1 + \frac{\alpha_s}{4\pi} C_F \left[\ln \frac{m_b^2}{\mu^2} - L\right] + \frac{\alpha_s}{4\pi} C_F \frac{1}{2} \frac{\Delta F_\perp}{V_-} \quad \text{where} \quad L = -\frac{2E}{m_B - 2E} \ln \frac{2E}{m_B}$$

"vertex" correction:
**parameter-free**

"spectator scattering":
mainly dependent on B meson LCDA
**but $a_s$ suppressed**

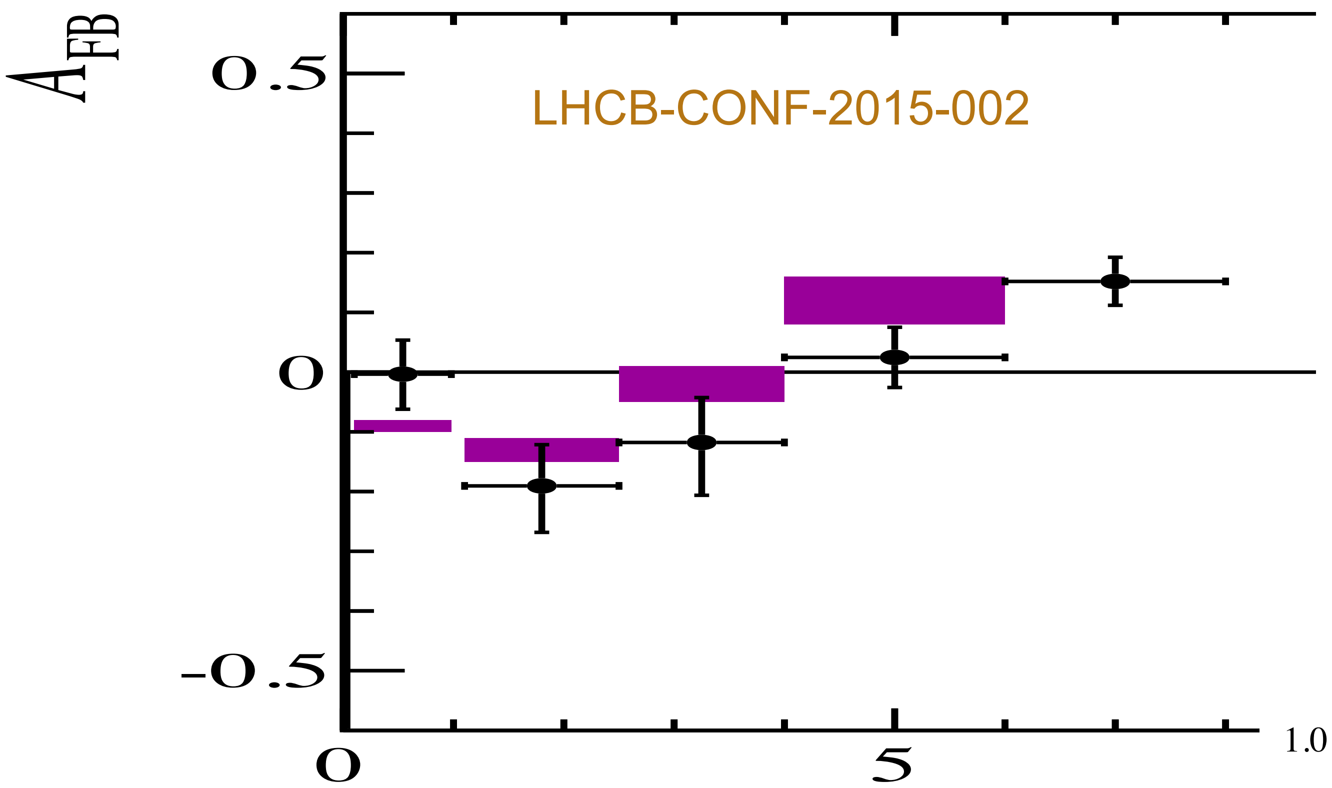
- Eliminates form factor dependence from some observables (eg $P_2$' and zero of $A_{FB}$) almost completely, up to power corrections

Descotes-Genon, Hofer, Matias, Virto

- pure HQ limit: $T_-(0)/V_-(0) \sim 1.05 > 1$ Beneke,Feldmann 2000

- compare to: $T_-(0)/V_-(0) = 0.94$ +/- 0.04 [D Straub, priv comm based on Bharucha, Straub, Zwicky 1503.05534]
 LCSR computation with correlated parameter variations.
 Difference consistent with power correction; remarkable 5% error

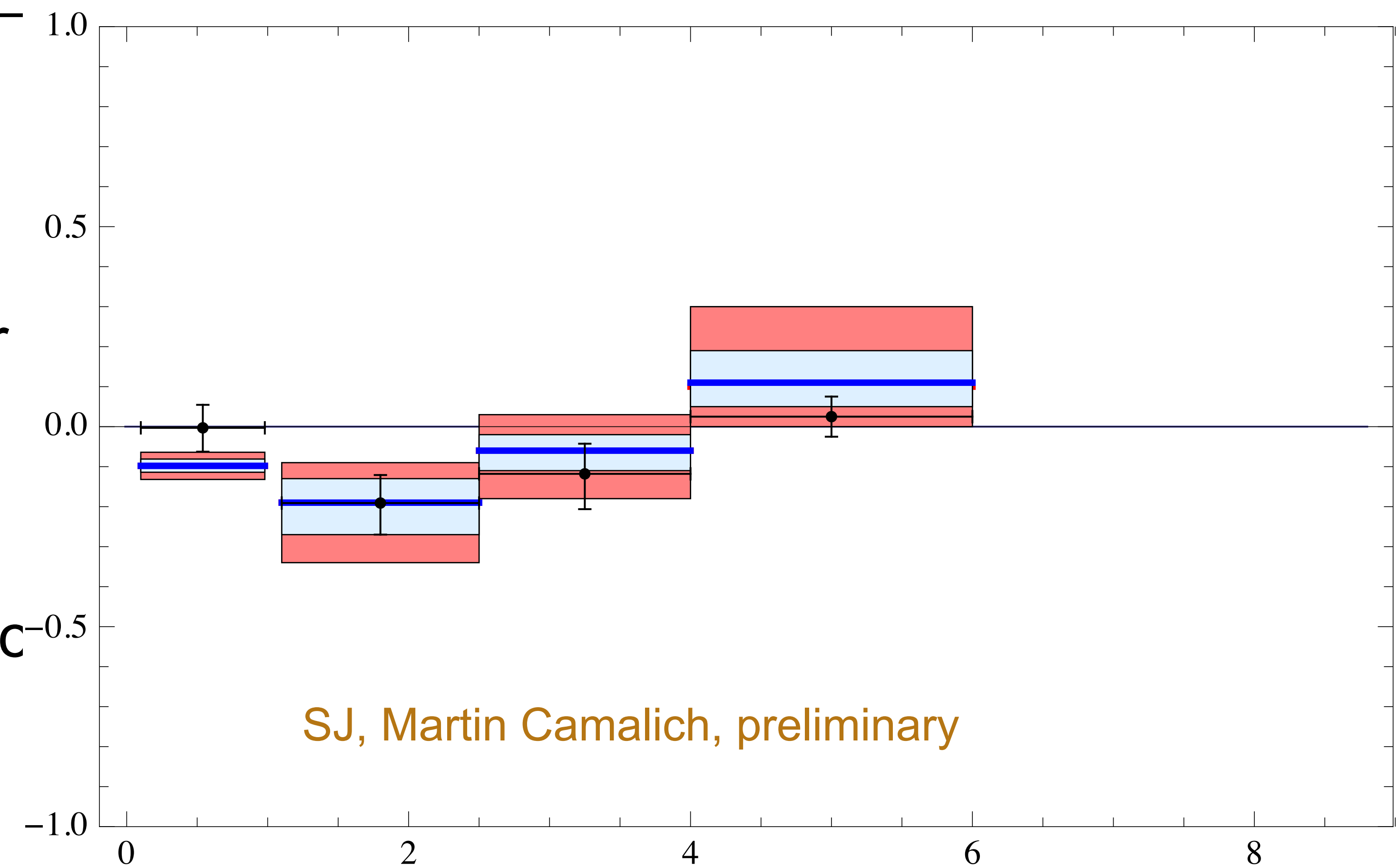
# Forward-backward asymmetry




LHCb Moriond 2015 (3 $fb^{-1}$)
downward shift of $A_{FB}$ relative to LCSR-based prediction
(Bharucha, Straub, Zwicky 2015)

Such a shift is largely equivalent to a **rightward shift** of the zero crossing.

Zero crossing in LCSR has been significantly lower than heavy-quark limit for many years (as low as <3 $GeV^2$)




blue line: pure heavy-quark limit, **no power corrections**

light blue: “68% Gaussian” theory error (including power corrections)

pink: full scan over all theory errors

Surprising that pure HQ limit appears to agree reasonably well with data !

“Clean” observables at present precision have noticeable form factor dependence

# General parameterisation of power corrections

SJ, Martin Camalich 2012

$$F(q^2) = F^\infty(q^2) + a_F + b_F q^2/m_B^2 + \mathcal{O}([q^2/m_B^2]^2)$$

heavy quark limit

Power corrections - parameterise

At most 1-2% over entire 0..6 GeV^2 range -> ignore

$a_F$, $b_F$ are $O(\Lambda/m_b)$
- varied at +/-10% of generic leading-power analogue (+/-0.03 and +/-0.1 respectively) for error bars on previous slides

One can eliminate two $a_F$ and $b_F$ by choice of two reference ("soft") form factors. **However,** unambiguous heavy-quark limit for form factor ratios (eg $T_-/V_-$): These are **invariant** under change of form factor scheme, as are **any observables**

Any calculation (eg LCSR) can be expressed in terms of the general parameterisation - but then one is using dynamical/model input beyond the heavy-quark expansion

Proposal ( Descotes-Genon et al 2014 ) to center ranges for $a_F$, $b_F$ around LCSR predictions (but replace the corresponding errors by ad hoc 10% ranges).

No theoretical justification given for this. **Practical** effect is to obtain predictions similar to LCSR - this is so by construction, and is not an independent check.

# Power corrections, scheme independence

SJ, Martin Camalich 1412.3183

Example

manifestly form-factor-scheme-independent

$$P_5' = P_5'|_\infty \Bigg( 1 + \frac{a_{V_-} - a_{T_-}}{\xi_\perp} \frac{m_B}{|\vec{k}|} \frac{m_B^2}{q^2} C_7^{\rm eff} \frac{C_{9,\perp} C_{9,\|} - C_{10}^2}{(C_{9,\perp}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\|})}$$

$$+ \frac{a_{V_0} - a_{T_0}}{\xi_\|} 2\, C_7^{\rm eff} \frac{C_{9,\perp} C_{9,\|} - C_{10}^2}{(C_{9,\|}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\|})}$$

$$+ 8\pi^2 \frac{\tilde{h}_-}{\xi_\perp} \frac{m_B}{|\vec{k}|} \frac{m_B^2}{q^2} \frac{C_{9,\perp} C_{9,\|} - C_{10}^2}{C_{9,\perp} + C_{9,\|}} + \text{further terms} \Bigg) + \mathcal{O}(\Lambda^2/m_B^2)$$

heavy-quark-limit result

("charm loop" power correction)

(**truncated** after 3 out of 11 independent power-correction terms!)

Many independent power-correction parameters appear.

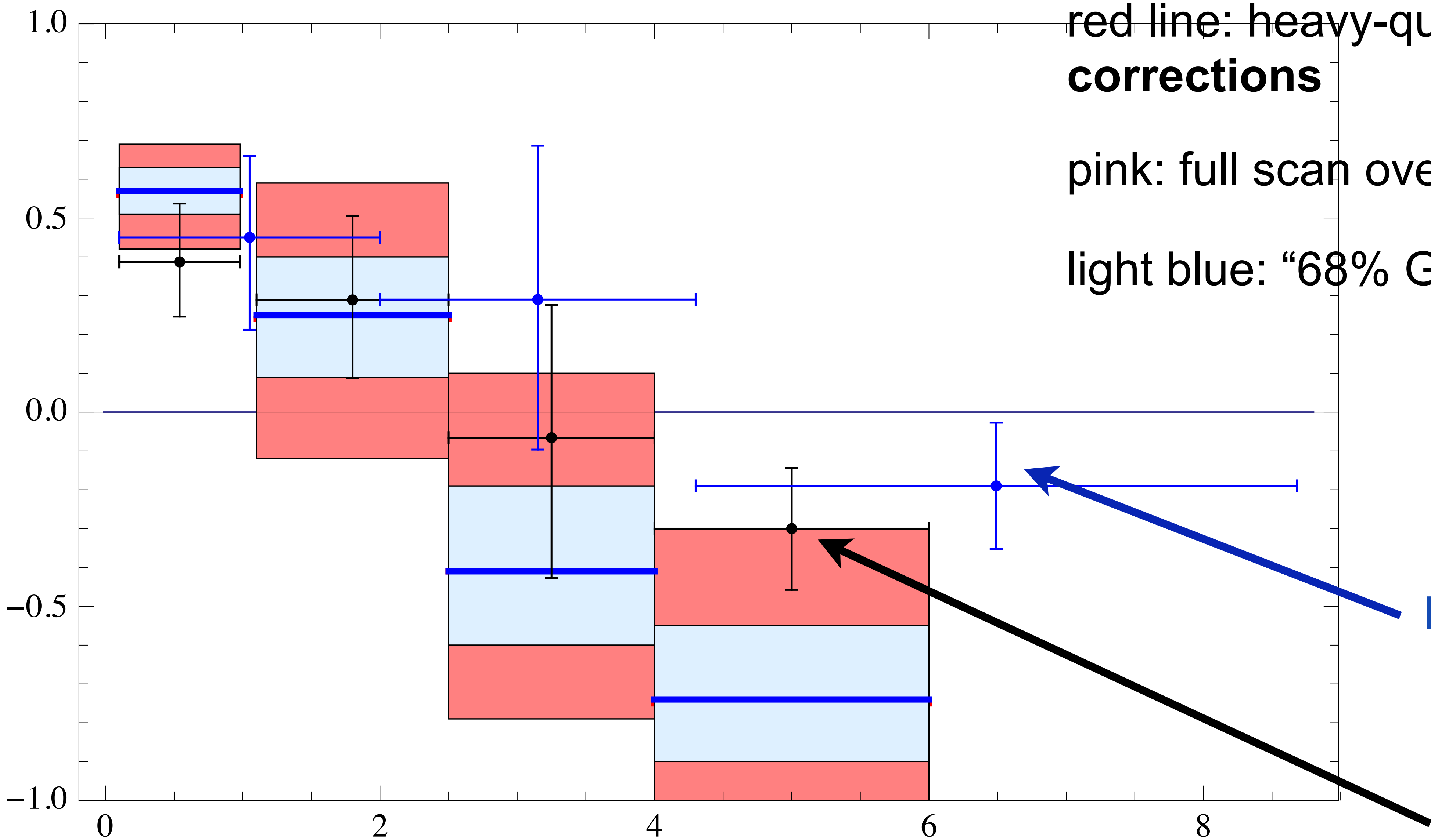
They appear only in form-factor-scheme-independent combinations.

Example: choose either $V_-$ as "soft" (reference) form factor, then $a_{V-}=0$,
or can choose $T_-$, then $a_{T-}=0$.
Because $V_-/T_-$ is fixed in QCD, the difference $(a_{V-} - a_{T-})$ agrees in both schemes, up to $O(\Lambda^2/m_b^2)$.

Numerical differences between different schemes are estimators of higher powers (beyond the truncated parameterisation).

# Angular observable $P_5'$

SJ, Martin Camalich, preliminary




red line: heavy-quark limit, **no power corrections**

pink: full scan over all theory errors

light blue: “68% Gaussian” theory error

LHCb 2013 (1 $fb^{-1}$)

LHCb Moriond 2015 (3 $fb^{-1}$)

(Ignore 6..8 GeV bin, above perturbative charm threshold and very close to resonances.)

For Gaussian errors [corresponding to what most authors employ], there is a noticeable deviation in a single bin; but also here less drastic than with LCSR-based theory

# Nonlocal term / charm loop

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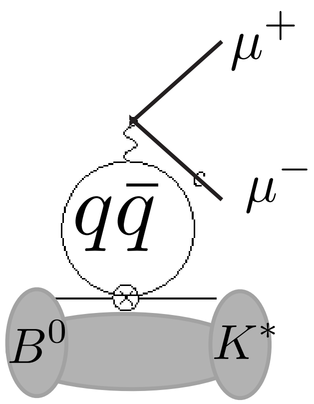
$$H_V(\lambda) \propto \tilde{V}_\lambda(q^2) C_9 - V_{-\lambda}(q^2) C_9' + \frac{2\, m_b m_B}{q^2} \left( \tilde{T}_\lambda(q^2) C_7 - \tilde{T}_{-\lambda}(q^2) C_7' \right) - \frac{16\, \pi^2 m_B^2}{q^2}\, h_\lambda(q^2)$$



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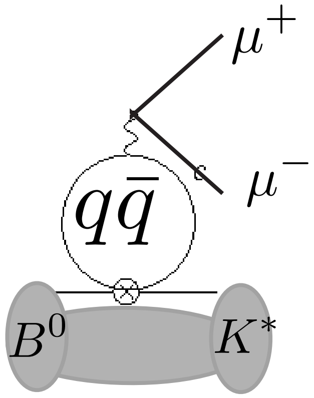


+ strong interactions!

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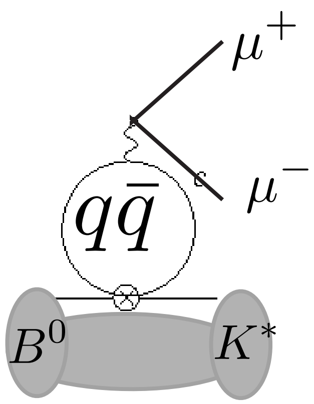


+ strong interactions!

more properly:

$$\frac{e^2}{q^2} L_V^\mu a_\mu^{\text{had}} = -i \frac{e^2}{q^2} \int d^4x e^{-iq\cdot x} \langle \ell^+ \ell^- | j_\mu^{\text{em,lept}}(x) | 0 \rangle \int d^4y\, e^{iq\cdot y} \langle M | j^{\text{em,had},\mu}(y) \mathcal{H}_{\text{eff}}^{\text{had}}(0) | \bar{B} \rangle$$

$$h_\lambda \equiv \frac{i}{m_B^2} \epsilon^{\mu *}(\lambda) a_\mu^{\text{had}}$$

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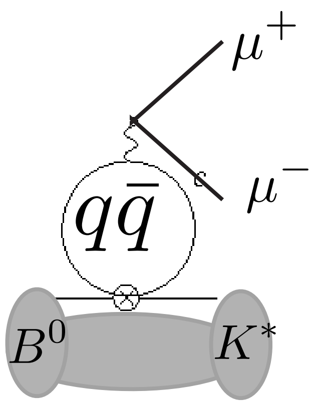


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nonlocal, nonperturbative, large normalisation ($V_{cb}^* V_{cs}$ $C_2$ )

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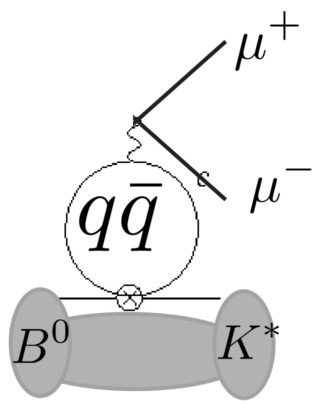


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nonlocal, nonperturbative, large normalisation ($V_{cb}^* V_{cs}$ $C_2$ )

traditional “ad hoc fix” : $C_9 \to C_9 + Y(q^2) = C_9^{\rm eff}(q^2)$ , $C_7 \to C_7^{\rm eff}$

“taking into account the charm loop”

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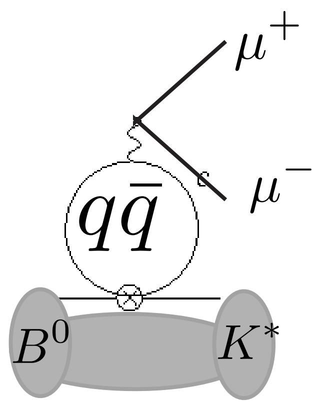


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nonlocal, nonperturbative, large normalisation ($V_{cb}^* V_{cs}$ $C_2$ )

traditional "ad hoc fix" : $C_9$ -> $C_9 + Y(q^2) = C_9^{\rm eff}(q^2)$ , "taking into account the charm loop"
$C_7$ -> $C_7^{\rm eff}$

* for $C_7^{\rm eff}$ this seems ok at lowest order (pure UV effect; scheme independence)
* for $C_9^{\rm eff}$ amounts to factorisation of scales ~ $m_b$ (, $m_c$ ,$q^2$) and Λ (soft QCD)
* not justified in large-N limit (broken already at leading logarithmic order)
* what about QCD corrections?
* not a priori clear whether this even gets one closer to the true result!

**only known justification** is a heavy-quark expansion in $\Lambda/m_b$ (just like inclusive decay is treated !)

Beneke, Feldmann, Seidel 2001, 2004

# Nonlocal term - another look

traditional “ad hoc fix” : $C_9 \to C_9 + Y(q^2) = C_9^{eff}(q^2)$ ,  $C_7 \to C_7^{eff}$

dominant effect: charm loop, proportional to ($z = 4\, m_c^2/q^2$)

$$-\frac{4}{9}\left(\ln\frac{m_q^2}{\mu^2} - \frac{2}{3} - z\right) - \frac{4}{9}(2+z)\sqrt{|z-1|}\begin{cases}\arctan\dfrac{1}{\sqrt{z-1}}, & z>1,\\ \ln\dfrac{1+\sqrt{1-z}}{\sqrt{z}} - \dfrac{i\pi}{2}, & z \leqslant 1\end{cases}$$

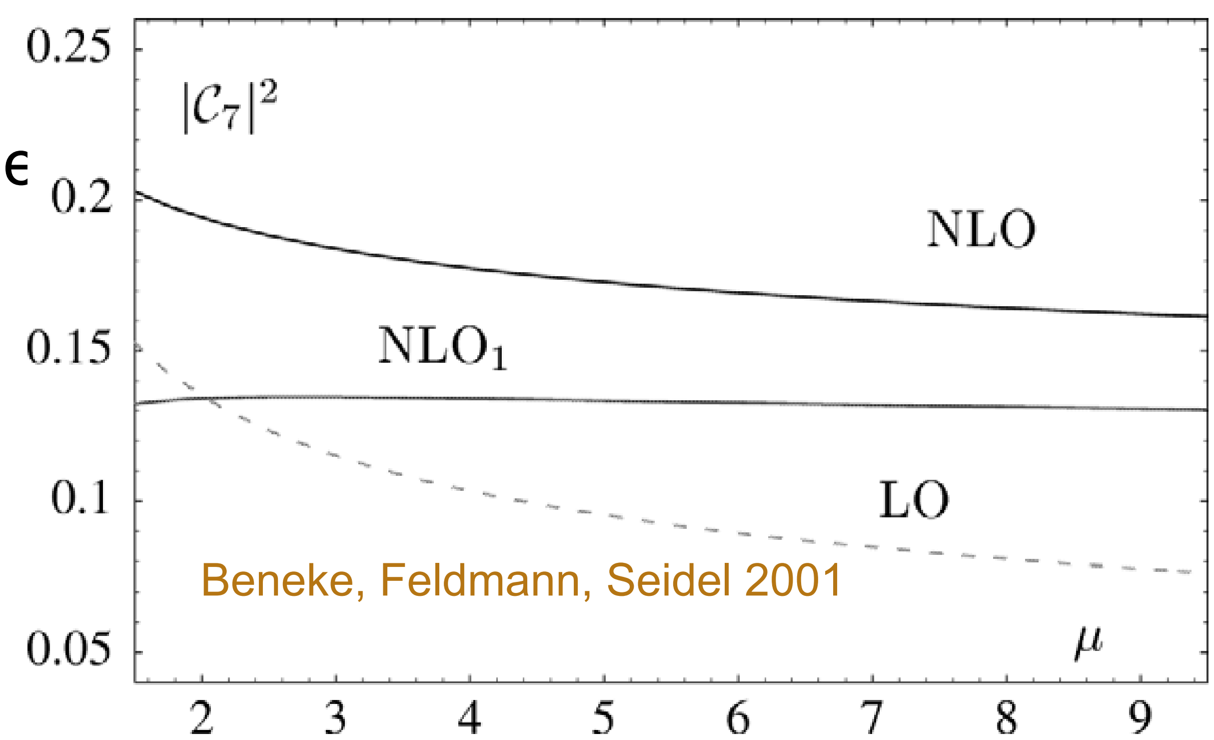
$$C_9^{\rm eff} = \begin{cases} 4.18|_{C_9} + (0.22+0.05i)|_Y & (m_c = m_c^{\rm pole} = 1.7{\rm GeV}) \\ 4.18|_{C_9} + (0.40+0.05i)|_Y & (m_c = m_c^{\overline{\rm MS}} = 1.2{\rm GeV}) \end{cases}$$

ie a 5% mass scheme ambiguity

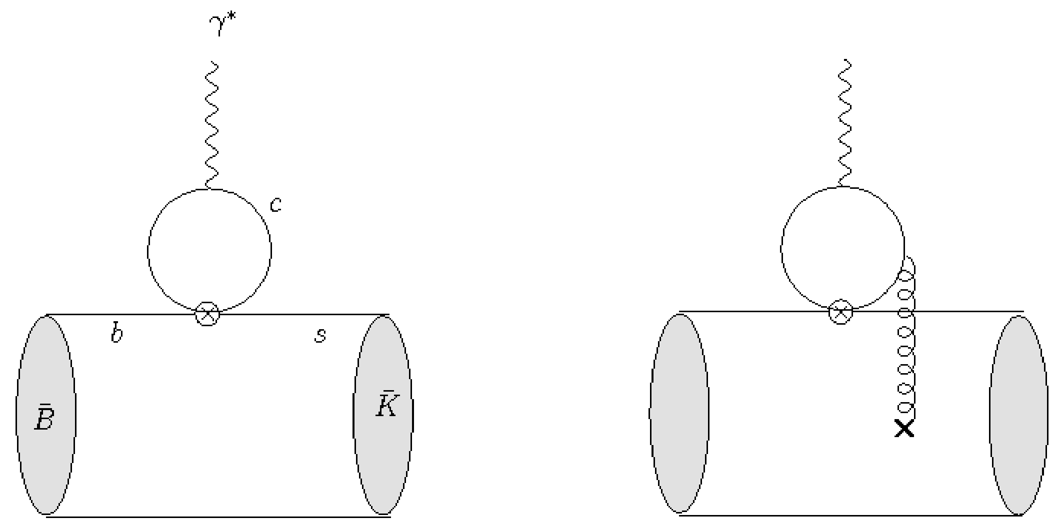
separately, one has a residual scale ambiguity of order 30% at the level of the decay amplitude

resolved in the heavy-quark expansion (to leading power)

Beneke, Feldmann, Seidel 2001, 2004



# Nonlocal terms:heavy-quark expansion




leading-power: factorises into perturbative kernels, form factors, LCDA's (including hard/hard-collinear gluon corrections to all orders)

$\alpha_s^0$ : $C_7 \rightarrow C_7^{eff}$

$C_9 \rightarrow C_9^{eff}(q^2)$

+ 1 annihilation diagram

$\alpha_s^1$ : further corrections to $C_7^{eff}(q^2)$ and $C_9^{eff}(q^2)$

(convergent) convolutions of hard-scattering kernels with meson light cone-distribution amplitudes

Beneke, Feldmann, Seidel 2001

state-of-the-art in phenomenology

unambigous (save for parametric uncertainties)

at subleading powers: breakdown of factorisation

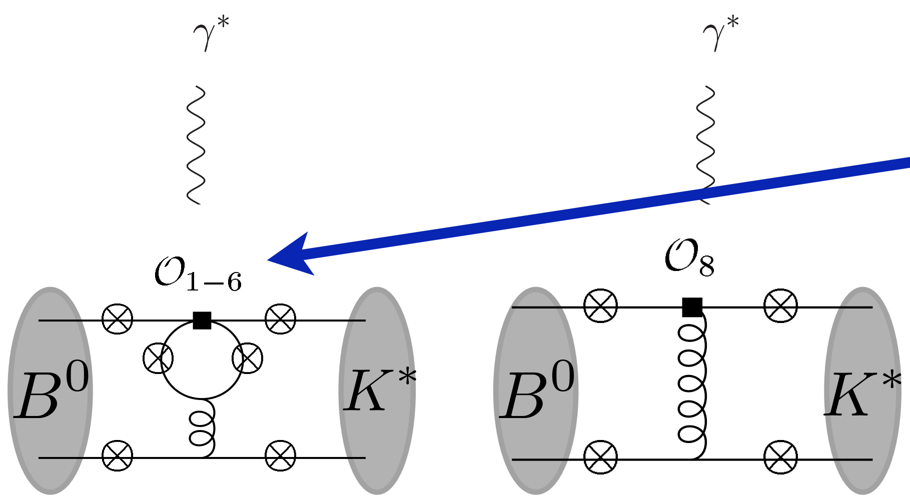
some contributions have been estimated as end-point divergent convolutions with a cut-off Kagan&Neubert 2001, Feldmann&Matias 2002

can perform light-cone OPE of charm loop & estimate resulting (nonlocal) operator matrix elements

Khodjamirian et al 2010

effective shifts of helicity amplitudes as large as ~10%

# New effect: spectator scattering




includes $Q_1^c$ , $Q_2^c$ - large Wilson coefficients

+ annihilation (+ "vertex corrections")

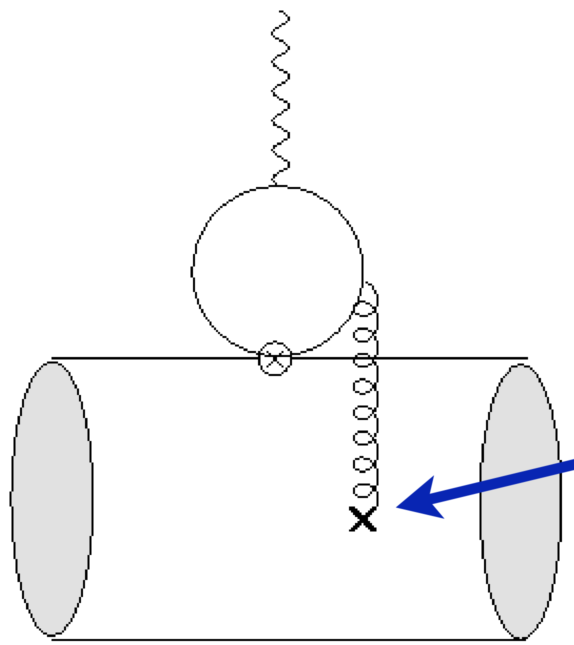
Beneke, Feldmann, Seidel 2001

leading-power: everything factorises into perturbative kernels, form factors, meson light-cone distribution amplitudes (including hard/hard-collinear gluon corrections to all orders)

$$h_\lambda = \int_0^1 du \phi_K^*(u) T(u, \alpha_s) + \mathcal{O}(\Lambda/m_b)$$

- leading power in the heavy quark limit - same as the vertex corrections going into $C_7^{\mathrm{eff}}$, $C_9^{\mathrm{eff}}$

# Long-distance charm loop



$$h_\lambda|_{c\bar{c}} = \frac{1}{m_B^2}\frac{2}{3}\epsilon^{\mu*}(\lambda)\int d^4y\, e^{iq\cdot y}\langle M|T[(\bar{c}\gamma^\mu c)(y)(C_1^c Q_1^c + C_2^c Q_2^c)(0)]|\bar{B}\rangle$$

consider soft gluon (in B rest frame)

From collinear factorisation viewpoint this represents the endpoint region, which is known to give a power-suppressed contribution

perform a “light-cone OPE”
(This is equivalent to expanding the charm loop, treating $\Lambda^2/(4\, m_c^2) \sim \Lambda/m_b$ ) Khodjamirian et al 2010

obtain

$$h_\lambda|_{c\bar{c},\mathrm{LD}} = \epsilon^{\mu*}(\lambda)\langle M(k,\lambda)|\tilde{\mathcal{O}}_\mu|\bar{B}\rangle$$

$$\tilde{\mathcal{O}}_\mu = \int d\omega I_{\mu\rho\alpha\beta}(q,\omega)\bar{s}_L\gamma^\rho\delta\Big(\omega - \frac{in_+\cdot D}{2}\Big)\tilde{G}^{\alpha\beta}b_L$$

(a nonlocal, light-cone operator)

need estimate of $\langle M(k,\lambda)|\tilde{\mathcal{O}}_\mu|\bar{B}\rangle$ (which goes into $H_V^\lambda$)

light-cone SR based on Khodjamirian et al 2010 for K* helicity amplitudes SJ, Martin Camalich 2012

**one outcome: two tests of right-handed dipol transitions remain clean**

for error estimate, introduce polynomial model in $q^2/(4m_c^2)$

# High-$q^2$ region (sketch)

- spectator scattering mechanism power-suppressed

- above open-charm (and perturbative-charm) thresholds

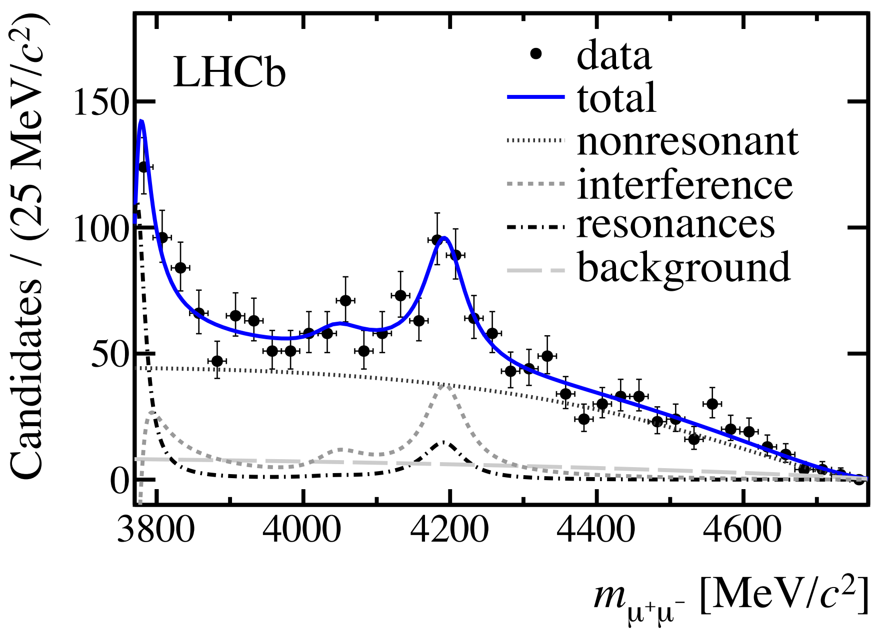
- however, for $q^2 >> 4m_c^2$, OPE at amplitude level

Grinstein, Pirjol 2004; Beylich, Buchalla, Feldmann 2011

Duality violation (≡ error beyond OPE)
- expected on general grounds for OPE above threshold

(Chibisov et al; Shifman 1990's)

- pronounced resonant structure observed




- difficult to quantify uncertainty due to this

Beylich, Buchalla, Feldmann 2011
(Chibisov et al; Shifman 1990's)
(Lyon, Zwicky 2013)

- like in low-$q^2$, probably best to stay away from the charm threshold region in looking for new physics

# Conclusions

* Experimental data paints an intriguing pattern of anomalies

* In my personal view: interesting enough to be taken seriously (also by model builders), but not conclusive yet

* Expect/require progress from LHCb via
   - more lepton universality tests
   - more data on angular distributions; precise measurement of $A_{FB}$ zero crossing, etc

   - $B_s$ -> mu mu ($B_s^*$ -> mu mu ?)  [also CMS]
   - do not forget about right-handed currents (electrons!)
         [-> see backup]

   - complementarity with Belle2 (electrons, inclusive decays)

* True (QCD) theory progress seems (very) hard, but at least we are accounting for all unknown contributions now.
Some recent conceptual advances in lattice regarding B->V form factors at physical point; prospects for phenomenology?

# BACKUP

# Optimised angular observables

=functions of the angular coefficients for which form factors drop out in the heavy quark limit if perturbative QCD corrections neglected.

Krueger,Matias 2005; Egede et al 2008
Becirevic, Schneider 2011
Matias, Mescia, Ramon, Virto 2012
Descotes-Genon et al 2012

E.g.

neglecting strong phase differences
[tiny; take into account in numerics]

$$P_1 \equiv \frac{I_3 + \bar{I}_3}{2(I_{2s} + \bar{I}_{2s})} = \frac{-2\,\mathrm{Re}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2} = 0$$

$$P_3^{CP} \equiv -\frac{I_9 - \bar{I}_9}{4(I_{2s} + \bar{I}_{2s})} = -\frac{\mathrm{Im}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2} = 0$$

(Melikhov 1998)
Krueger, Matias 2002
Lunghi, Matias 2006
Becirevic, Schneider 2011
Becirevic, Kou, et al 2012

$$P_5' = \frac{\mathrm{Re}[(H_V^- - H_V^+)H_A^{0*} + (H_A^- - H_A^+)H_V^{0*}]}{\sqrt{(|H_V^0|^2 + |H_A^0|^2)(|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2)}} = \frac{C_{10}\left(C_{9,\perp} + C_{9,\parallel}\right)}{\sqrt{(C_{9,\parallel}^2 + C_{10}^2)(C_{9,\perp}^2 + C_{10}^2)}}$$

**in SM**, neglecting power corrections and pert. QCD corrections

where

$$C_{9,\perp} = C_9^{\mathrm{eff}}(q^2) + \frac{2\,m_b\,m_B}{q^2}\,C_7^{\mathrm{eff}}$$

$$C_{9,\parallel} = C_9^{\mathrm{eff}}(q^2) + \frac{2\,m_b\,E}{q^2}\,C_7^{\mathrm{eff}}$$

$C_7$ and $C_9$ opposite sign
destructive interference enhances vulnerability to anything that violates the large-energy form factor relations (or more generally underestimated errors on form facors

much less of an issue in than to $P_1$ or $P_3^{CP}$ than eg in $P_5'$ (and others)

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$$P_5' = \frac{\mathrm{Re}[(H_V^- - H_V^+)H_A^{0*} + (H_A^- - H_A^+ \ldots}{\sqrt{(|H_V^0|^2 + |H_A^0|^2)(|H_V^+|^2 + |H_V^-|^2 + |H \ldots}}$$

Two approximate null tests of the SM

What are the leading corrections?

where

$$C_{9,\perp} = C_9^{\mathrm{eff}}(q^2) + \frac{2\, m_b\, m_B}{q^2} C_7^{\mathrm{eff}}$$

$$C_{9,||} = C_9^{\mathrm{eff}}(q^2) + \frac{2\, m_b\, E}{q^2} C_7^{\mathrm{eff}}$$

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# RH current probes

Extending to BSM Wilson coefficients, have

neglecting strong phase differences
[tiny; take into account in numerics]

close to $q^2 = 0$ (photon
pole dominance)

$$P_1 \equiv \frac{I_3 + \bar{I}_3}{2(I_{2s} + \bar{I}_{2s})} = \frac{-2\,\mathrm{Re}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2} \approx 2\frac{\mathrm{Re}(C_7 C_7'^*)}{|C_7|^2 + |C_7'|^2}$$

$$P_3^{CP} \equiv -\frac{I_9 - \bar{I}_9}{4(I_{2s} + \bar{I}_{2s})} = -\frac{\mathrm{Im}(H_V^+ H_V^{-*} + H_A^+ H_A^{-*})}{|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2} \approx \frac{\mathrm{Im}(C_7 C_7'^*)}{|C_7|^2 + |C_7'|^2}$$

Lunghi, Matias 2006
Becirevic, Schneider 2011
Becirevic, Kou, et al 2012
SJ, Martin Camalich 2012,2014

- **double** suppression $T_+(q^2) = \mathcal{O}(q^2/m_B^2) \times \mathcal{O}(\Lambda/m_b)$

- extra suppression of LD contribution to $H_V^+$ (model by effective helicity-dependent $C_7$ (or $C_9$) shift, within range established by power counting)

SJ, Martin Camalich 2012,2014

Helicity hierarchy survives power corrections
and is highly effective close to $q^2$=0

# Power corrections: analytical

SJ, Martin Camalich 1412.3183

Compare

$$P_5' = P_5'|_\infty \left(1 + \frac{a_{V_-} - a_{T_-}}{\xi_\perp} \frac{m_B}{|\vec{k}|} \frac{m_B^2}{q^2} C_7^{\rm eff} \frac{C_{9,\perp}C_{9,\parallel} - C_{10}^2}{(C_{9,\perp}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\parallel})} + \frac{a_{V_0} - a_{T_0}}{\xi_\parallel} 2\, C_7^{\rm eff} \frac{C_{9,\perp}C_{9,\parallel} - C_{10}^2}{(C_{9,\parallel}^2 + C_{10}^2)(C_{9,\perp} + C_{9,\parallel})} + 8\pi^2 \frac{\tilde{h}_-}{\xi_\perp} \frac{m_B}{|\vec{k}|} \frac{m_B^2}{q^2} \frac{C_{9,\perp}C_{9,\parallel} - C_{10}^2}{C_{9,\perp} + C_{9,\parallel}} + \text{further terms}\right) + \mathcal{O}(\Lambda^2/m_B^2)$$

(truncated after 3 out of 11 independent power-correction terms!)
also, dependence on soft form factors reappears at PC level

and

$$P_1 = \frac{1}{C_{9,\perp}^2 + C_{10}^2} \frac{m_B}{|\vec{k}|} \left( -\frac{a_{T_+}}{\xi_\perp} \frac{2\, m_B^2}{q^2} C_7^{\rm eff} C_{9,\perp} - \frac{a_{V_+}}{\xi_\perp} (C_{9,\perp} C_9^{\rm eff} + C_{10}^2) - \frac{b_{T_+}}{\xi_\perp} 2 C_7^{\rm eff} C_{9,\perp} - \frac{b_{V_+}}{\xi_\perp} \frac{q^2}{m_B^2} (C_{9,\perp} C_9^{\rm eff} + C_{10}^2) + 16\pi^2 \frac{h_+}{\xi_\perp} \frac{m_B^2}{q^2} C_{9,\perp} \right) + \mathcal{O}(\Lambda^2/m_B^2).$$

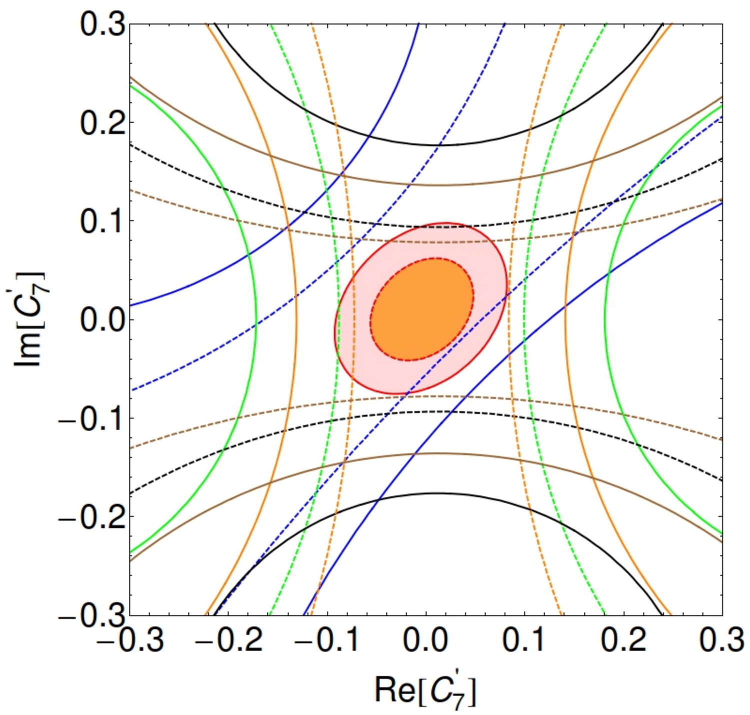
(complete expression)

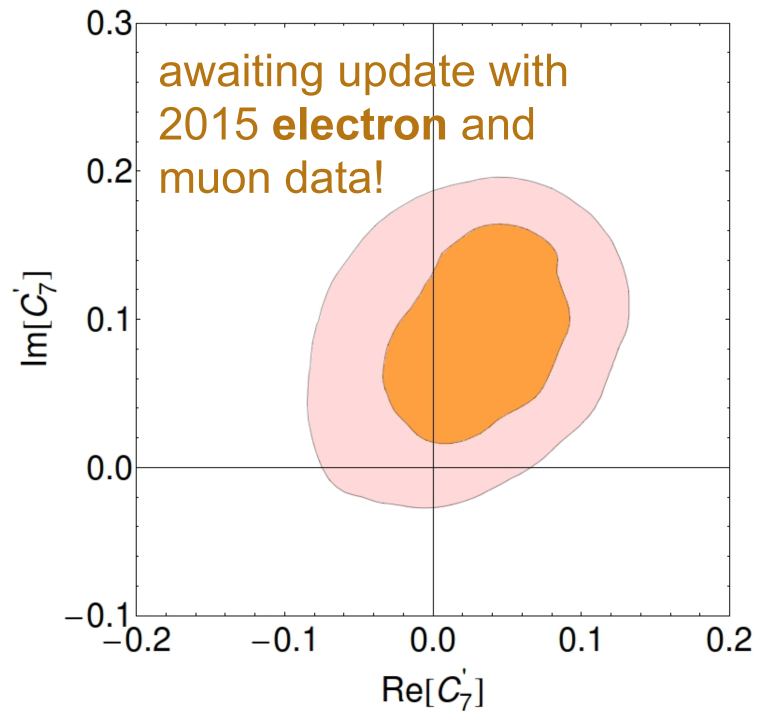
Further notice that $a_{T+}$ vanishes as $q^2$->0, $h_+$ helicity suppressed [will show], and the other three terms lacks the photon pole.

Hence $P_1$ **much** cleaner than $P_5$', especially at very low $q^2$

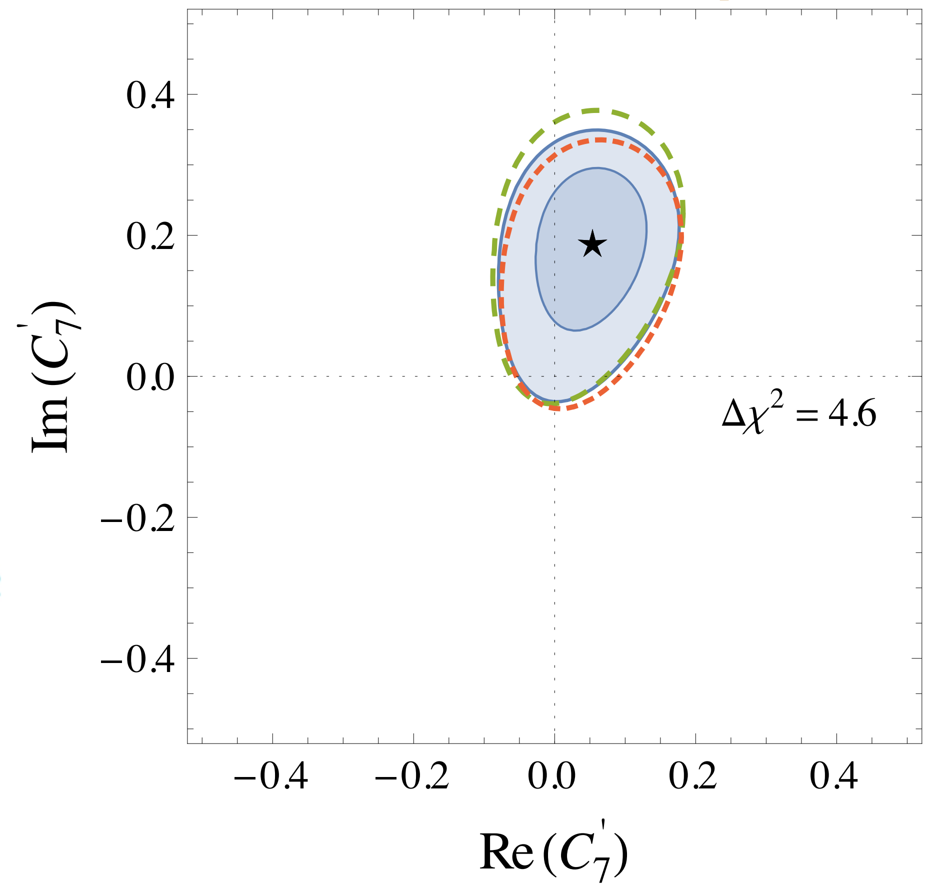
# Status/prospects

SJ, Martin Camalich
1412.2183




Altmannshofer, Straub
1411.3163v3 [update including Moriond 2015 muon data]




$$S \simeq \frac{2\mathrm{Im}(e^{-2i\beta} C_7 C_7')}{|C_7|^2 + |C_7'|^2},$$

$$P_1 \simeq \frac{2\mathrm{Re}(C_7 C_7')}{|C_7|^2 + |C_7'|^2},$$

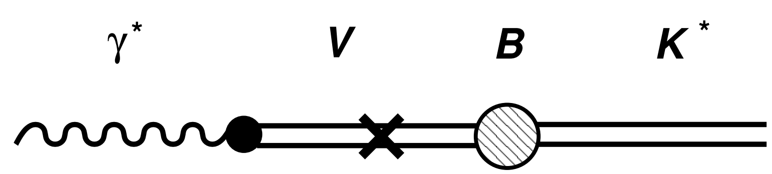
$$P_3^{\mathrm{CP}} \simeq \frac{2\mathrm{Im}(C_7 C_7')}{|C_7|^2 + |C_7'|^2}$$

- Left: assuming $\sigma_{P_i} = 0.25$ for muons and electrons, no theory errors
- Middle: Profile likelihood for 2014 data (1sigma and 95% CL)
- Right: post-Moriond fit including new muon data
- excellent sensitivity to right-handed currents remains with conservative treatment of QCD uncertainties

# Light-quark contributions

Operators without charm have strong charm or CKM suppression; power corrections should be negligible.

However, they generate (mild) resonance structure even below the charm threshold, presumably "duality violation"
Presumably ρ,ω,φ most important; use vector meson dominance supplemented by heavy-quark limit B➔VK* amplitudes




$$\tilde{a}_\mu^{\mathrm{had,\,lq}} = \int d^4x\, e^{-iq\cdot x} \sum_{P,P'} \langle 0|j_\mu^{\mathrm{em,lq}}(x)|P'\rangle\langle P'(x)|P(0)\rangle\langle \bar{K}^* P|\,\mathcal{H}_{\mathrm{eff}}^{\mathrm{had}}(0)|\bar{B}\rangle$$

estimate **uncertainty** from difference between VMD model and the subset of heavy-quark limit diagrams corresponding to intermediate V states.

Helicity hierarchies in **hadronic** B decays prevent large uncertainties in $H_V^+$ from this source, too.