

Gravitational wave background from Standard Model physics

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FOR FUNDAMENTAL PHYSICS

In collaboration with Mikko Laine

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GWs from equilibrium sources

- GWs can be produced from eq. too. Well known Weinberg
- In thermal eq. particle scatter \Rightarrow GW production
- Naive power counting: for momentum $k > T$

$$\Gamma \sim \alpha \frac{T^3}{m_{\text{Pl}}^2} e^{-k/T}$$

with some **internal plasma coupling**, the **gravitational coupling** and a **Boltzmann suppression**

- Since $k \sim 3T$ Γ must be small. Is that always true?

In this talk

- Go beyond this usual assumption
- Use a modern, consistent Thermal Field Theory framework
- Try to give reliable estimates for $T > 160 \text{ GeV}$
- In particular, concentrate on the IR: for $k \ll T$ collective phenomena enter the rate and change the previous estimate

Outline

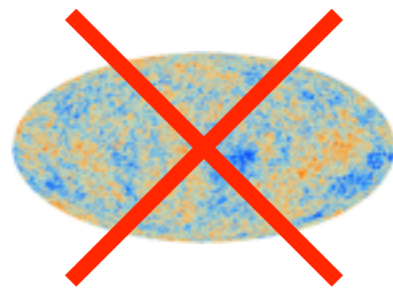
- ✓ Introduction and motivation
- Overview and formalism
- The IR rate and the viscosity of plasmas
- The $k \sim T$ rate
- Embedding the rates in cosmology: limits and prospects for detection

Overview



Equilibrium production

- GWs produced from an equilibrium plasma, but *not in equilibrium with it*.
- Similar to photon production from the QCD plasma in heavy-ion collisions. Common aspect: reinteraction (rescatterings / absorptions) and backreaction (cooling) negligible. In the photon case because $\alpha_{\text{EM}} \ll \alpha_s$



- There $\langle n_{\text{em}} \rangle = 0$ and $\langle \mathbf{J}_{\text{em}} \rangle = 0$, but thermal fluctuations \Rightarrow charge and current fluctuations \Rightarrow photons

$$\frac{d\Gamma_{\gamma}(\mathbf{k})}{d^3\mathbf{k}} = \frac{1}{(2\pi)^3 2k} \sum_{\lambda} \epsilon_{\mu,\mathbf{k}}^{(\lambda)} \epsilon_{\nu,\mathbf{k}}^{(\lambda)*} \int_{\mathcal{X}} e^{i\mathcal{K}\cdot\mathcal{X}} \langle J_{\text{em}}^{\mu}(0) J_{\text{em}}^{\nu}(\mathcal{X}) \rangle$$

Photon production

$$\frac{d\Gamma_\gamma(\mathbf{k})}{d^3\mathbf{k}} = \frac{1}{(2\pi)^3 2k} \sum_\lambda \epsilon_{\mu,\mathbf{k}}^{(\lambda)} \epsilon_{\nu,\mathbf{k}}^{(\lambda)*} \int_{\mathcal{X}} e^{i\mathbf{k}\cdot\mathcal{X}} \langle J_{\text{em}}^\mu(0) J_{\text{em}}^\nu(\mathcal{X}) \rangle$$

- For $k \gtrsim T$ things go as you'd expect (but a very tricky calculation to get the actual numbers)

$$\frac{d\Gamma_\gamma(\mathbf{k})}{d^3\mathbf{k}} \stackrel{k \gtrsim T}{\sim} \frac{\alpha_{\text{EM}} \alpha_s \ln(1/\alpha_s) T^2 e^{-k/T}}{(2\pi)^3 k}$$

LO Arnold Moore Yaffe (AMY) [JHEP0111](#) [JHEP0112](#) (2001)

NLO JG Hong Lu Kurkela Moore Teaney [JHEP1305](#) (2013)

- For $k \ll T$ the amplitude of J fluctuations is related to the **conductivity** of the plasma, a collective phenomenon

$$\frac{d\Gamma_\gamma(\mathbf{k})}{d^3\mathbf{k}} \stackrel{k \lesssim \alpha_s^2 T}{\sim} \frac{2T \sigma}{(2\pi)^3 k} \sim \frac{\alpha_{\text{EM}} T^2}{(2\pi)^3 k \alpha_s^2 \ln(1/\alpha_s)}$$

AMY [JHEP0011](#) (2000), [JHEP0305](#) (2003)

Graviton production

- Start from textbooks: TT gauge, Minkowski background (cosmological expansion later)

$$\ddot{h}_{ij}^{\text{TT}} - \nabla^2 h_{ij}^{\text{TT}} = 16\pi G T_{ij}^{\text{TT}} \quad E_{\text{GW}} = \frac{1}{32\pi G} \int_{\mathbf{x} \in V} [\dot{h}_{ij}^{\text{TT}}(t, \mathbf{x})]^2$$

- h^{TT} superposition of forward and backward propagating GWs. By taking average over oscillations rewrite E as

$$\langle\langle E_{\text{GW}} \rangle\rangle = \frac{1}{64\pi G} \int_{\mathbf{x} \in V} \left\{ [\dot{h}_{ij}^{\text{TT}}(t, \mathbf{x})]^2 + |\nabla h_{ij}^{\text{TT}}(t, \mathbf{x})|^2 \right\}$$

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- h^{TT} superposition of forward and backward propagating GWs. By taking average over oscillations rewrite E as

$$H \equiv \langle\langle E_{\text{GW}} \rangle\rangle = \frac{1}{64\pi G} \int_{\mathbf{x} \in V} \left\{ [\dot{h}_{ij}^{\text{TT}}(t, \mathbf{x})]^2 + |\nabla h_{ij}^{\text{TT}}(t, \mathbf{x})|^2 \right\}$$

- **Canonical form!** (Up to trivial normalization) Then do the same as for photons (but taking ρ_{GW} rather than Γ_{γ})

$$\frac{d\rho_{\text{GW}}}{dt d^3\mathbf{k}} = \frac{4\pi G}{(2\pi)^3} \sum_{\lambda} \epsilon_{ij,\mathbf{k}}^{\text{TT}(\lambda)} \epsilon_{mn,\mathbf{k}}^{\text{TT}(\lambda)*} \int_{\mathcal{X}} e^{i\mathbf{k}\cdot\mathcal{X}} \langle T^{ij}(0) T^{mn}(\mathcal{X}) \rangle$$

Graviton production

$$\frac{d\rho_{\text{GW}}}{dt d^3\mathbf{k}} = \frac{4\pi G}{(2\pi)^3} \sum_{\lambda} \epsilon_{ij,\mathbf{k}}^{\text{TT}(\lambda)} \epsilon_{mn,\mathbf{k}}^{\text{TT}(\lambda)*} \int_{\mathcal{X}} e^{i\mathbf{k}\cdot\mathcal{X}} \langle T^{ij}(0) T^{mn}(\mathcal{X}) \rangle$$



Take a sum over the polarizations with $k_{\parallel}z$

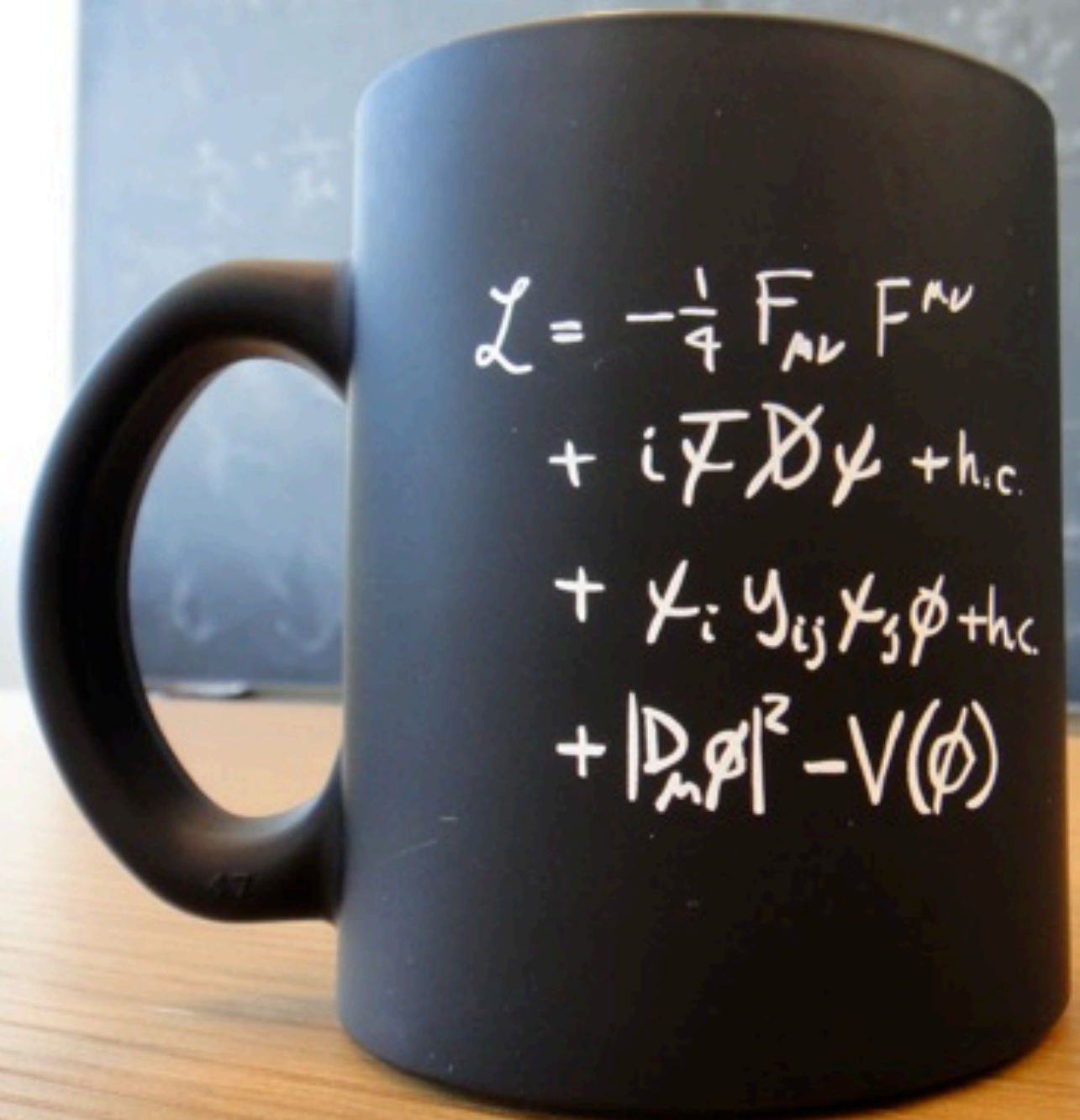


Rotate T^{11} - T^{22} into T^{12}

$$\frac{d\rho_{\text{GW}}}{dt d \ln k} = \frac{8k^3}{\pi m_{\text{Pl}}^2} \int_{\mathcal{X}} e^{ik(t-z)} \langle T_{12}(0) T_{12}(\mathcal{X}) \rangle$$

- The same results can be obtained with a purely classical derivation

GW rate for $k \ll T$ at $T > 160 \text{ GeV}$



The image shows a black mug with white text on a wooden desk. The text on the mug is a Lagrangian equation. In the background, a chalkboard is visible with some faint, illegible writing.

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + i\bar{\psi} \not{D} \psi + \text{h.c.} \\ & + \chi_i Y_{ij} \chi_j \phi + \text{h.c.} \\ & + |D_\mu \phi|^2 - V(\phi)\end{aligned}$$

Hydrodynamic limit

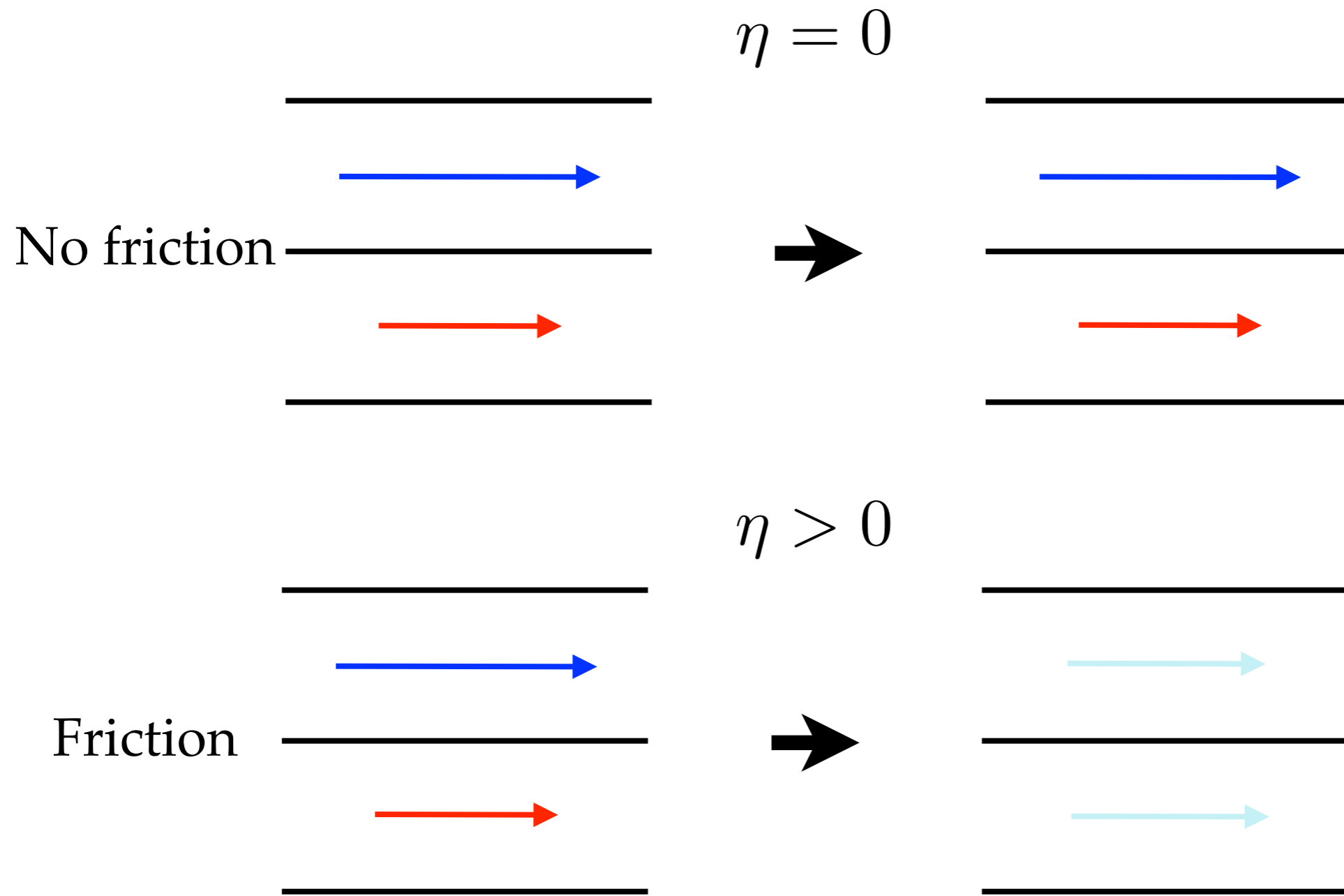
- Field theories admit a long-wavelength hydrodynamical limit. Hydrodynamics: Effective Theory based on a gradient expansion of the flow velocity
- For hydro fluctuations with local flow velocity \mathbf{v} around an eq. state (with temp. T), at first order in the gradients and in \mathbf{v}

$$T^{00} = e, \quad T^{0i} = (e + p)v^i$$

$$T^{ij} = (p - \zeta \nabla \cdot \mathbf{v})\delta^{ij} - \eta \left(\partial_i v^j + \partial_j v^i - \frac{2}{3} \delta^{ij} \nabla \cdot \mathbf{v} \right)$$

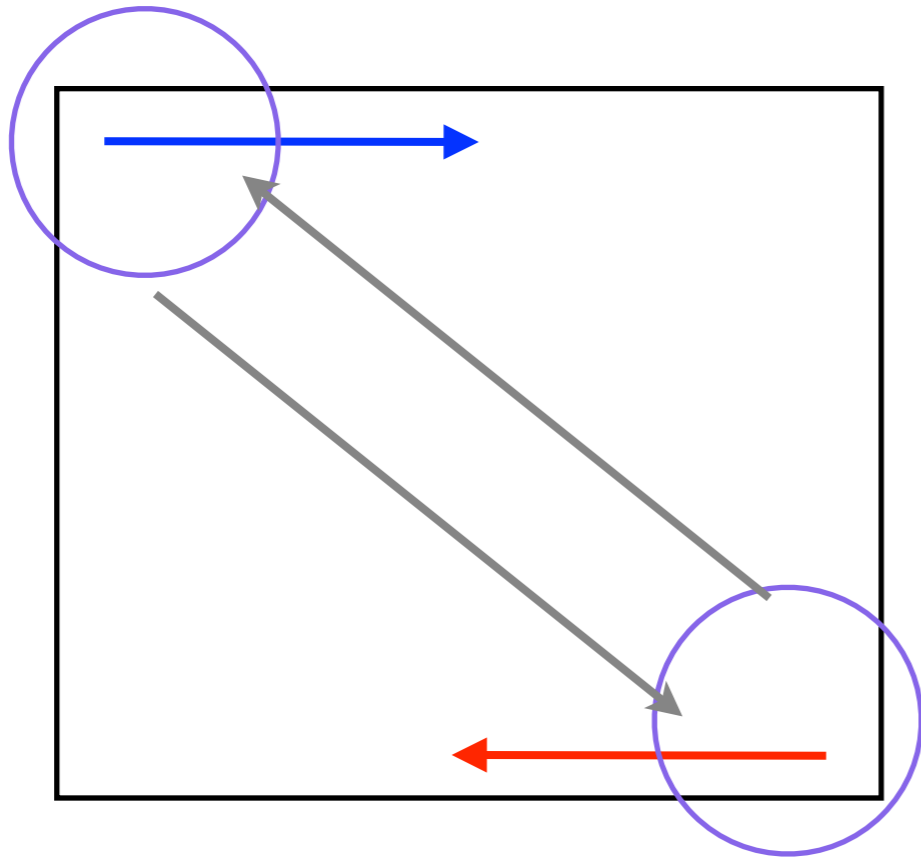
Navier-Stokes hydro, two *transport coefficients*: **bulk** and **shear viscosity**

The shear viscosity

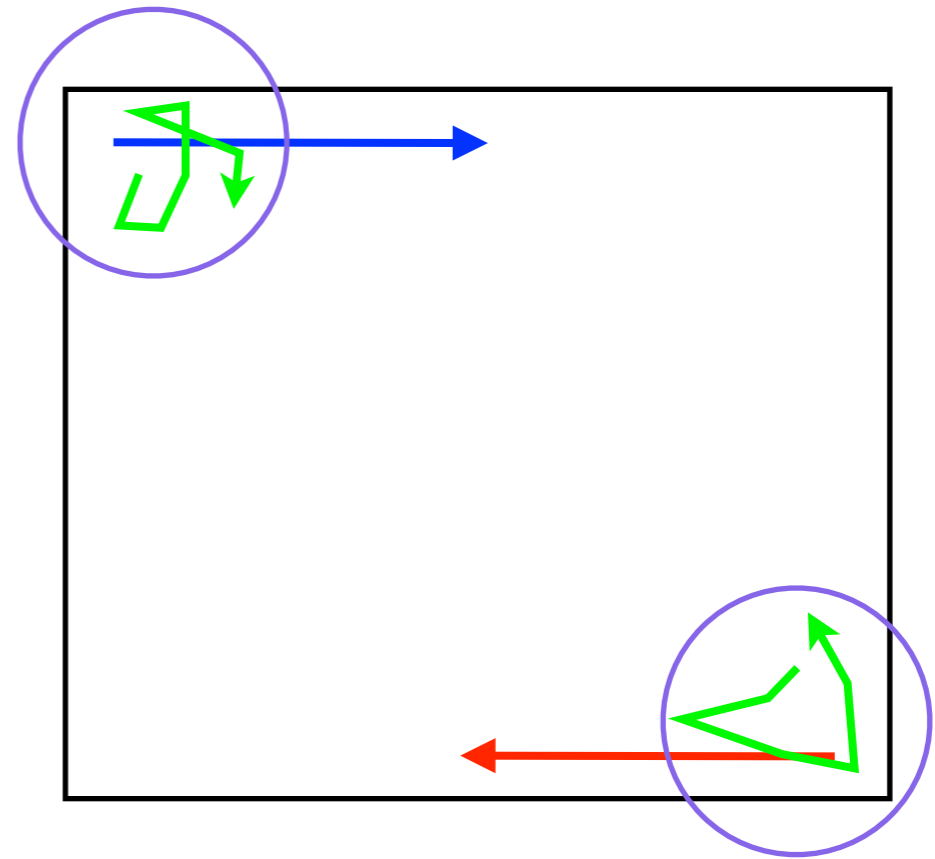


- Finite shear viscosity smears out flow differences (diffusion)

Estimating η : counterintuitive?



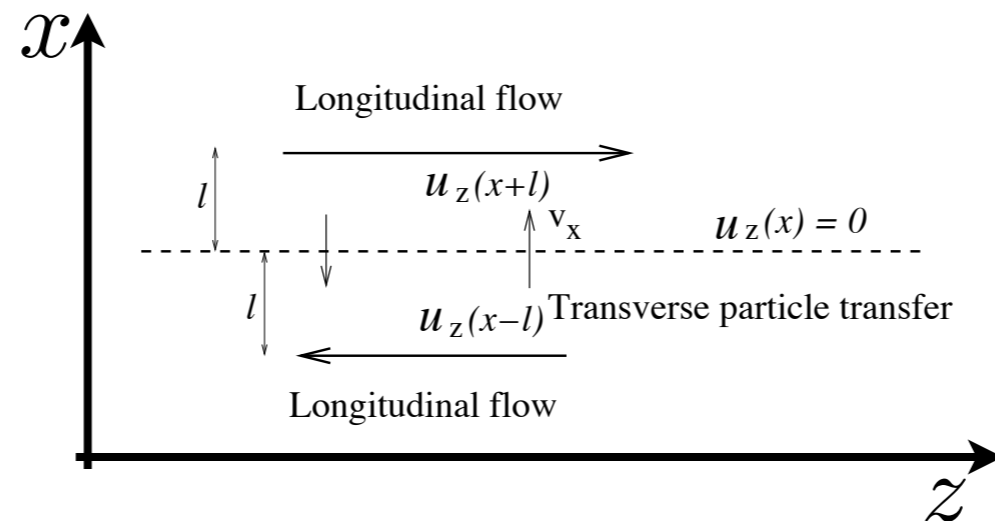
- Weak coupling: long distances between collisions, easy diffusion. **Large η**



- Strong coupling: short distances between collisions, little diffusion. **Small η**

Estimating η

- u flow velocity, v_x microscopical velocity of particles



- $T^{0z} = (e+p)u^0 u^z$ diffuses along x with $v^x = u^x/u^0$. Net change $(e+p)v^x u^0 (u^z(x - l_{\text{mfp}}) - u^z(x + l_{\text{mfp}})) \approx -2(e+p)v^x u^0 l_{\text{mfp}} \partial_x u^z(x) \sim -\eta u^0 \partial_x u^z(x)$

- Using $e+p = sT$ and in the high- T limit ($v^x \sim 1$)

$$\frac{\eta}{s} \sim T l_{\text{mfp}}$$

Estimating η

- (Mean free path)⁻¹ \sim cross section \times density

$$\frac{\eta}{s} \sim T l_{\text{mfp}} \sim \frac{T}{n\sigma} \sim \frac{1}{T^2 \sigma}$$

- Cross section in a perturbative gauge theory (T only scale*)

$$\sigma \sim \frac{g^4}{T^2} \quad \frac{\eta}{s} \sim \frac{1}{g^4}$$

- * Coulomb divergences and screening scales ($m_D \sim gT$) in gauge theories

$$\sigma \sim \frac{g^4}{T^2} \ln(1/g) \quad \frac{\eta}{s} \sim \frac{1}{g^4 \ln(1/g)}$$

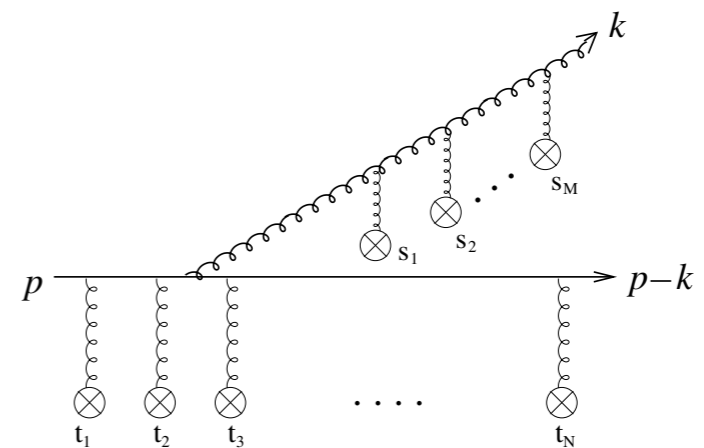
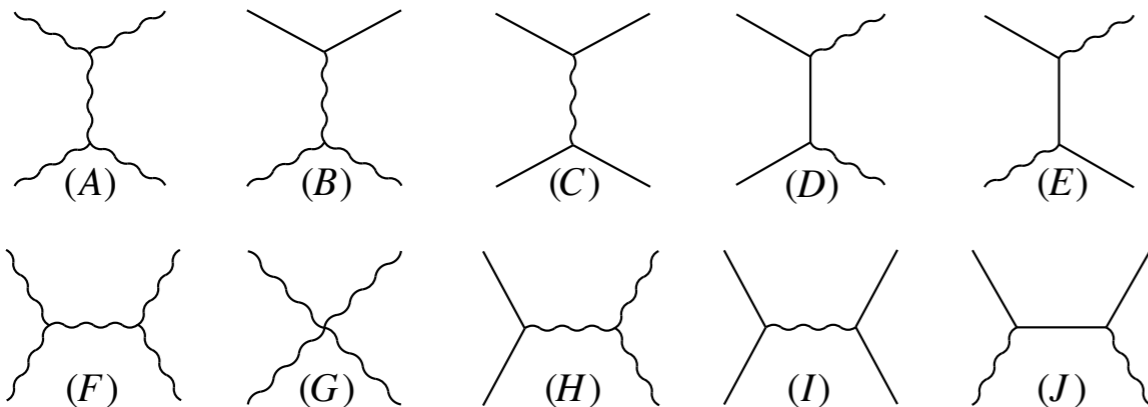
Computing η

- Kubo formula (S TT part of T)

$$\eta = \frac{1}{20} \lim_{\omega \rightarrow 0} \frac{1}{\omega} \int d^4x e^{i\omega t} \langle [S^{ij}(t, \mathbf{x}), S^{ij}(0, \mathbf{0})] \rangle \theta(t)$$

- Not practical at weak coupling: use effective kinetic theory with $2 \leftrightarrow 2$ and $1 \leftrightarrow 2$ processes [AMY \(2000-2003\)](#)

$$(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}}) f(t, \mathbf{x}, \mathbf{p}) = C^{2 \leftrightarrow 2}[f] + C^{1 \leftrightarrow 2}[f]$$

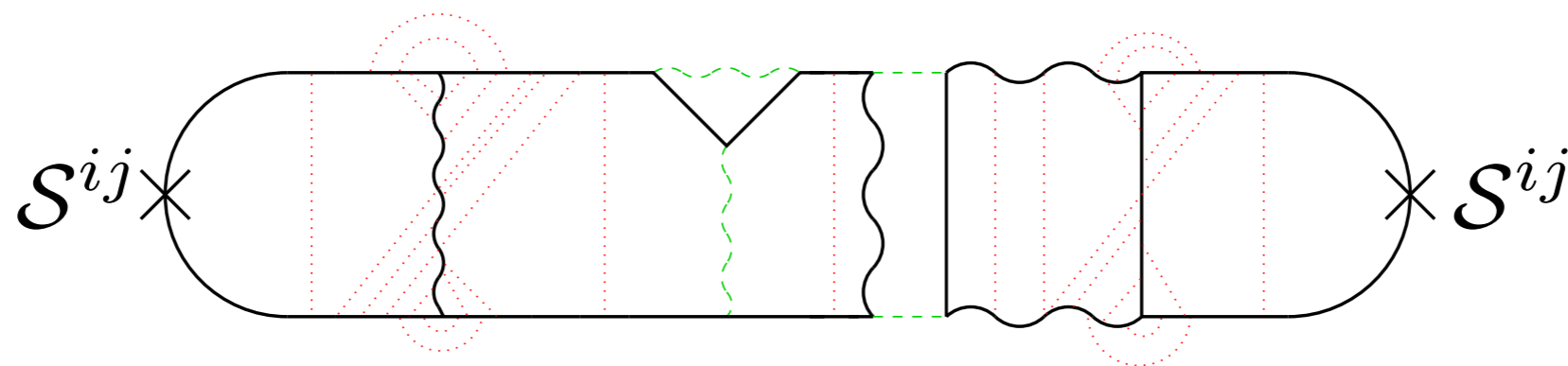


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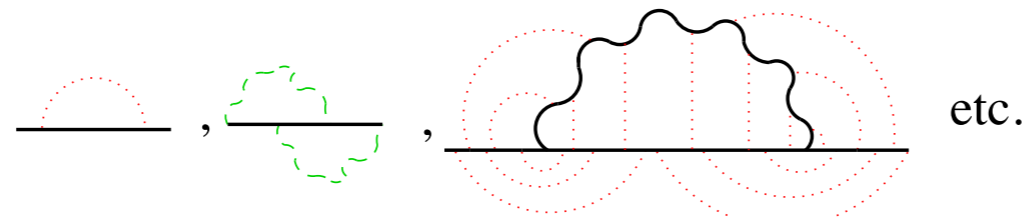
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----- Hard off-shell

..... Soft, spacelike, gauge boson, HTL resummed

———— Hard on-shell, resummed with diagrams of form



Computing η

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$$\eta = \frac{1}{20} \lim_{\omega \rightarrow 0} \frac{1}{\omega} \int d^4x e^{i\omega t} \langle [S^{ij}(t, \mathbf{x}), S^{ij}(0, \mathbf{0})] \rangle \theta(t)$$

- Not practical at weak coupling: use effective kinetic theory with $2 \leftrightarrow 2$ and $1 \leftrightarrow 2$ processes [AMY \(2000-2003\)](#)
- For the SM at $T > 160$ GeV η is dominated by the slowest processes, those involving right-handed leptons only

$$\eta \simeq \frac{16T^3}{g_1^4 \ln(5T/m_{D1})} \quad \rightarrow \quad \eta \simeq 400 T^3$$

g_1 hypercharge coupling with screening mass $m_{D1} = \sqrt{11/6} g_1 T$

Only a leading-log estimate, no complete LO for $T > 160$ GeV
[AMY \(2000-2003\)](#)



Hydrodynamic limit

- 4-momentum conservation for a perturbation along z :
decoupling of v^1 and v^2

$$\partial_0 T^{0j} + \partial_i T^{ij} = 0 \quad \Rightarrow \quad \mathbf{v}_\perp(t, \mathbf{k}) = \mathbf{v}_\perp(0, \mathbf{k}) e^{-\eta k^2 t / (e+p)}$$

- Now look at the T^{0i} correlator (operator ordering irrelevant in the soft limit), $i', j' = \{1, 2\}$

$$\begin{aligned} & \frac{1}{V} \int_{-\infty}^{\infty} dt e^{i\omega t} \left\langle \frac{1}{2} \{ T^{0i'}(t, \mathbf{k}), T^{0j'}(0, -\mathbf{k}) \} \right\rangle \\ &= \frac{\frac{2\eta k^2}{e+p}}{\omega^2 + \frac{\eta^2 k^4}{(e+p)^2}} \int_{\mathbf{x} \in V} e^{-i\mathbf{k} \cdot \mathbf{x}} \left\langle T^{0i'}(0, \mathbf{x}) T^{0j'}(0, \mathbf{0}) \right\rangle \end{aligned}$$



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- For small k this becomes the **momentum susceptibility**

$$\chi_{\mathbf{p}} = e + p$$



Hydrodynamic limit

- Use a Ward identity to go from $T^{0i'}$ to $T^{3i'}$

$$\int_{\mathcal{X}} e^{i(\omega t - kz)} \left\langle \frac{1}{2} \{T^{3i'}(\mathcal{X}), T^{3j'}(0)\} \right\rangle \stackrel{\omega, k \lesssim \alpha^2 T}{=} \frac{2\eta T \omega^2 \delta^{i'j'}}{\omega^2 + \frac{\eta^2 k^4}{(e+p)^2}}$$

- What we want (T^{12}) is different but related. Since v^1 and v^2 are uncoupled from the EOMS, their fluctuations are uncorrelated in spacetime $\Rightarrow \omega, k$ independent

$$\left\langle \frac{1}{2} \{T_{i'j'}^{\text{TT}}(t_1, \mathbf{x}_1), T_{k'l'}^{\text{TT}}(t_2, \mathbf{x}_2)\} \right\rangle = \Phi_{i'j'k'l'} \delta(t_1 - t_2) \delta^{(3)}(\mathbf{x}_1 - \mathbf{x}_2)$$

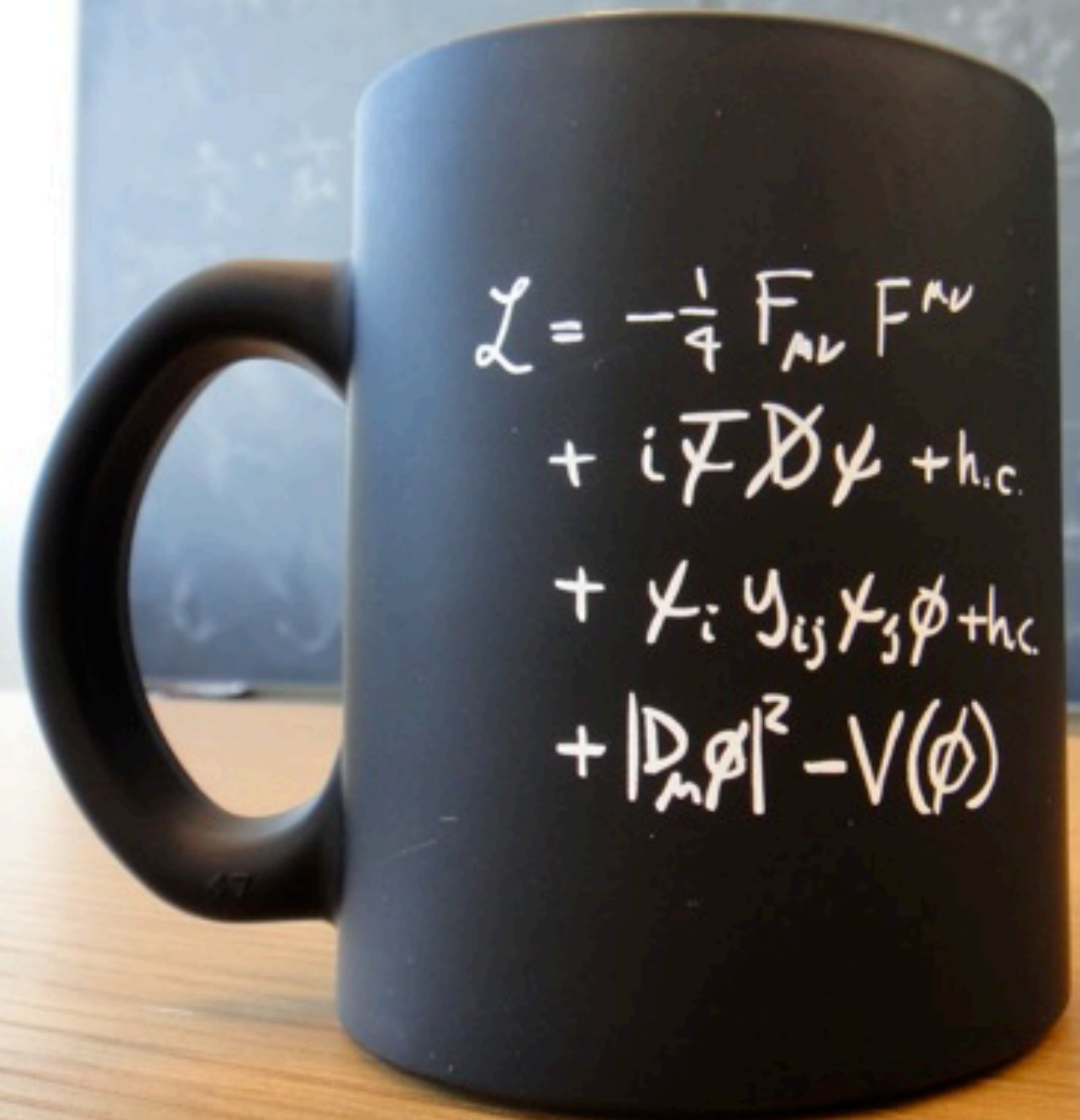
- Setting $\omega=k$ and sending $k \rightarrow 0$ restores 3D symmetry, so that by comparing

$$\lim_{k \rightarrow 0} \int_{\mathcal{X}} e^{ik(t-z)} \left\langle \frac{1}{2} \{T_{12}(\mathcal{X}), T_{12}(0)\} \right\rangle = 2\eta T$$

$$\frac{d\rho_{\text{GW}}}{dt d \ln k} \stackrel{k \lesssim \alpha^2 T}{=} \frac{16k^3 \eta T}{\pi m_{\text{Pl}}^2}$$

Obtainable formally by linear response [Hong Teaney \(2010\)](#)

GW rate for $k \sim T$ at $T > 160 \text{ GeV}$

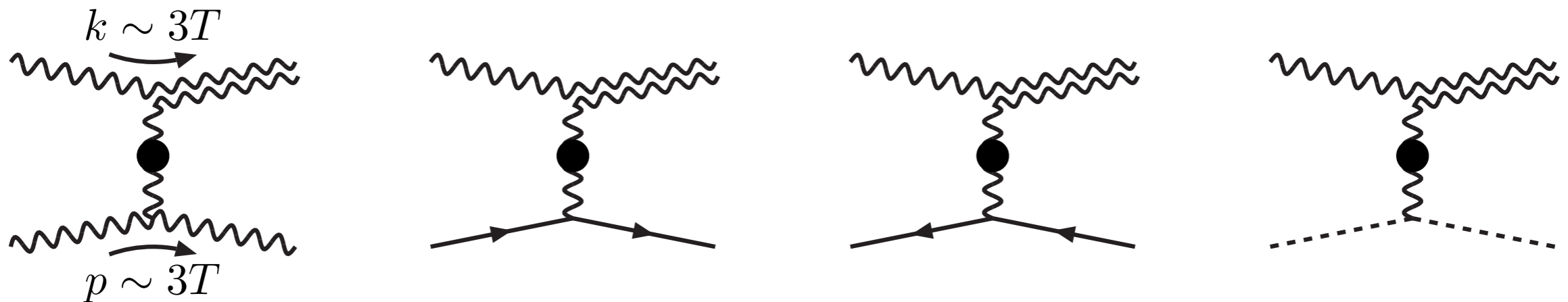


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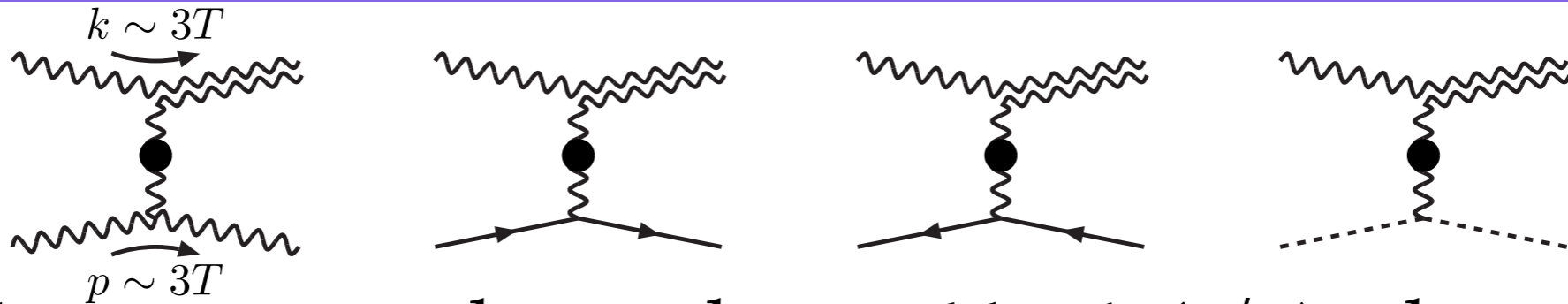
A leading-log estimate

- A complete leading-order calculation for $k \sim T$ is not easy: requires all $2 \leftrightarrow 2$ scatterings between SM particles
- However, scatterings with intermediate gauge bosons are IR sensitive: logarithmic divergence with bare propagators



- The cure is **Hard Thermal Loop resummation** (collective physics: screening, plasma oscillations and Landau damping)

A leading-log estimate



- These diagrams are then enhanced by $\ln(1/g)$. The coefficient of this term is easy to determine: a *leading-log calculation*

$$\frac{d\rho_{\text{GW}}}{dt d \ln k} = \frac{2k^4 T n_B(k)}{\pi^2 m_{\text{Pl}}^2} \left\{ \sum_{i=1}^3 d_i m_{\text{D}i}^2 \ln \frac{5T}{m_{\text{D}i}} + \mathcal{O}\left(g^2 T^2 \chi\left(\frac{k}{T}\right)\right) \right\}$$

- d_i multiplicities of the gauge groups ($d_1=1, d_2=3, d_3=8$), $m_{\text{D}i}$ Debye masses $m_{\text{D}1}^2 = 11g_1^2 T^2/6, m_{\text{D}2}^2 = 11g_2^2 T^2/6, m_{\text{D}3}^2 = 2g_3^2 T^2$
Non-logarithmic unknown part
- Bonus: all fundamental¹ forces in one equation

Cosmological implications



Summary

- Our computations can be summarized as

$$\frac{d\rho_{\text{GW}}}{dt d \ln k} = \frac{16k^3 \eta T}{\pi m_{\text{Pl}}^2} \phi\left(\frac{k}{T}\right)$$

with

$$\phi\left(\frac{k}{T}\right) \simeq \begin{cases} 1 & , \quad k \lesssim \alpha^2 T \\ \frac{k f_{\text{B}}(k)}{8\pi\eta} \sum_{i=1}^3 d_i m_{\text{Di}}^2 \left(\ln \frac{5T}{m_{\text{Di}}} + \mathcal{O}(1) \right) & , \quad k \gtrsim 3T \end{cases}$$

Embedding in Hubble expansion

- Take as reference temperature $T_0 \equiv 160$ GeV (EW crossover)
- Since $\rho_{\text{GW}}(t) = \int_{\mathbf{k}} k f(t, k)$, with f GW phase space distribution

$$(\partial_t + 4H)\rho_{\text{GW}}(t) = \int_{\mathbf{k}} R(T, k) = \frac{32\pi}{m_{\text{Pl}}^2} \int_{\mathbf{k}} 32\pi\eta T \phi(k/T)$$

- Normalize by $s^{4/3}$ to get rid of H and integrate

$$\frac{\rho_{\text{GW}}(t_0)}{s^{4/3}(t_0)} = \int_{t_{\min}}^{t_0} dt \int_{\mathbf{k}} \frac{R(T, k)}{s^{4/3}(t)} = \int_{T_0}^{T_{\max}} dT \int_{\mathbf{k}} \frac{R(T, k)}{TH(T)3c_s^2(T)s^{4/3}(T)}$$

at initial time t_{\min} (maximum temperature T_{\max}) no thermally produced GWs present

Embedding in Hubble expansion

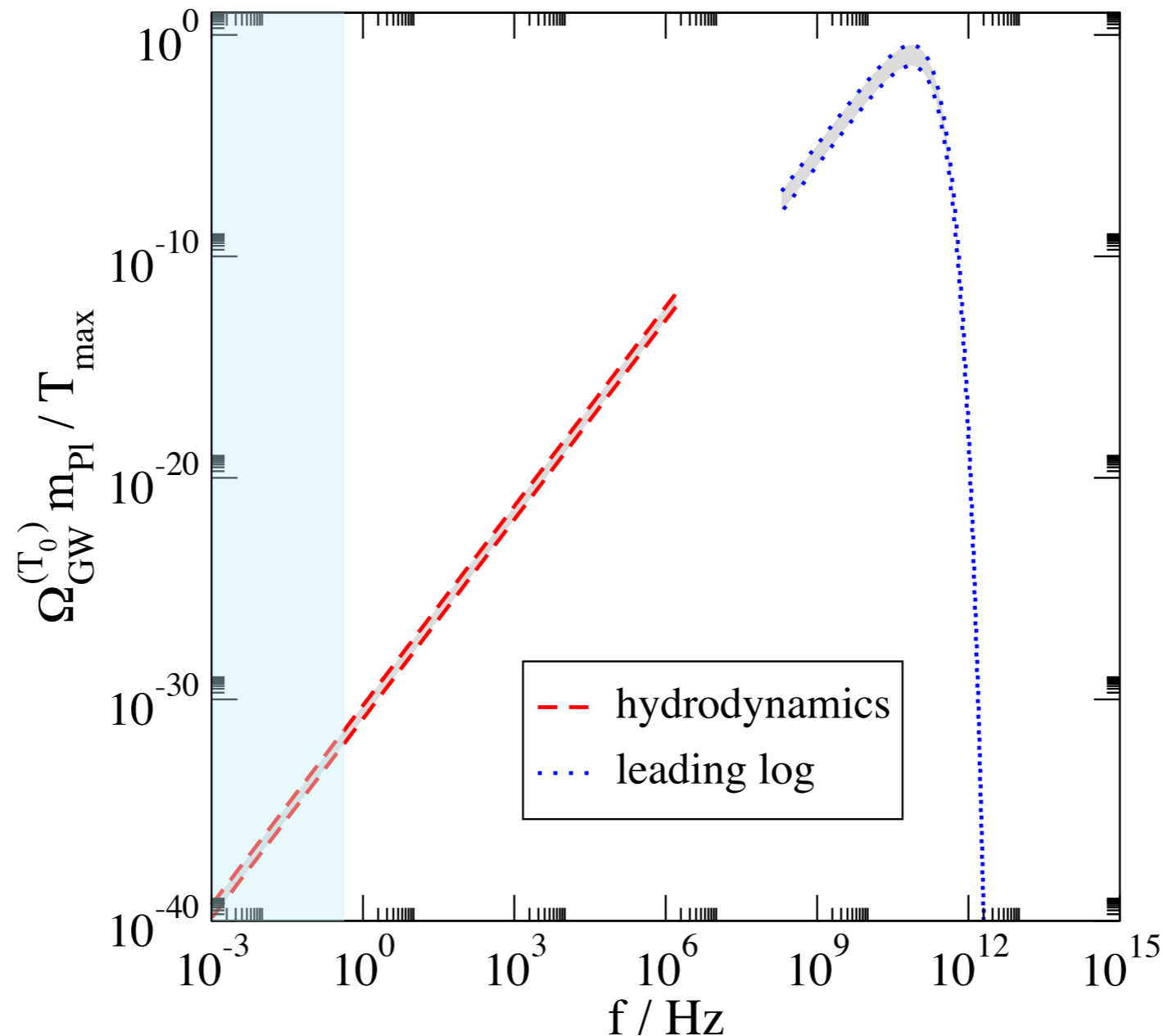
- Finally, redshift momenta to k_0 at T_0

$$\begin{aligned}\Omega_{\text{GW}}(k_0) &\equiv \frac{1}{e(T_0)} \frac{d\rho_{\text{GW}}}{d \ln k_0} \\ &= \frac{8k_0^3 s^{1/3}(T_0)}{m_{\text{Pl}} \sqrt{6\pi^3} e(T_0)} \int_{T_0}^{T_{\text{max}}} dT \frac{\eta(T)}{c_s^2(T) s^{1/3}(T) e^{1/2}(T)} \phi\left(\frac{k_0}{T} \left[\frac{s(T)}{s(T_0)}\right]^{\frac{1}{3}}\right)\end{aligned}$$

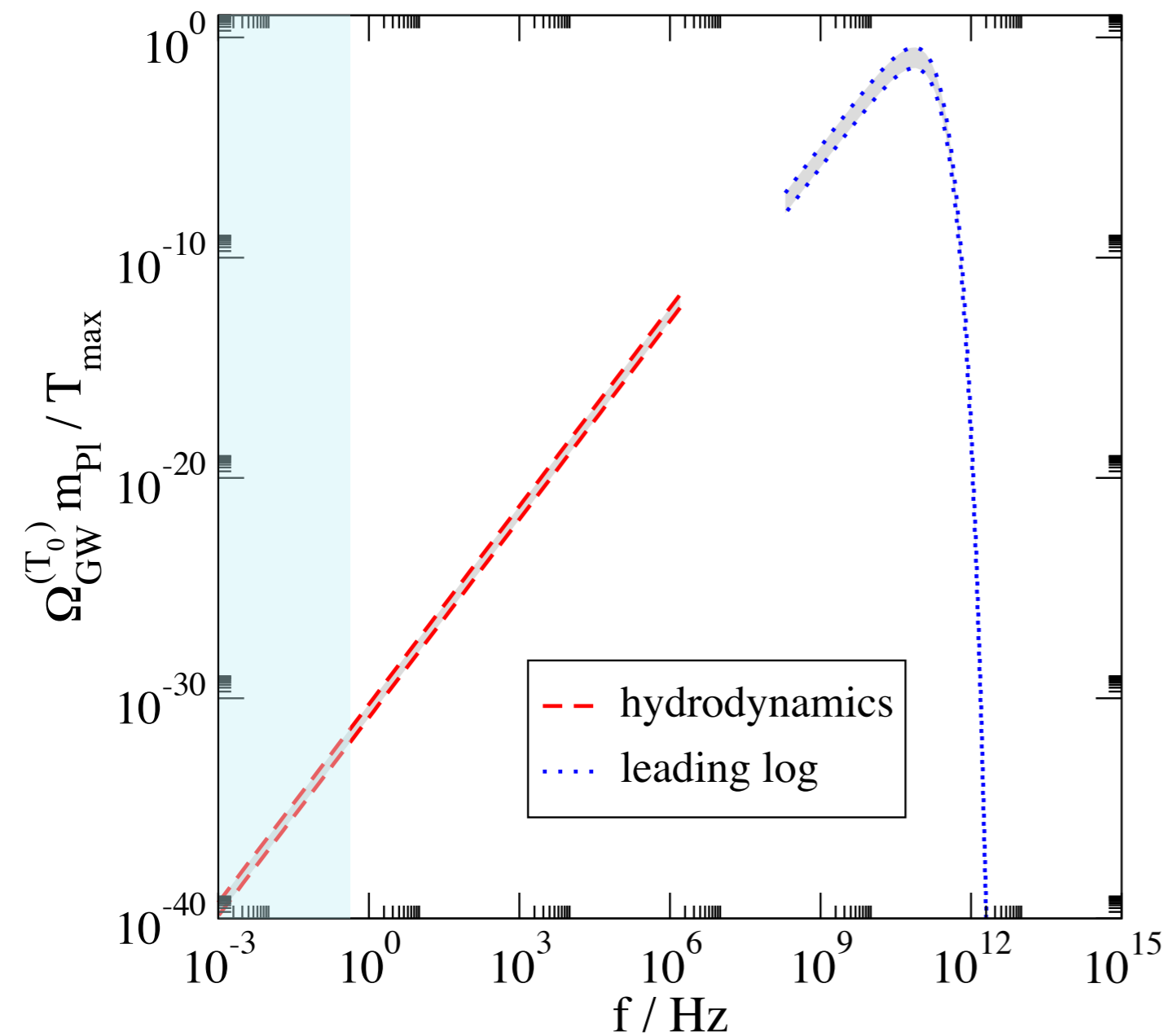
- Approximate form for $c_s^2 = 1/3$ and dimensional scaling $s = \hat{s}T^3$, $\eta = \hat{\eta}T^3$, $e = \hat{e}T^4$

$$\Omega_{\text{GW}}(k_0) \simeq \frac{24\hat{\eta}}{\sqrt{6\pi^3}\hat{e}^3} \frac{T_{\text{max}}}{m_{\text{Pl}}} \frac{k_0^3}{T_0^3} \phi\left(\frac{k_0}{T_0}\right)$$

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The bands for the **hydrodynamic** and **leading-log** results correspond to **varying $\eta=100\dots400$** and to **varying the constant $O(1)$ within the range $0\dots10$** . The couplings were fixed at a scale $\mu = \pi T$ with $T \simeq 10^6$ GeV. For obtaining the current day energy fraction the result needs to be multiplied by $\Omega_{\text{rad}} \sim 5 \times 10^{-5}$



- If for instance $T_{\max}=10^6$ GeV (horizon radius \sim eLISA arm length) then ~ 40 orders of magnitude below sensitivity
- The peak is in the μ -wave range, as the CMB, since it happens for $k \sim 3T_{\max}$ and redshifts at decoupling to $k \sim 3T_{\text{dec}}(3.9 / 106.75)^{1/3} \sim T_{\text{dec}}$. Interesting for future exps?

- In other words the thermal background continues to grow with k for 10+ decades after the peak eLISA frequency
- If compared to EWPT sources, this peak will eventually overtake their rapidly falling spectra

The total energy

- The peak at $k \sim 3T_{\max}$ also implies that the total energy might not be so negligible

$$\int d \ln k_0 \Omega_{\text{GW}}(k_0) \simeq \frac{24\hat{\eta}}{\pi\sqrt{6\pi\hat{e}^3}} \frac{T_{\max}}{m_{\text{Pl}}T_0^3} \int_0^\infty dk_0 k_0^2 \phi\left(\frac{k_0}{T_0}\right) \simeq \frac{24}{\pi\sqrt{6\pi\hat{e}^3}} \left(8 \dots \frac{\hat{\eta}}{3}\right) \frac{T_{\max}}{m_{\text{Pl}}}$$

- Parametrizing our ignorance of $\phi\left(\frac{k_0}{T_0}\right)$ with **two limits** (lead-log...hydro)
- GWs are constrained not to carry as much energy as one relativistic d.o.f.

Smith Pierpaoli Kamionkowski **PRL97** (2006) Henrot-Versille *et al* *Class. Quant. Grav.* **32** (2015)

The total energy

- At $T_0 \sim 160$ GeV we must then require

$$\frac{24}{\pi \sqrt{6\pi \hat{e}^3}} \left(8 \dots \frac{\hat{\eta}}{3} \right) \frac{T_{\max}}{m_{\text{Pl}}} \ll \frac{1}{100}$$

- This can be used to constrain T_{\max} . For $\hat{e} \sim 35$, $\hat{\eta} \sim 400$ we have $T_{\max} \lesssim 10^{17} \dots 10^{18}$ GeV
- Not a stringent constraint (reheating temperatures above 10^{16} GeV excluded in standard inflation), but could be sharpened by knowing more about $\phi\left(\frac{k_0}{T_0}\right)$

Conclusions

- We have shown how to set up the determination of the equilibrium contribution to gravitational waves
- We have determined it at **leading order** in the infrared: it is related to the **shear viscosity** of the EW plasma, which is not small
- We have obtained a **leading-log** estimate for $k \sim T$, coming from **scatterings of thermal plasma constituents**

Conclusions

- The resulting Ω_{GW} is tiny in the eLISA window, but it **peaks in the GHz range**, where it would overtake non-equilibrium EWPT sources
- The best observational prospect is in **future hi-freq. exps**
- This **thermal background is however guaranteed to be present** and, since its production spans many decades, the **associated total energy is not small**
- This energy can be used to (weakly) **constrain the highest temperature of the radiation epoch**
- Estimates could be sharpened with a full leading-order calculation