The University of Nottingham
UNITED KINGDOM · CHINA · MALAYSIA

GRAVITY WAVES: INFLATION AND BEYOND

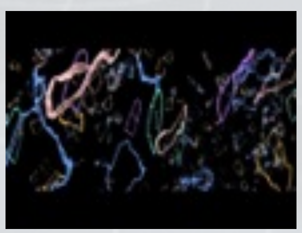
Paul Saffin

Outline

- before the Hot-Big-Bang

- inflation

- direct detection of cosmological GWs



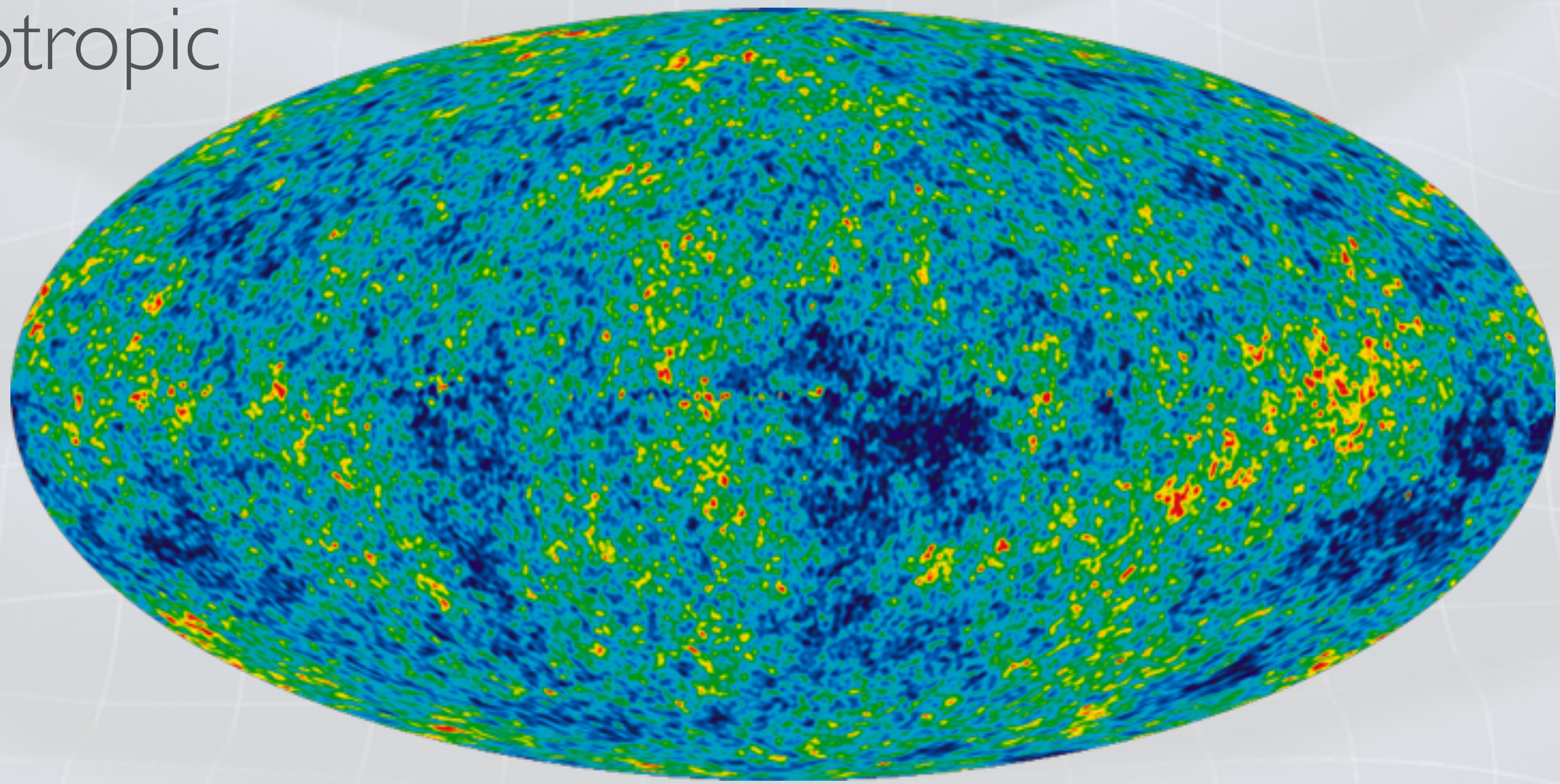
- Pre-Big-Bang

- the end of inflation
  - preheating
  - oscillons, Q-balls

- conclusions

the early Universe was:

- homogeneous
- isotropic

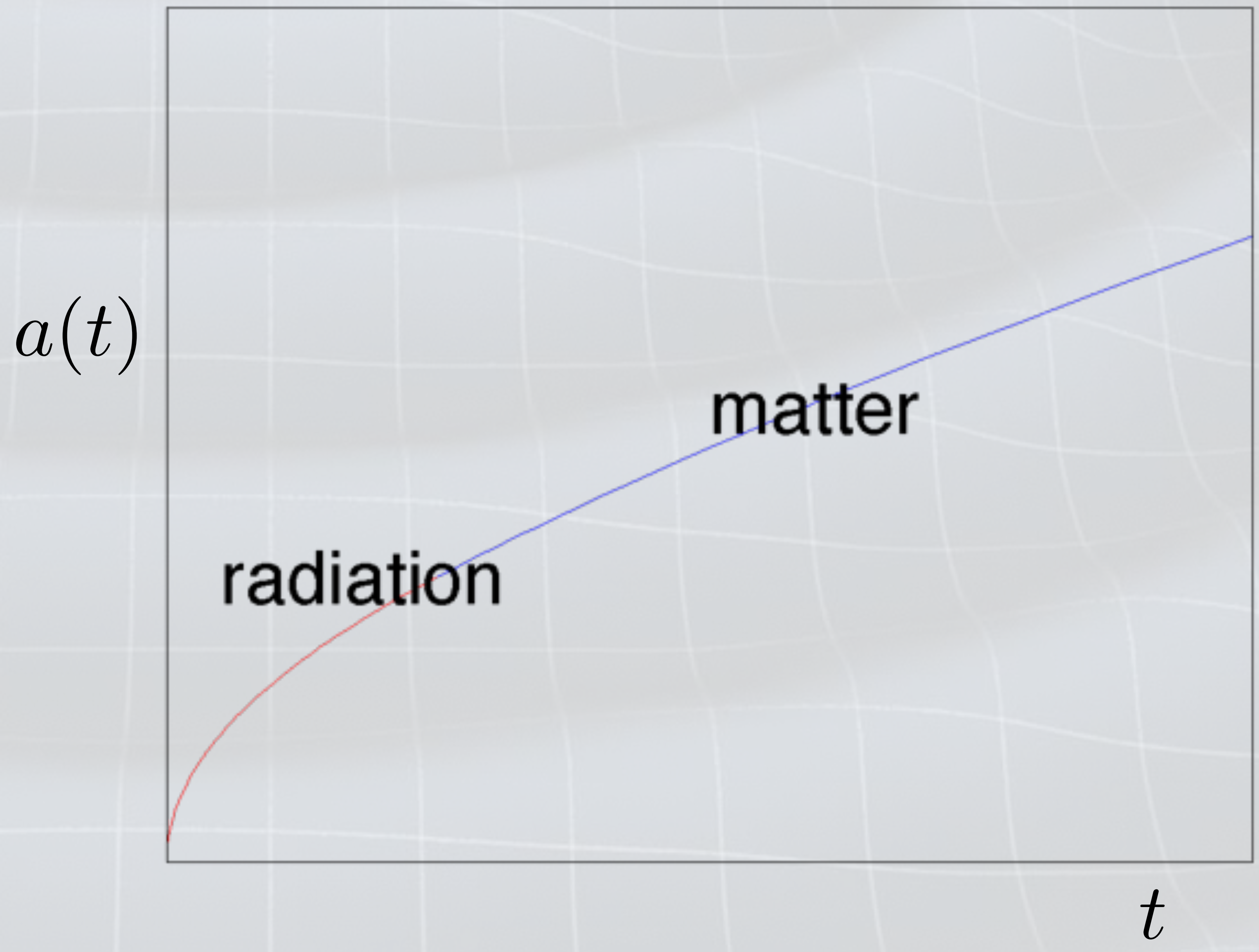


CMB temperature fluctations are $1 : 10^5$

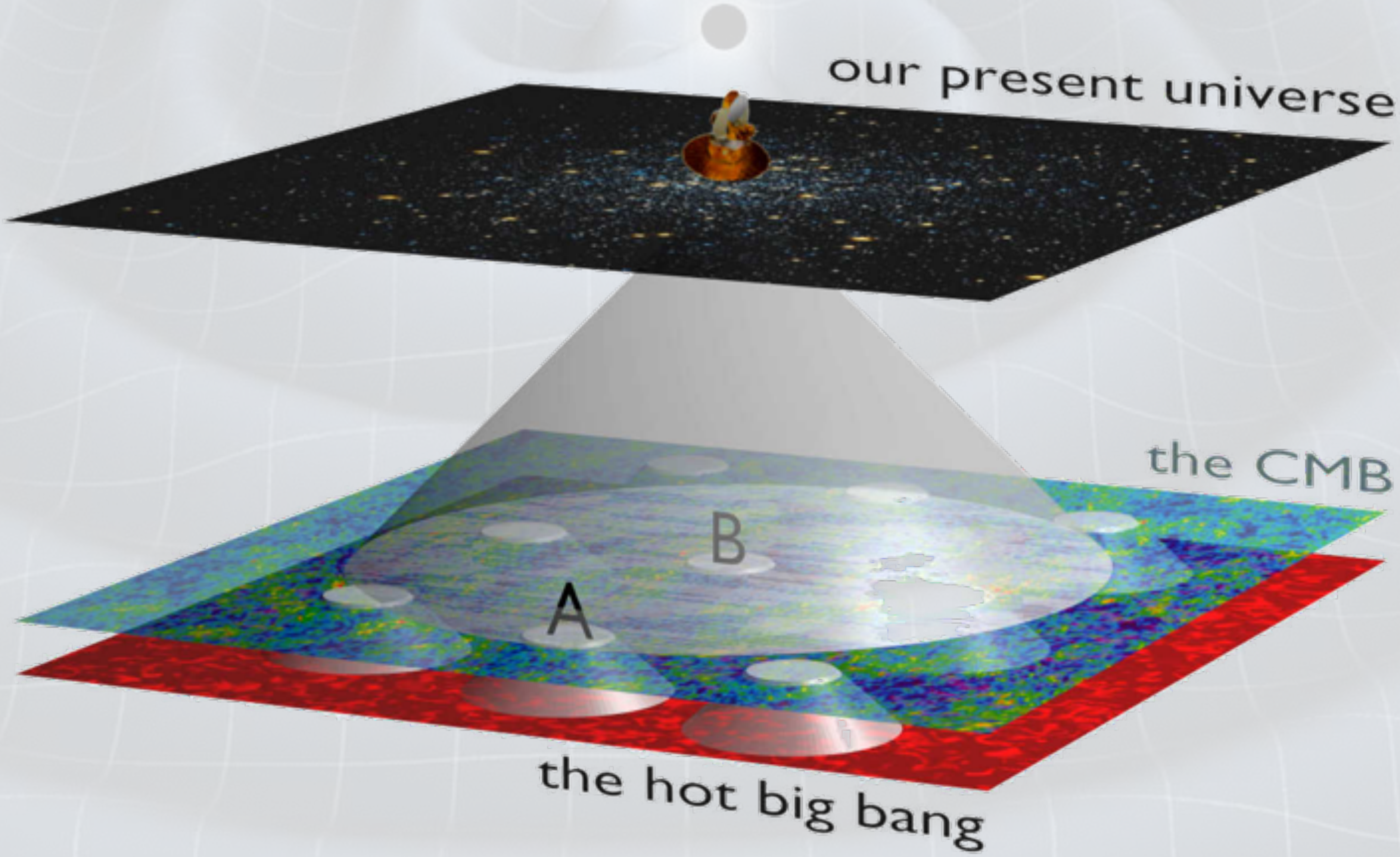
Horizon Problem

$$ds^2 = -dt^2 + a(t)^2 \left[dx^2 + dy^2 + dz^2\right]$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$$



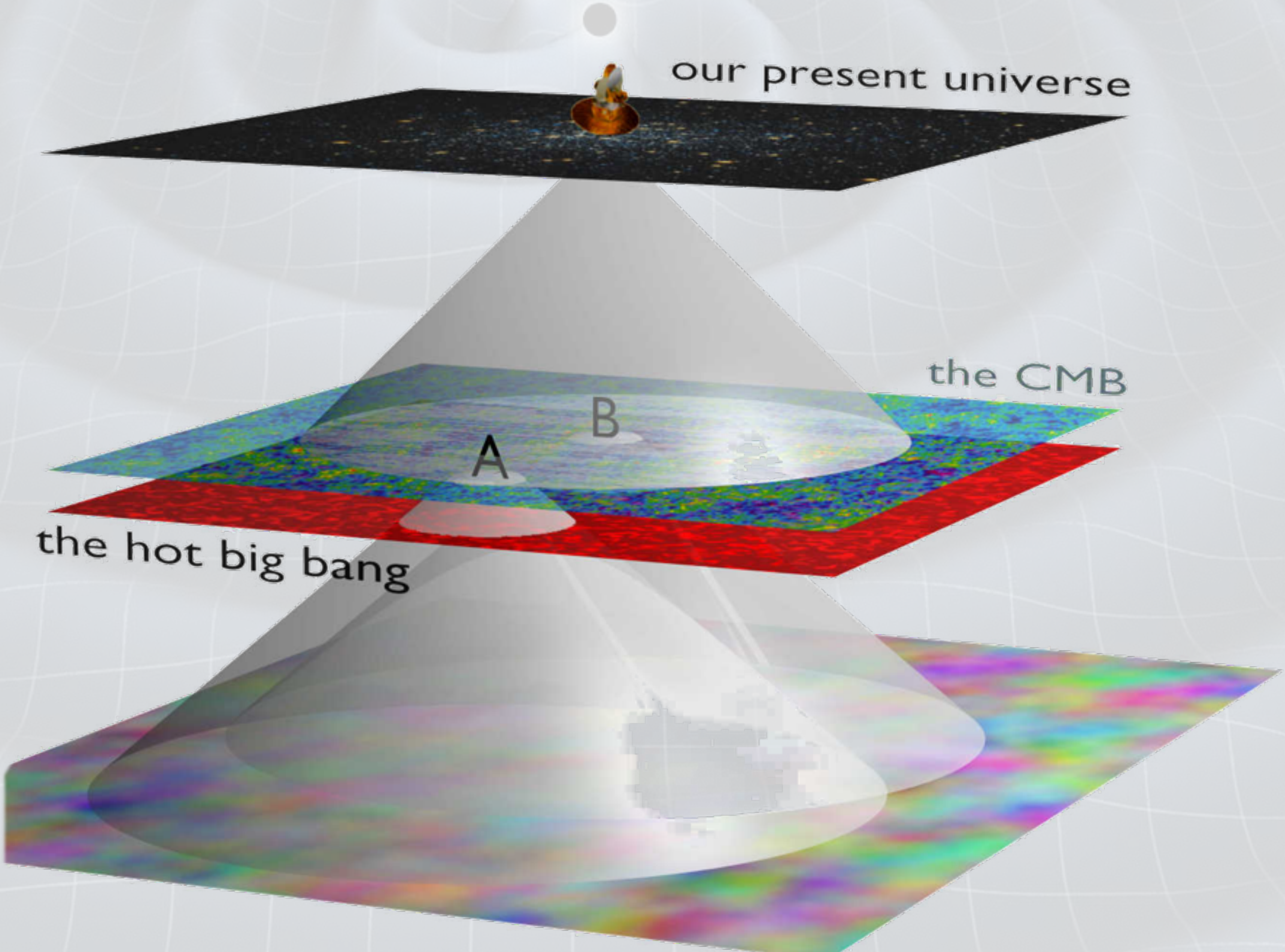
Horizon Problem



causal contact for $\theta < 2^\circ$

Wang arXiv:1303.1523

Horizon Problem



Wang arXiv:1303.1523

$$ds^2 = -dt^2 + a(t)^2 \left(dx^2 + dy^2 + dz^2\right)$$

$$\eta = \int \frac{dt}{a} = \int \frac{da}{a}\frac{1}{aH}$$ = particle horizon

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$$a(\eta) \propto \begin{cases} \eta & radiation \\ \eta^2 & dust \end{cases}$$

$$p = w\rho$$

$$\frac{d}{dt}\left(\frac{1}{aH}\right) \propto (1+3w)$$

$$ds^2 = -dt^2 + a(t)^2 \left(dx^2 + dy^2 + dz^2\right)$$

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$$\frac{d}{dt}\left(\frac{1}{aH}\right) \propto (1 + 3w)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(1 + 3w)\rho$$

$$ds^2 = -dt^2 + a(t)^2 \left(dx^2 + dy^2 + dz^2\right)$$

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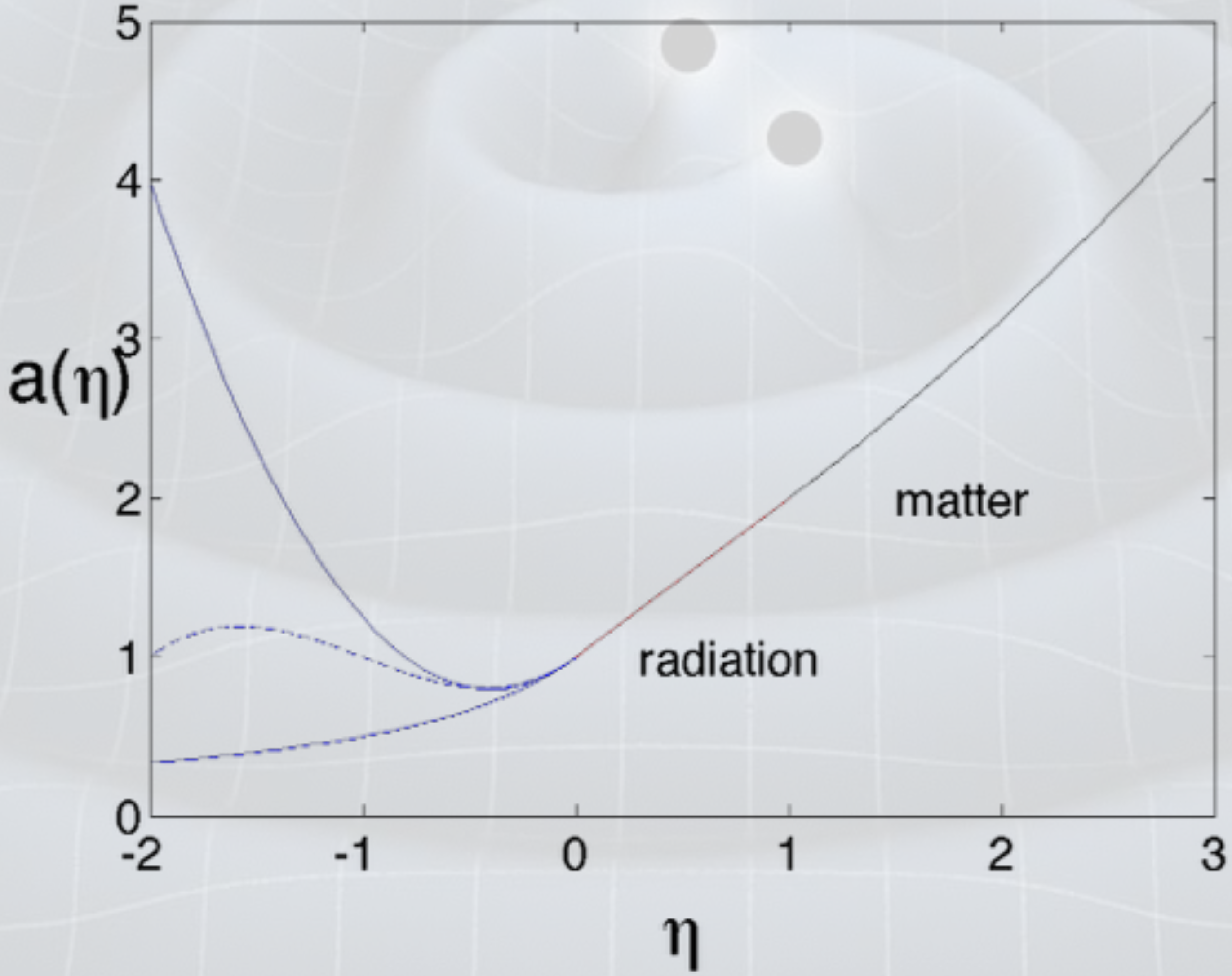
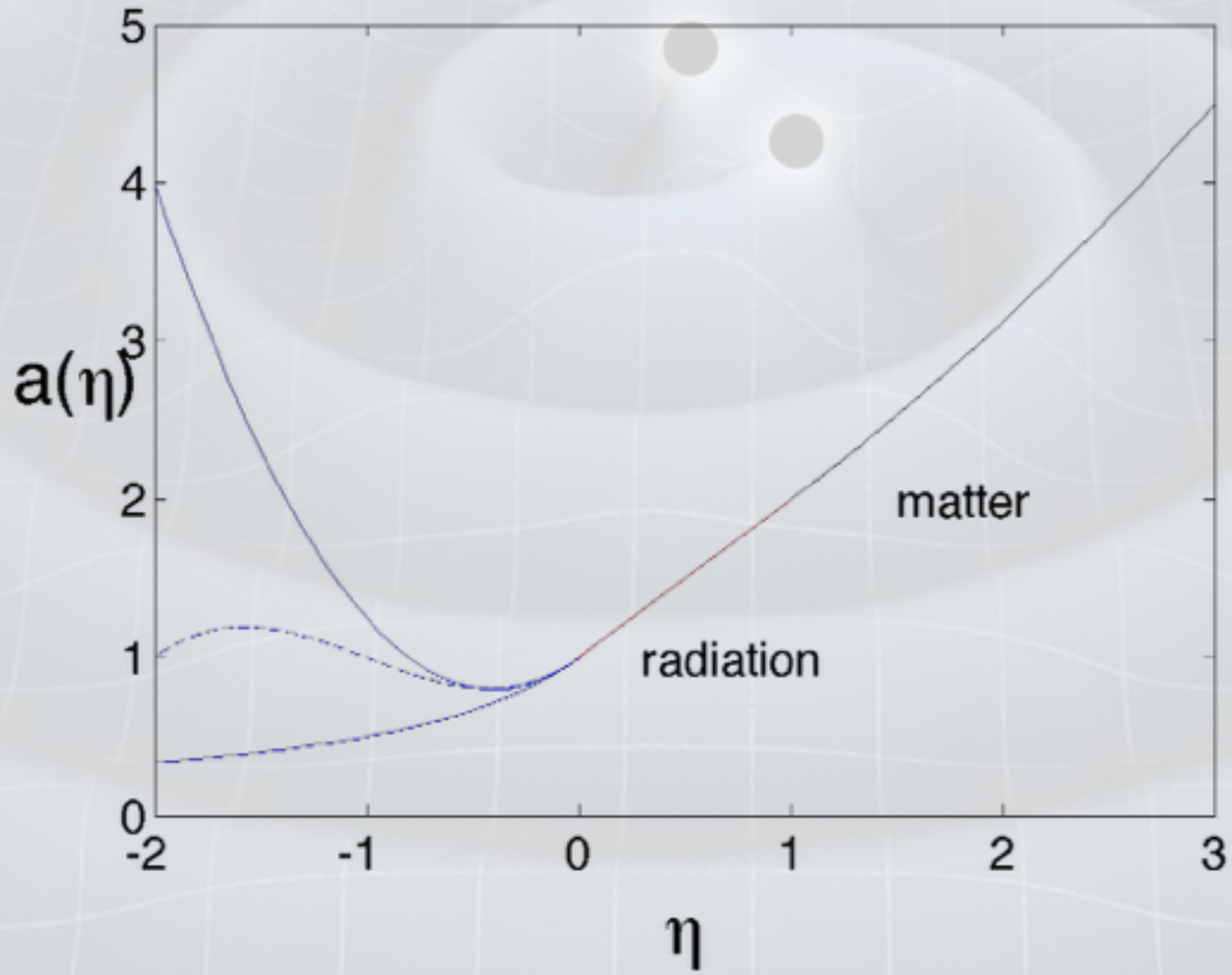
$$p = w\rho \qquad \frac{d}{dt}\left(\frac{1}{aH}\right) \propto (1+3w) \qquad \frac{\ddot{a}}{a} = -\frac{1}{6}(1+3w)\rho$$

flatness problem: $$1 - \Omega = -\frac{k}{(aH)^2}$$

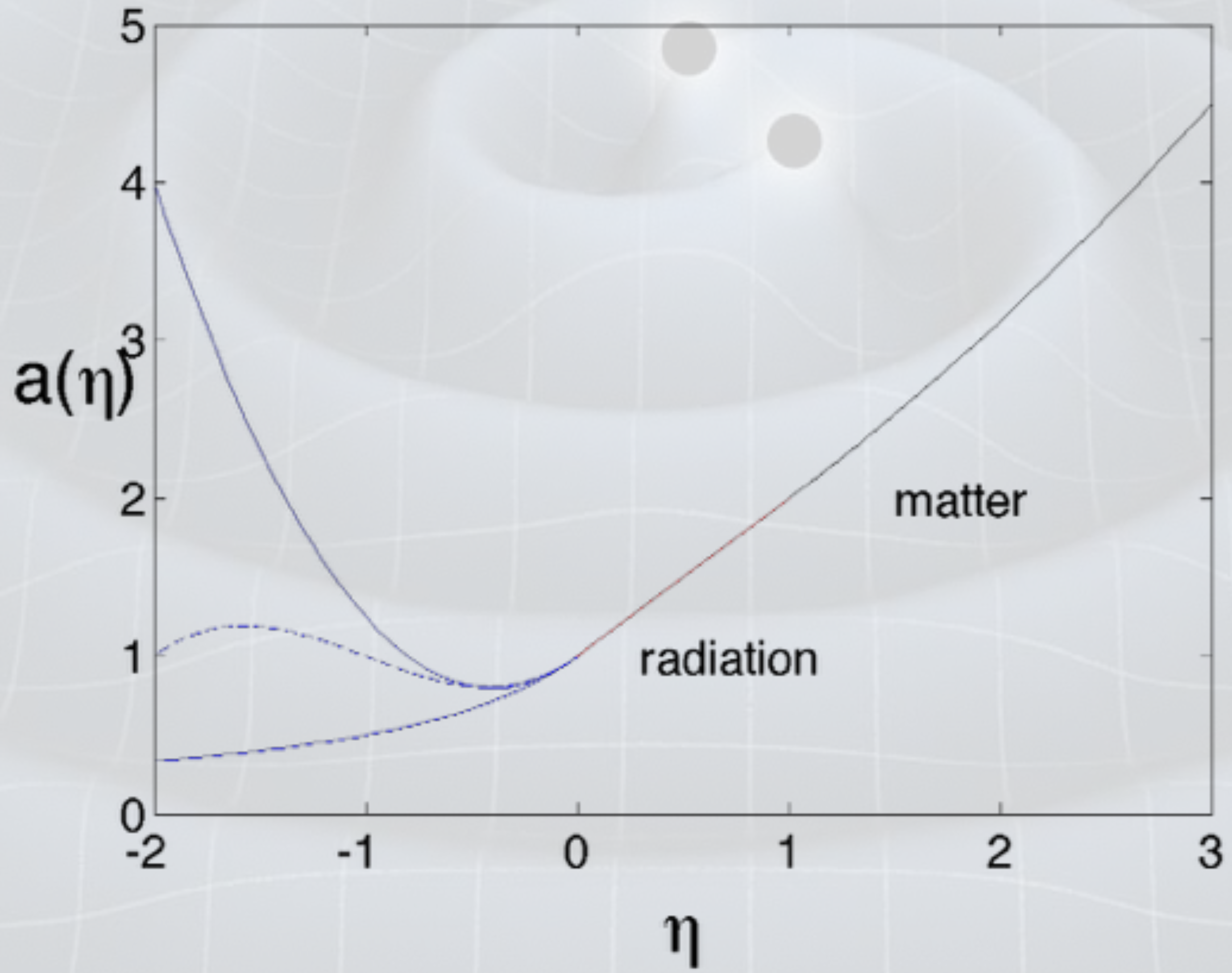


a(η)
η
matter
radiation
0
1
2
3
4
5
-2
-1
0
1
2
3



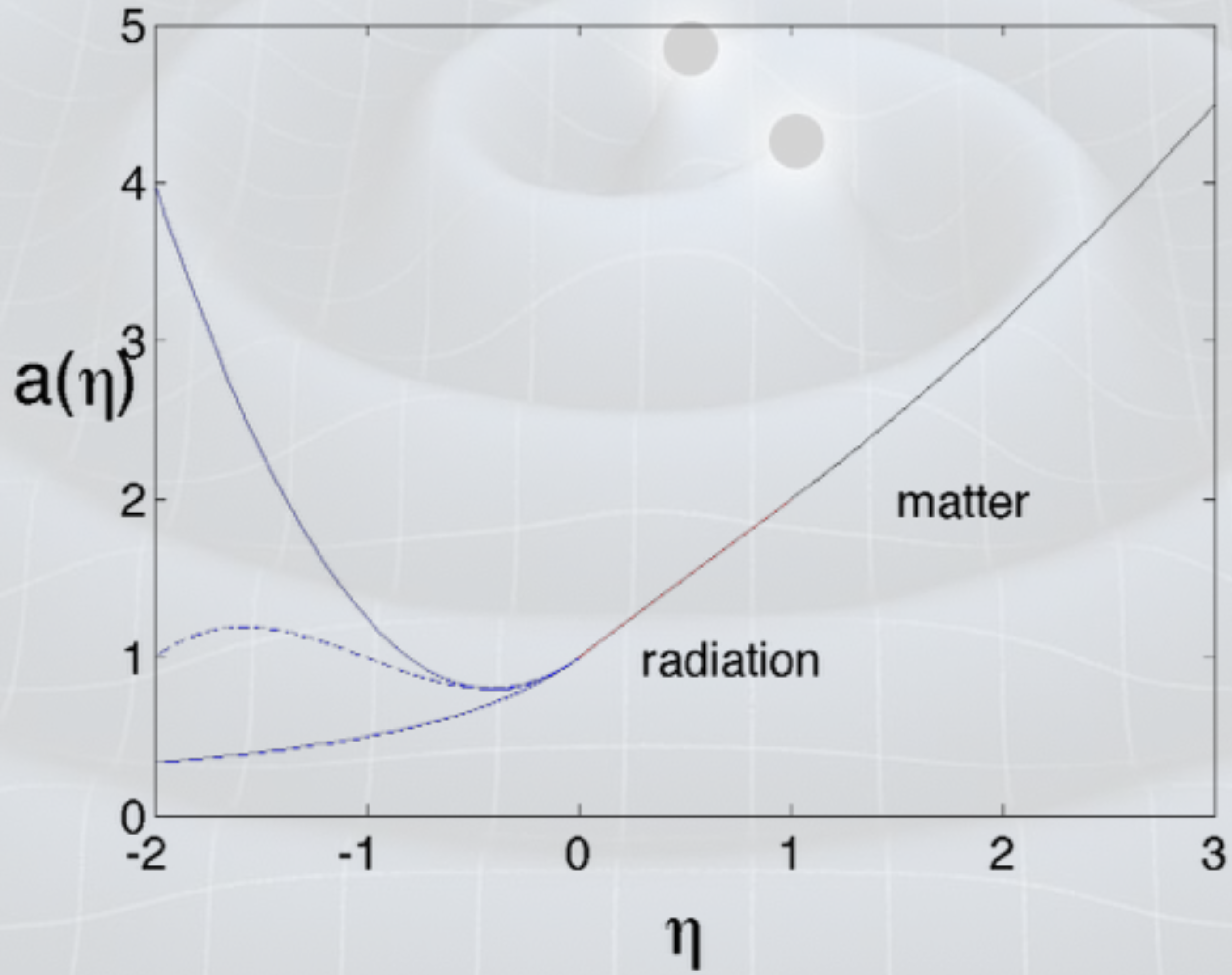
deSitter

$$a(t) = e^{Ht},\ a(\eta) = -\frac{1}{H\eta} \Rightarrow \frac{1}{\mathcal{H}} = -\eta$$



deSitter $a(t) = e^{Ht},\ a(\eta) = -\frac{1}{H\eta} \Rightarrow \frac{1}{\mathcal{H}} = -\eta$

power-law inflation $a(t) = \kappa t^2,\ a(\eta) = \frac{1}{\kappa\eta^2} \Rightarrow \frac{1}{\mathcal{H}} = -\eta/2$



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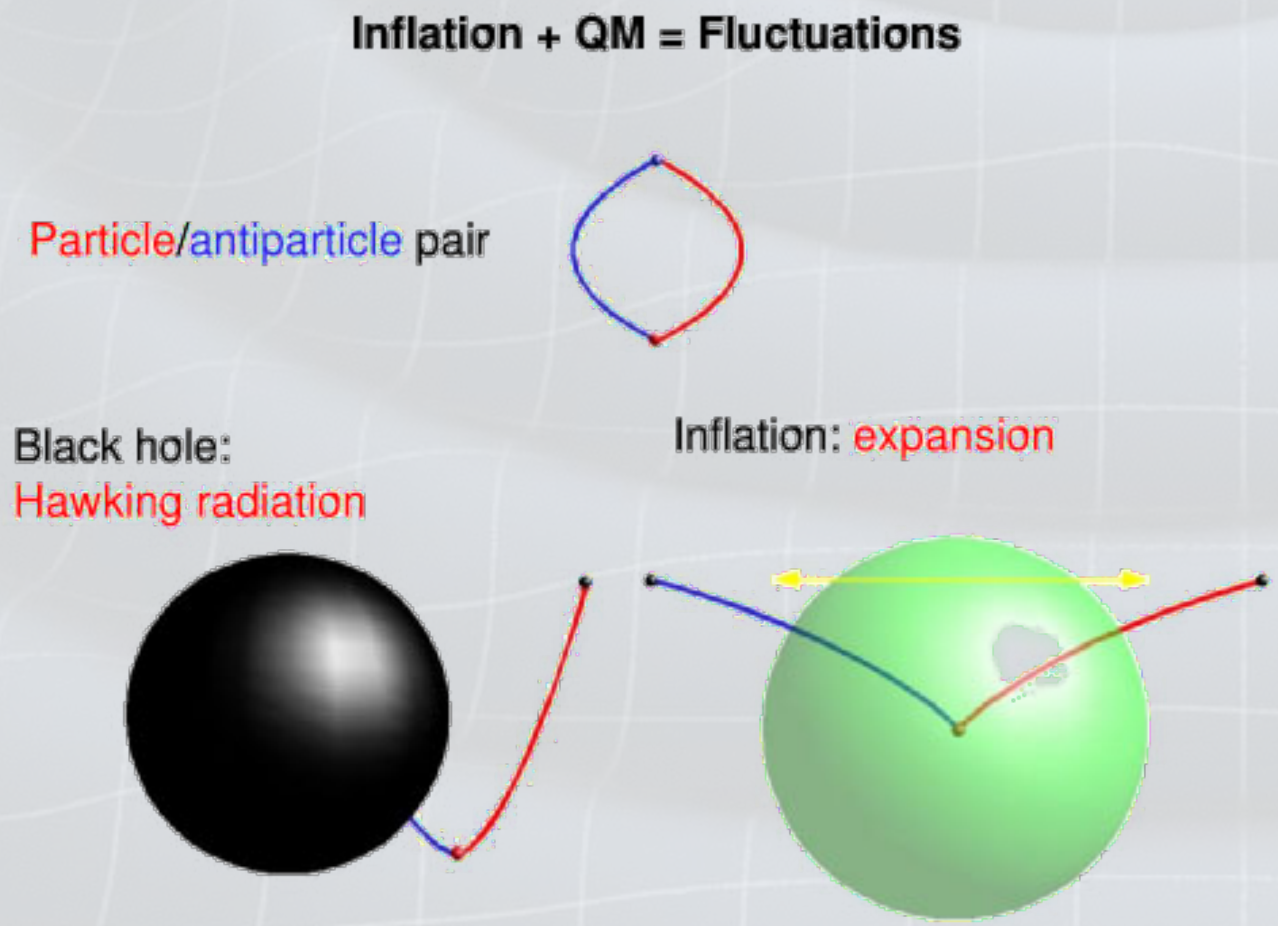
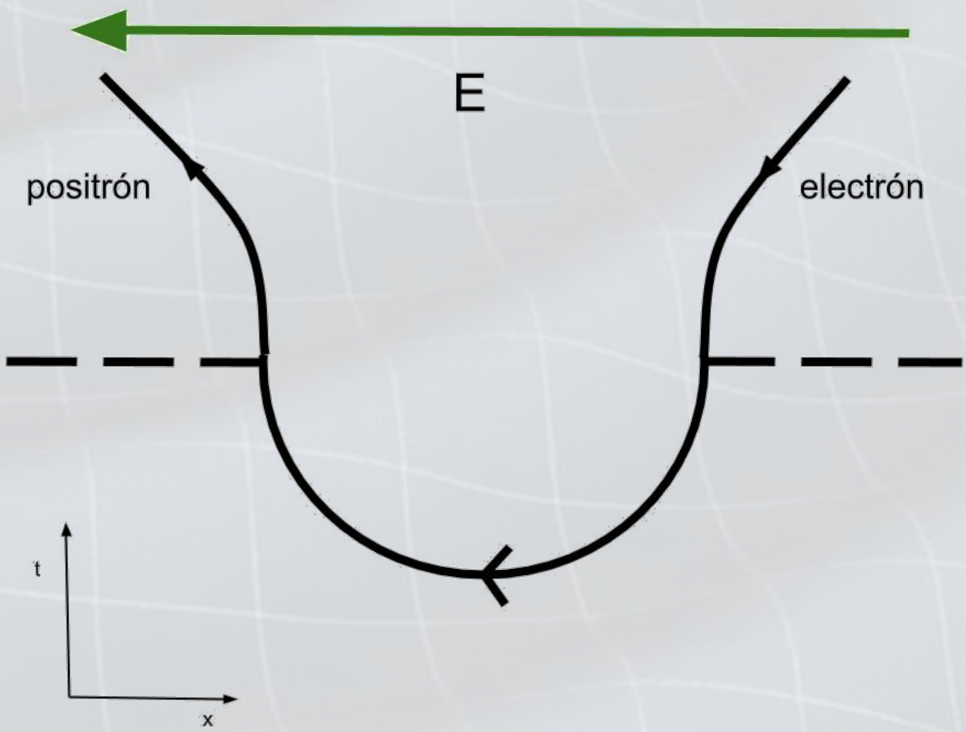
power-law inflation $a(t) = \kappa t^2,\ a(\eta) = \frac{1}{\kappa\eta^2} \Rightarrow \frac{1}{\mathcal{H}} = -\eta/2$

collapsing dust $a(t) = \kappa(-t)^{2/3},\ a(\eta) = \frac{\kappa^3}{9}\eta^2 \Rightarrow \frac{1}{|\mathcal{H}|} = |\eta/2|$

Fluctuations

- the Universe accelerates, and creates particles
- analogue with Hawking effect
- analogue with Schwinger effect





$$E_{Sch} \simeq 10^{18} V m^{-1}$$

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \nabla^2\delta\phi = 0$$

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \nabla^2\delta\phi = 0$$

$$u = a\delta\phi$$

$$u_k'' + \left(k^2 - \frac{a''}{a}\right)u_k = 0$$

- Mukhanov, JETP. Lett 41 (1985)
- Sasaki, Prog. The. Phys. 76 (1986)

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vacuum fluctuations on small scales, $k^2 >> \left|\frac{a''}{a}\right|$

$$u_k \to e^{-ik\eta}/\sqrt{k}$$

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vacuum fluctuations on small scales, $k^2 >> \left|\frac{a''}{a}\right|$

$$u_k \to e^{-ik\eta}/\sqrt{k}$$

$$a \sim t^p \sim \eta^{\frac{1}{2}-\nu}$$

$$\frac{a''}{a} = \frac{\nu^2 - \frac{1}{4}}{\eta^2}$$

$$\nu = \frac{3}{2} + \frac{1}{p-1}$$

(deSitter corresponds to $p \to \infty$)

- Mukhanov, JETP. Lett 41 (1985)
- Sasaki, Prog. The. Phys. 76 (1986)

general mode function

$$u_k = \sqrt{|k\eta|}\left[u_+ H^{(1)}_{|\nu|}(|k\eta|) + u_- H^{(2)}_{|\nu|}(|k\eta|)\right]$$

N.B. $H^2 \propto \eta^{2\nu-3}$

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and fixes the large scale (late time) $|k\eta| << 1$ behaviour

$$u_k(|k\eta| << 1) \sim \frac{1}{\sqrt{k}}|k\eta|^{1/2-|\nu|}$$

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$$u_k(|k\eta| << 1) \sim \frac{1}{\sqrt{k}}|k\eta|^{1/2-|\nu|}$$

giving a scalar power spectrum for deSitter, $\nu = 3/2$, that is scale invariant

$$P_{\delta\phi} \simeq k^{-3}\frac{H^2}{M_P^2}|k\eta|^{3-2|\nu|}$$

N.B. $H^2 \propto \eta^{2\nu-3}$

$$r = \frac{\Delta_h^2}{\Delta_s^2} \qquad \Delta^2 = \frac{k^3}{2\pi^2}\mathcal{P}$$

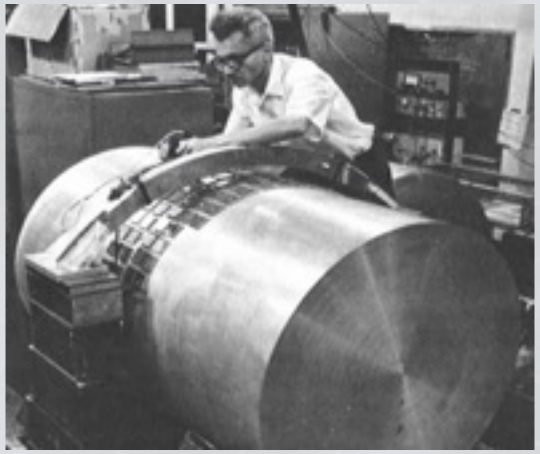
the observed scalar amplitude at large scales is $\Delta_s^2 \simeq 10^{-9}$

$$\left(\frac{H_{inf}}{6 \times 10^{-5} M_p}\right)^2 \simeq \frac{r}{0.12}$$

for slow-roll inflation $H^2 \simeq \dfrac{V}{3M_p^2}$

$$V \simeq \left(1.9 \times 10^{16} GeV\right)^4 \frac{r}{0.12}$$

- Planck arXiv:1502.0211



Direct Detection

$$\rho_{gw} = 2 \int \frac{d^3k}{(2\pi)^2} 2\pi f \; n_f = 16\pi^2 \int d(\ln f) f^4 \; n_f$$

$$\Omega_{gw}(f) = \frac{1}{\rho_c} \frac{d\rho_{gw}}{d \ln f} = \frac{16\pi^2 n_f f^4}{\rho_c}$$

$$u_k'' + \left(k^2 - \frac{a''}{a}\right) u_k = 0 \qquad\qquad u = a\delta\phi$$

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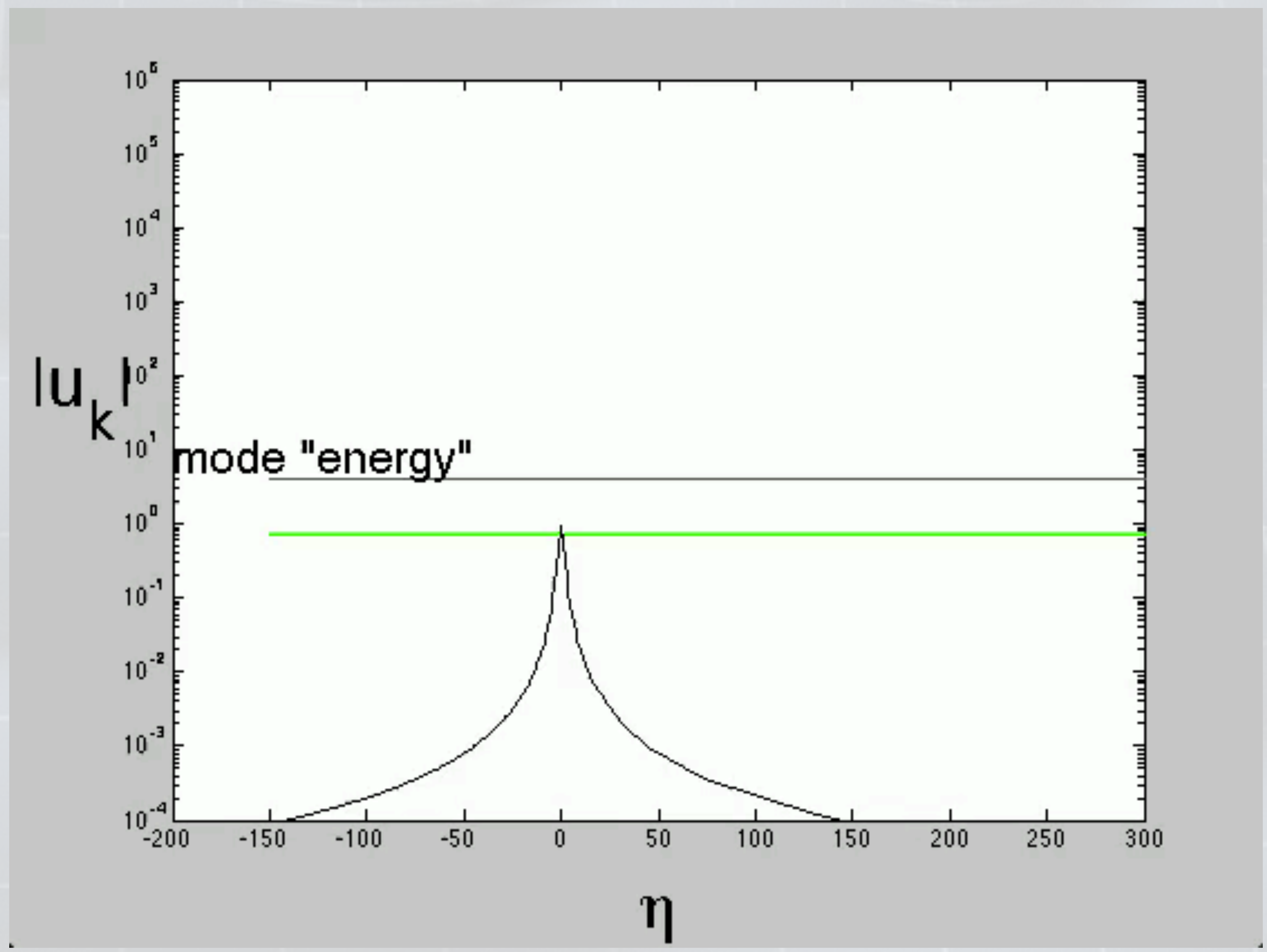

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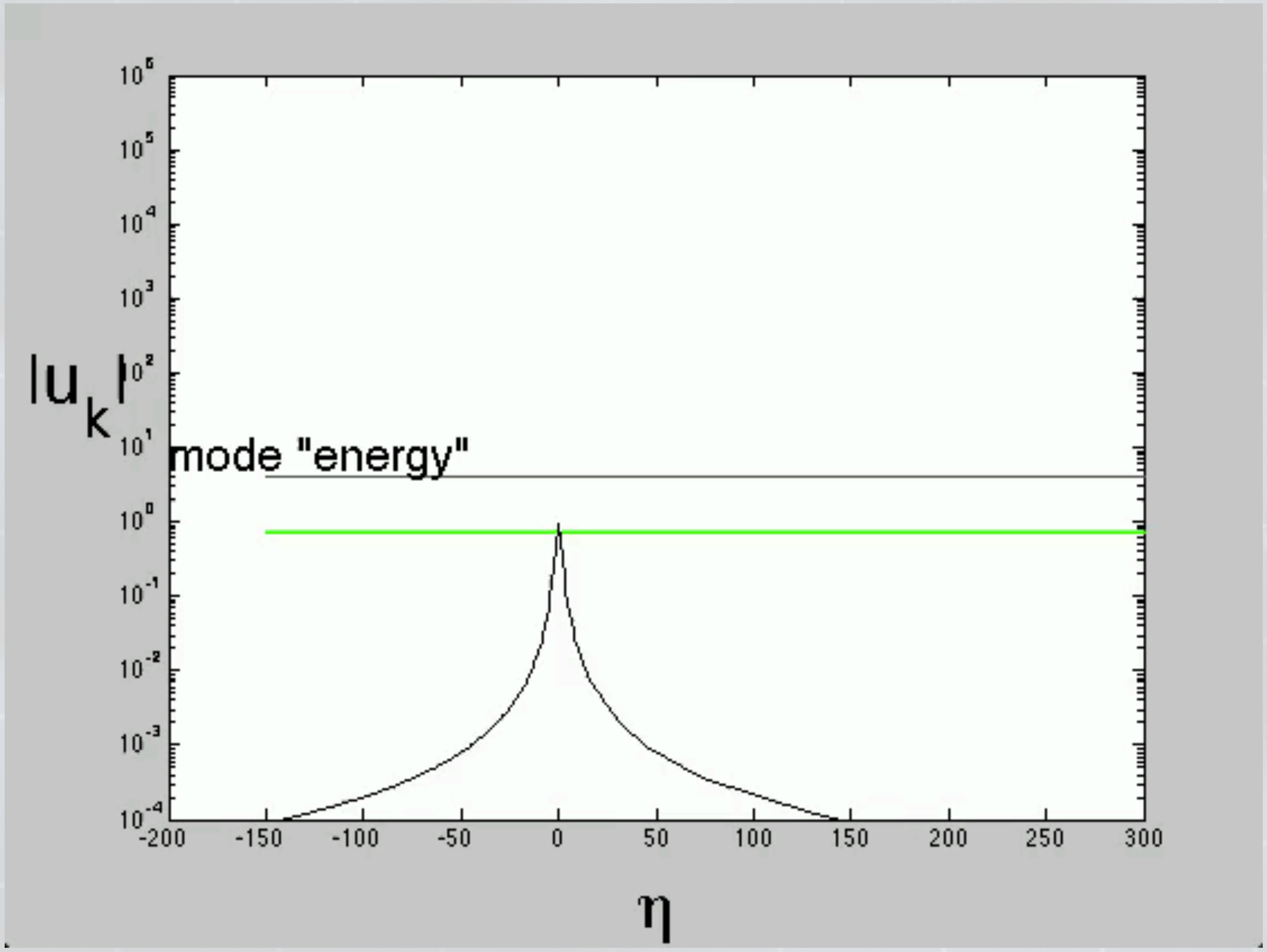
modes with $k < aH \sim \frac{a''}{a}$

evolve as

$$u_k(\eta) = C_k a(\eta) + D_k a(\eta) \int^{\eta} \frac{d\eta'}{a^2(\eta')}$$



deSitter to radiation particle production:

$$\delta\phi_k = \frac{1}{a(\eta)}\left(1 - \frac{i}{k\eta}\right)e^{-ik\eta} \qquad a(\eta) = -\frac{1}{H\eta}$$

$$\delta\phi_k = \frac{1}{a(\eta)}\left(\alpha_k e^{-ik\eta} + \beta_k e^{ik\eta}\right) \qquad a(\eta) = \frac{1}{H\eta_1^2}(\eta - 2\eta_1)$$

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matching the solutions at the transition gives

$$\alpha_k = 1 - \frac{i}{k\eta_1} - \frac{1}{2k^2\eta_1^2} \qquad \beta_k = \frac{1}{2k^2\eta_1^2}$$

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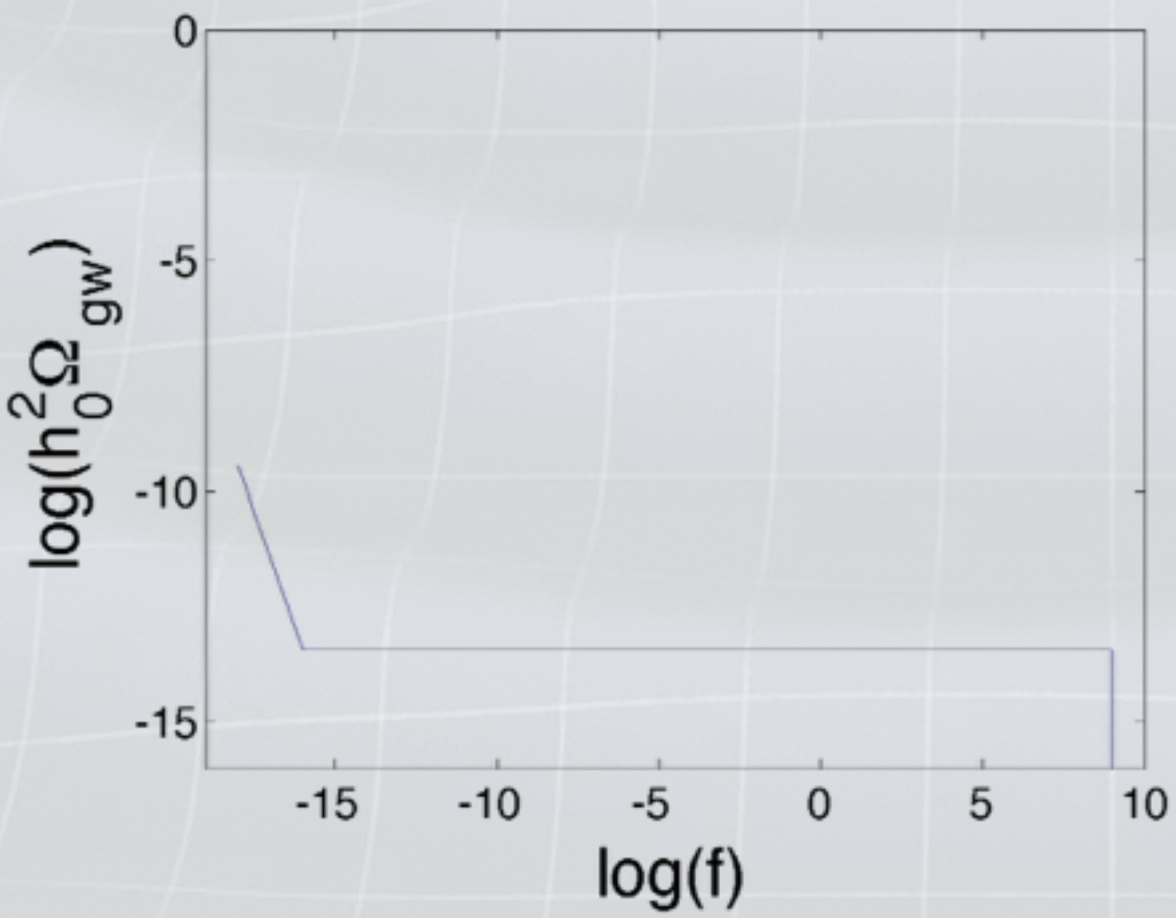
$$n_k = |\beta_k|^2 = \frac{1}{4k^4\eta_1^4}$$

$$\Omega_{gw}(f) = \frac{1}{\rho_c}\frac{d\rho_{gw}}{d\ln f} = \frac{16\pi^2 n_f f^4}{\rho_c}$$

deSitter to radiation particle production:

$$k\eta_1 = 2\pi f a_0 |\eta_1| = \frac{2\pi f}{H_{inf}} \frac{a_0}{a_1} = \frac{2\pi f}{H_{inf}} \left(\frac{t_0}{t_{eq}}\right)^{2/3} \left(\frac{t_{eq}}{t_1}\right)^{1/2}$$

$$h_0^2 \Omega_{gw}(f) \simeq 10^{-13} \left(\frac{H}{10^{-4} M_p}\right)^2$$



nucleosynthesis has:

$$h_0^2 \Omega_{gw} < 10^{-5}$$

COBE has:

$$H_{inf} < 6 \times 10^{-5} M_p$$

~deSitter inflation?

$$P_{\delta\phi} \simeq k^{-3}\frac{H^2}{M_P^2}\left|k\eta\right|^{3-2|\nu|}$$

$$a \propto t^p \qquad \nu = \frac{3}{2} + \frac{1}{p-1} \qquad a \sim \eta^{\frac{p}{1-p}} \sim \eta^{\frac{1-2\nu}{2}}$$

- Wands gr-qc/9809062
- Finelli, Brandenberger hep-th/0112249
- Cai, Qiu, Brandenberger, Zhang arXiv:0810.4677
- Cai, Brandenberger, Zhang arXiv:1101.0822
- Cai, Quintin, Saradakis, Wilson-Ewing arXiv:1404.4364

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can get a flat spectrum from collapsing dust,

$$p = \frac{2}{3},\ \nu = -\frac{3}{2}$$

- Wands gr-qc/9809062
- Finelli, Brandenberger hep-th/0112249
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- Cai, Brandenberger, Zhang arXiv:1101.0822
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~deSitter inflation?

string cosmology

$$ds^2 = -dt^2 + a^2(t)d\underline{x}^2 + b^2(t)d\underline{y}^2$$

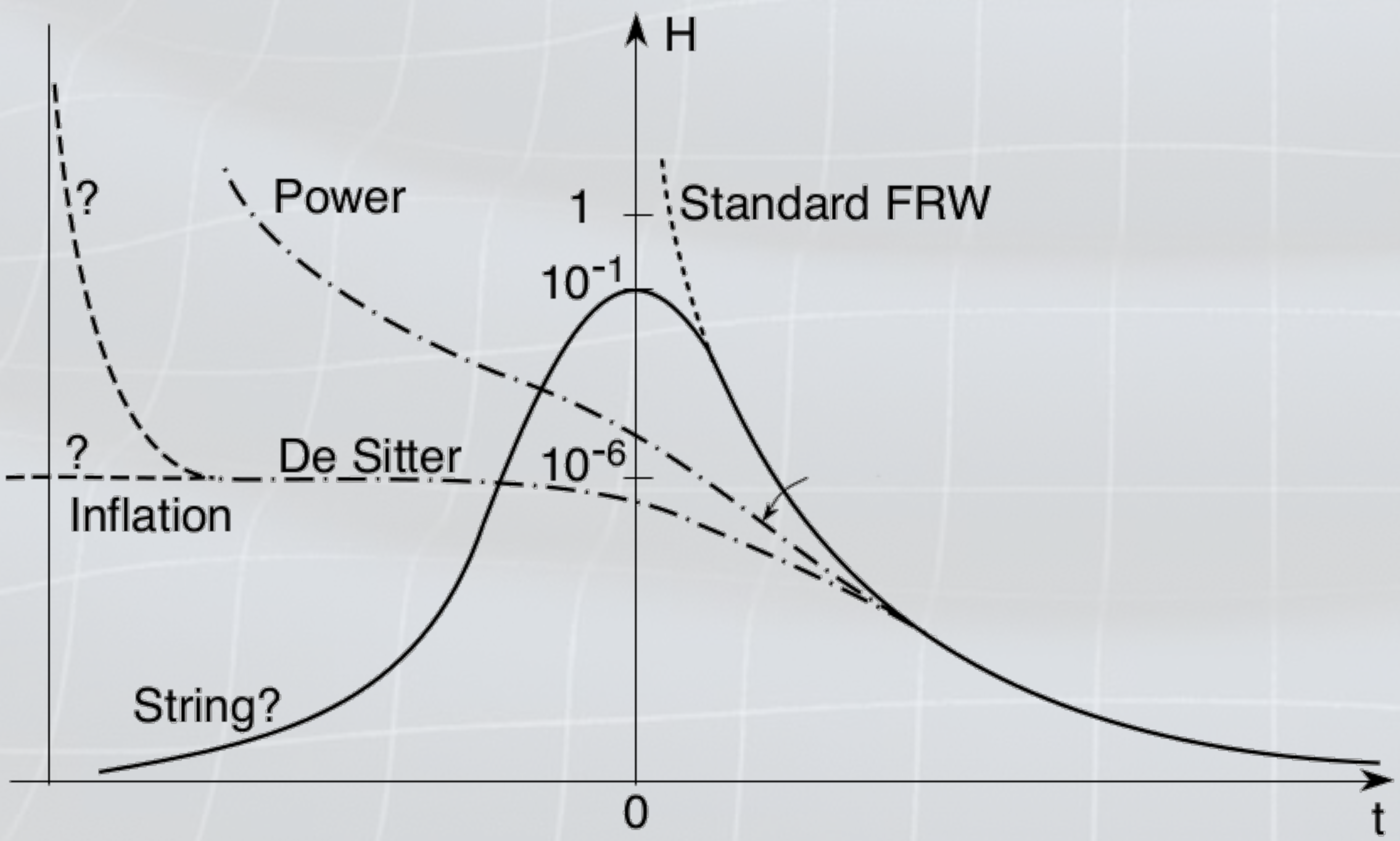
$$S = \frac{1}{2l_p^{d+n-1}} \int d^{d+n+1}x\sqrt{-g}e^{-\phi}\left[R + (\nabla\phi)^2 - V(\phi)\right]$$

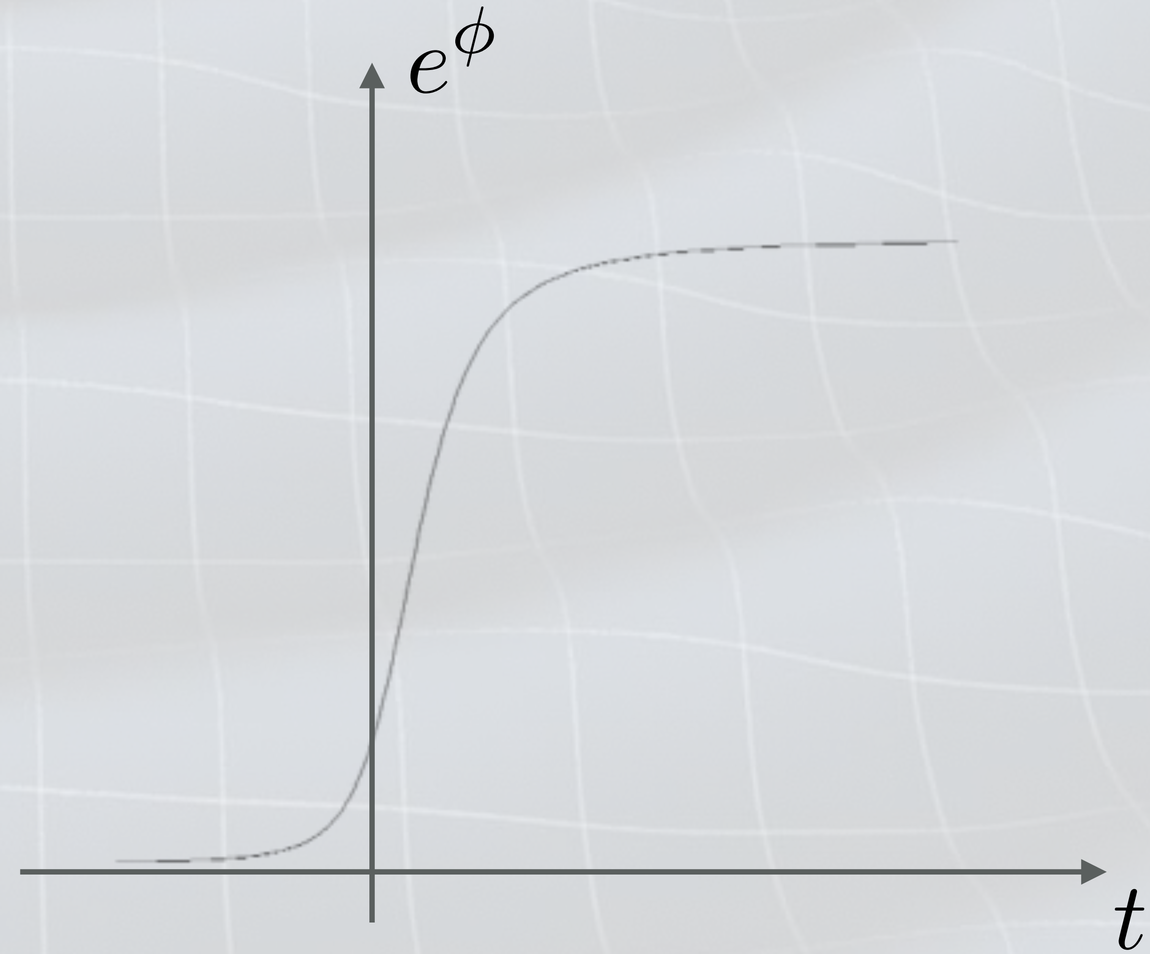
n = number of compact dimensions

string cosmology

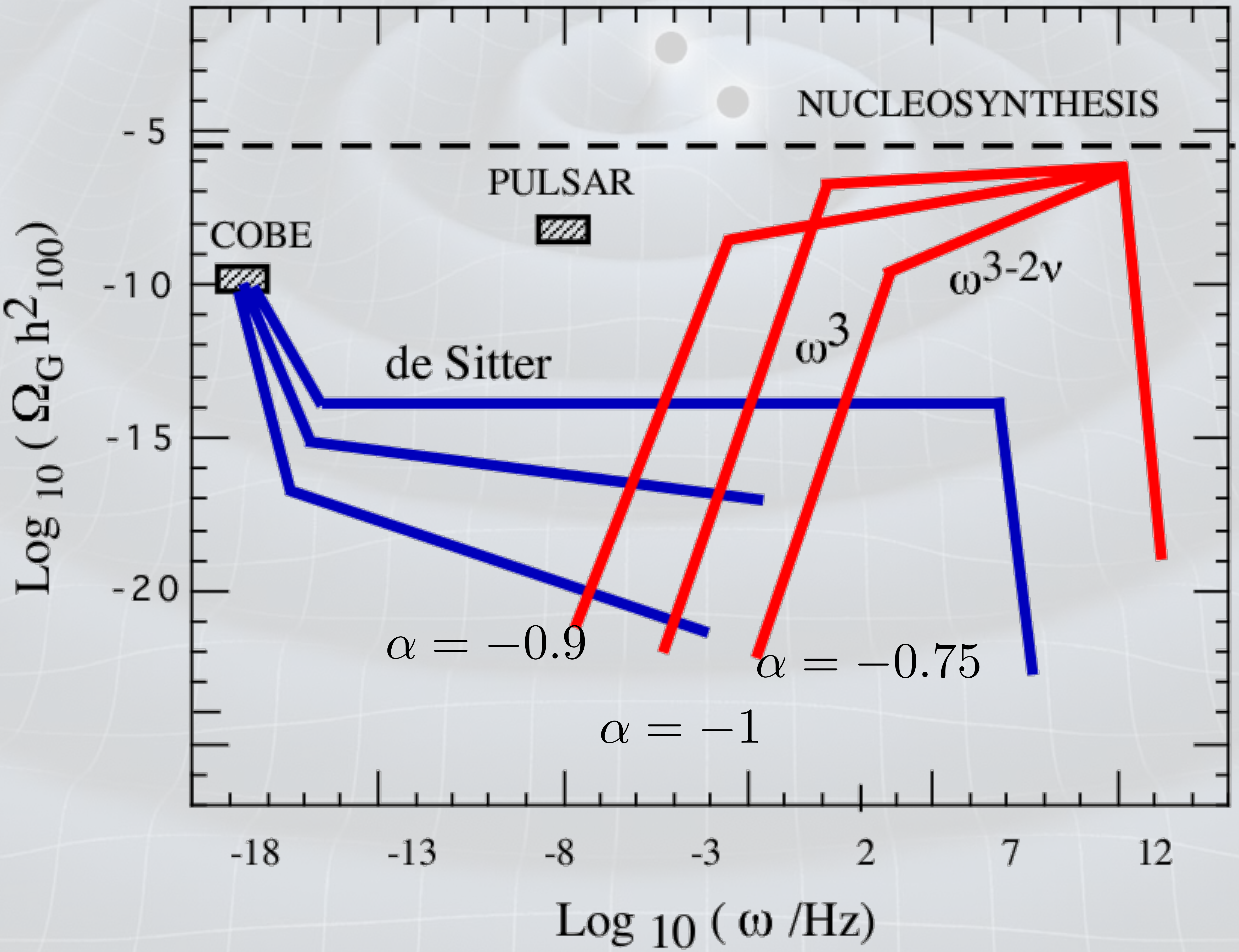
$$a(t) \to a^{-1}(-t), \quad \phi(t) \to \phi(-t) - 6\ln[a(-t)]$$

$$H > 0,\ \dot{H} > 0,\ \dot{\phi} > 0 \qquad \to \qquad H > 0,\ \dot{H} < 0, \dot{\phi} = 0$$



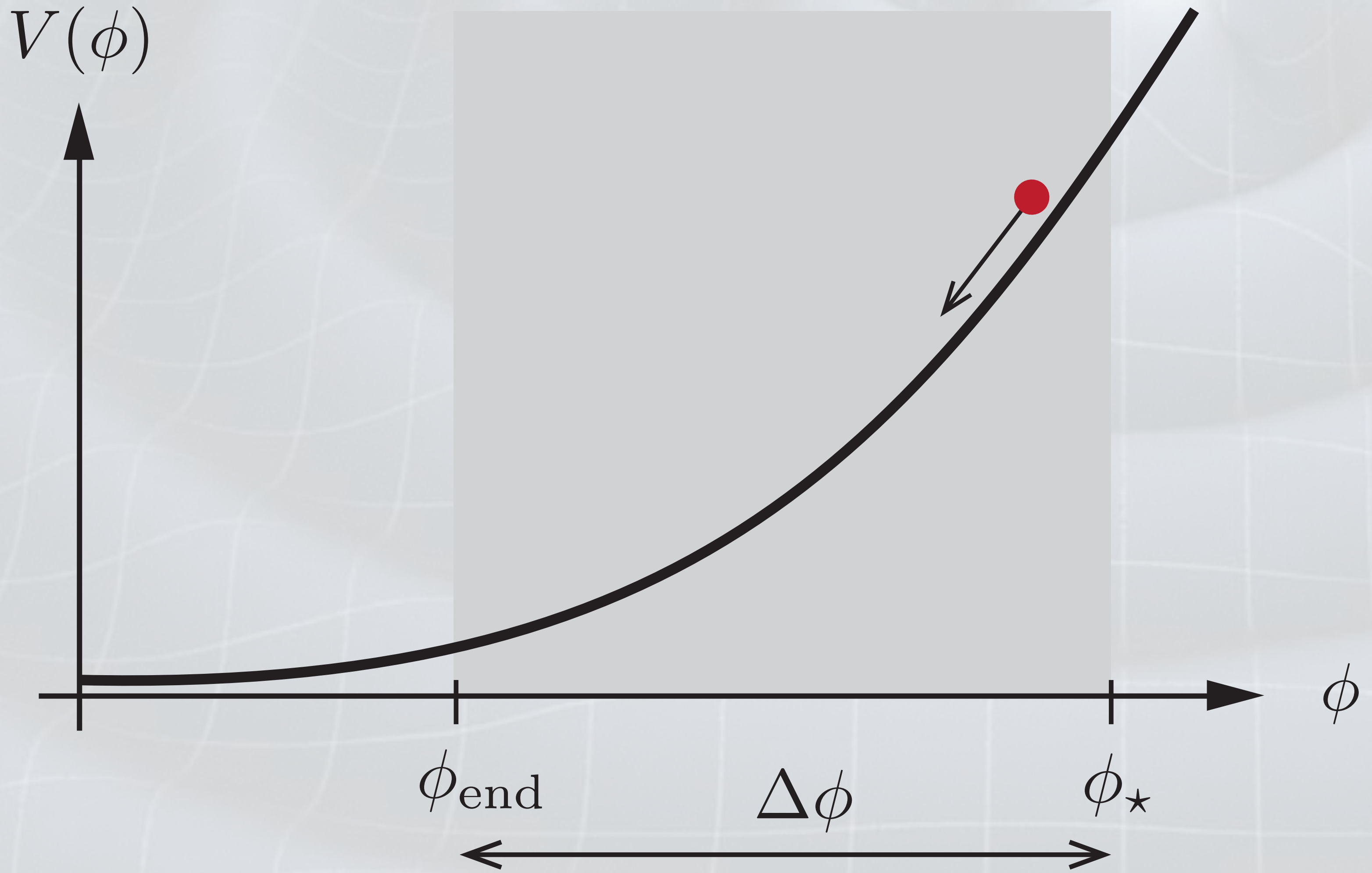


Veneziano (1991)



- Gasperini, Veneziano 2002

when Hot Big Bang starts



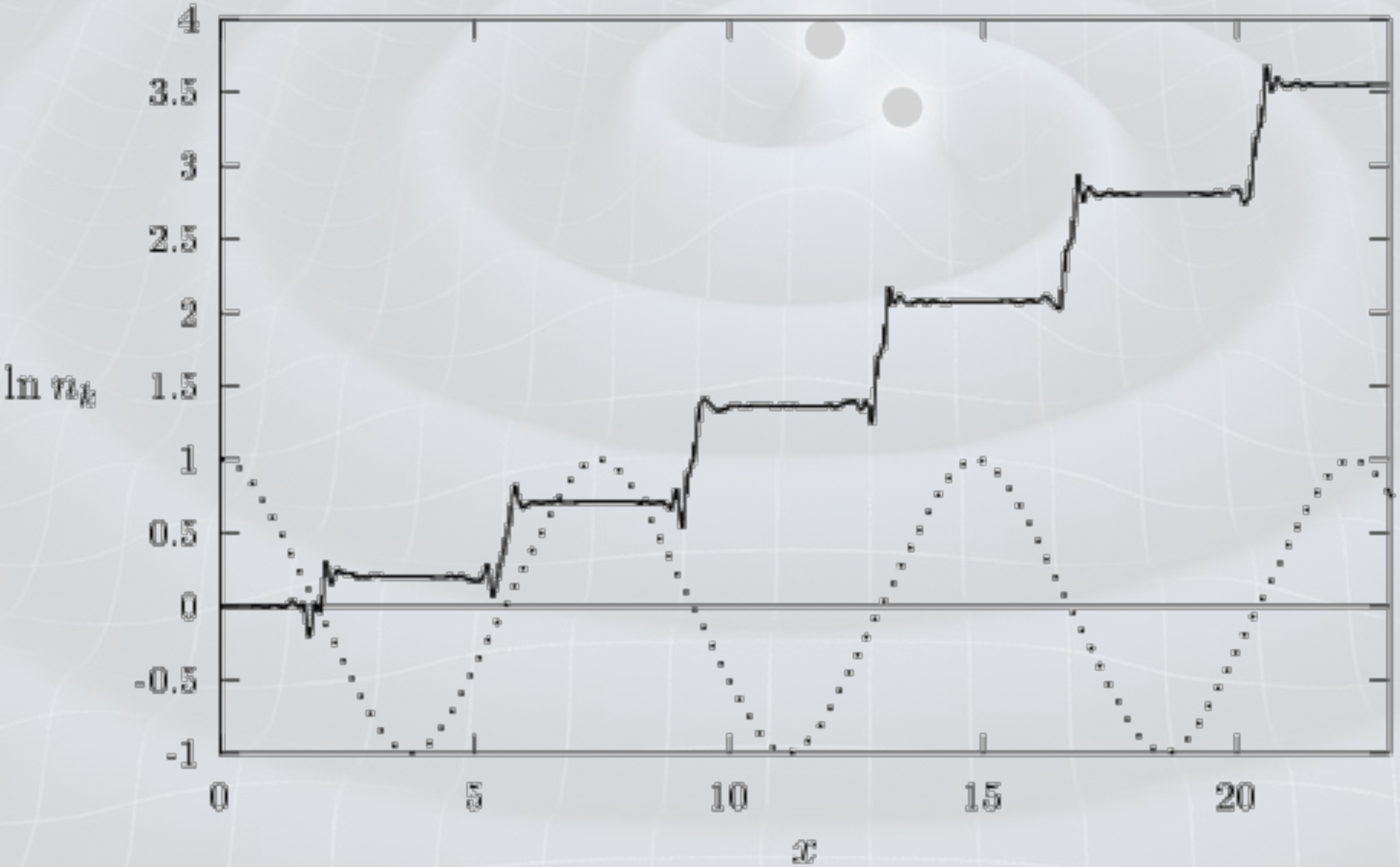
$$H^2 = \frac{1}{3M_p^2}\left[\frac{1}{2}\dot{\phi}^2 + V\right]$$

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

now include a matter field, χ

$$V(\phi,\chi) = \frac{\lambda}{4}\phi^4 + \frac{g^2}{2}\phi^2\chi^2$$

$$m_\chi^2\big|_{eff} = g^2\phi^2$$

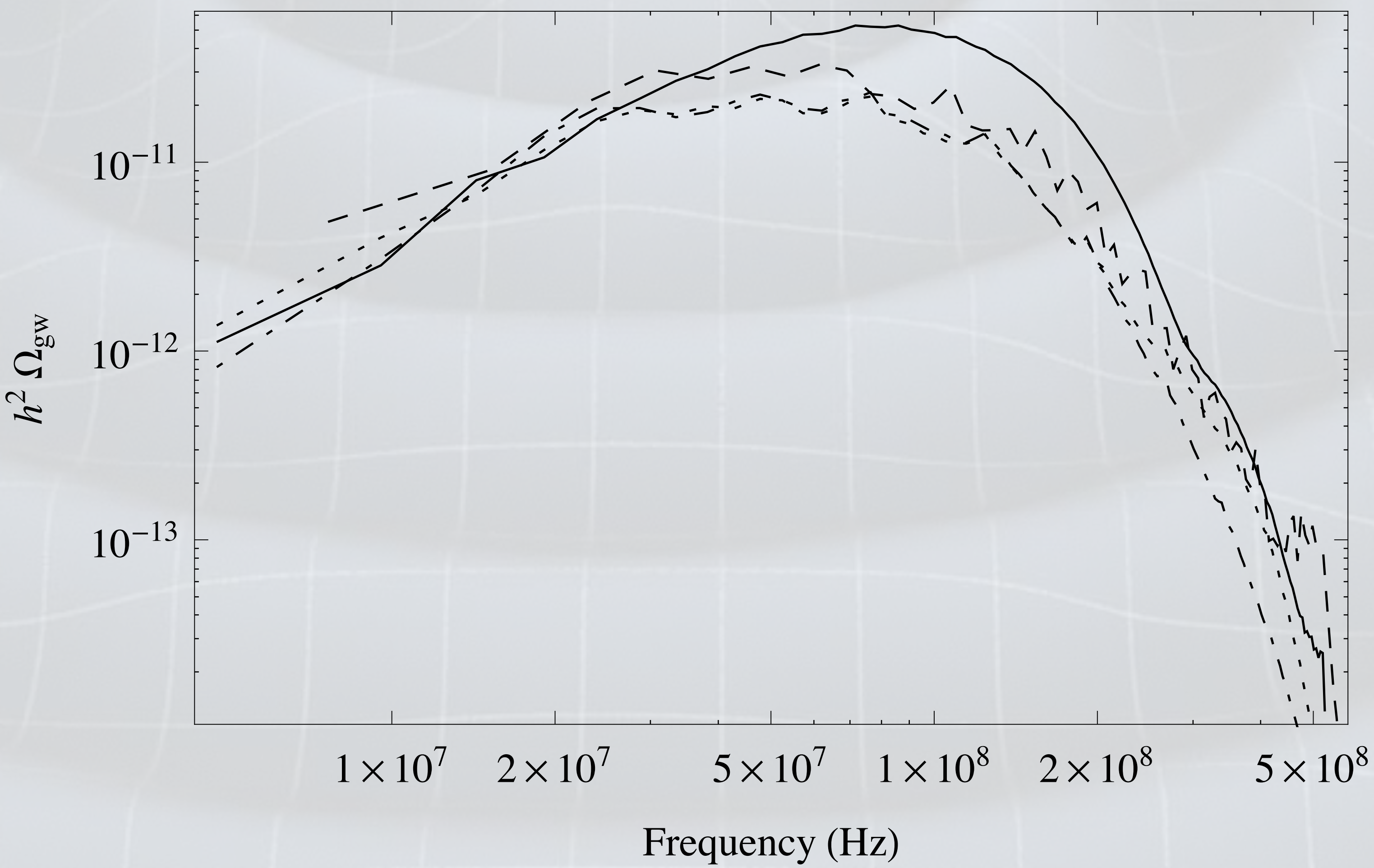


$$\frac{d^2 X_k}{d\tilde{t}^2} + \left(K^2 + q f^2\right) X_k = 0$$

$$K = \frac{k}{\sqrt{\lambda}\phi_0} \qquad q = \frac{g^2}{\lambda}$$

- Kofman, Linde, Starobinsky hep-th/9405187, hep-ph/9704452
- Greene, Kofman, Linde, Starobinsky hep-ph/9705347

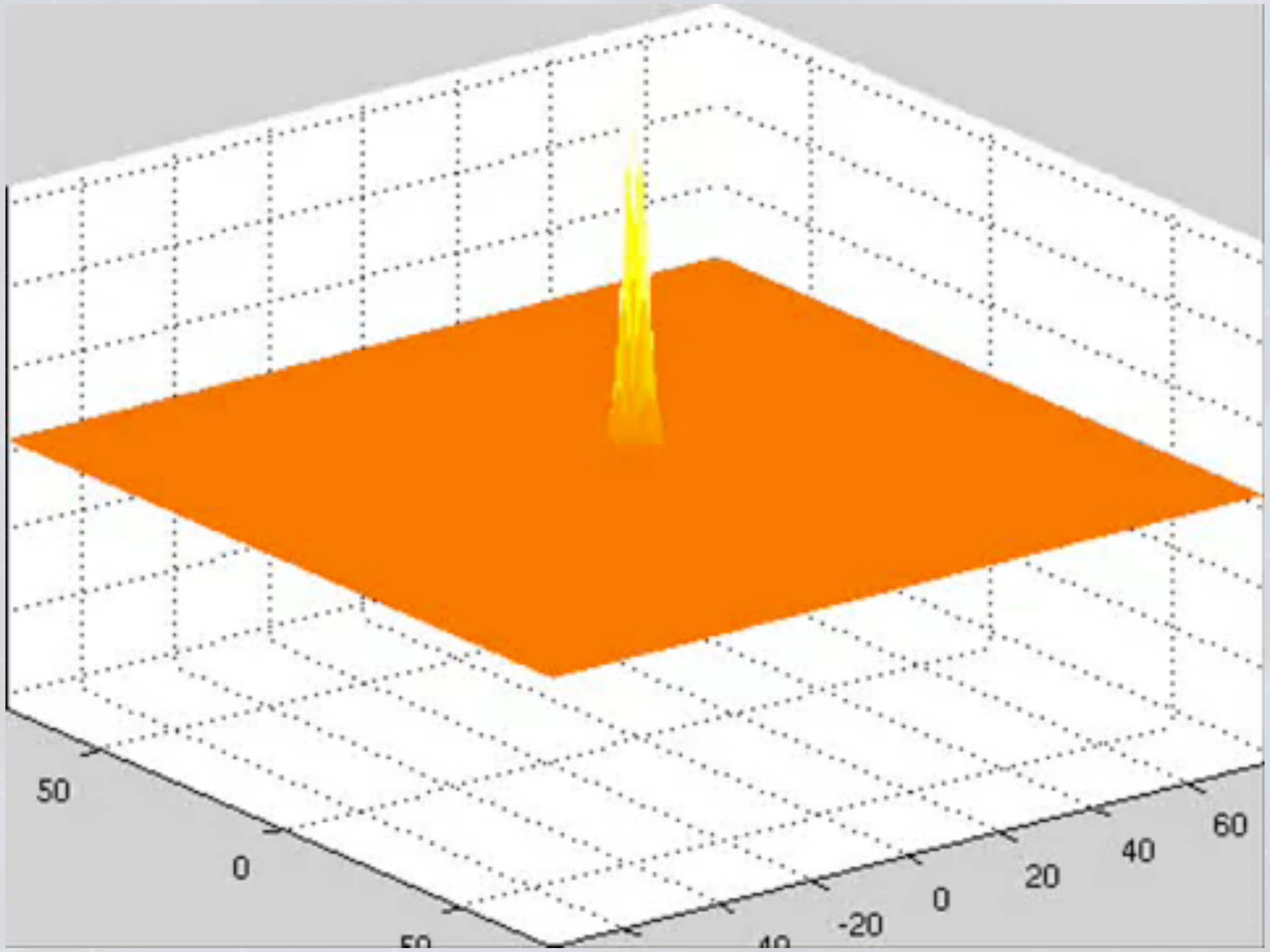
typical spectrum for parametric resonance, preheating



$$\lambda = 10^{-14},\ \phi_0 \sim 0.3 M_P,\ q = \frac{g^2}{\lambda} = 120$$

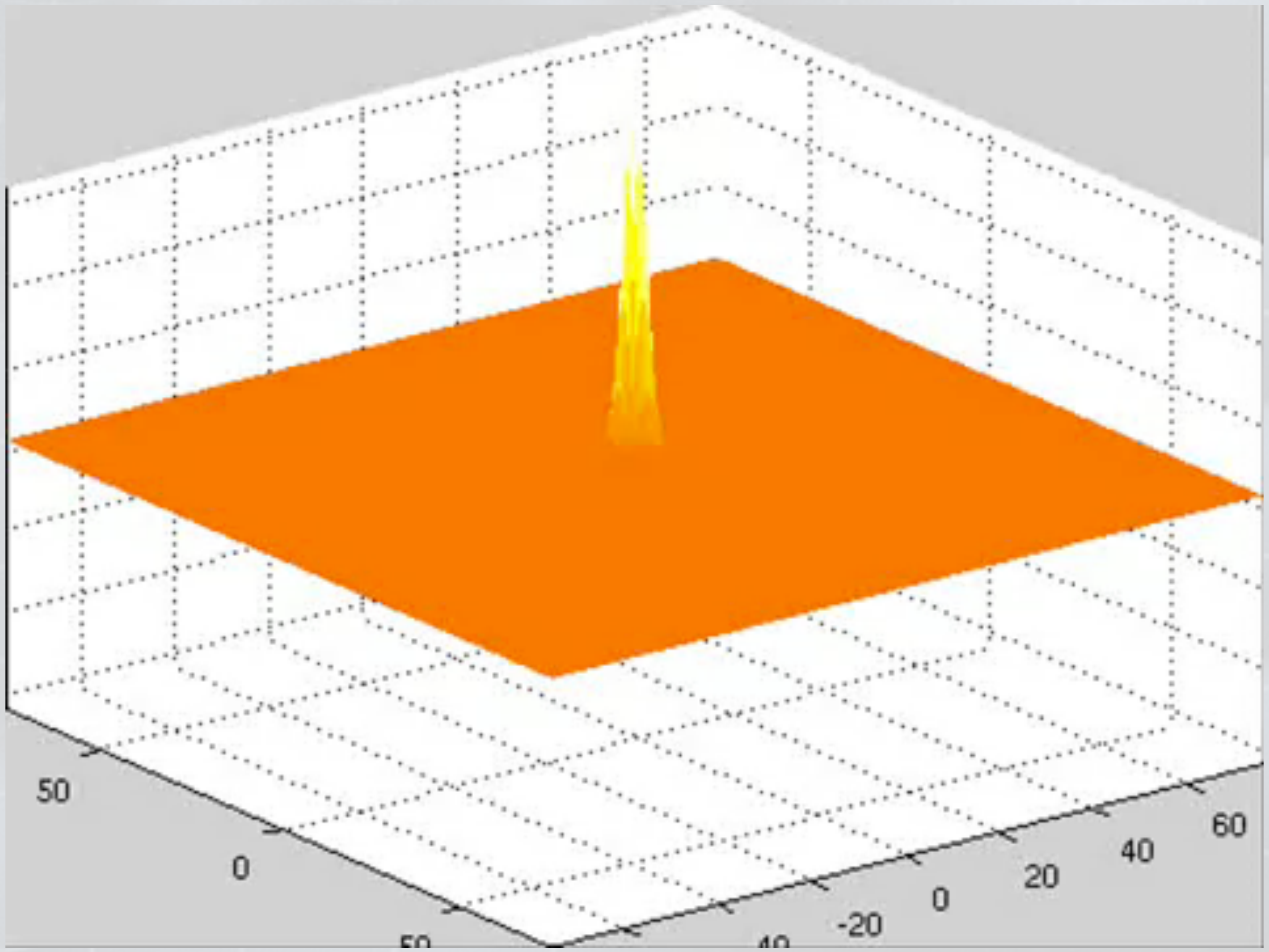
- Klebnikov, Tkachev hep-ph/9701423
- Dufaux, Bergman, Felder, Kofman, Uzan arXiv:0707.0875
- Easther, Giblin, Lim arXiv:0712.2991

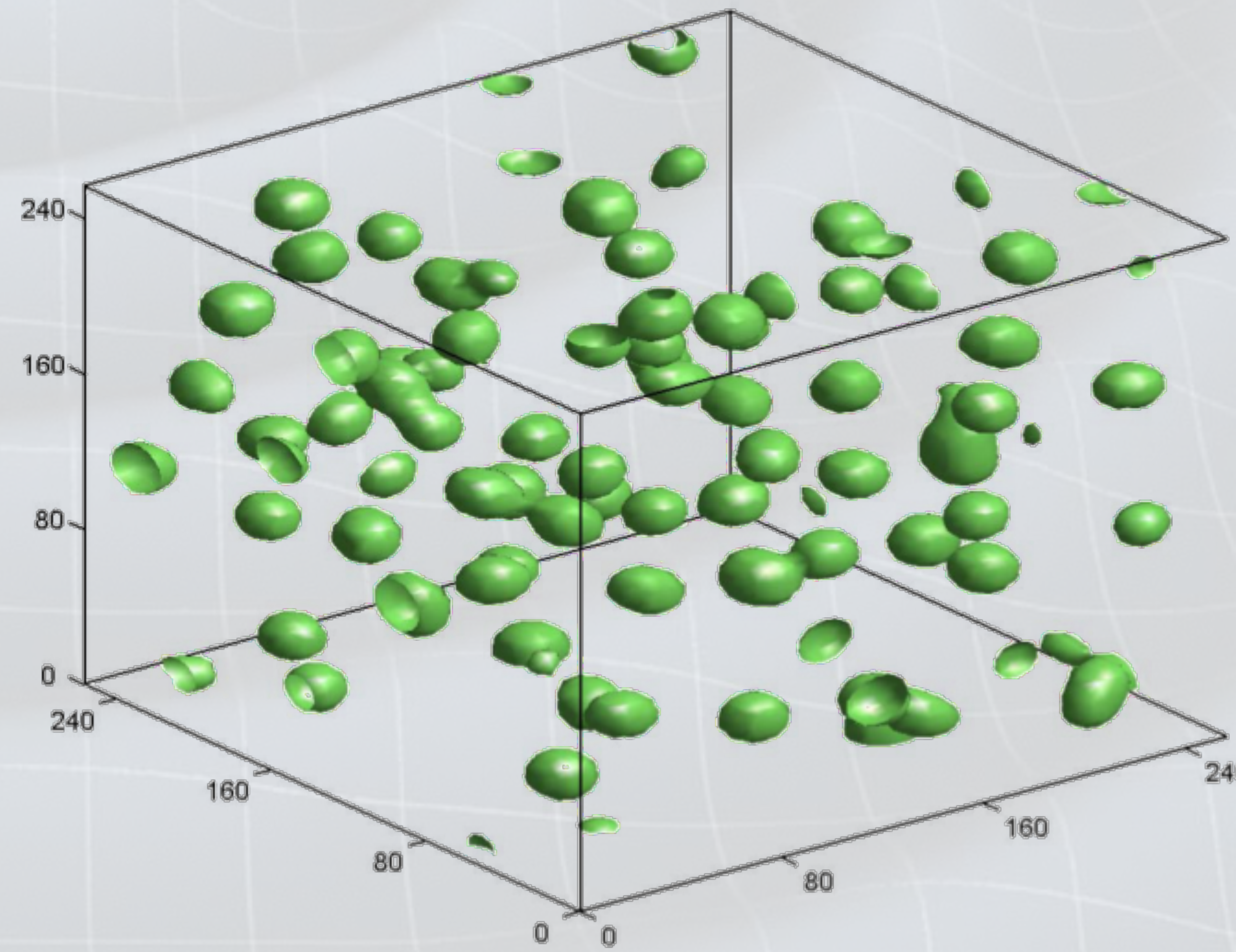
gravitational waves from oscillons



- Gleiser hep-ph/9308279
- Zhou, Copeland, Easther, Finkel, Mou, Saffin arXiv:1304.6094

gravitational waves from oscillons





$$V(\phi) = m^2 M^2 \left[\sqrt{1 + \phi^2/M^2} - 1\right]$$

$$V(\phi) : \frac{1}{2} m^2 \phi^2 \to m^2 M \phi$$

$$M = 10^{-2} M_P, \; m = 10^{-5} M_P$$

- Gleiser hep-ph/9308279
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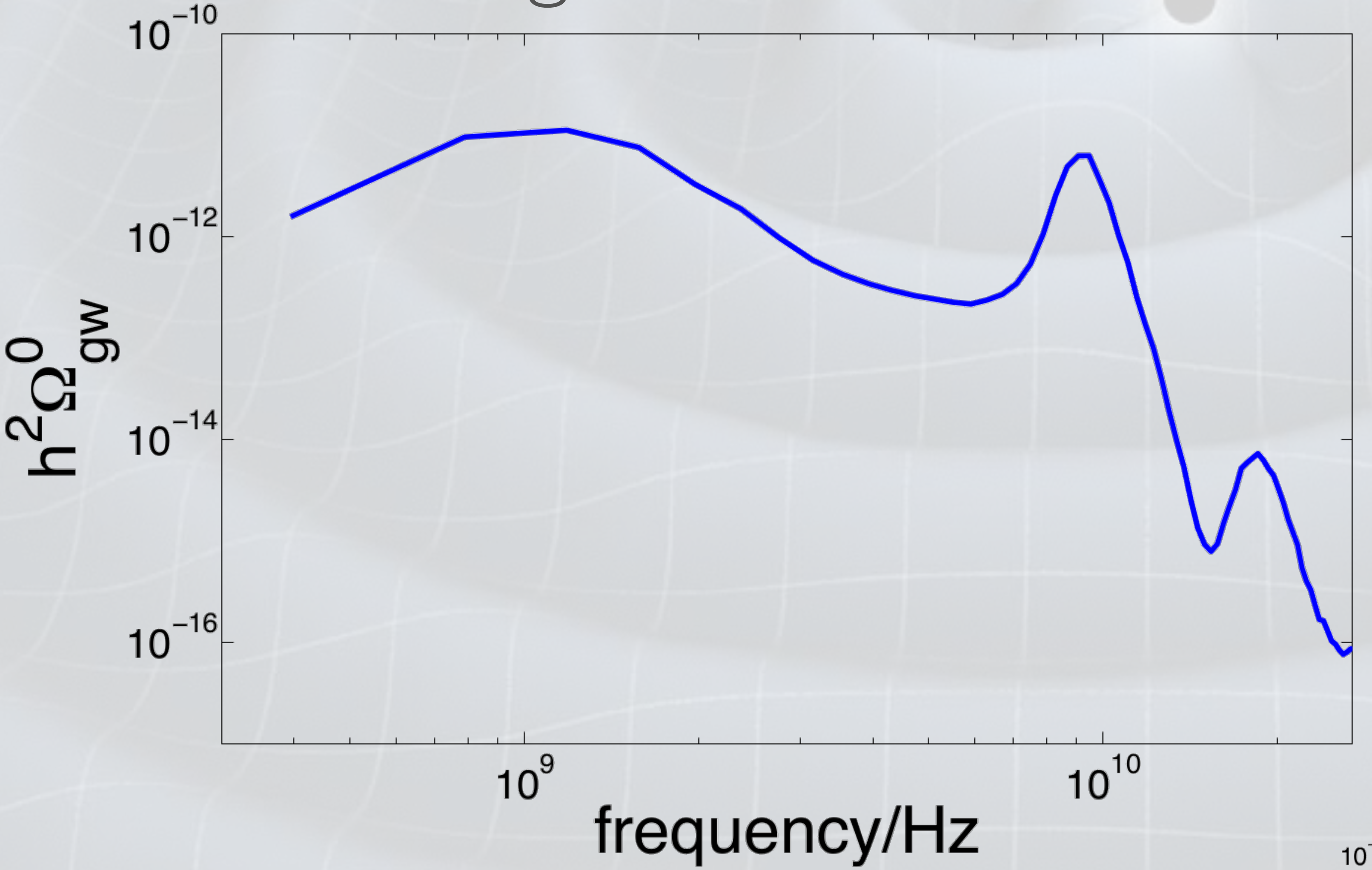
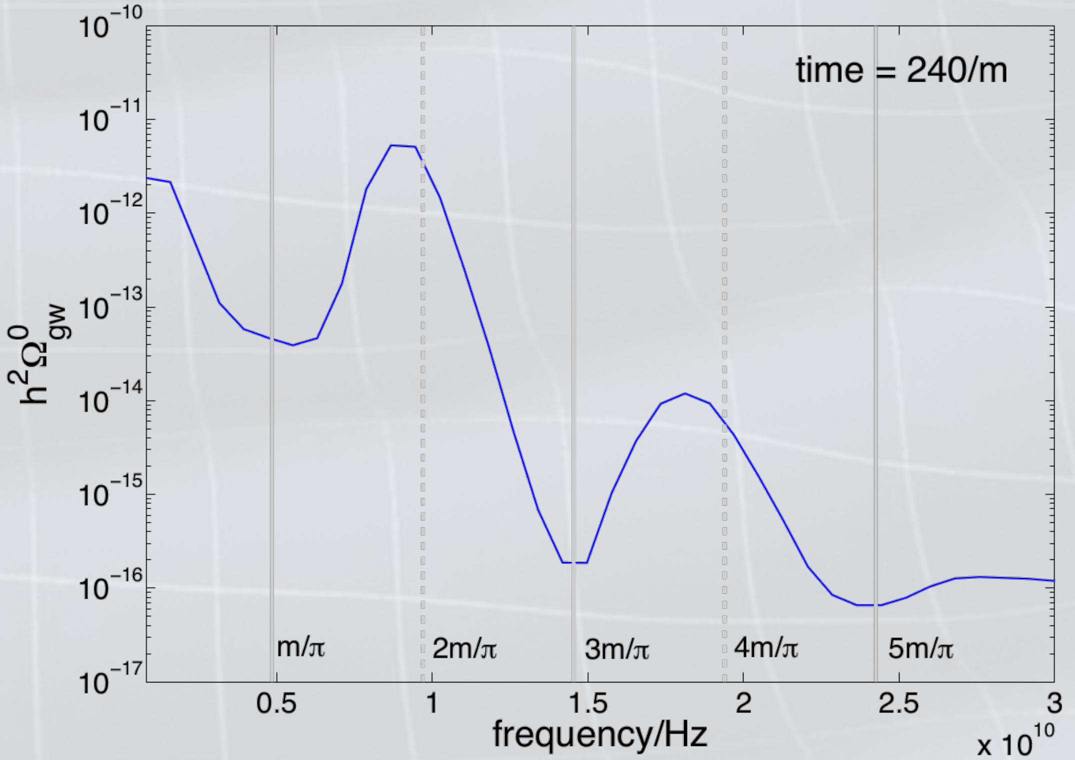
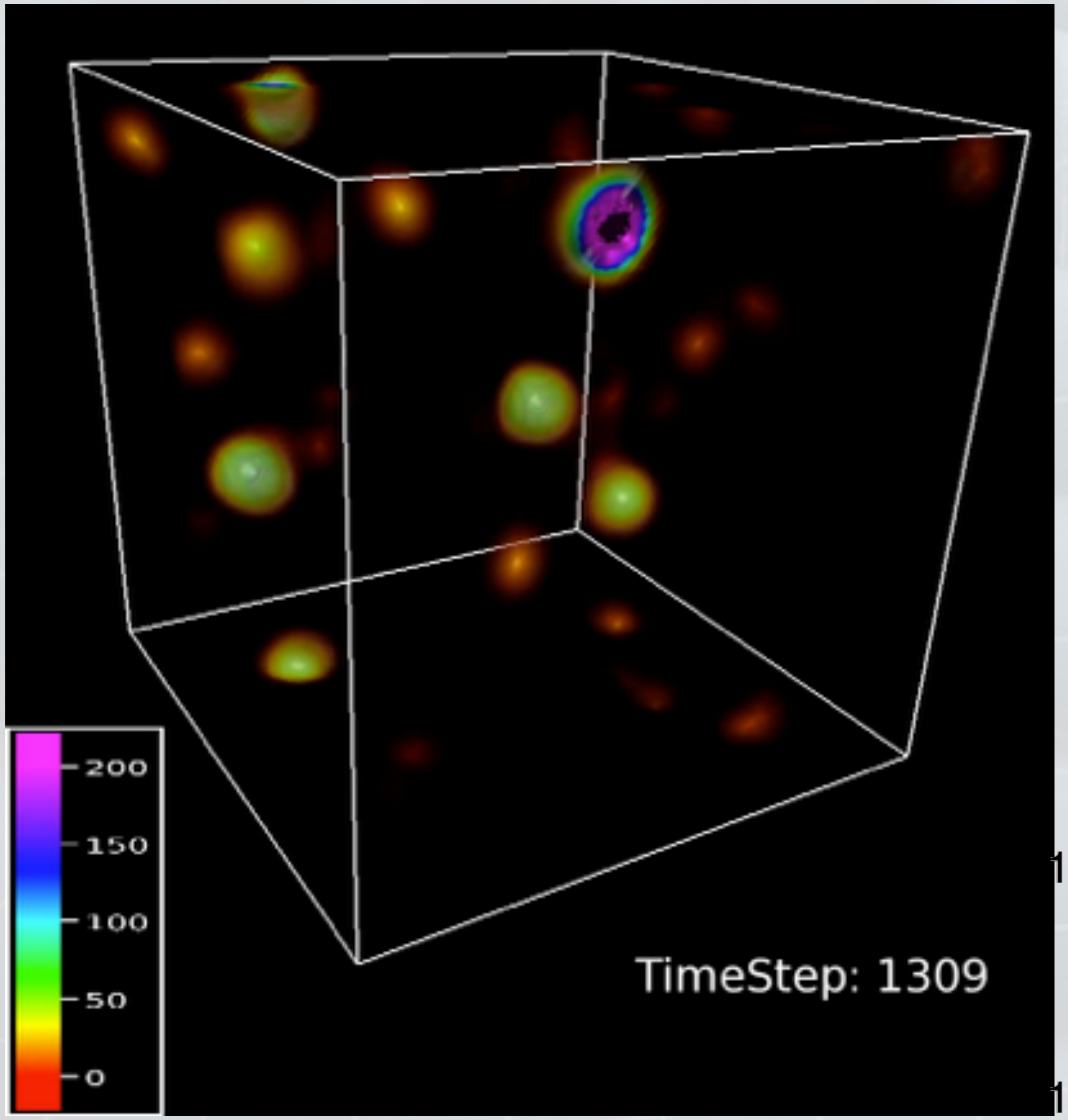
gravitational waves from oscillons

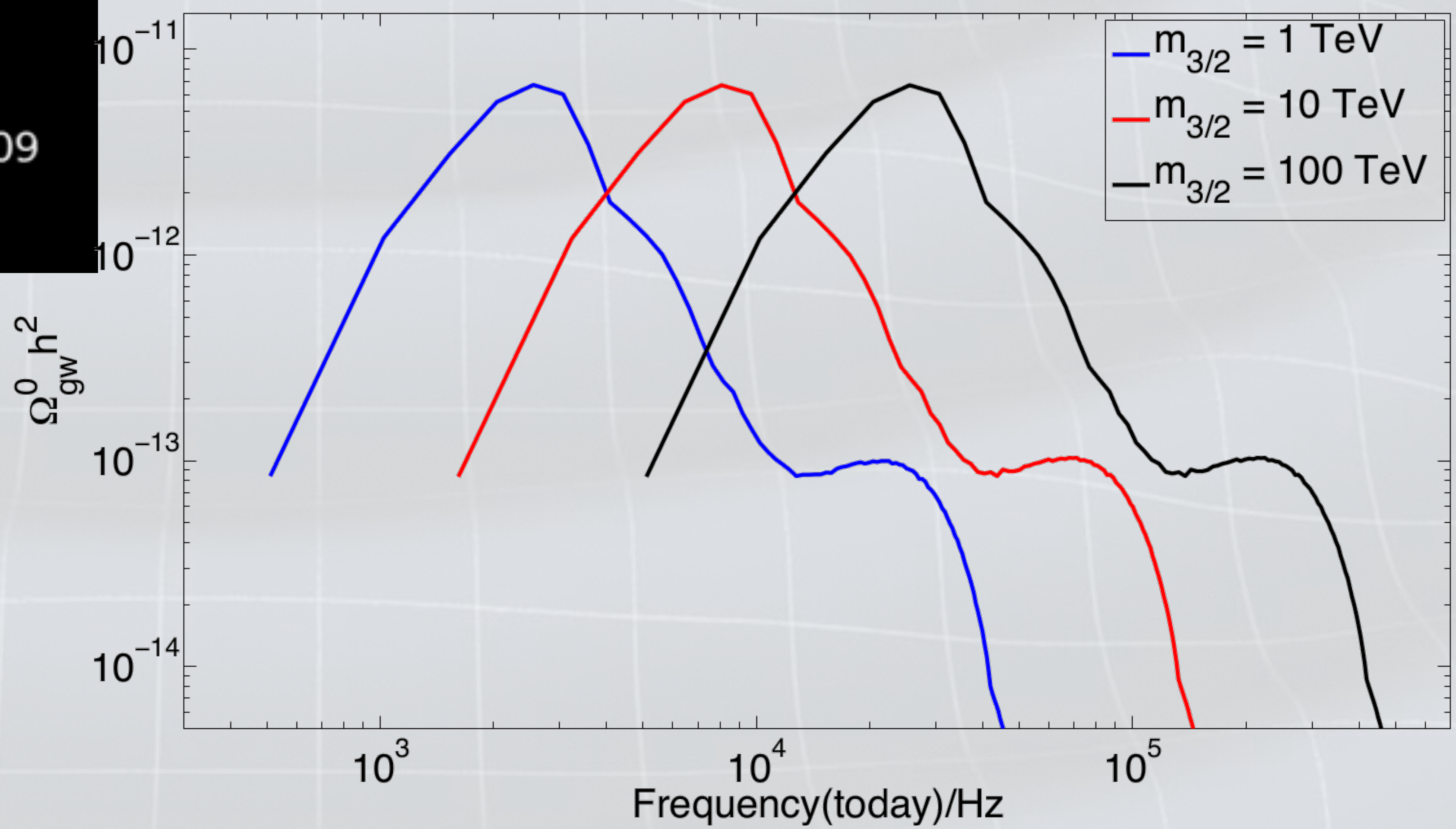




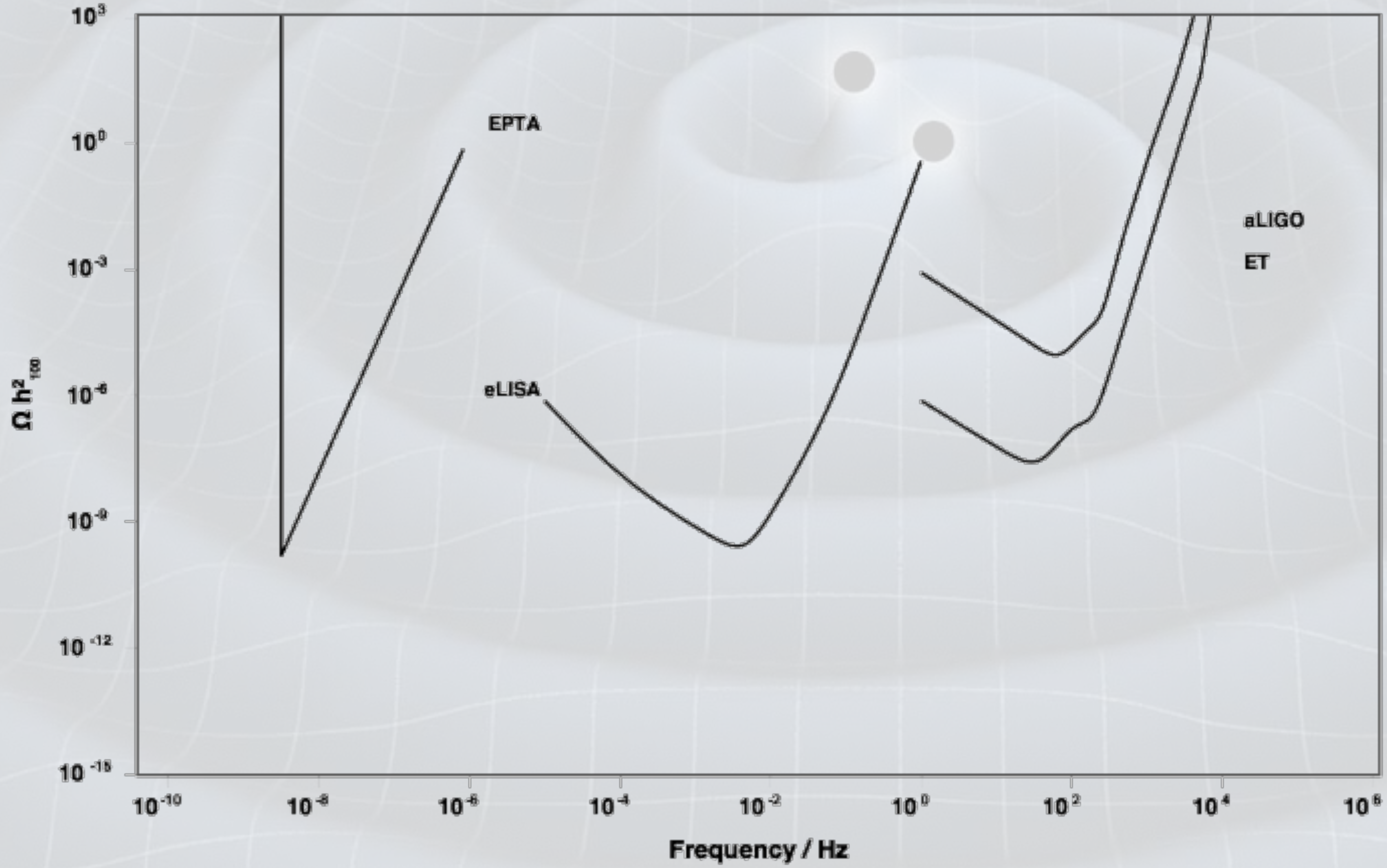
- Zhou, Copeland, Easther, Finkel, Mou, Saffin arXiv:1304.6094

gravitational waves from fragmenting condensates (Q-balls)





Zhou arXiv:1501.01217



rhcole.com/apps/GWplotter

high frequency detectors:

- Goryachev, Tobar - acoustic cavities, MHz-GHz, arXiv:1410.2334
- Arvanitaki, Geraci - optically levitated sensors, MHz, arXiv:1207.5320
- Cruise, Ingley - 100MHz, Class. Quant. Grav. 23
- INFN Genoa -
- Kawamura Japan - 100MHz
- www.GravWave.com

Conclusions

- we need to know the gravitational wave spectrum

Conclusions

- we need to know the gravitational wave spectrum
  - direct access to pre-hot-big-bang physics
  - spectrum not contaminated by scalars
  - discriminate between inflation and pre-big-bang
  - reheating (big bang) physics impacts the spectrum