Neutrino Oscillations

Inês Terrucha Marta Reis

Instituto Superior Técnico

Hands on Quantum Mechanics

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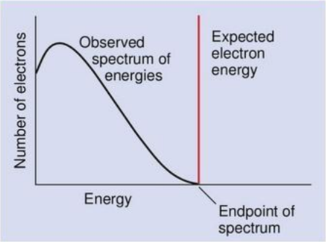
A little bit of History

1899 - Nuclear β decay was discovered by E.Rutherford

1991 - Lise Meitner and Otto Hahn performed an experiment showing that the energies of electrons emitted by beta decay had a continuous rather than discrete spectrum.



1930 - W.Pauli suggested that in addition to electrons and protons, it was also emited an extremely light neutral particle which he called the 'neutron'

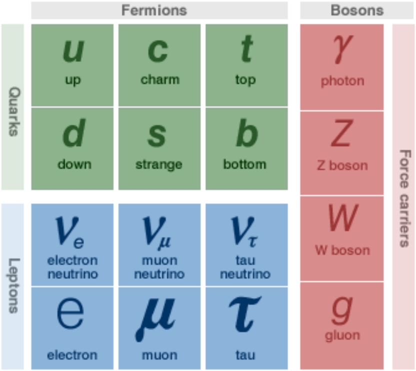
$$n \rightarrow p + e^{-} + \overline{\nu}_e$$

1932 - E.Chadwick discovered the neutron

1934 - Enrico Fermi renamed Pauli's 'neutron' to neutrino

1956 - Discovery of a particle fitting the expected characteristics of the neutrino is announced by Clyde Cowan and Fred Reines. This neutrino is later determined to be the partner of the electron
1962- Discovery of the muon neutrino at Brookhaven National Laboratory and CERN
1968 - Discovery of the electron neutrino produced by the Sun
1978 - Discovery of the tau lepton at SLAC, existence of tau neutrino theorized
2000 - Discovery of the tau neutrino by the DONUT collaboration

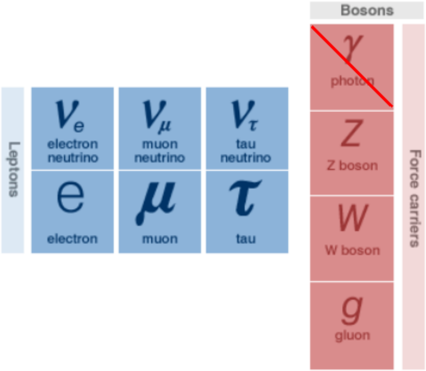
Neutrinos in the Standard Model



- Electrically neutral
- Half-integer spin

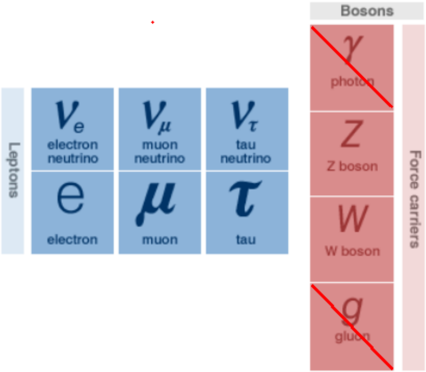
- Zero mass
- Come in three flavors
- Each flavor is also associated with an antiparticle, called an antineutrino, which also has no electric charge and half-integer spin
- All neutrinos are left-handed, and all antineutrinos are right-handed.

Neutrinos in the Standard Model



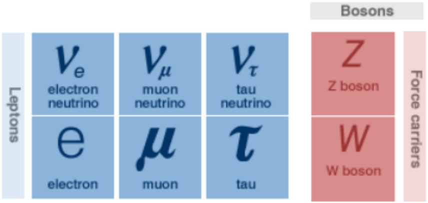
- Neutrinos do not carry any electric charge so they are not affected by the electromagnetic force that acts on charged particles

Neutrinos in the Standard Model



- Since they are leptons they are not affected by the strong force that acts on particles inside atomic nuclei

Neutrinos in the Standard Model

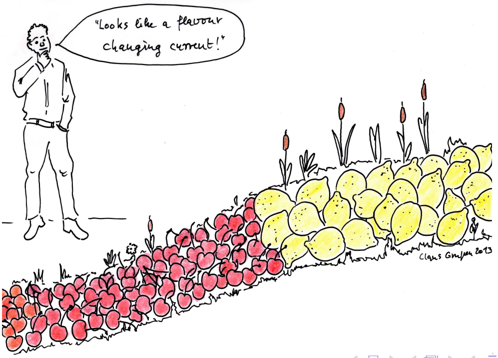


- Neutrinos are therefore affected only by the weak subatomic force.
- The weak force is a very short-range interaction. Thus, neutrinos typically pass through normal matter unimpeded and undetected.

Neutrino Oscillations

What is it?

- Neutrino oscillation is a quantum mechanical phenomenon whereby a neutrino created with a specific lepton flavor (electron, muon or tau) can later be measured to have a different flavor. The probability of measuring a particular flavor for a neutrino varies periodically as it propagates through space.



- Neutrino oscillation implies that the neutrino has a non-zero mass, which was not included as part of the original Standard Model of particle physics.

"Roger! I said oscillate—*not* osculate!"

Neutrino Oscillations

Why do we study it?

- Neutrinos travel with ultrarelativistic velocity and the oscillation lenght is very long. Using neutrino oscillation it is possible to study a very low mass scale regime which other experiments struggle to reach. The neutrino oscillations is not incorporated in the SM, it is expected that new physics will evolve from studies of neutrino oscillations.

The Infamous "Neutrino-burglar"

Neutrinos vs Antineutrinos

- The left-handed neutrinos α ($\alpha = e, \tau, \mu$) can be described in terms of the right-handed massive neutrinos k in a superposition through the contributions of the elements of the mixing matrix U:

$$|\nu_\alpha\rangle = \sum_k U^*_{\alpha k} |\nu_k\rangle \qquad (\alpha = e, \mu, \tau)$$

- Similarly, the antineutrinos with flavor α ($\alpha = e, \tau, \mu$) are superpositions of massive antineutrinos with weights proportional to the conjugated elements of the mixing matrix:

$$|\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle \qquad (\alpha = e, \mu, \tau)$$

Time evolution - Resolution of the Schrödinger Equation

- We want to know how our neutrinos and antineutrinos behave in time. We start by knowing how the massive neutrinos do, by solving the Schrödinger equation:

$$i \frac{\mathrm{d}}{\mathrm{d}t} |\nu_k(t)\rangle = \mathscr{H} |\nu_k(t)\rangle$$

- Knowing that the eigenvalues of H are:

$$E_k = \sqrt{\vec{p}^2 + m_k^2}$$

- We finally obtain:

$$|\nu_k(t)\rangle = e^{-iE_k t} |\nu_k\rangle$$

Time evolution - α in terms of β

We now want to write our left-handed neutrinos in terms of other left-handed neutrinos to understand the probability of one becoming another through time.

- We do that by making use of the unitarity relation:

$$U^\dagger U = \mathbf{1} \quad \Longleftrightarrow \quad \sum_\alpha U^*_{\alpha k} U_{\alpha j} = \delta_{jk}$$

- And with some mathematical manipulation...

$$|\nu_\alpha(t)\rangle = \sum_{\beta=e,\mu,\tau} \left(\sum_k U^*_{\alpha k}\, e^{-iE_k t}\, U_{\beta k} \right) |\nu_\beta\rangle$$

Transition Amplitude and Probability

After gathering these tools we can now finally calculate the amplitude and the corresponding transition probability of our flavor neutrinos:

$$A_{\nu_\alpha \to \nu_\beta}(t) \equiv \langle \nu_\beta | \nu_\alpha(t) \rangle = \sum_k U_{\alpha k}^* \, U_{\beta k} \, e^{-iE_k t}$$

$$P_{\nu_\alpha \to \nu_\beta}(t) = \left| A_{\nu_\alpha \to \nu_\beta}(t) \right|^2 = \sum_{k,j} U_{\alpha k}^* \, U_{\beta k} \, U_{\alpha j} \, U_{\beta j}^* \, e^{-i(E_k - E_j)t} \, .$$

Oscillation Phase

Because neutrinos and antineutrinos are ultrarelativistic particles, there are some approximations we can make to facilitate in terms of experimental quantities.

- Our dispersion relation can be approximated to: $E_k \simeq E + \frac{m_k^2}{2E}$ in which case we end up with: $E_k - E_j \simeq \frac{\Delta m_{kj}^2}{2E}$
- The light-ray approximation lets us do: $t = L$

Which gives us an oscillation phase:

$$\Phi_{kj} = -\frac{\Delta m_{kj}^2 \, L}{2 \, E}$$

Oscillation Probabilities: Neutrino vs antineutrino

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \sum_{k,j} U^*_{\alpha k}\, U_{\beta k}\, U_{\alpha j}\, U^*_{\beta j}\, \exp\left(-i\,\frac{\Delta m^2_{kj} L}{2E}\right)$$

vs.

$$P_{\bar\nu_\alpha \to \bar\nu_\beta}(L,E) = \sum_{k,j} U_{\alpha k}\, U^*_{\beta k}\, U^*_{\alpha j}\, U_{\beta j}\, \exp\left(-i\,\frac{\Delta m^2_{kj} L}{2E}\right)$$

Other convenient ways of writing the probabilities...

More mathematical manipulation step-by-step:

1. Separate the sums into $\sum_{k=j}$ and $2 \times \sum_{k>j}$;
2. Eliminate the imaginary part of the second term:

$$\sum_{k \neq j} \Im\mathfrak{m}\left[U^*_{\alpha k}\, U_{\beta k}\, U_{\alpha j}\, U^*_{\beta j}\right] = 0 \qquad (\alpha \neq \beta)$$

3. Use the unitarity relation: $U\,U^\dagger = \mathbf{1} \quad \Longleftrightarrow \quad \sum_k U_{\alpha k}\, U^*_{\beta k} = \delta_{\alpha\beta}$ to write:

$$\sum_k |U_{\alpha k}|^2\, |U_{\beta k}|^2 = \delta_{\alpha\beta} - 2 \sum_{k>j} \Re\mathfrak{e}\left[U^*_{\alpha k}\, U_{\beta k}\, U_{\alpha j}\, U^*_{\beta j}\right]$$

Oscillation Probabilities: Neutrino vs antineutrino - Reformulated

We are left with:

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \delta_{\alpha\beta} - 2\sum_{k>j} \mathfrak{Re}\left[U^*_{\alpha k}\, U_{\beta k}\, U_{\alpha j}\, U^*_{\beta j}\right] \left[1 - \cos\left(\frac{\Delta m^2_{kj} L}{2E}\right)\right]$$
$$+ 2\sum_{k>j} \mathfrak{Im}\left[U^*_{\alpha k}\, U_{\beta k}\, U_{\alpha j}\, U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right) ,$$

Which after some trigonometric changes becomes...

Oscillation Probabilities: Neutrino vs antineutrino

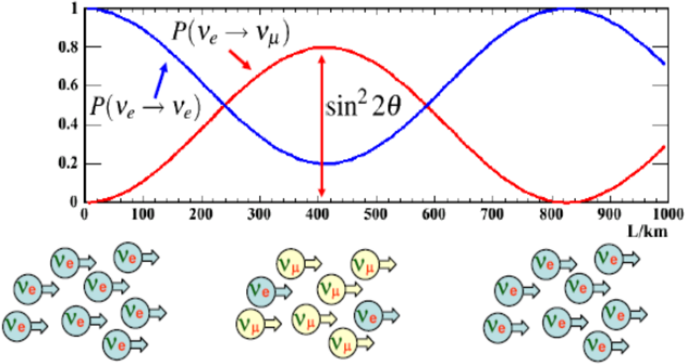
$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \delta_{\alpha\beta} - 4 \sum_{k>j} \Re\mathfrak{e}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin^2\left(\frac{\Delta m^2_{kj} L}{4E}\right)$$
$$+ 2 \sum_{k>j} \Im\mathfrak{m}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)$$

and

$$P_{\bar\nu_\alpha \to \bar\nu_\beta}(L,E) = \delta_{\alpha\beta} - 4 \sum_{k>j} \Re\mathfrak{e}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin^2\left(\frac{\Delta m^2_{kj} L}{4E}\right)$$
$$- 2 \sum_{k>j} \Im\mathfrak{m}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)$$

Oscillation Probabilities: a visual example

•e.g. $\Delta m^2 = 0.003\,\mathrm{eV}^2, \quad \sin^2 2\theta = 0.8, \quad E_\nu = 1\,\mathrm{GeV}$



The End!

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