Oscilação de neutrinos

2ª Apresentação – Parte 1

Modelo de dois neutrinos aplicado ao caso geral

HonQM 2015

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Matrizes de Mistura

3 Neutrinos

$$U = VQ\,, \quad Q = \mathrm{diag}\left(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}\right)$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

2 Neutrinos

$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

Large Δm^2 dominance

$$P^{\text{eff}}_{\nu_\alpha \to \nu_\beta}(L,E) = \delta_{\alpha\beta} - 4 \left[\delta_{\alpha\beta} \widetilde{\sum_k} |U_{\alpha k}^*|^2 - \left| \widetilde{\sum_k} U_{\alpha k}^* U_{\beta k} \right|^2 \right] \sin^2\left(\frac{\Delta m^2_{N1} L}{4E} \right)$$

Active small Δm^2

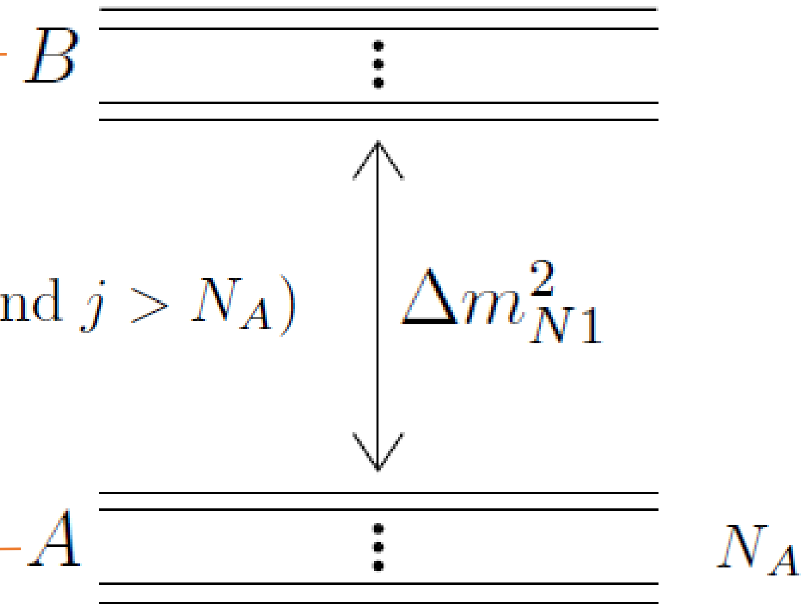
$$\begin{aligned} P^{\text{eff}}_{\nu_\alpha \to \nu_\beta}(L,E) = \delta_{\alpha\beta} \\ & - 2 \left(\sum_{j \leq N_{A_1}} \sum_{k=N_{A_1}+1}^{N_A} \mathfrak{Re}\left[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \right] \right) \left[1 - \cos\left(\frac{\Delta m^2_{N_A 1} L}{2E} \right) \right] \\ & + 2 \left(\sum_{j \leq N_{A_1}} \sum_{k=N_{A_1}+1}^{N_A} \mathfrak{Im}\left[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \right] \right) \sin\left(\frac{\Delta m^2_{N_A 1} L}{2E} \right) \\ & - 2 \sum_{j \leq N_A} \sum_{k > N_A} \mathfrak{Re}\left[U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \right] . \end{aligned} \quad (7.145)$$

Δm^2 grande dominante

$|\Delta m^2_{N1}| \gg |\Delta m^2_{kj}| \quad \text{for} \; k, j > N_A$

$|\Delta m^2_{kj}| \simeq |\Delta m^2_{N1}| \quad \text{for} \quad (k \leq N_A \text{ and } j > N_A)$

$|\Delta m^2_{N1}| \gg |\Delta m^2_{kj}| \quad \text{for} \quad k, j \leq N_A$



$\nu_1, \ldots, \nu_{N_A} \in A \text{ and } \nu_{N_A+1}, \ldots, \nu_N \in B$

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \left| \sum_k U_{\alpha k}^* U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E}\right) \right|^2$$

$$||$$

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \left| \sum_{k \leq N_A} U_{\alpha k}^* U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E}\right) \right.$$

$$\left. + \sum_{k > N_A} U_{\alpha k}^* U_{\beta k} \exp\left(-i \frac{\Delta m_{k1}^2 L}{2E}\right) \right|^2$$

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \left| \sum_{k \leq N_A} U^*_{\alpha k} U_{\beta k} \exp\left(-i \frac{\Delta m^2_{k1} L}{2E}\right) + \sum_{k > N_A} U^*_{\alpha k} U_{\beta k} \exp\left(-i \frac{\Delta m^2_{k1} L}{2E}\right) \right|^2$$

$$=$$

$$P^{\text{eff}}_{\nu_\alpha \to \nu_\beta}(L, E) = \left| \sum_{k \leq N_A} U^*_{\alpha k} U_{\beta k} + \exp\left(-i \frac{\Delta m^2_{N1} L}{2E}\right) \sum_{k > N_A} U^*_{\alpha k} U_{\beta k} \right|^2$$

$$+ \sum_{k > N_a} U^*_\alpha U_\beta - \sum_{k > N_a} U^*_\alpha U_\beta$$

$$\sum_{k \leq N_a} U^*_\alpha U_\beta + \sum_{k > N_a} U^*_\alpha U_\beta = \sum_{k=1}^{N} U^*_\alpha U_\beta = \delta_{\alpha\beta}$$

$$P^{\text{eff}}_{\nu_\alpha \to \nu_\beta}(L, E) = \left| \delta_{\alpha\beta} - \left[1 - \exp\left(-i \frac{\Delta m^2_{N1} L}{2E}\right)\right] \widetilde{\sum_k} U^*_{\alpha k} U_{\beta k} \right|^2$$

$$P^{\text{eff}}_{\nu_\alpha\to\nu_\beta}(L,E) = \delta_{\alpha\beta} - 4\left[\delta_{\alpha\beta}\widetilde{\sum_k}|U^*_{\alpha k}|^2 - \left|\widetilde{\sum_k} U^*_{\alpha k}\,U_{\beta k}\right|^2\right]\sin^2\left(\frac{\Delta m^2_{N1}L}{4E}\right)$$

$\alpha \neq \beta$

$$P^{\text{eff}}_{\nu_\alpha\to\nu_\beta}(L,E) = 4\left|\widetilde{\sum_k} U^*_{\alpha k}\,U_{\beta k}\right|^2 \sin^2\left(\frac{\Delta m^2_{N1}L}{4E}\right)$$

$$P^{\text{eff}}_{\nu_\alpha\to\nu_\beta}(L,E) = \sin^2 2\vartheta^{\text{eff}}_{\alpha\beta}\,\sin^2\left(\frac{\Delta m^2_{N1}L}{4E}\right)$$

$$\sin\vartheta^{\text{eff}}_{\alpha\beta} = \frac{1}{\sqrt{2}}\left(1 \pm \sqrt{1 - 4\left|\widetilde{\sum_k} U^*_{\alpha k}\,U_{\beta k}\right|^2}\right)^{1/2}$$

$\alpha = \beta$

$$P^{\text{eff}}_{\nu_\alpha\to\nu_\alpha}(L,E) = 1 - 4\left(\widetilde{\sum_k}|U_{\alpha k}|^2\right)\left(1 - \widetilde{\sum_k}|U_{\alpha k}|^2\right)\sin^2\left(\frac{\Delta m^2_{N1}L}{4E}\right)$$

$$P^{\text{eff}}_{\nu_\alpha\to\nu_\alpha}(L,E) = 1 - \sin^2 2\vartheta^{\text{eff}}_{\alpha\alpha}\,\sin^2\left(\frac{\Delta m^2_{N1}L}{4E}\right)$$

$$\sin^2 2\vartheta^{\text{eff}}_{\alpha\alpha} = 4\left(\widetilde{\sum_k}|U_{\alpha k}|^2\right)\left(1 - \widetilde{\sum_k}|U_{\alpha k}|^2\right)$$

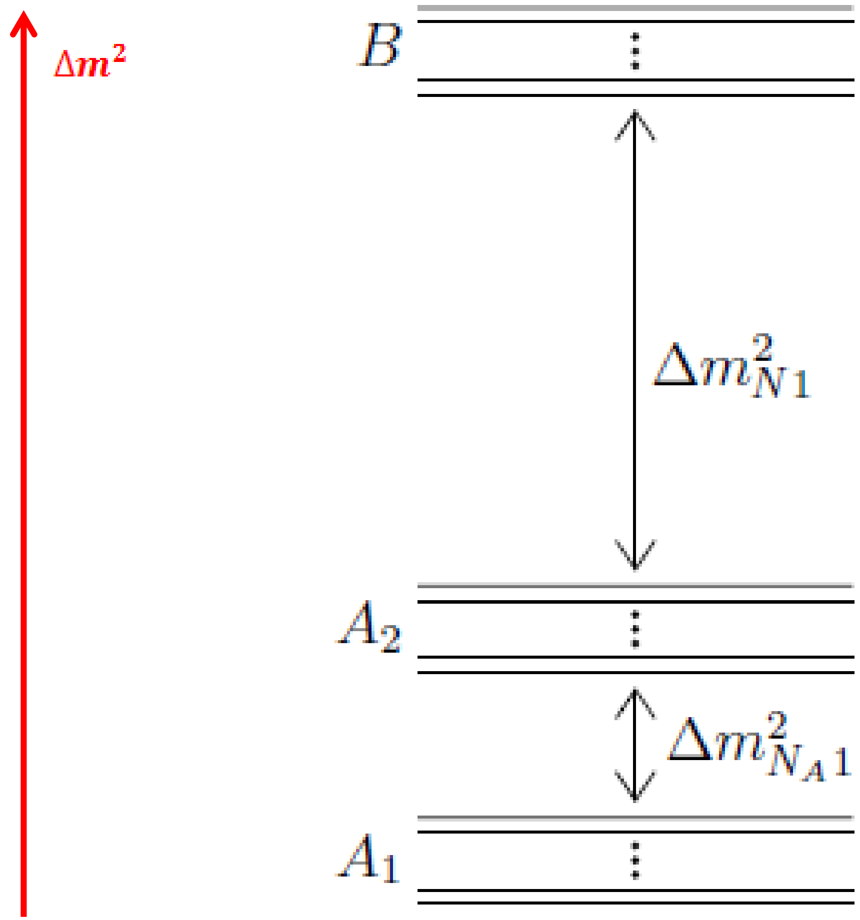
$$\sin\vartheta_{\alpha\beta}^{\text{eff}} = \frac{1}{\sqrt{2}}\left(1 \pm \sqrt{1 - 4\left|\widetilde{\sum_k} U_{\alpha k}^* U_{\beta k}\right|^2}\right)^{1/2}$$

$$\sin^2 2\vartheta_{\alpha\alpha}^{\text{eff}} = 4\left(\widetilde{\sum_k}|U_{\alpha k}|^2\right)\left(1 - \widetilde{\sum_k}|U_{\alpha k}|^2\right)$$

$$\sin^2 2\vartheta_{\alpha\beta}^{\text{eff}} = 4\,|U_{\alpha N}|^2\,|U_{\beta N}|^2 \quad (\alpha \neq \beta)\,,$$

$$\sin^2 2\vartheta_{\alpha\alpha}^{\text{eff}} = 4\,|U_{\alpha N}|^2\left(1 - |U_{\alpha N}|^2\right)$$

Agora com três grupos



Calcular probabilidades

$$
\begin{aligned}
P^{\text{eff}}_{\nu_\alpha \to \nu_\beta}(L,E) = \delta_{\alpha\beta} \\
& - 2 \left(\sum_{j \le N_{A_1}} \sum_{k = N_{A_1}+1}^{N_A} \Re\mathfrak{e}\left[U^*_{\alpha k} \, U_{\beta k} \, U_{\alpha j} \, U^*_{\beta j} \right] \right) \left[1 - \cos\left(\frac{\Delta m^2_{N_A 1} L}{2E} \right) \right] \\
& + 2 \left(\sum_{j \le N_{A_1}} \sum_{k = N_{A_1}+1}^{N_A} \Im\mathfrak{m}\left[U^*_{\alpha k} \, U_{\beta k} \, U_{\alpha j} \, U^*_{\beta j} \right] \right) \sin\left(\frac{\Delta m^2_{N_A 1} L}{2E} \right) \\
& - 2 \sum_{j \le N_A} \sum_{k > N_A} \Re\mathfrak{e}\left[U^*_{\alpha k} \, U_{\beta k} \, U_{\alpha j} \, U^*_{\beta j} \right] .
\end{aligned}
$$

Calcular probabilidades

Sobrevivência (α=β):

$$P^{\text{eff}}_{\nu_\alpha \to \nu_\alpha}(L,E) = \left(1 - \sum_{k>N_A} |U_{\alpha k}|^2\right)^2 P^{(N_A,1)}_{\nu_\alpha \to \nu_\alpha}(L,E) + \left(\sum_{k>N_A} |U_{\alpha k}|^2\right)^2$$

$$P^{(N_A,1)}_{\nu_\alpha \to \nu_\alpha}(L,E) = 1 - \sin^2 2\vartheta^{\text{eff}}_{\alpha\alpha} \, \sin^2\left(\frac{\Delta m^2_{N_A 1} L}{4E}\right)$$

Semelhante à oscilação de dois neutrinos!

Calcular probabilidades

O ângulo efectivo é dado por:

$$\sin^2 2\vartheta_{\alpha\alpha}^{\text{eff}} = 4\,\frac{\left(\sum_{k\leq N_{A_1}} |U_{\alpha k}|^2\right)\left(\sum_{k=N_{A_1}+1}^{N_A} |U_{\alpha k}|^2\right)}{\left(\sum_{k=1}^{N_A} |U_{\alpha k}|^2\right)^2}$$

Vamos fazer uma verificação à probabilidade calculada

Calcular probabilidades

$$P^{\text{eff}}_{\nu_\alpha \to \nu_\alpha}(L,E) = \left(1 - \sum_{k>N_A} |U_{\alpha k}|^2\right)^2 P^{(N_A,1)}_{\nu_\alpha \to \nu_\alpha}(L,E) + \left(\sum_{k>N_A} |U_{\alpha k}|^2\right)^2$$

Imaginemos que o grupo B deixa de existir…

Oscilação tipo dois neutrinos!

Calcular probabilidades

Transição (α≠β):

$$P^{\text{eff}}_{\nu_\alpha \to \nu_\beta}(L,E) = 4\left|\sum_{k \le N_{A_1}} U^*_{\alpha k} U_{\beta k}\right|^2 \sin^2\left(\frac{\Delta m^2_{N_A 1} L}{4E}\right)$$

$$+ 4\left(\sum_{j \le N_{A_1}} \sum_{k > N_A} \mathfrak{Re}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right]\right) \sin^2\left(\frac{\Delta m^2_{N_A 1} L}{4E}\right)$$

$$+ 2\left(\sum_{j \le N_{A_1}} \sum_{k > N_A} \mathfrak{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right]\right) \sin\left(\frac{\Delta m^2_{N_A 1} L}{2E}\right)$$

$$+ 2\left|\sum_{k > N_A} U^*_{\alpha k} U_{\beta k}\right|^2 .$$