With LHC, Particle Physics is entering a New Golden Age

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Open, Fundamental Questions in Particle Physics

- What is the Origin of Matter?
  What happened to the anti-matter?
- How to understand the mass spectrum of quarks and leptons?
- How to understand the replication of Fermion families?
- The Higgs sector? Multiple Higgs?

- CP Violation?
- Dark Matter?
- How to combine Quantum Mechanics with Gravitation?
- Grand unification?
- Proton decay?
- Neutrinos? Why are the masses so low? Why is leptonic mixing large, in contrast with small quark mixing?

A descoberta da <u>Mecânica Quântica</u>
foi a maior revolução na
Ciência, no Século XX <u>e uma</u>
<u>das maiores descobertas em Ciência,</u>
<u>de todos os tempos.</u>

As desigualdades de Bell, completam o conhecimento profundo da Mecânica Quântica, esclarecendo os aspectos mais subtis da Mecânica Quântica.

A descoberta das desigualdades de Bell é por muitos considerada um dos maiores avanços em Física, depois da Mecânica Quântica e Relatividade

Correlações de Spin e

Desigualdades de Bell

Referência: J. J. Sakurai

Modern Quantum Mechanics

Com Foreword de John S. Bell

(CERN)

Consider a two-electron system in a spin-singlet state, that is with total spin zero. Recall that:

$$S=1 \begin{cases} |\uparrow\uparrow\rangle \\ \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \\ |\downarrow\downarrow\rangle \end{cases} ; \quad S=0 \rightarrow \frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right]$$

Sakurai's notation:

$$|\text{spin singlet}\rangle = \frac{1}{\sqrt{2}}\left[|\hat{z}+;\hat{z}-\rangle - |\hat{z}-;\hat{z}+\rangle\right]$$

Suppose we make a measurement on the spin component of one of the electrons. Clearly, there is a 1/2 probability of obtaining either up or down because the composite system may be in $|\hat{z}+;\hat{z}-\rangle$ or $|\hat{z}-;\hat{z}+\rangle$ with equal probability.

But if one of the components is shown to be in the spin-up state, the other is necessarily in the spin-down state and vice-versa.

It is remarkable that this kind of correlation can persist even if the 2 particles are well separated and have ceased to interact, provided that as they fly apart, there is no change in their spin states. This is certainly the case for a $J=0$ system desintegrating spontaneously into two spin $1/2$ particles with no relative angular momentum, because angular momentum conservation must hold in the desintegration process.

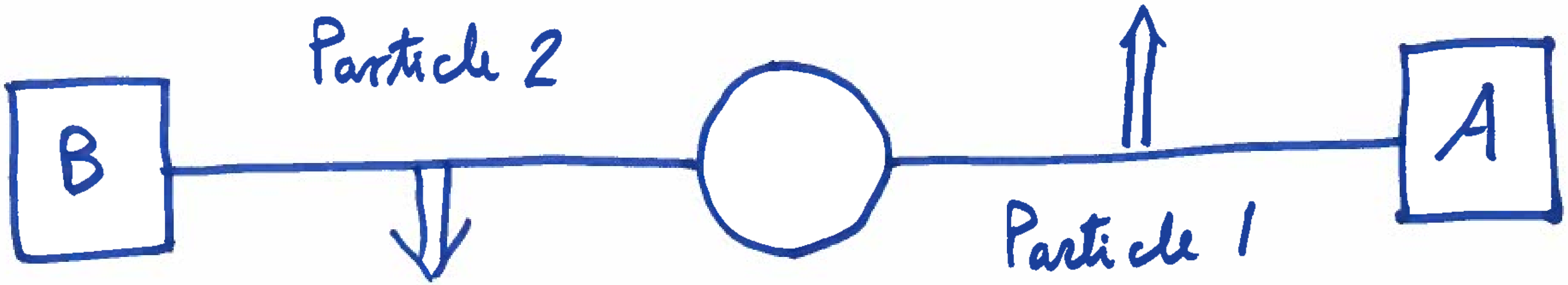
Example: rare decay of the η meson into a muon pair:

$$\eta \to \mu^+ \mu^-$$

Unfortunately the branching ratio for this decay is only 6×10^{-6}!

The correlations described previously might not be so peculiar, so strange. One may say: " It is just like an urn known to contain one black ball and one white ball. When we blindly pick one of them, there is 50-50 chance of getting black or white. But if the first ball we pick is black then we can predict with certainty that the second ball will be white!
It turns out that the analogy is too simple!

The actual quantum mechanical situation is far more sophisticated than that. This is because observers may choose to measure S_x (or any other component) in place of S_z.



Recall that for a single spin 1/2 system, the S_x eigenkets and S_z eigenkets are related as follows:

$$|\hat{x}\pm\rangle = \frac{1}{\sqrt{2}}\left(|\hat{z}+\rangle \pm |\hat{z}-\rangle\right)$$

$$|\hat{z}\pm\rangle = \frac{1}{\sqrt{2}}\left(|\hat{x}+\rangle \pm |\hat{x}-\rangle\right)$$

Returning now to our composite system, it is obvious that we can rewrite the spin-singlet ket as:

$$|\text{spin singlet}\rangle = \frac{1}{\sqrt{2}}\left(|\hat{x}-;\hat{x}+\rangle - |\hat{x}+;\hat{x}-\rangle\right)$$

Summary of Quantum mechanical Correlations:

1. If A measures S_z and B measures S_x, there is a completely random correlation between the two measurements

2. If A measures S_x and B measures S_x, there is 100% (opposite sign) correlation between the two measurements.

3. If A makes no measurement, B measurements show random results

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Spin comp. meas. by A	A result	Spin c. meas. by D	D result
z	+	z	−
z	−	x	+
x	−	z	−
x	−	z	+
z	+	x	−
x	+	x	−
z	+	x	+
x	−	x	+
z	−	z	+
z	−	x	−
x	+	z	+
x	+	z	−

These considerations show that the outcome of B's measurements appear to depend on what kind of measurement A decides to perform. Notice that A and B can be miles apart with no possibility of communications or mutual interactions.

Many physicists have felt uncomfortable with this interpretation of spin-correlation measurements.

Einstein Locality Principle

" But ~~on~~ one supposition we should, in my opinion, absolutely hold fast: The real factual situation of the system S_2 is independent of what is done with system S_1, which is spatially separated from the former.

Some have argued that the "difficulties" encountered here are inherent in the probabilistic interpretation of quantum mechanics and that the dynamic behaviour at the microscopic level appears probabilistic only because some yet unknown parameters the so-called hidden variables were not specified

Question: How to distinguish through an experiment Quantum Mechanics from a theory with hidden variables?

Let us derive Bell's inequality within the framework of a simple model conceived by E.P. Wigner that incorporates the essential features of hidden variables theories. Proponents of this model agree that it is impossible to determine S_x and S_z simultaneously. However, when we have a large number of spin 1/2 particles, we assign a certain fraction of them to have the following property:

If S_z is measured we obtain + with certainty

If S_x is measured we obtain − with certainty

A particle satisfying this property is said to belong to type :

$$(\hat{z}+ \, ; \, \hat{x}-)$$

This can be generalized to three unit vectors $\hat{a}, \hat{b}, \hat{c}$ which are in general not mutually orthogonal

Notation : Particle of type $(\hat{a}_-, \hat{b}_+, \hat{c}_+)$:

this means that if $\vec{S}.\hat{a}$ is measured, one obtains $(-)$ with certainty, if $\vec{S}.\hat{b}$ is measured, one obtains $(+)$ with certainty and if $\vec{S}.\hat{c}$ is measured one obtains $(+)$ with certainty.

There must be perfect matching in the sense that the other particle necessarily belongs to type $(\hat{a}_+, \hat{b}_-, \hat{c}_-)$, to ensure zero total angular momentum.

Population	Particle 1	Particle 2
N_1	$(\hat{a}_+, \hat{b}_+, \hat{c}_+)$	$(\hat{a}_- \hat{b}_- \hat{c}_-)$
N_2	$(\hat{a}_+ \hat{b}_+ \hat{c}_-)$	$(\hat{a}_- \hat{b}_- \hat{c}_+)$
N_3	$(\hat{a}_+ \hat{b}_- \hat{c}_+)$	$(\hat{a}_- \hat{b}_+ \hat{c}_-)$
N_4	$(\hat{a}_+ \hat{b}_- \hat{c}_-)$	$(\hat{a}_- \hat{b}_+ \hat{c}_+)$
N_5	$(\hat{a}_- \hat{b}_+, \hat{c}_+)$	$(\hat{a}_+ \hat{b}_- \hat{c}_-)$
N_6	$(\hat{a}_- \hat{b}_+ \hat{c}_-)$	$(\hat{a}_+ \hat{b}_- \hat{c}_+)$
N_7	$(\hat{a}_- \hat{b}_- \hat{c}_+)$	$(\hat{a}_+ \hat{b}_+ \hat{c}_-)$
N_8	$(\hat{a}_- \hat{b}_- \hat{c}_-)$	$(\hat{a}_+ \hat{b}_+ \hat{c}_+)$

Qual é a probabilidade de o observador A medir $\vec{S}_1 \cdot \hat{a}$ e obter + e o observador B medir $\vec{S}_2 \cdot \hat{b}$ e obter + ?

$$P(\hat{a}_+, \hat{b}_+) = \frac{N_3 + N_4}{\sum_{i=1}^{8} N_i}$$

Do mesmo modo:

$$P(\hat{a}_+ ; \hat{c}_+) = \frac{N_2 + N_4}{\sum_{i=1}^{8} N_i} \; ; \qquad P(\hat{c}_+, \hat{b}_+) = \frac{N_3 + N_7}{\sum_{i=1}^{8} N_i}$$

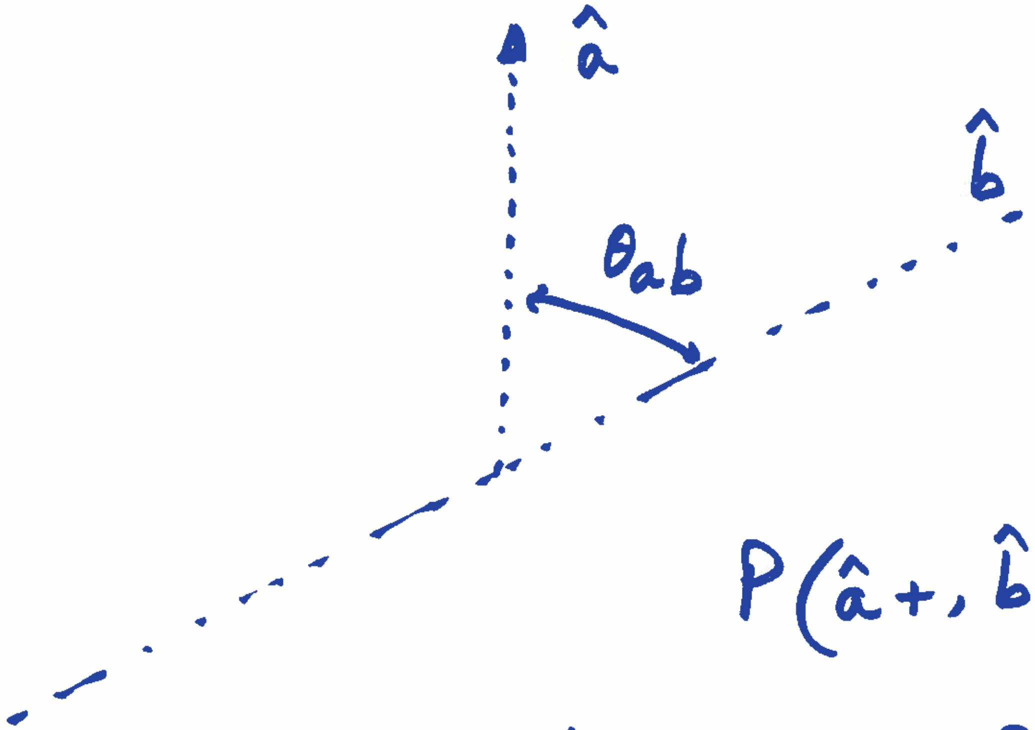
É óbvio que $\quad N_3 + N_4 \leq (N_2 + N_4) + (N_3 + N_7)$

Ou seja:

$$P(\hat{a}+;\hat{b}+) \leq P(\hat{a}+;\hat{c}_+) + P(\hat{c}+;\hat{b}+)$$

Isto é um exemplo de uma desigualdade de Bell, válida numa teoria de variáveis escondidas. Importante: para certas configurações dos eixos $\hat{a}, \hat{b}, \hat{c}$ a Mecânica Quântica viola esta desigualdade de Bell!!

Cálculo de $P(\hat{a}+, \hat{b}+)$ na Mecânica Quântica:

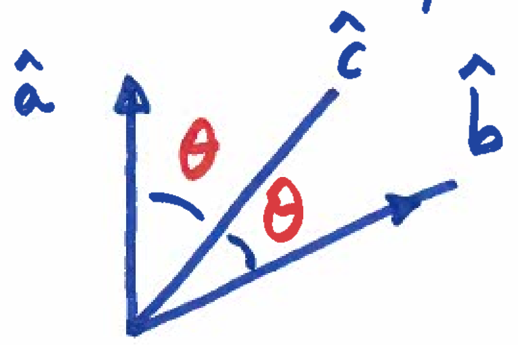


$$P(\hat{a}+, \hat{b}+) = \frac{1}{2} \sin^2\left(\frac{\theta_{ab}}{2}\right)$$

Portanto a desigualdade de Bell, diz-nos que:

$$\sin^2\left(\frac{\theta_{ab}}{2}\right) \leq \sin^2\left(\frac{\theta_{ac}}{2}\right) + \sin^2\left(\frac{\theta_{cb}}{2}\right)$$

Para simplificar, escolhamos $\hat{a}, \hat{b}, \hat{c}$ no mesmo plano em que $\hat{c}$ bisecta $\hat{a}$ e $\hat{b}$



$$\theta_{ab} = 2\theta \; ; \; \theta_{ac} = \theta_{cb} = \theta$$

A desigualdade de Bell é violada para

$$0 < \theta < \pi/2$$

Por exemplo para $\theta = \pi/4$, obtemos:

$$0.50 \leq 0.292$$