Partículas idênticas em M.Q.

Supomos que cada partícula está num determinado estado $\psi_1(x)$ e $\psi_2(x)$. Em M.Q. este novo estado é caracterizado por

$$|\psi_1 \otimes \psi_2\rangle \equiv |\psi_1(x)\rangle \otimes |\psi_2(x)\rangle$$

Significado: <u>Produto de Kronecker</u>

$$|\psi_1(x)\rangle = c_i|i\rangle \quad e \quad |\psi_2(x)\rangle = c_j|j\rangle$$

Podem ser o mesmo espaço-vectorial ou em geral podem ser diferentes (partículas n̄ idênticas)

$$|\psi_1 \otimes \psi_2\rangle = c_i c_j \, |i\rangle \otimes |j\rangle$$

A nova base é dada por $\{|i\rangle \otimes |j\rangle\}$ e qualquer vector do espaço é dado por

$$W = \omega_k |k\rangle = w_{ij} \, |i\rangle \otimes |j\rangle$$

$$\left.\begin{array}{l} i = 1, \ldots, n_1 \\ j = 1, \ldots, n_2 \end{array}\right\} \quad k = 1, \ldots, n_1 \times n_2$$

Existe uma correspondência $k \leftrightarrow (i,j)$

ex: $|1\rangle = |\uparrow\uparrow\rangle$, $|2\rangle = |\uparrow\downarrow\rangle$, $|3\rangle = |\downarrow\uparrow\rangle$, $|4\rangle = |\downarrow\downarrow\rangle$

Como actuam os operadores lineares?

$$U(g) \otimes U'(g)\big(|i\rangle \otimes |j\rangle\big) = \big(U(g)|i\rangle\big) \otimes \big(U'(g)\,|j\rangle\big)$$

produto interno?

$$\langle k|k'\rangle = \langle i|i'\rangle\langle j|j'\rangle$$

logo $|k|^2 = \langle i|i\rangle\langle j|j\rangle = 1$ pois admitimos

$\langle i|i'\rangle = \delta_{ii'}$

$\langle j|j'\rangle = \delta_{jj'}$

$$(U(g)\otimes U'(g))\,|i\rangle\otimes|j\rangle = M(g)_{i'i\,j'j}\,|i'\rangle\otimes|j'\rangle$$
$$= M(g)_{k'k}\,|k'\rangle$$

$$\boxed{\dim(M(g)) = n_1\times n_2}$$

Caso

SU(2) spin $\{|i\rangle\} = \{|\uparrow\rangle, |\downarrow\rangle\}$

reais nestes

$\hat{J}_3|\uparrow\rangle = +\frac{1}{2}$, $\hat{J}_3|\downarrow\rangle = -\frac{1}{2}$ $\quad U(t_1,t_2,t_3) = \exp\left(i\,t_i\frac{\sigma_i}{2}\right)$

$$\boxed{|\uparrow\downarrow\rangle = |\uparrow\rangle\otimes|\downarrow\rangle}$$

. First rule: The diagonal generators define the different vector basis and it is aditive.

$\hat{J}_3\otimes\hat{J}_3\,|\uparrow\rangle\otimes|\downarrow\rangle = (J_3|\uparrow\rangle)\otimes(J_3|\downarrow\rangle)$ (Álgebra)

or $\left(e^{-it\hat{J}_3}\otimes e^{-it\hat{J}_3}\right)|\uparrow\rangle\otimes|\downarrow\rangle = \left(e^{-i\frac{1}{2}t}e^{+i\frac{1}{2}t}\right)\left(|\uparrow\rangle\otimes|\downarrow\rangle\right) =$

$$= e^{-it\left(+\frac{1}{2}-\frac{1}{2}\right)}\left(|\downarrow\rangle\otimes|\downarrow\rangle\right)$$

$\hat{J}^2 = ?$ $\quad \hat{J}^2|\uparrow\downarrow\rangle$? não está definido.

R: O produto de Kronecker é em geral irredutível.

$$|S\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) \quad \text{singleto } j=0$$

$$\left.\begin{array}{l} |T_1\rangle = |\uparrow\uparrow\rangle \\ |T_2\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \\ |T_3\rangle = |\downarrow\downarrow\rangle \end{array}\right\} \text{tripleto } j=1$$

irreps

Em termos de componentes

$$|\psi\rangle \to \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

em SO(2)

$$\psi_i \longrightarrow \psi_i' = U(g)_{ij}\,\psi_j$$

dizemos que se transforma como um vector

$\psi^\dagger \chi$ é um invariante se ψ, χ forem vectores (transf.)

$$\psi^\dagger \chi \Rightarrow \psi'^\dagger \chi' = \psi^\dagger \underbrace{U(g)^\dagger U(g)}_{\mathbb{1}} \chi = \psi^\dagger \chi$$

Da mesma forma que podemos escrever os elementos do produto de Kronecker:

$$\psi_{ij} \equiv \psi_i \otimes \psi_j$$

e então temos

$$\psi_{ij}' = \psi_i' \otimes \psi_j' = \left(U(g)_{ii'}\, U(g)_{jj'}\right) \psi_{i'} \otimes \psi_{j'}$$

podemos também definir a representação conjugada

$$\psi^i = (\psi_i)^* \quad \text{e neste caso}$$

$$\psi^{i'} = \psi_\delta^\gamma\, (U(g))^*_{i\delta} = \psi^\delta\, U(g)^{i\gamma}$$

a relação $U(g)^\dagger U(g) = \mathbb{1}$ escreve-se como

$$\psi^i \chi_i \Rightarrow \psi'^i \chi'_i = \big(\underbrace{U(g)^{ik}\, U(g)_{ij}}_{\delta_{kj}}\big)\, \psi^k \otimes \chi_j =$$

$$= \psi^k \chi_k$$

$$U(g)^{ik}\, U(g)_{ij} = \delta^k_j$$

$$\varepsilon^{ij} \longrightarrow \varepsilon'^{ij} = U^{ii'} U^{jj'} \varepsilon^{i'j'} = |U|\, \varepsilon^{ij} = \underline{\underline{\varepsilon^{ij}}}$$

invariante

em $SU(n)$ temos

$\varepsilon^{\overbrace{ij\cdots k}^{n}}$ é um invariante

$SU(5)$ temos $\quad \psi^{ij}\, \psi^{kl}\, \psi^m\, \varepsilon_{ijklm}$

é um invariante

$$\psi_S^{ij} = \frac{1}{\sqrt{2}}\left(\psi^{ij} + \psi^{ji}\right)$$

$$\psi_A^{ij} = \frac{1}{\sqrt{2}}\left(\psi^{ij} - \psi^{ji}\right)$$

é fácil de ver que $\quad \psi^{ij} = \psi_A^{ij} + \psi_S^{ij}$

e $\quad \psi'^{ij}_{S/A} = U(g)_{ii'}\, U(g)_{jj'}\, \psi^{i'j'}_{S/A}$

verificamos que $\left\{ \begin{array}{l} \psi'^{ji}_S = +\, \psi'^{ij}_S \\ \psi'^{ji}_A = -\, \psi'^{ij}_A \end{array} \right)$ permutação

$\{\psi_A\}, \{\psi_S\}$ são sub. espaços invariantes logo

$\boxed{4 = 3 + 1}$

Diagramas de Young

$$\Psi = \psi_1(x) \otimes \psi_2 \otimes \psi_3(x) = \Psi(1,2,3)$$

$$S_3 = \{ e, (12), (13), (23), \overset{=(123)}{(13)(12)}, \overset{=(132)}{(12)(13)} \}$$

$$P_{(12)}\Psi(1,2,3) = \Psi(2,1,3)$$

Simetrizador $\quad \begin{array}{|c|c|c|}\hline \ & \ & \ \\ \hline\end{array} = \mathbb{1} + P_{(12)} + P_{(13)} + P_{(23)} + P_{(123)} + P_{(132)}$

Anti-simetrizador $\quad \begin{array}{|c|}\hline \ \\ \hline \ \\ \hline \ \\ \hline\end{array} = \mathbb{1} - P_{(12)} - P_{(13)} - P_{(23)} + P_{(123)} + P_{(132)}$

$\begin{array}{|c|c|c|}\hline \ & \ & \ \\ \hline\end{array} \sim 1 \qquad \begin{array}{|c|c|c|}\hline \ & \ & \ \\ \hline\end{array} \sim 1'$

$\begin{array}{|c|c|}\hline \ & \ \\ \hline \ \\ \cline{1-1}\end{array} \sim 2 \longrightarrow$ cada linha não pode ter mais caixas com a anterior.

• Cada caixa representa uma partícula num estado

$$\begin{array}{|c|c|}\hline i & j \\ \hline k \\ \cline{1-1}\end{array} = \hat{A}_{ik}\hat{S}_{ij}\Psi(1,2,3)$$

$$\hat{A}_{ij} = \mathbb{1} - P_{(ij)}$$

$$\hat{S}_{ij} = \mathbb{1} + P_{(ij)}$$

$$\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1}\end{array}\Psi(1,2,3) = \Psi(1,2,3) + \Psi(2,1,3) - \Psi(3,2,1) - \Psi(2,3,1)$$

$\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1}\end{array}$ e $\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1}\end{array}$ são dois permutados que não se podem transformar um no outro ~ 2

As irreps de S_3 são $1, 1'$ e 2

SU(3)

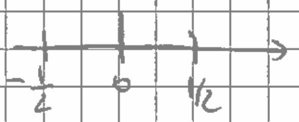
← Standard tableau

como em SU(2) $\quad \psi_A^{ij} \sim 1 = \begin{array}{|c|}\hline \, \\ \hline \, \\ \hline\end{array} \to \begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline\end{array}$

$\psi_S \sim 3 = \begin{array}{|c|c|}\hline \, & \, \\ \hline\end{array} \to \begin{array}{|c|c|}\hline 1 & 1 \\ \hline\end{array} + \begin{array}{|c|c|}\hline 1 & 2 \\ \hline\end{array} , \begin{array}{|c|c|}\hline 2 & 2 \\ \hline\end{array}$

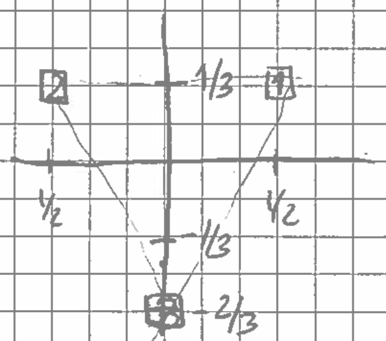
$\qquad\qquad\qquad\quad \downarrow \qquad\quad \downarrow \qquad\quad \downarrow$

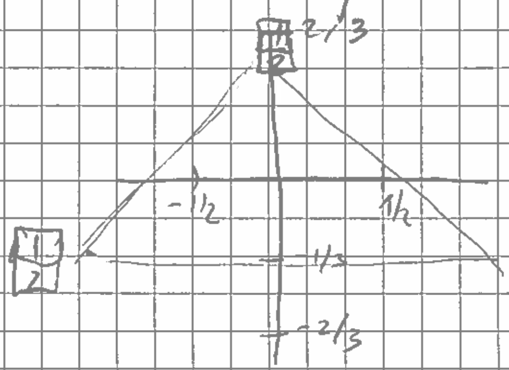
$\qquad\qquad\qquad -\frac{1}{2} \qquad\ \ 0 \qquad\ \ +\frac{1}{2}$



no caso de SU(3) temos:

$$T_8 = \begin{pmatrix} +\frac{1}{3} & & \\ & +\frac{1}{3} & \\ & & -\frac{2}{3} \end{pmatrix} \qquad T_3 = \begin{pmatrix} \frac{1}{2} & & \\ & -\frac{1}{2} & \\ & & 0 \end{pmatrix}$$





$\begin{array}{|c|c|}\hline 1 & 1 \\ \hline\end{array} , \begin{array}{|c|c|}\hline 1 & 2 \\ \hline\end{array} , \begin{array}{|c|c|}\hline 1 & 3 \\ \hline\end{array} , \begin{array}{|c|c|}\hline 2 & 2 \\ \hline\end{array} \ \begin{array}{|c|c|}\hline 2 & 3 \\ \hline\end{array} \ \begin{array}{|c|c|}\hline 3 & 3 \\ \hline\end{array} \sim 6$

$\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline\end{array} , \begin{array}{|c|}\hline 1 \\ \hline 3 \\ \hline\end{array} , \begin{array}{|c|}\hline 2 \\ \hline 3 \\ \hline\end{array} \sim 3$

em S_n não se usam índices repetidos, pelo que temos menos possibilidades $\quad d_Y|_{S_n} \leq d_Y|_{SU(n)}$

casos mistos

$\begin{array}{|c|c|}\hline 1 & 1 \\ \hline 2 \\ \cline{1-1}\end{array} \quad (\frac{1}{2}, 1) \quad$ tem 8 estad