17th international workshop on advanced computing and analysis techniques in physics research (ACAT)
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Deconvolving the detector from an observed signal in Fourier space

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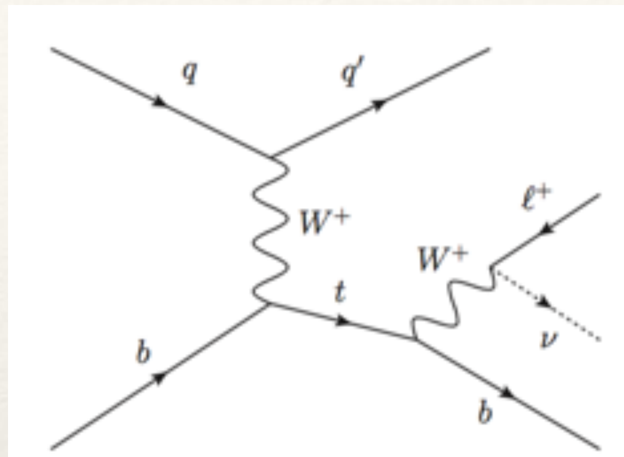
Including a numerical recipe!

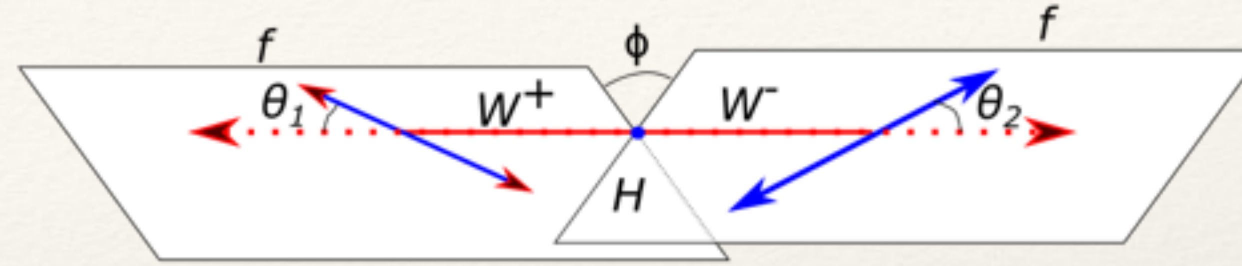
Introducing the main characters

1. A Signal, e.g:



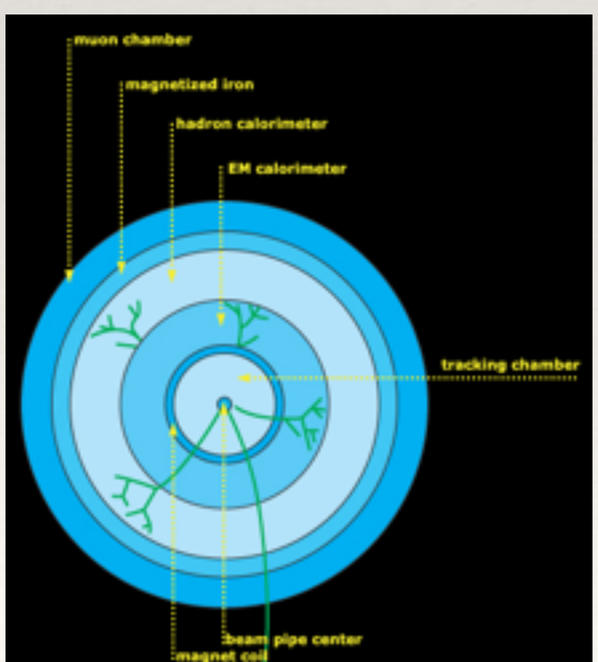
single top *t*-channel

,



Heavy $H \rightarrow W^{+}W^{-}$, lepton + jets

2. A particle detector



3. The *convolution theorem*.

The signal

The purpose of the analysis is to measure an angular distribution $d^n\Gamma / d\Omega^n$.

The reason for studying the angular distribution is: sensitivity to coupling constants in the decay, particularly anomalous couplings, which would be a sign of new physics.

$$\mathcal{L}_{Wtb} = -\frac{g}{\sqrt{2}}\bar{b}\gamma^{\mu}(V_L P_L + V_R P_R)tW_{\mu}^{-} - \frac{g}{\sqrt{2}}\bar{b}\frac{i\sigma^{\mu\nu}q_{\nu}}{M_W}(g_L P_L + g_R P_R)tW_{\mu}^{-} + h.c.$$

The Wtb vertex.

Nucl.Phys.B840:349-378,2010

$$\mathcal{L}_{HWW} = m_W^2\left(\sqrt{2}G_F)^{1/2}(1 - \frac{g^2 v}{2\Lambda^2}f_{\Phi,2})\right)HW_{\mu}^{+}W^{-\mu} + \frac{g^2 v}{2\Lambda^2}\frac{f_W}{2}(W_{\mu\nu}^{+}W^{-\mu}\partial^{\nu}H + h.c) - \frac{g^2 v}{2\Lambda^2}f_{WW}W_{\mu\nu}^{+}W^{-\mu\nu}$$

The HWW vertex.

arXiv:1505.05516

In particular we concentrate on signatures with a single neutrino in the final sate:

* Resolution effects are large.
* But the final state can be fully, if not precisely, reconstructed.

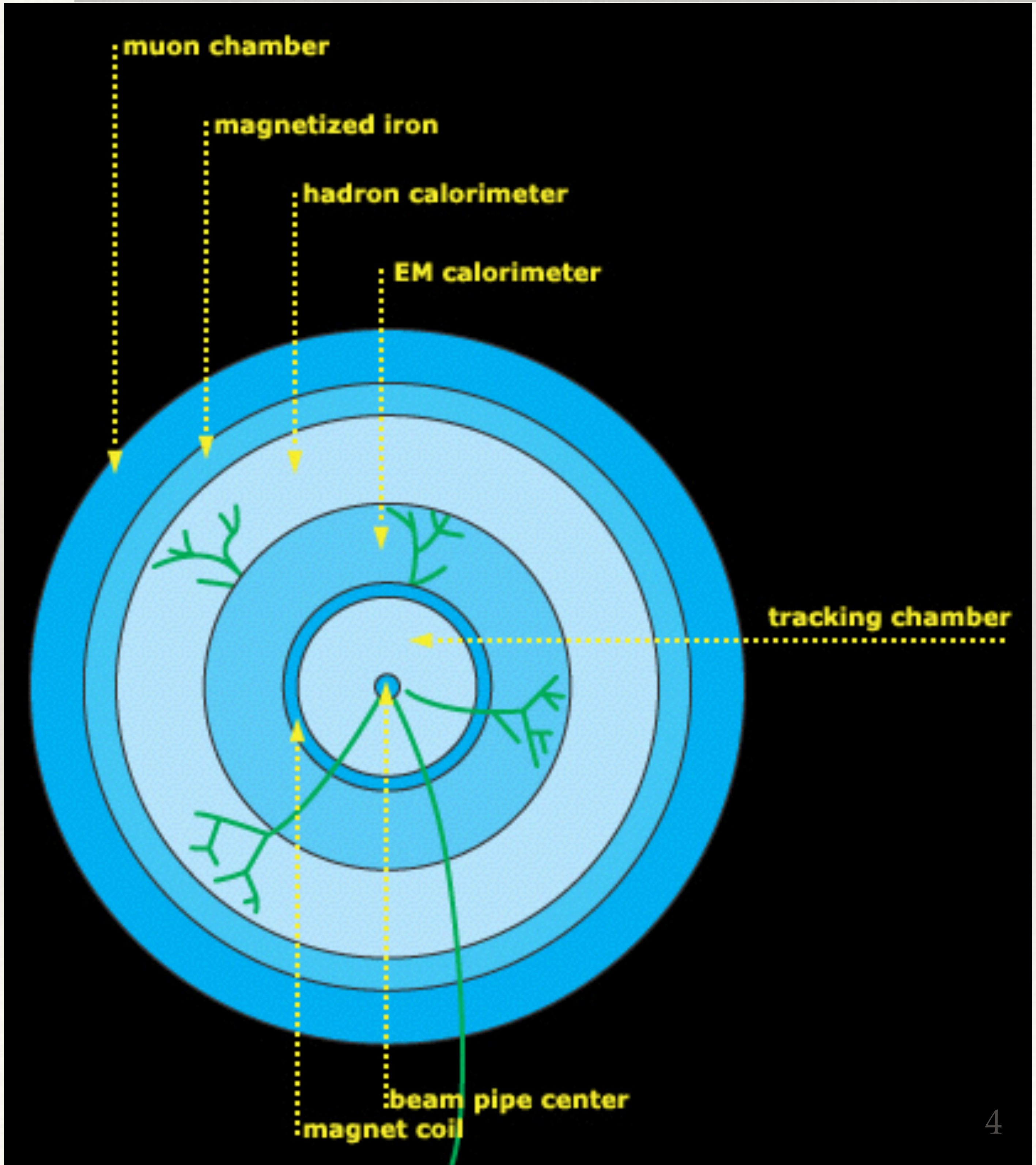
Single top t-channel w/ leptonic decay, √s=14 TeV generated with PROTOS.

https://jaguilar.web.cern.ch/jaguilar/protos/manual.ps

H(200 GeV) ->W^+W^- , lepton+jets mode, √s=14 TeV generated with PYTHIA8.

Comput. Phys.Commun. 191 (2015) 159

The detector



We demonstrate a technique with a simple detector simulation:

Very simple smearing of the neutrino energy and direction are applied:

$\sigma(E_T) = 0.5\sqrt{E_T}$. in both x, y directions.

Neutrino p_z obtained from the W mass constraint; quadratic ambiguity solved by optimizing the reconstructed top (Higgs) mass.

Cuts applied on p_T of lepton, and jets E_T^{miss}, lepton isolation, detector acceptance.

These isolate the lepton + jets signal.

The convolution theorem

$$\widetilde{f \star g} = \sqrt{2\pi} \cdot \tilde{f} \cdot \tilde{g}$$

The Fourier transform of the convolution of two functions is the product of their Fourier transforms.

Used in signal processing. Proof is simple:

$$(f \star g)(t) \equiv \int f(x) g(t-x) dx$$

Define convolution

$$= \int \left[\frac{1}{\sqrt{2\pi}} \int \tilde{f}(\omega) e^{-i\omega x} d\omega \right] \cdot \left[\frac{1}{\sqrt{2\pi}} \int \tilde{g}(\omega') e^{-i\omega'(t-x)} d\omega' \right] dx$$

Express in terms of Fourier transforms

$$= \int d\omega \int d\omega' \left[\tilde{f}(\omega) \cdot \tilde{g}(\omega') \right] e^{-i\omega'(t)} \left[\frac{1}{2\pi} \int e^{i(\omega' - \omega)x} dx \right]$$

Rearrange

$$= \int d\omega \int d\omega' \left[\tilde{f}(\omega) \cdot \tilde{g}(\omega') \right] e^{-i\omega'(t)} \delta(\omega' - \omega)$$

Discern a δ-function

$$= \int d\omega \left[\tilde{f}(\omega) \cdot \tilde{g}(\omega) \right] e^{-i\omega'(t)}$$

Collapse an integral

$$\widetilde{f \star g} = \sqrt{2\pi} \cdot \tilde{f} \cdot \tilde{g}$$

Conclude…

Formulated more abstractly, we can imagine that convolution theorem works with any set of basis functions, not just complex exponentials.

$$
\begin{aligned}
(f \star g)(t) &\equiv \int f(x) g(t-x) dx \\
&= \sum_x \langle f|x\rangle\langle x-t|g\rangle \\
&= \sum_{x,k,k'} \langle f|k\rangle\langle k|x\rangle\langle x-t|k'\rangle\langle k'|g\rangle \\
&= \sum_{x,k,k'} \langle f|k\rangle\langle k|x\rangle\langle x|k'\rangle\langle k'|g\rangle e^{ik't} \\
&= \sum_{k,k'} \langle f|k\rangle \delta_{k,k'} \langle k'|g\rangle e^{ik't} \\
&= \sum_k \langle f|k\rangle\langle k|g\rangle e^{ikt} \\
&= \int \tilde{f}\tilde{g} e^{ikt} dk
\end{aligned}
$$

$$\widetilde{f \star g} = \sqrt{2\pi} \cdot \tilde{f} \cdot \tilde{g}$$

Convolution on a sphere

Mathematische Annalen
December 1917, Volume 78, Issue 1, pp 398-404

Über orthogonal-invariante Integralgleichungen.

Von

E. Hecke in Basel.

Origin of the Funk-Hecke theorem.

218 IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 25, NO. 2, FEBRUARY 2003

Lambertian Reflectance and Linear Subspaces

Ronen Basri, *Member, IEEE*, and David W. Jacobs, *Member, IEEE*

First practical application?

BASRI AND JACOBS: LAMBERTIAN REFLECTANCE AND LINEAR SUBSPACES

Fig. 1. The same face, under two different lighting conditions.

The above paper addresses the issue of facial recognition under different lighting conditions.

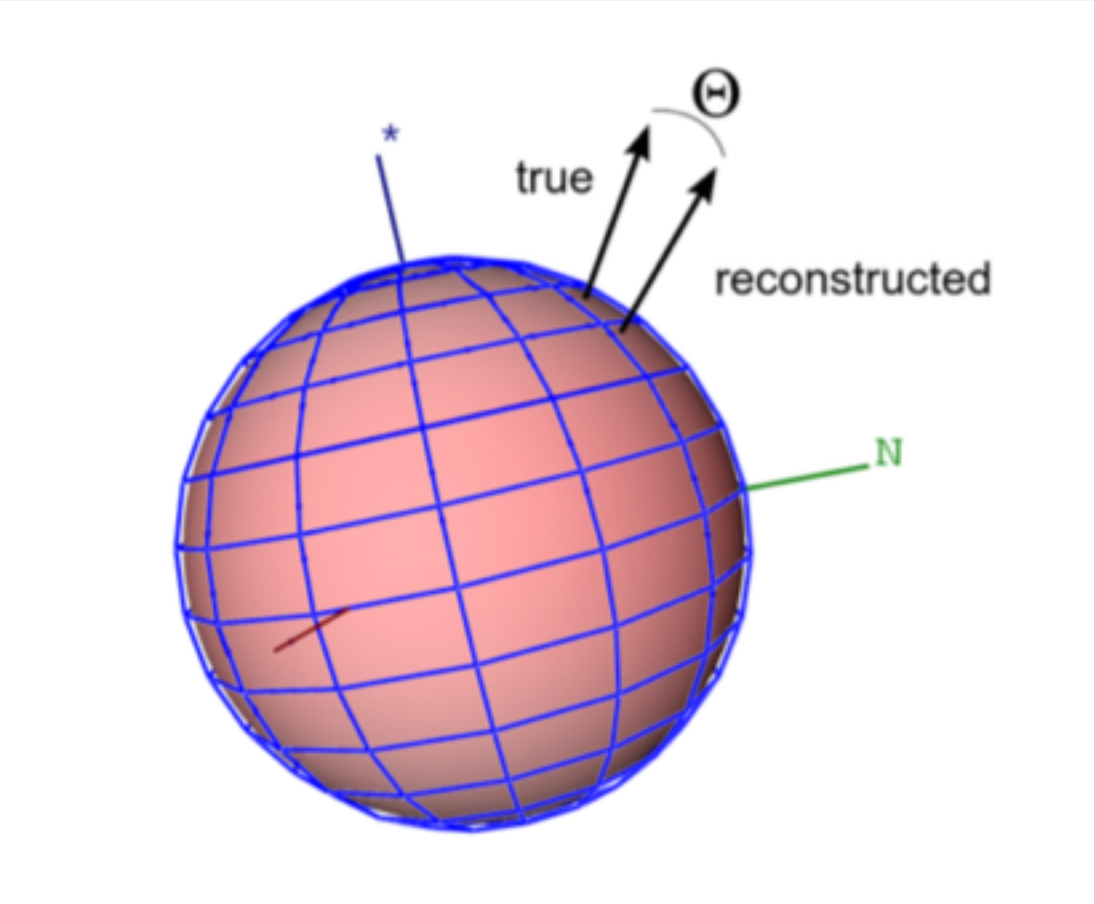
Sound familiar?

Angular analog: the Funk-Hecke theorem

Describes the effect of isotropic angular smearing on an angular distribution $d\Gamma/d\Omega$.

signal $$\rho(\theta,\phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} c_l^m Y_l^m(\theta,\phi)$$

detector effects $$\mathcal{R}(\Theta) = \sum_{l=0}^{M} r_l P_l(\cos\Theta)$$



$$(\rho\star\mathcal{R})(\theta,\phi) = \sum_{l=0}^{M}\sum_{m=-l}^{l} d_l^m Y_l^m(\theta,\phi)$$

where $$d_l \equiv \frac{2}{2l+1} r_l c_l^m \qquad \text{(no summation)}$$

The proof of this theorem is not hard either.

But the theorem is not sufficient for our purposes and we will have to "roll our own"

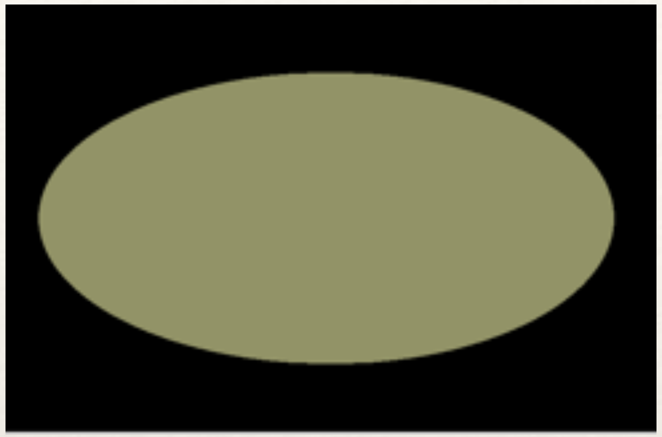
We use an orthogonal function called an *M*-Function, a function of three angles built from spherical harmonics:

$$
\begin{aligned}
M_{k,l}^{m}(\theta_1, \theta_2, \phi) &= \sqrt{2\pi} Y_k^m(\theta_1, 0) Y_l^m(\theta_2, \phi) \\
&= \sqrt{2\pi} Y_k^m(\theta_1, \phi) Y_l^m(\theta_2, 0)
\end{aligned}
$$

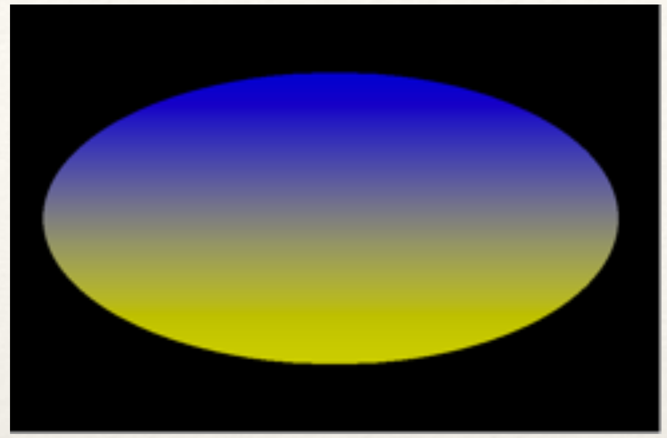
These functions:

- are orthogonal
- are complete
- obey *Gaunt's theorem.*

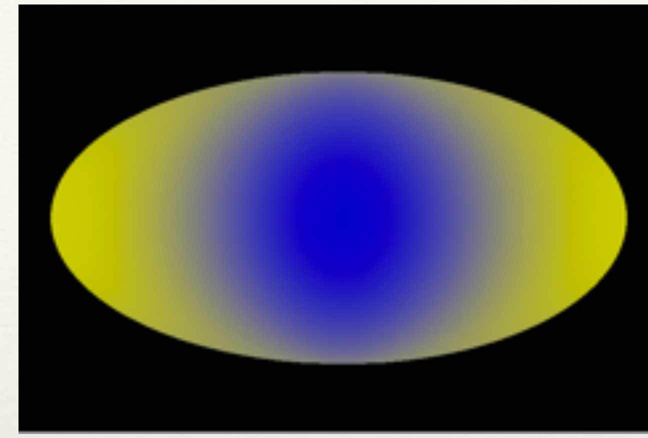
Here is a visual picture of *M*-function projections:



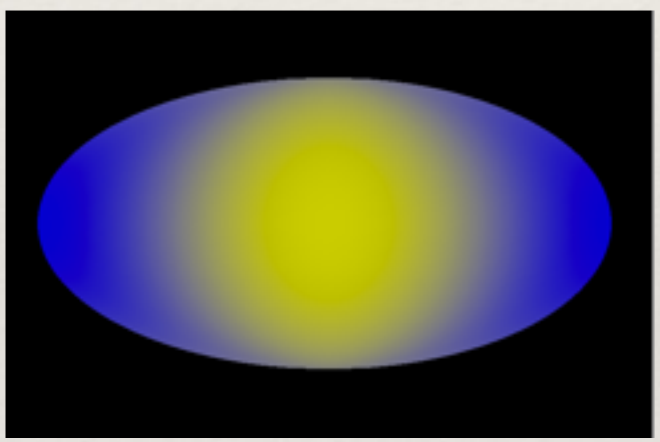
$a_{000}(s)$ $a_{100}(a)$



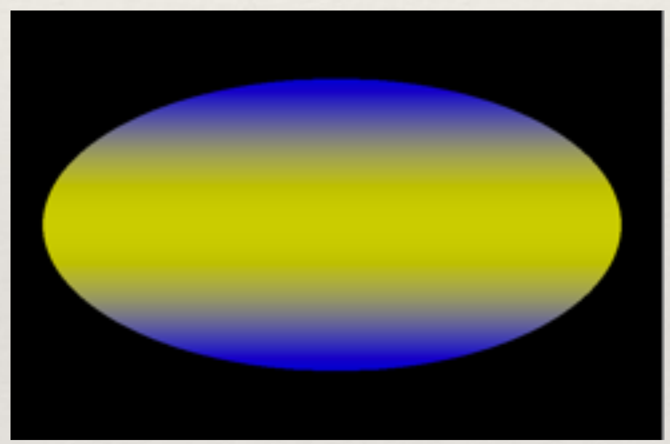
$a_{010}(s)$ $a_{110}(a)$



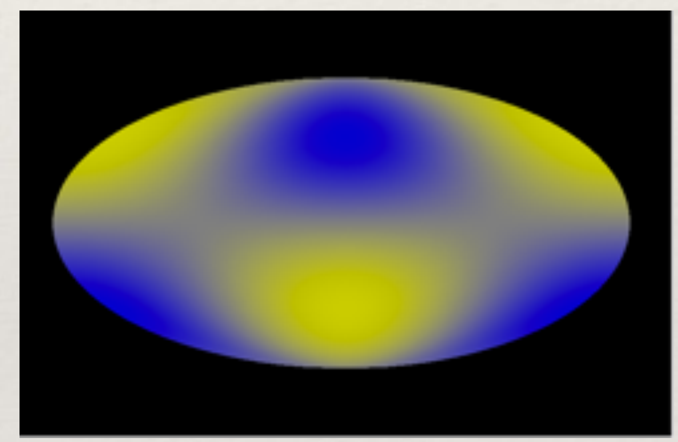
$a_{111}(a)$



$a_{1,1,-1}(a)$

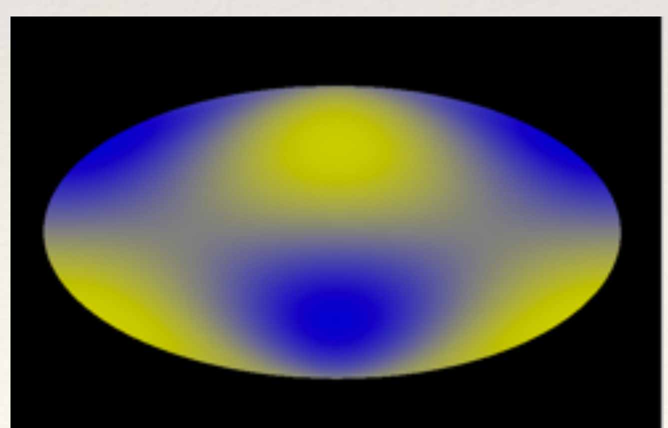


$a_{0,2,0}$ (s)



$a_{1,2,1}(a)$

Difficult to display three variables in a graph. We show θ_2 vs ϕ in these plots.



$a_{1,2,-1}(a)$

The qualifiers (a) and(s) mean that the distribution is asymmetric or symmetric in θ_1

Properties of M-functions

Orthogonality: $$\int M_{k,l}^{m}(\theta_1,\theta_2,\phi) M_{k',l'}^{m'*}(\theta_1,\theta_2,\phi) d\Omega^M = \delta_{k,k'}\delta_{l,l'}\delta_{m,m'}$$

where $$d\Omega^M = \sin\theta_1 \sin\theta_2 d\theta_1 d\theta_2 d\phi$$

Complex conjugation $$M_{k',l'}^{m'*}(\theta_1,\theta_2,\phi) = M_{k,l}^{-m}(\theta_1,\theta_2,\phi)$$

Gaunt's theorem: $$M_{k,l}^{m}(\theta_1,\theta_2,\phi) M_{k',l'}^{m'}(\theta_1,\theta_2,\phi) = W_{k,l,k',l',L,K}^{m,m',M} M_{K,L}^{M}(\theta_1,\theta_2,\phi)$$

I.E, if I have to multiply two M-functions, I can write the product as a sum of M-functions.

The *known* coefficients $W_{k,l,k',l',L,K}{}^{m,m',M}$ can be expressed in terms of Gaunt coefficients and the Gaunt coefficients in Clebsch-Gordan coefficients: gory details in the hidden slide.

Gory details, Gaunt expansion

$$W_{k,l,k',l',L,K}^{m,m',M} = \sqrt{2\pi} G_{k,k',K}^{m,m',M} G_{l,l',L}^{m,m',M}$$

$$G_{l,l',L}^{m,m',M} = \sqrt{\frac{(2l+1)(2l'+1)}{4\pi(2L+1)}} C_{l,l',L}^{m,m',M} C_{l,l',L}^{0,0,0}$$

where :

$G_{l,l',L}^{m,m',M}$ are Gaunt Coefficients

$C_{l,l',L}^{m,m',M}$ are Clebsch – Gordan Coefficients

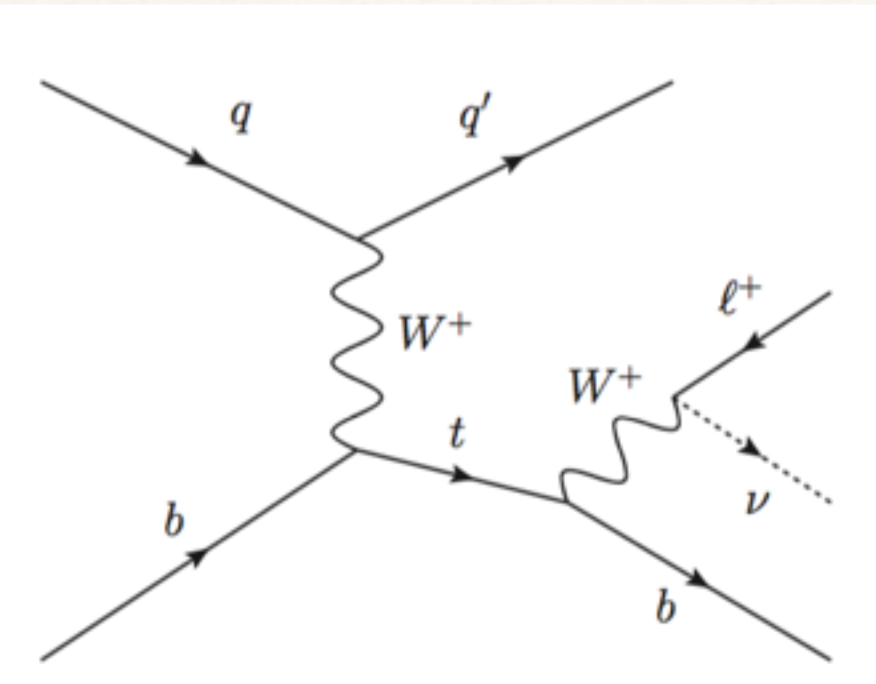
Nature arranges for some important processes to have a simple form.

$$\rho(\theta_1, \theta_2, \phi) \equiv \frac{1}{\Gamma} \frac{d\Gamma(\theta_1, \theta_2, \phi)}{d\Omega^M}$$

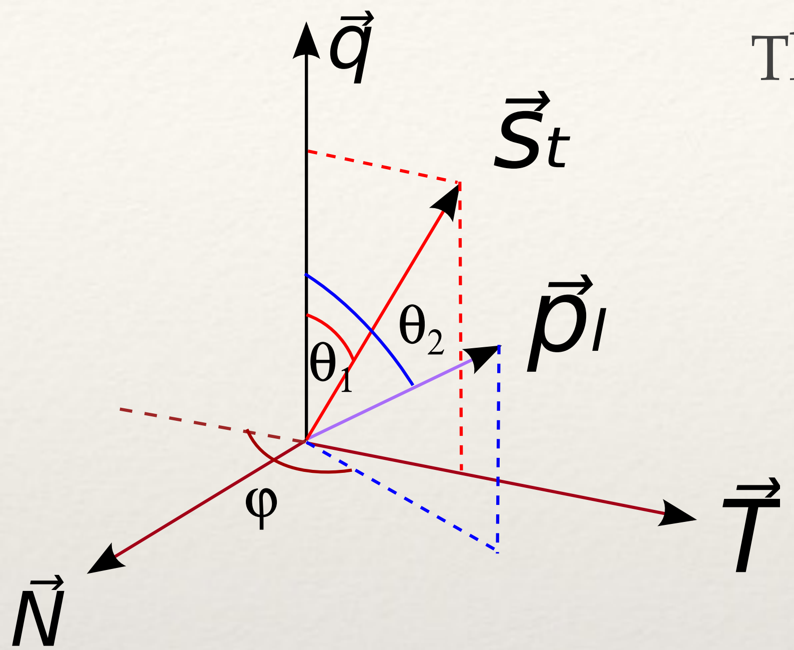
$$= a^m_{k,l} M^m_{k,l}(\theta_1, \theta_2, \phi)$$

Summation implied, finite series.

Single top t-channel production & decay



Single top t-channel production & decay



Three angles are involved

The triple differential decay rate:

$$\rho(\theta_1, \theta_2, \phi) \equiv \frac{1}{\Gamma} \frac{d\Gamma(\theta_1, \theta_2, \phi)}{d\Omega^M}$$

$$= a^m_{k,l} M^m_{k,l}(\theta_1, \theta_2, \phi)$$

Nucl.Phys.B840:349-378,2010

$$a^0_{0,1} = + \frac{\sqrt{3}}{2} \left(|A_{1,\frac{1}{2}}|^2 - |A_{-1,-\frac{1}{2}}|^2 \right)$$

$$a^0_{0,2} = + \frac{1}{2\sqrt{5}} \left(|A_{1,\frac{1}{2}}|^2 - 2|A_{0,\frac{1}{2}}|^2 - 2|A_{0,-\frac{1}{2}}|^2 + |A_{-1,-\frac{1}{2}}|^2 \right)$$

$$a^0_{1,0} = + P \frac{1}{\sqrt{3}} \left(|A_{1,\frac{1}{2}}|^2 - |A_{0,\frac{1}{2}}|^2 + |A_{0,-\frac{1}{2}}|^2 - |A_{-1,-\frac{1}{2}}|^2 \right)$$

$$a^0_{1,1} = + P \frac{1}{2} \left(|A_{1,\frac{1}{2}}|^2 + |A_{-1,-\frac{1}{2}}|^2 \right)$$

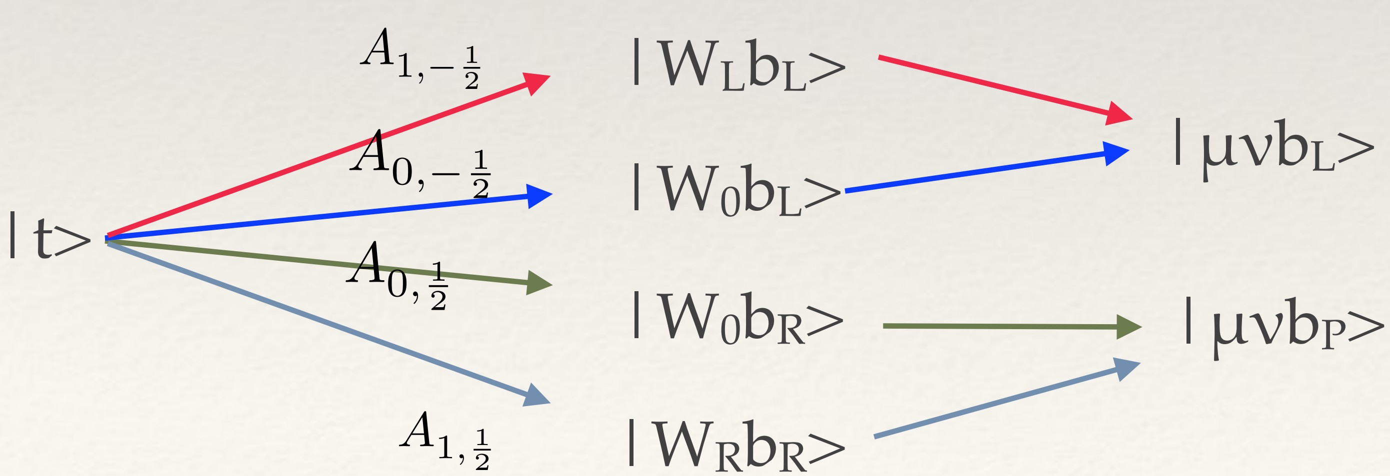
$$a^0_{1,2} = + P \frac{1}{2\sqrt{15}} \left(|A_{1,\frac{1}{2}}|^2 + 2|A_{0,\frac{1}{2}}|^2 - 2|A_{0,-\frac{1}{2}}|^2 - |A_{-1,-\frac{1}{2}}|^2 \right)$$

$$a^1_{1,1} = - P \frac{1}{\sqrt{2}} \left(A_{1,\frac{1}{2}} A^*_{0,\frac{1}{2}} + A^*_{-1,-\frac{1}{2}} A_{0,-\frac{1}{2}} \right)$$

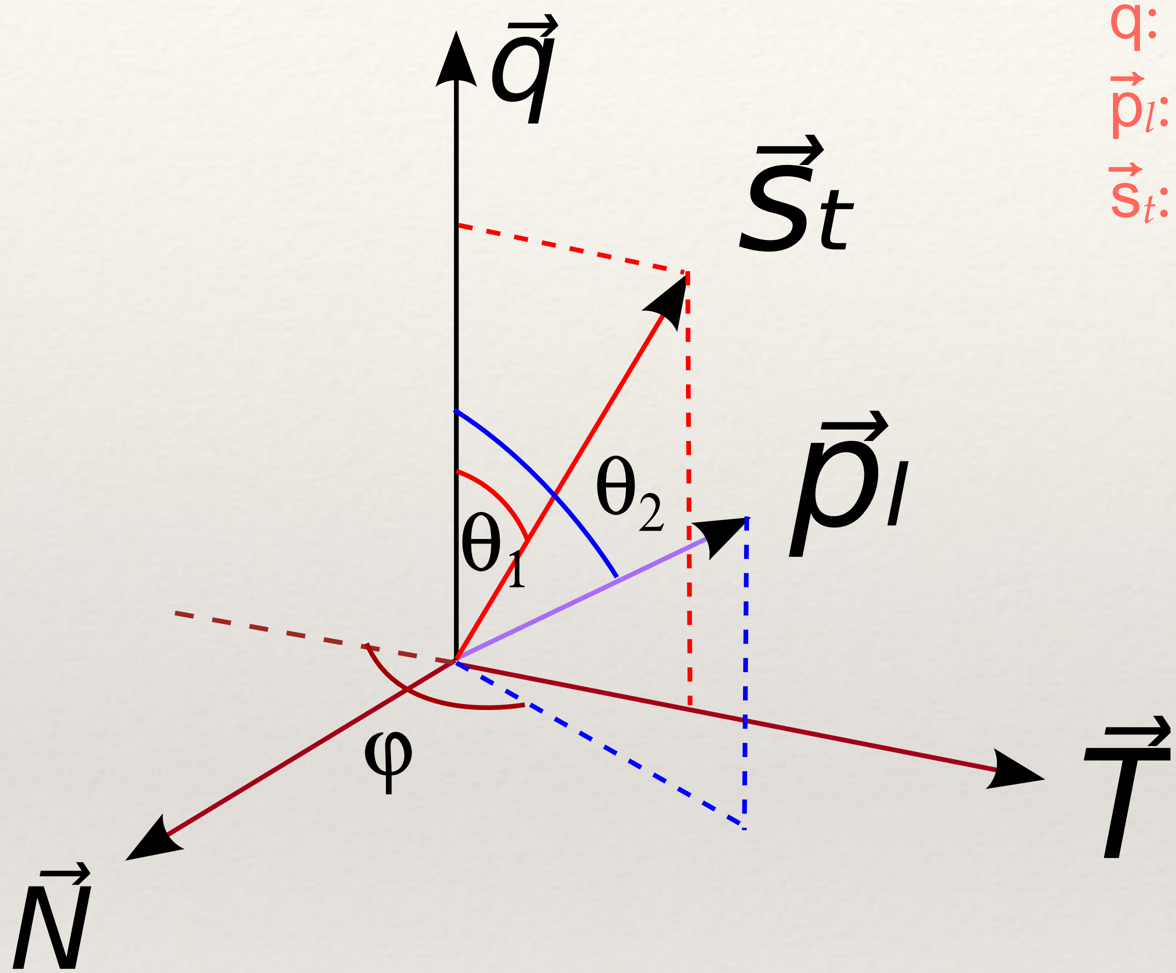
$$a^{-1}_{1,1} = - P \frac{1}{\sqrt{2}} \left(A^*_{1,\frac{1}{2}} A_{0,\frac{1}{2}} + A_{-1,-\frac{1}{2}} A^*_{0,-\frac{1}{2}} \right)$$

$$a^1_{1,2} = - P \frac{1}{\sqrt{10}} \left(A_{1,\frac{1}{2}} A^*_{0,\frac{1}{2}} - A^*_{-1,-\frac{1}{2}} A_{0,-\frac{1}{2}} \right)$$

$$a^{-1}_{1,2} = - P \frac{1}{\sqrt{10}} \left(A^*_{1,\frac{1}{2}} A_{0,\frac{1}{2}} - A_{-1,-\frac{1}{2}} A^*_{0,-\frac{1}{2}} \right)$$



More gory details



$\vec{q}$: W momentum, top rest frame
$\vec{p}_l$: lepton momentum, W rest frame
$\vec{s}_t$: polarization axes (spectator quark)

There are three angle, two polar

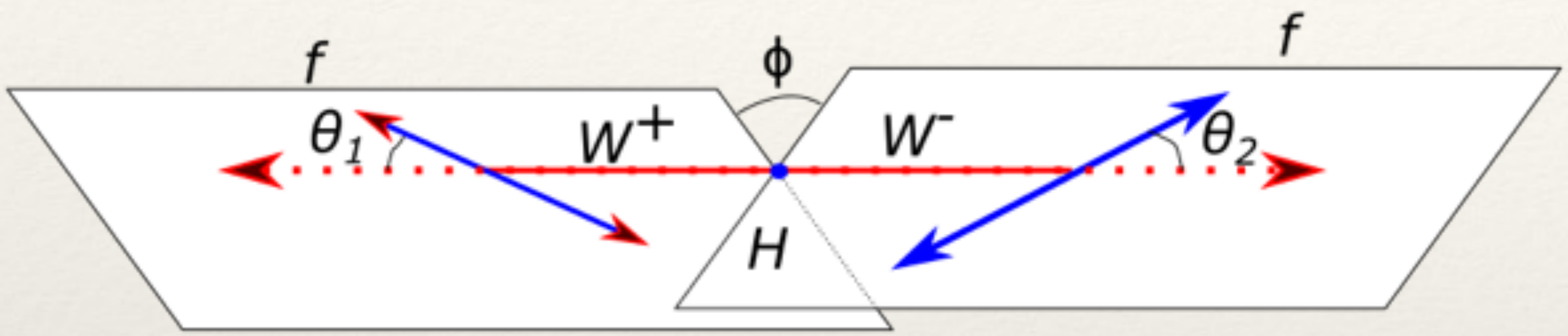
$$\cos\theta_1 \equiv \hat{q} \cdot \hat{s}_t$$

$$\cos\theta_2 \equiv \hat{q} \cdot \hat{p}_l$$

$$\tan\phi = \frac{\hat{p}_l \times (\hat{q} \times (\hat{q} \times \hat{s}_t))}{\hat{q} \cdot (\hat{p}_l \times \hat{s}_t)}$$

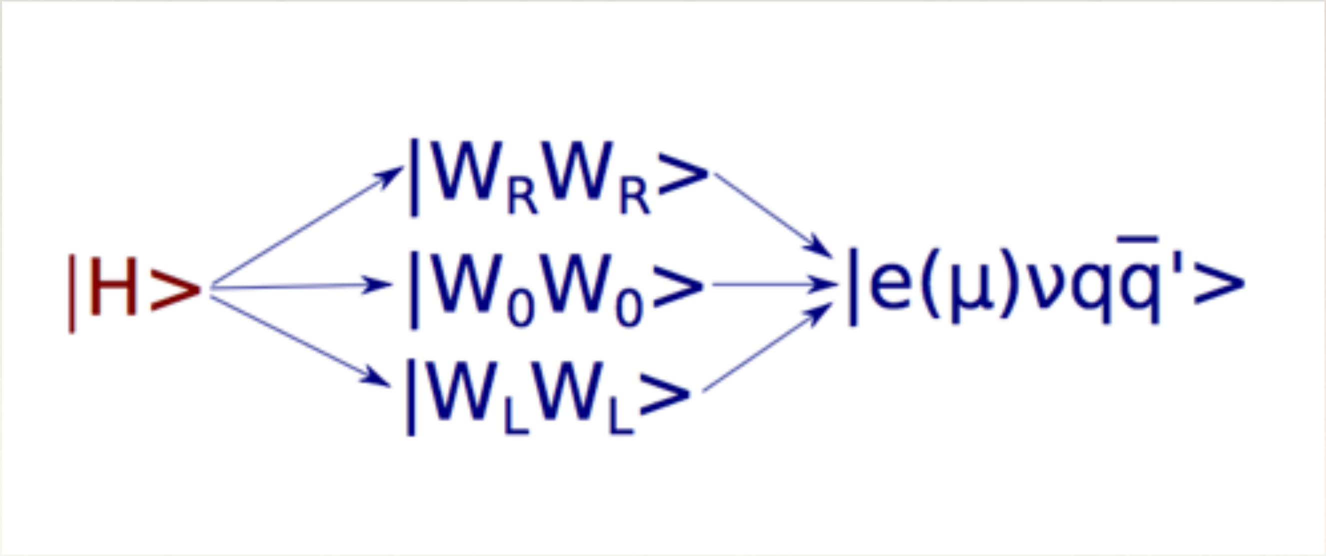
Nucl.Phys.B840:349-378,2010

Heavy Higgs decay to two vector bosons:



$$\rho(\theta_1, \theta_2, \phi) \equiv \frac{1}{\Gamma}\frac{d\Gamma(\theta_1, \theta_2, \phi)}{d\Omega^M}$$

$$= a^m_{k,l} M^m_{k,l}(\theta_1, \theta_2, \phi)$$



$$a^0_{0,0} = \frac{1}{\sqrt{8\pi}}(|A_R|^2 + |A_L|^2 + |A_0|^2)$$

$$a^0_{0,1} = a^0_{1,0} = \sqrt{\frac{3}{32\pi}}(|A_L|^2 - |A_R|^2)$$

$$a^0_{0,2} = a^0_{2,0} = \frac{1}{\sqrt{160\pi}}(|A_R|^2 + |A_L|^2 - 2|A_0|^2)$$

$$a^0_{1,1} = \frac{3}{\sqrt{128\pi}}(|A_R|^2 + |A_L|^2)$$

$$a^0_{2,1} = a^0_{1,2} = \sqrt{\frac{3}{640\pi}}(|A_L|^2 - |A_R|^2)$$

$$a^0_{2,2} = \frac{1}{40\sqrt{2\pi}}(|A_R|^2 + |A_L|^2 + 4|A_0|^2)$$

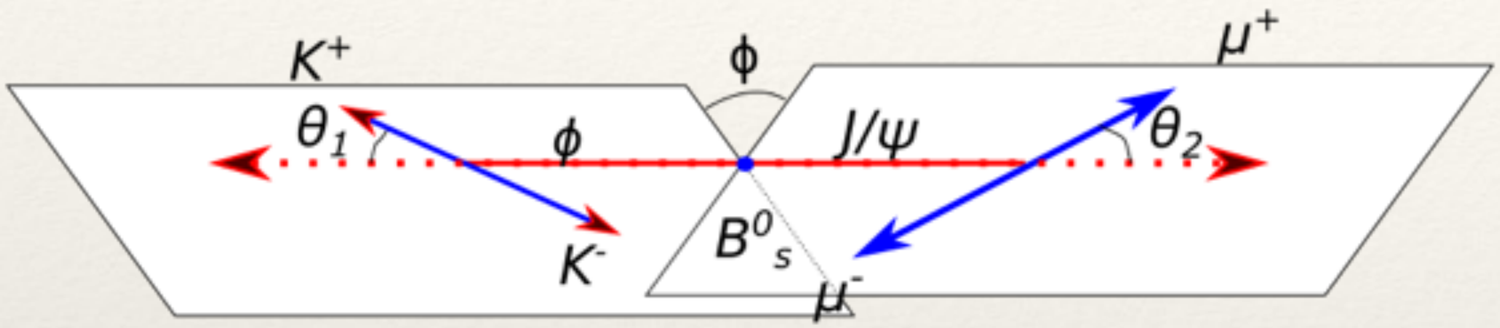
$$a^1_{1,1} = a^{-1*}_{1,1} = \frac{3}{\sqrt{128\pi}}(A_0 A^*_R + A_L A^*_0)$$

$$a^1_{1,2} = a^1_{2,1} = a^{-1*}_{1,2} = a^{-1*}_{2,1} = \frac{3}{\sqrt{640\pi}}(A_L A^*_0 - A_0 A^*_R)$$

$$a^1_{2,2} = a^{-1*}_{2,2} = \frac{3}{40\sqrt{2\pi}}(A_0 A^*_R + A_L A^*_0)$$

$$a^2_{2,2} = a^{-2*}_{2,2} = \frac{3}{20\sqrt{2\pi}} A_L A^*_R$$

And for B physics aficionados, $B^0_s \to J/\psi\phi$



$$\rho(\theta_1, \theta_2, \phi) \equiv \frac{1}{\Gamma} \frac{d\Gamma(\theta_1, \theta_2, \phi)}{d\Omega^M}$$

$$= a^m_{k,l} M^m_{k,l}(\theta_1, \theta_2, \phi)$$

$$a^0_{0,0} = \frac{1}{\sqrt{8\pi}} \left(|A_0|^2 + |A_{||}|^2 + |A_\perp|^2\right)$$

$$a^0_{0,2} = \frac{1}{\sqrt{160\pi}} \left(|A_0|^2 + |A_{||}|^2 - 2|A_\perp|^2\right)$$

$$a^0_{2,0} = \frac{1}{\sqrt{40\pi}} \left(2|A_0|^2 - |A_{||}|^2 - |A_\perp|^2\right)$$

$$a^0_{2,2} = \frac{1}{20\sqrt{2\pi}} \left(2(|A_0|^2 + |A_\perp|^2) - |A_{||}|^2\right)$$

$$a^1_{1,2} = a^{-1*}_{1,2} = -\frac{27}{256}\sqrt{\frac{\pi}{10}} \left(A_\perp A^*_{||} - A_{||} A^*_\perp\right)$$

$$a^1_{2,2} = a^{-1*}_{2,2} = \frac{3i}{40\sqrt{\pi}} \left(-A^*_0 A^*_\perp + A_0 A^*_\perp\right)$$

$$a^2_{2,2} = a^{-2*}_{2,2} = \frac{3}{40\sqrt{2\pi}} \left(|A_0|^2 - 2|A_{||}|^2\right)$$

Journal-ref: Eur.Phys.J.C6:647-662,1999

Obtaining the coefficients:

Suppose: $$\rho(\theta_1, \theta_2, \phi) = a^m_{k,l} M^m_{kl}(\theta, \theta^*, \phi^*)$$

Formally: $$a^m_{k,l} = \int \rho(\theta_1, \theta_2, \phi) M^{m*}_{kl}(\theta_1, \theta_2, \phi) d\Omega^M$$

Consider how to evaluate this integral using Monte Carlo integration:

* generate data according $\varrho(\theta_1,\theta_2,\phi)$.
* take the average value of $M_{kl}{}^{m^*}$ for the so-generated dataset. $$a^m_{kl} = \langle M^{m*}_{k,l}(\theta_1, \theta_2, \phi) \rangle$$

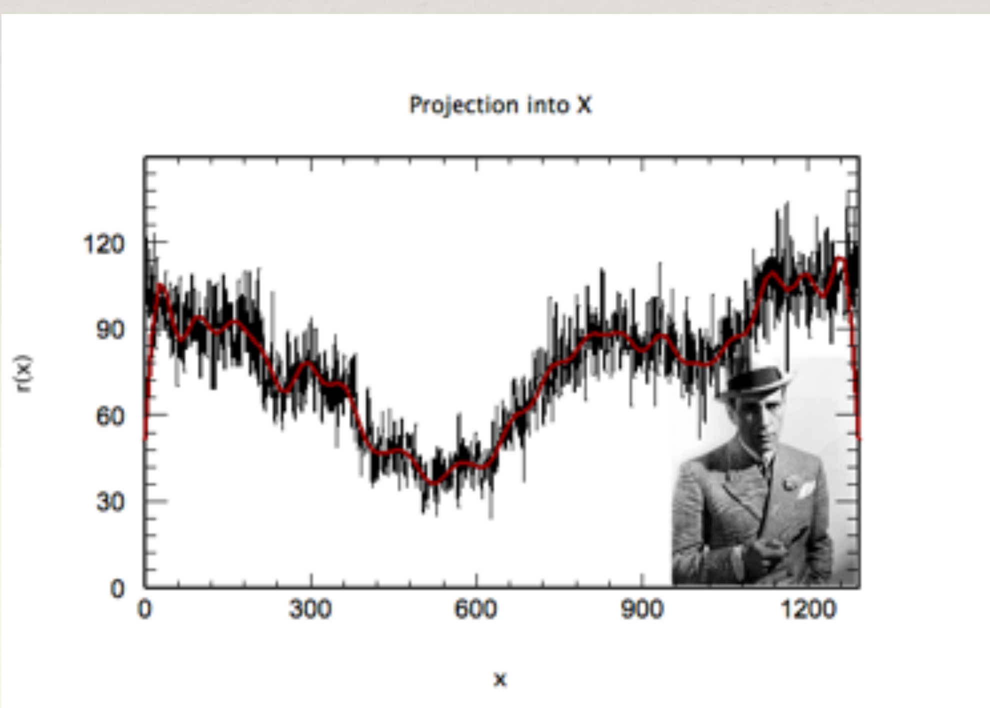
Notice:

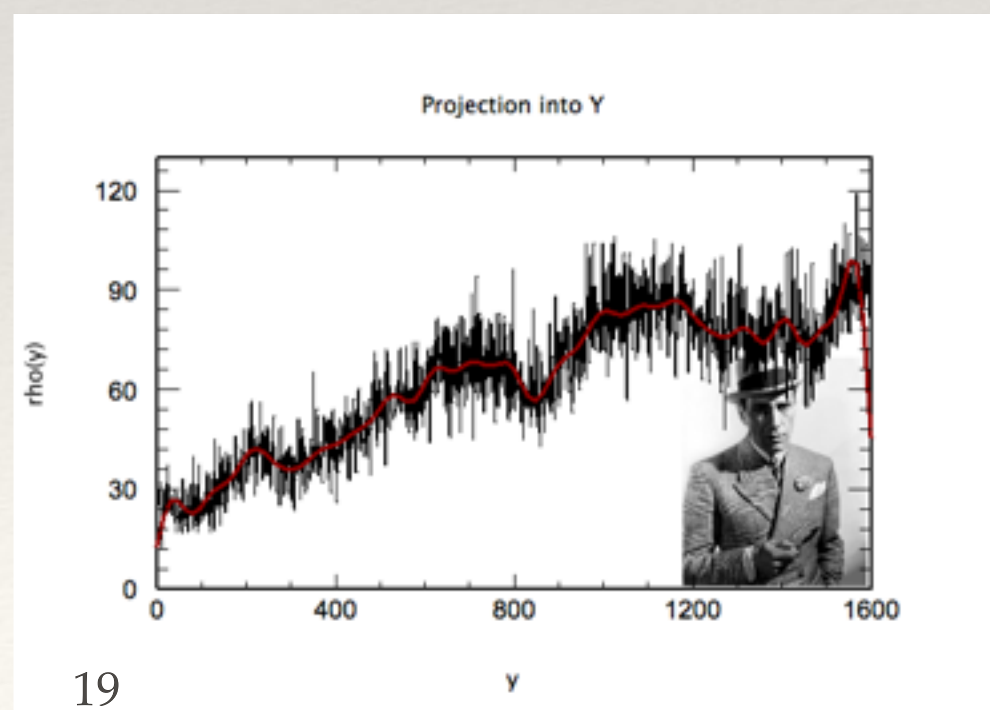
* You do not need to know ϱ.
* All you need for this is the dataset. Good: that's all you have anyway.

And:

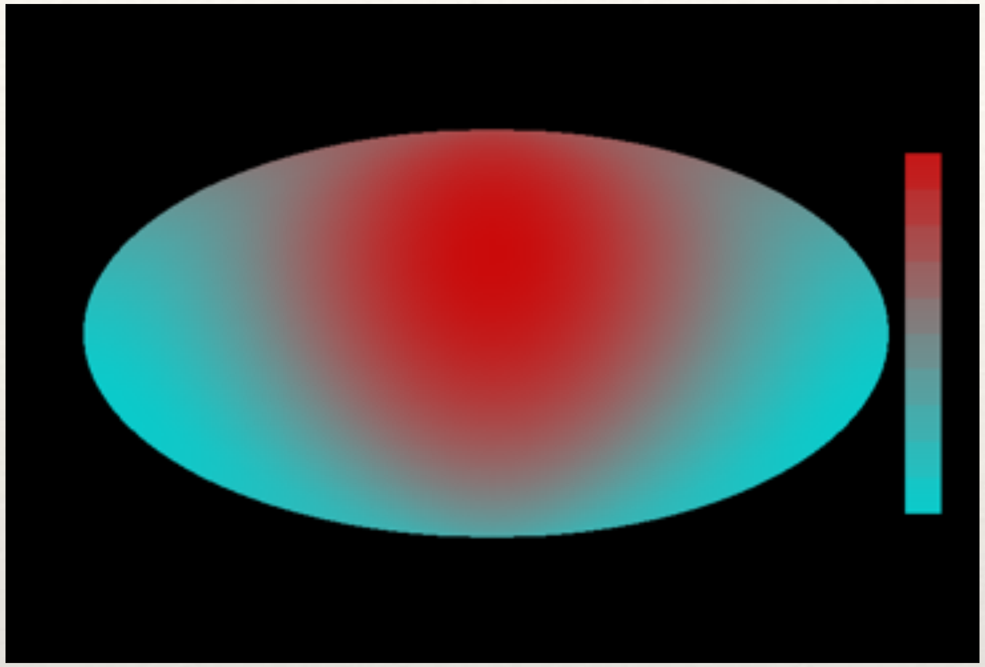
* You can also obtain errors, correlations, a full covariance matrix!

Here is a demonstration using harmonic basis functions in a rectangular space:

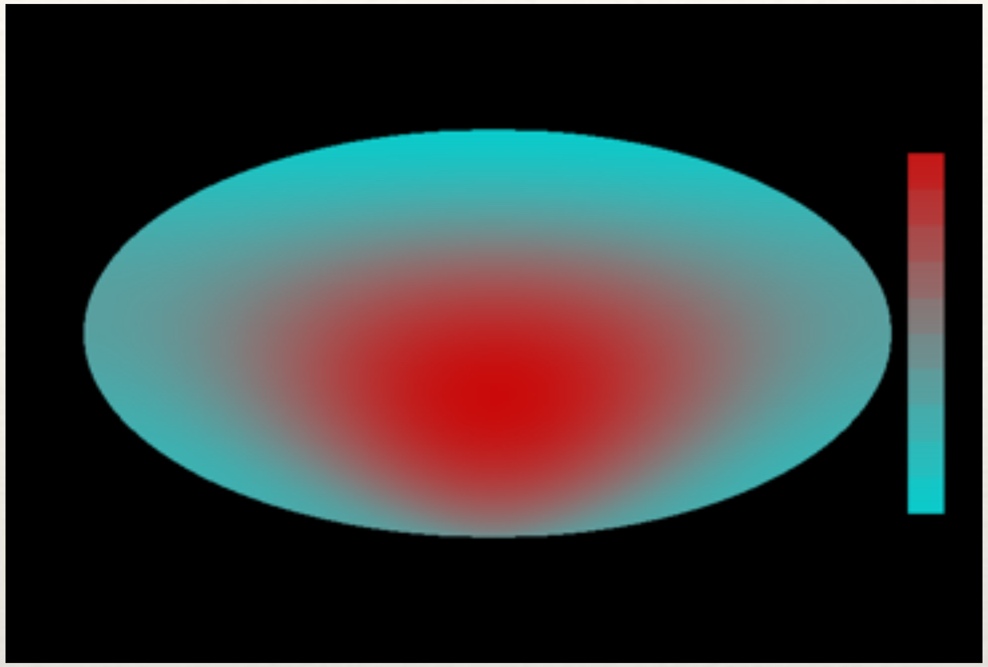




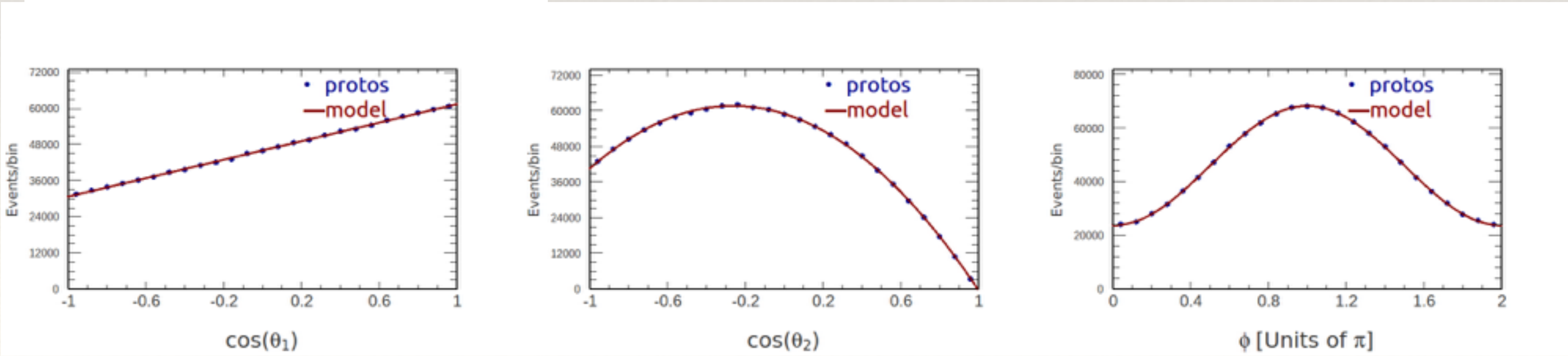
Single top t-channel



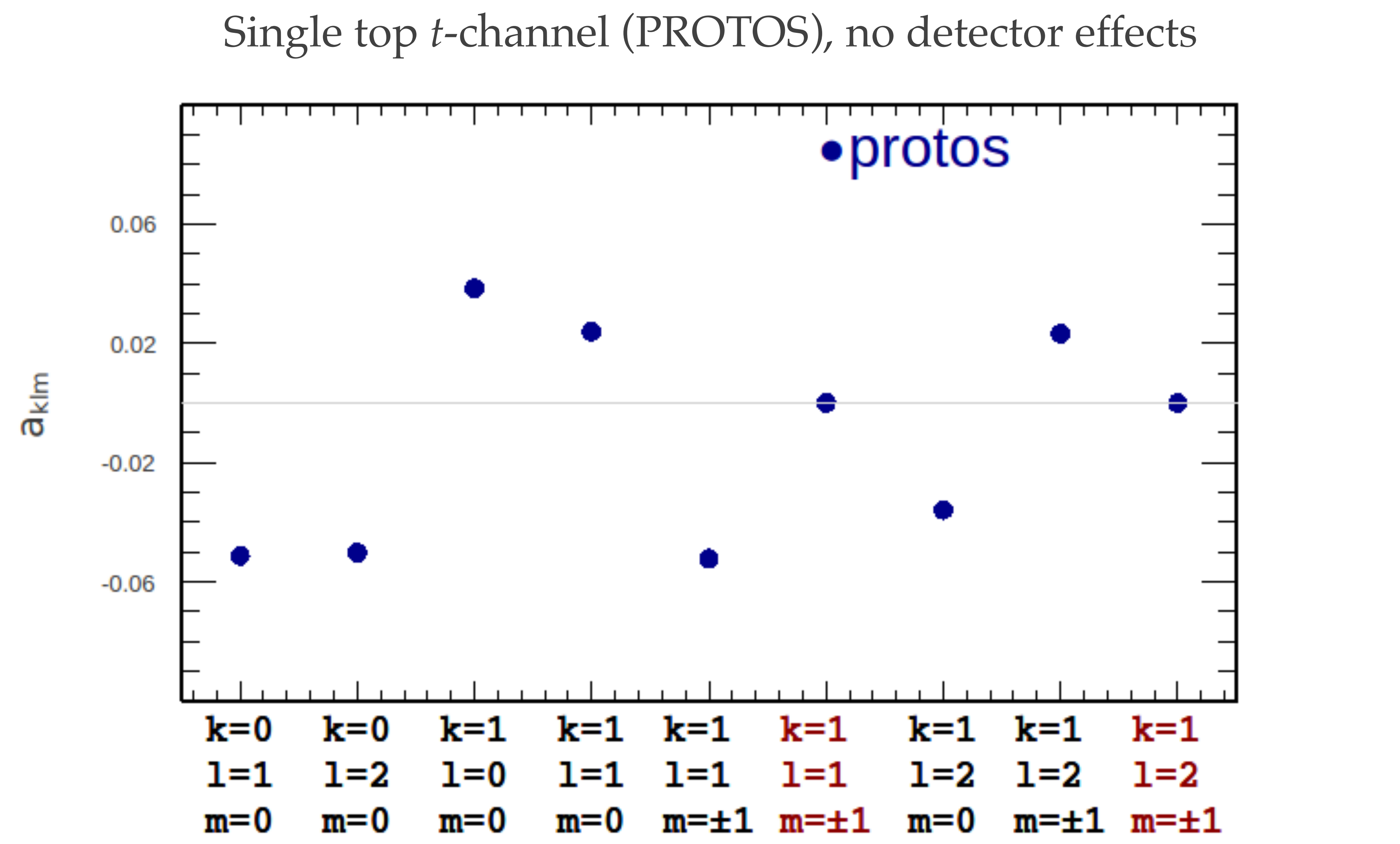
θ_1 vs. ϕ



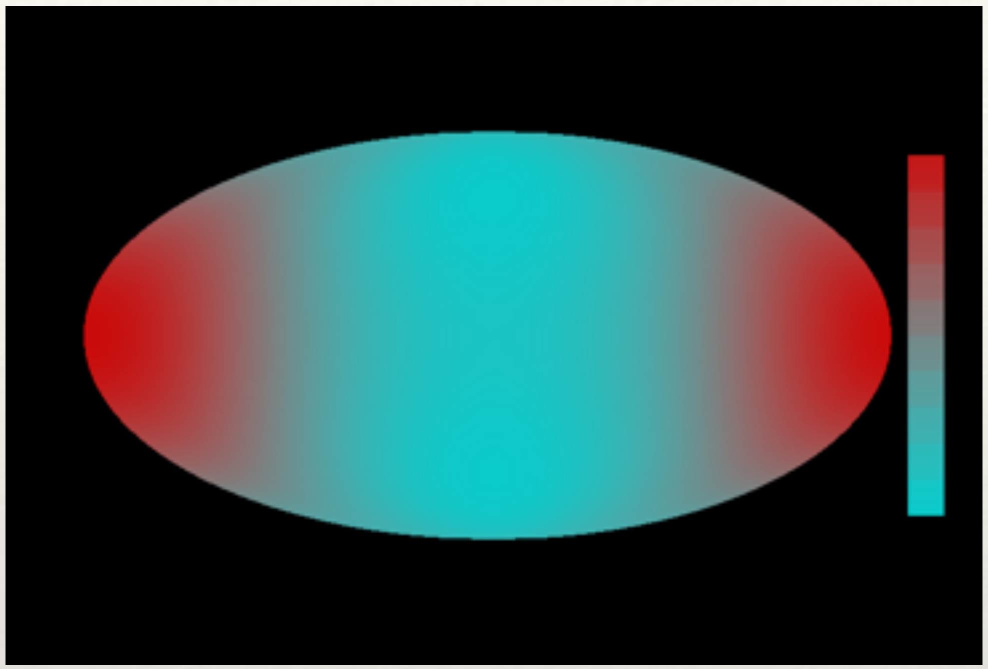
θ_2 vs. ϕ



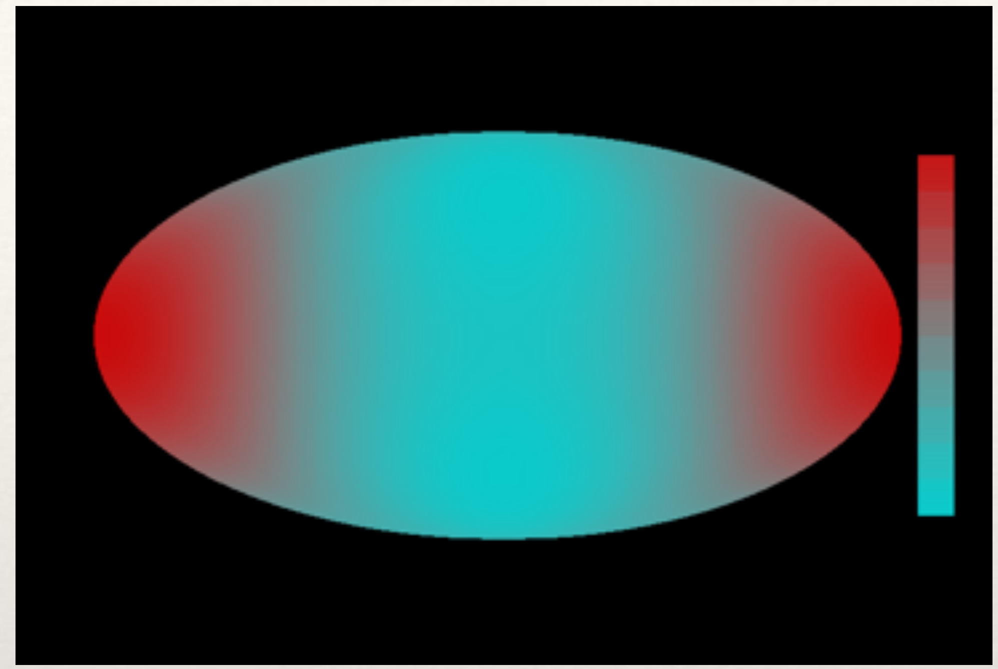
Finite number of coefficients describe the shape in the absence of detector effects



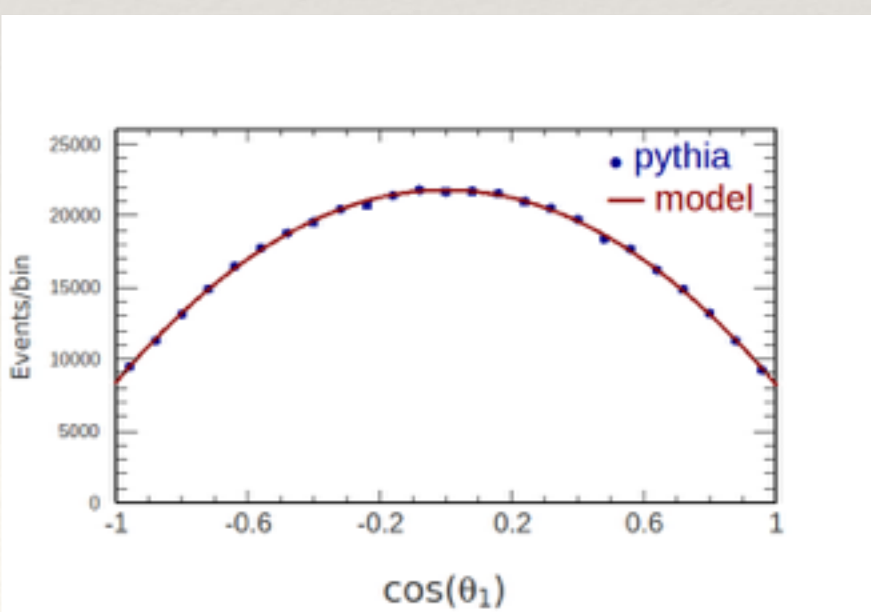
$H \rightarrow W^+W^-$

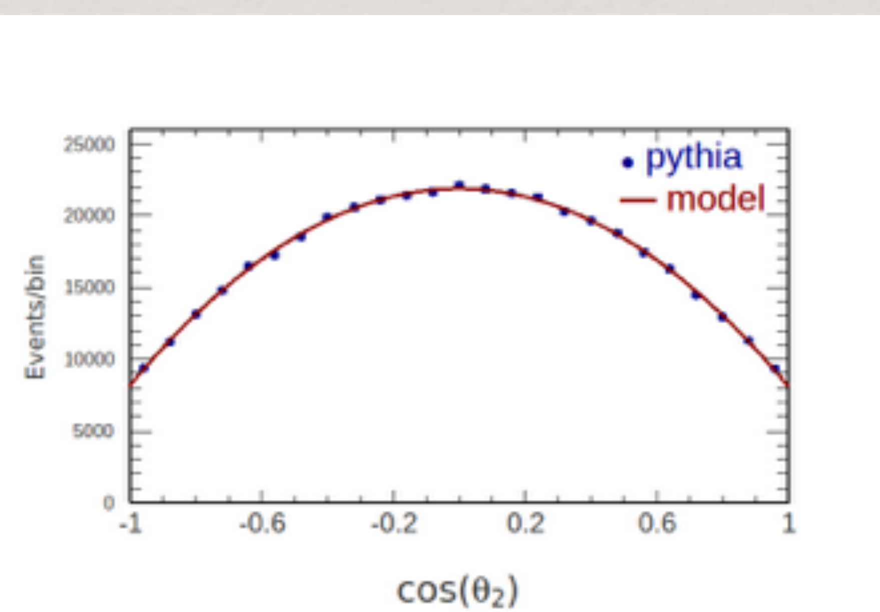


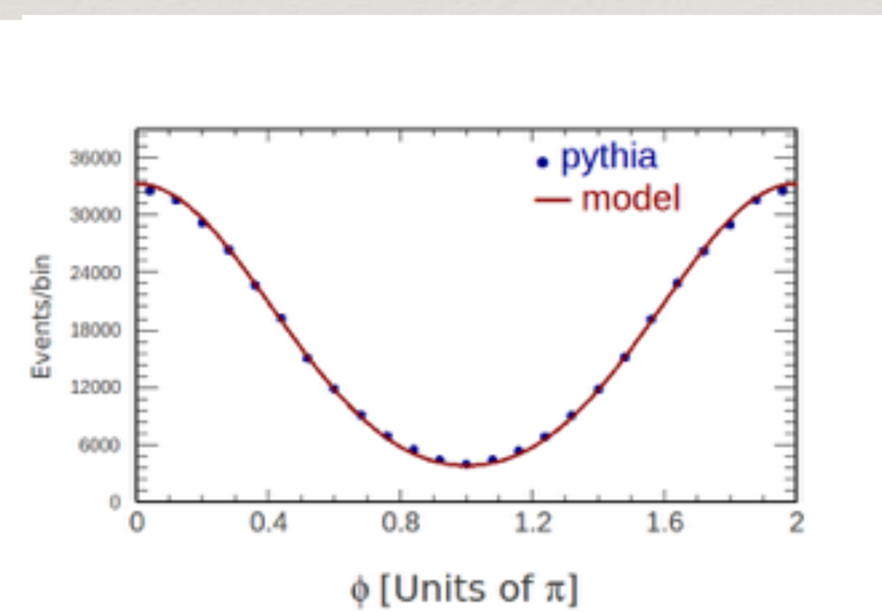
θ_1 vs. ϕ



θ_2 vs. ϕ

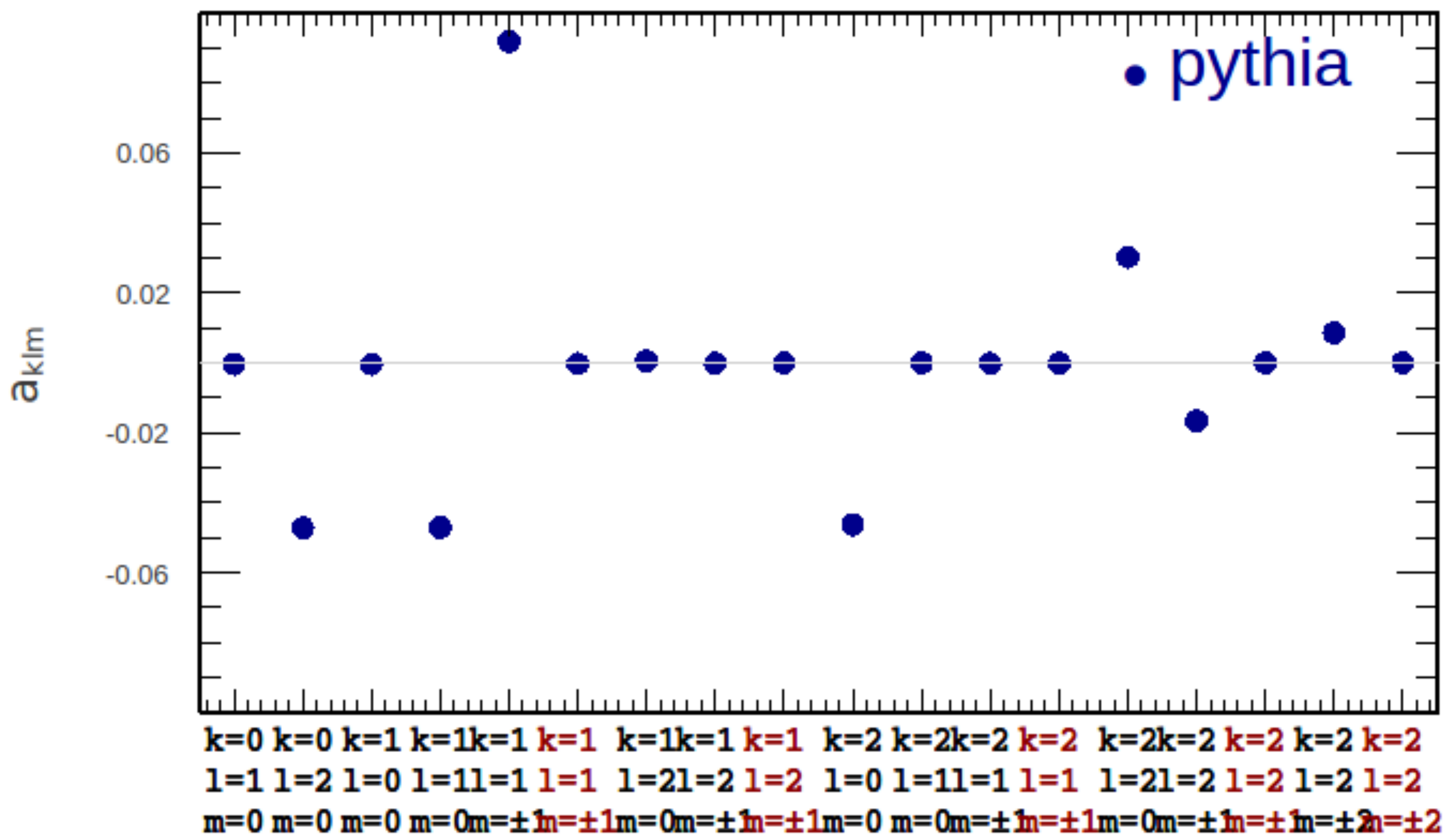




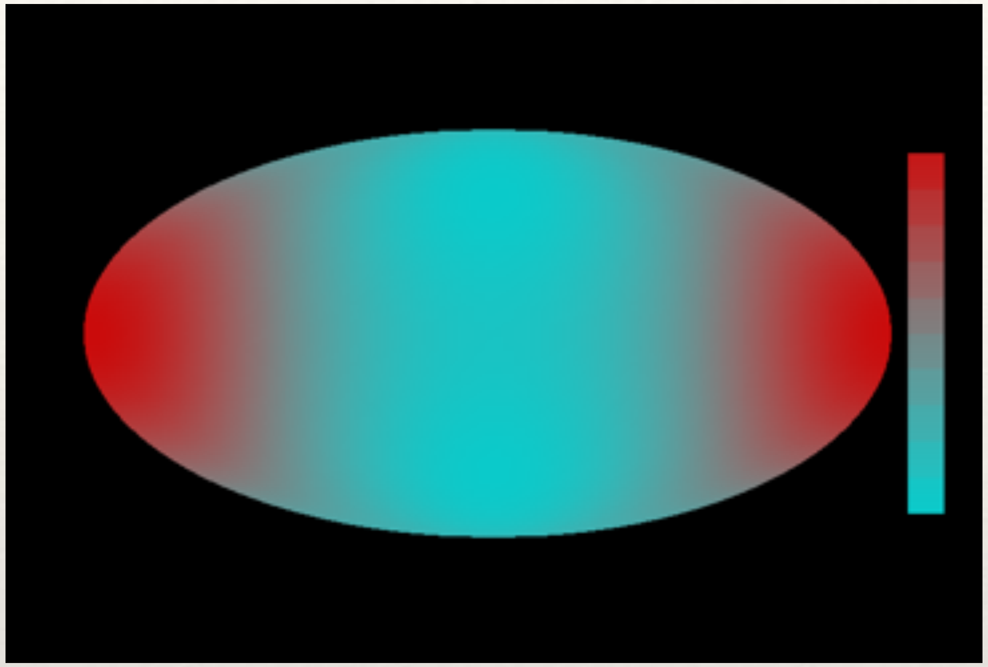


Finite number of coefficients describe the shape in the absence of detector effects

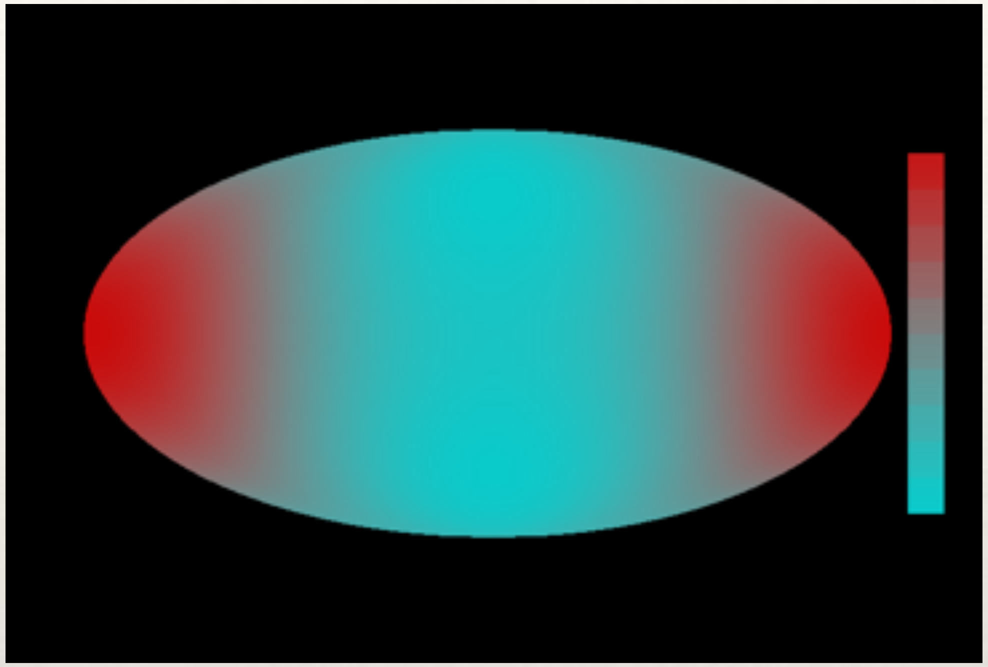
$H \to W^+W^-$ (PYTHIA), no detector effects



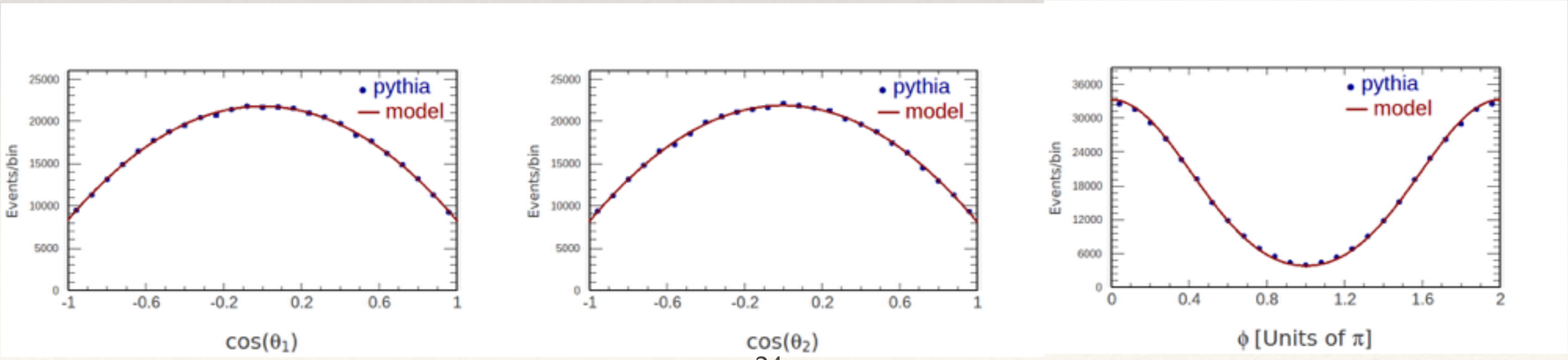
$H \rightarrow W^+W^-$ (pythia truth level)



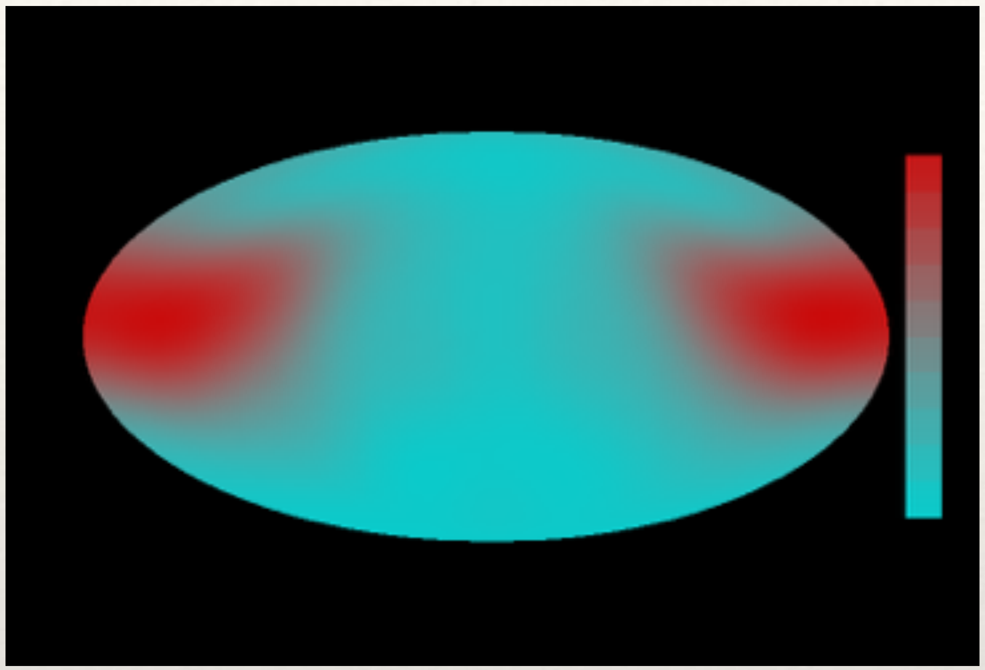
θ_1 vs. ϕ



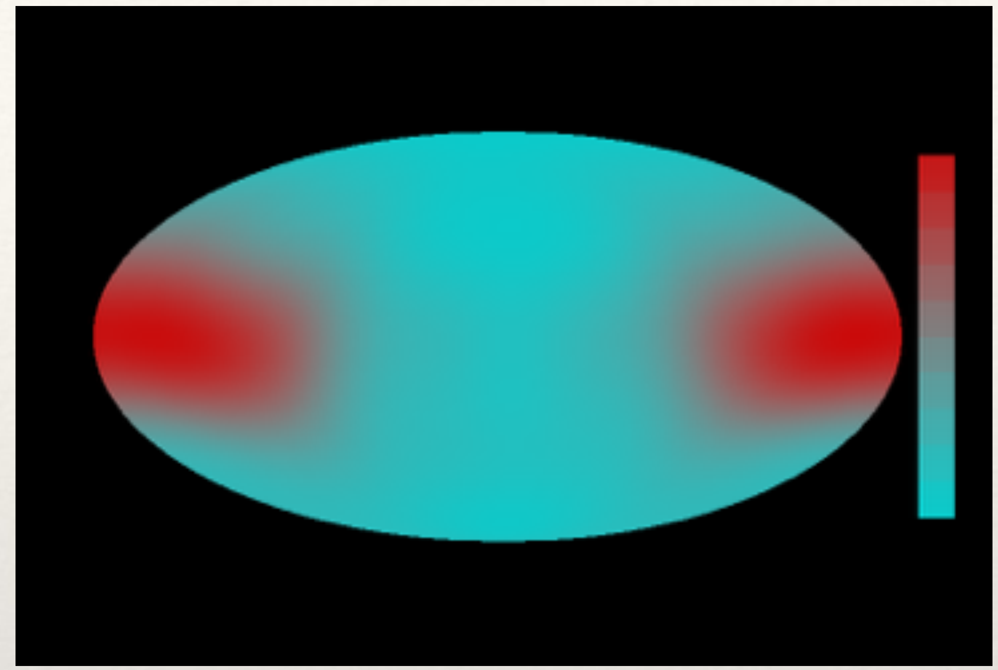
θ_2 vs. ϕ



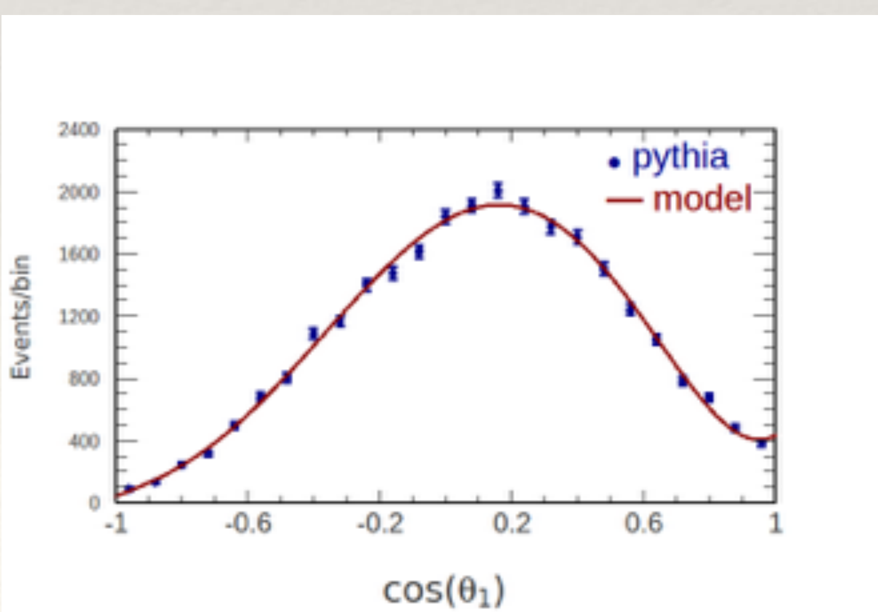
$H \rightarrow W^+W^-$ (reconstruction level)

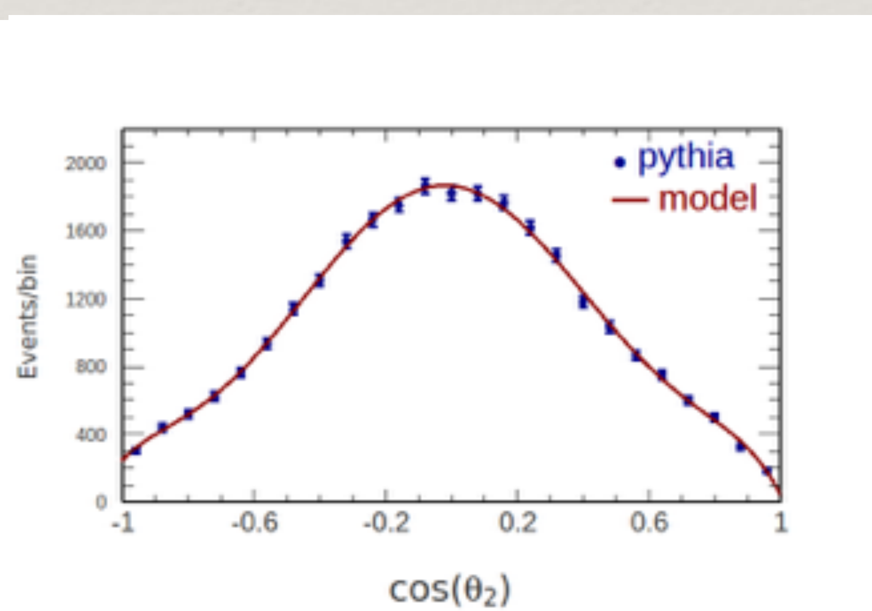


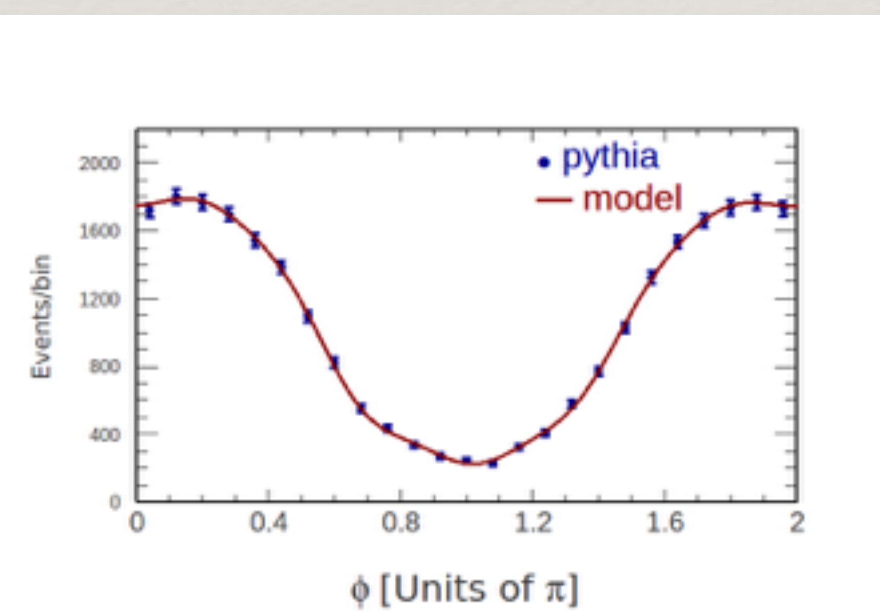
θ_1 vs. ϕ



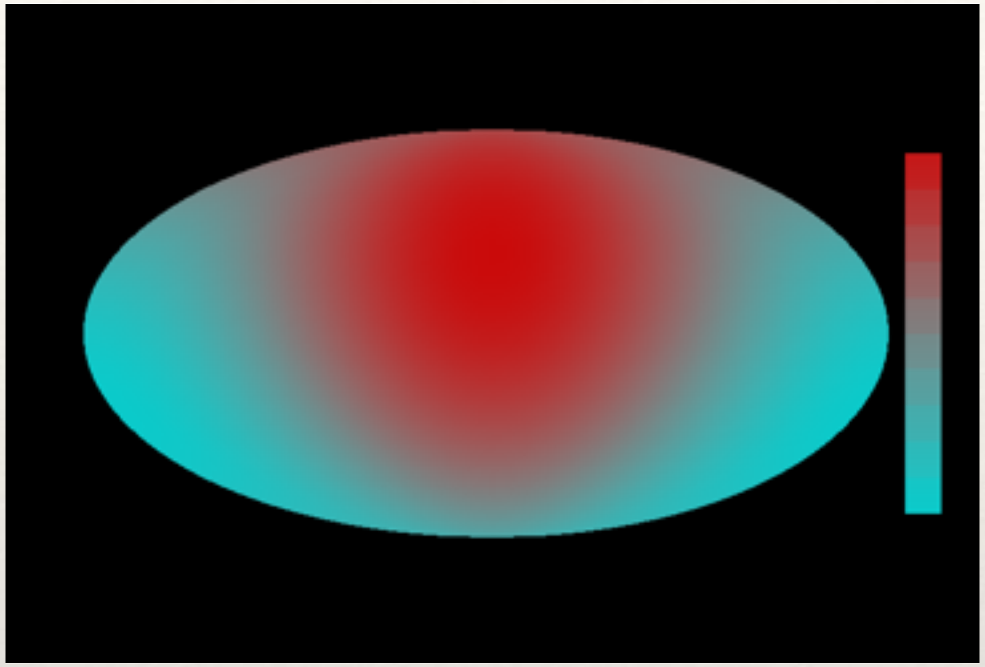
θ_2 vs. ϕ



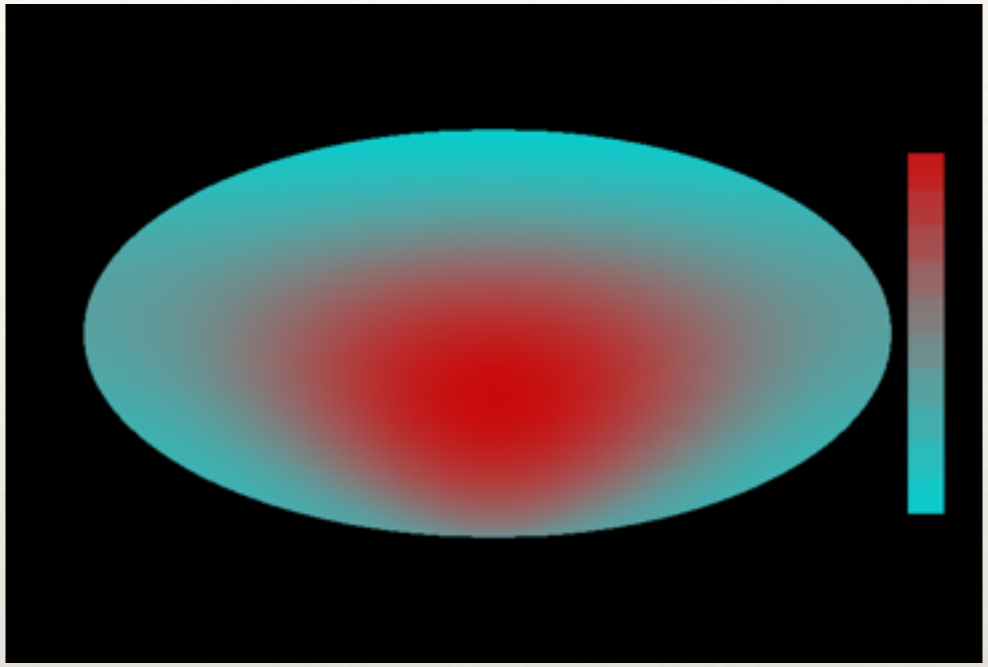




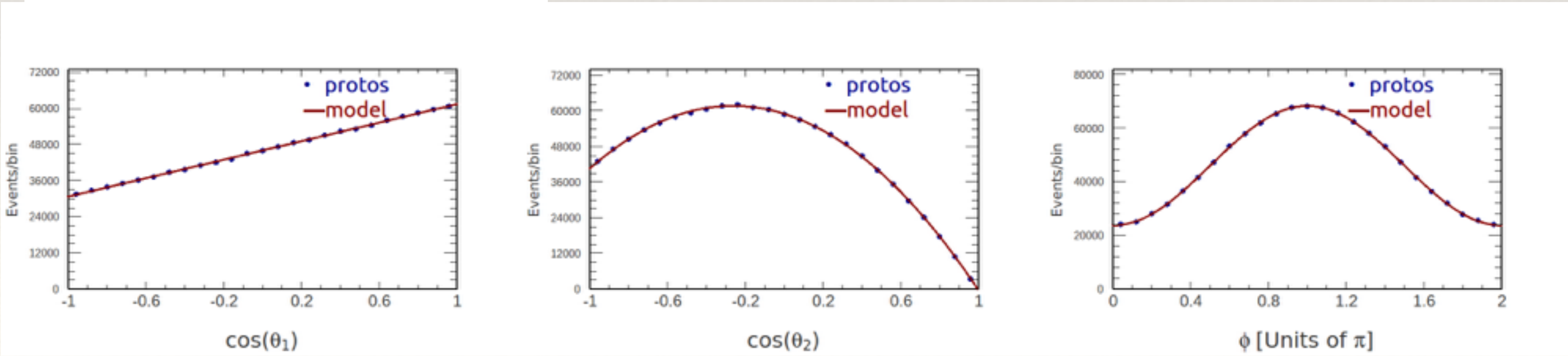
Single top t-channel (protos truth level)



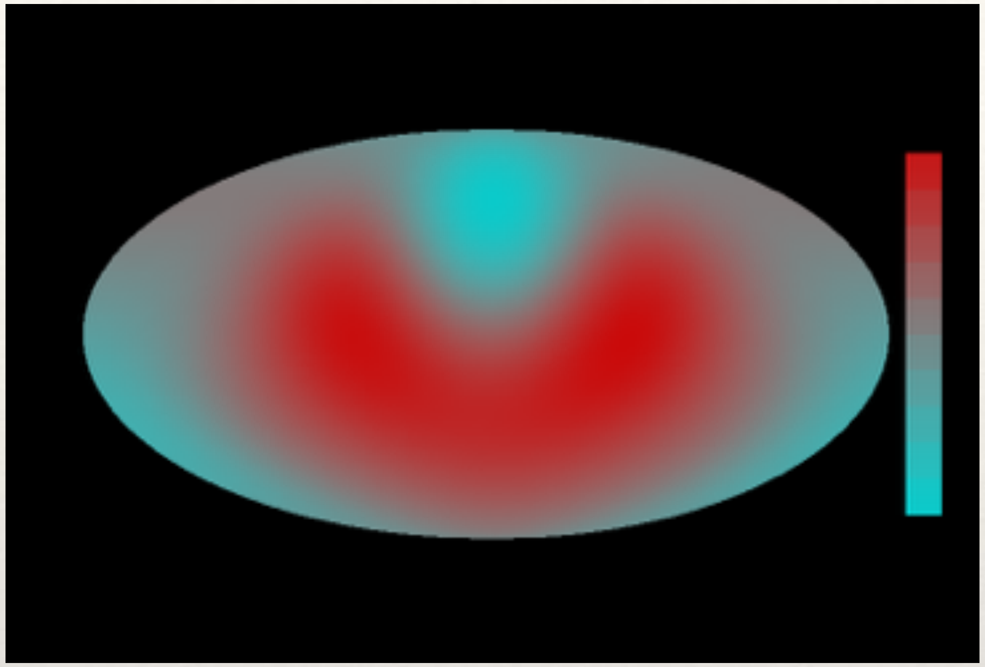
θ_1 vs. ϕ



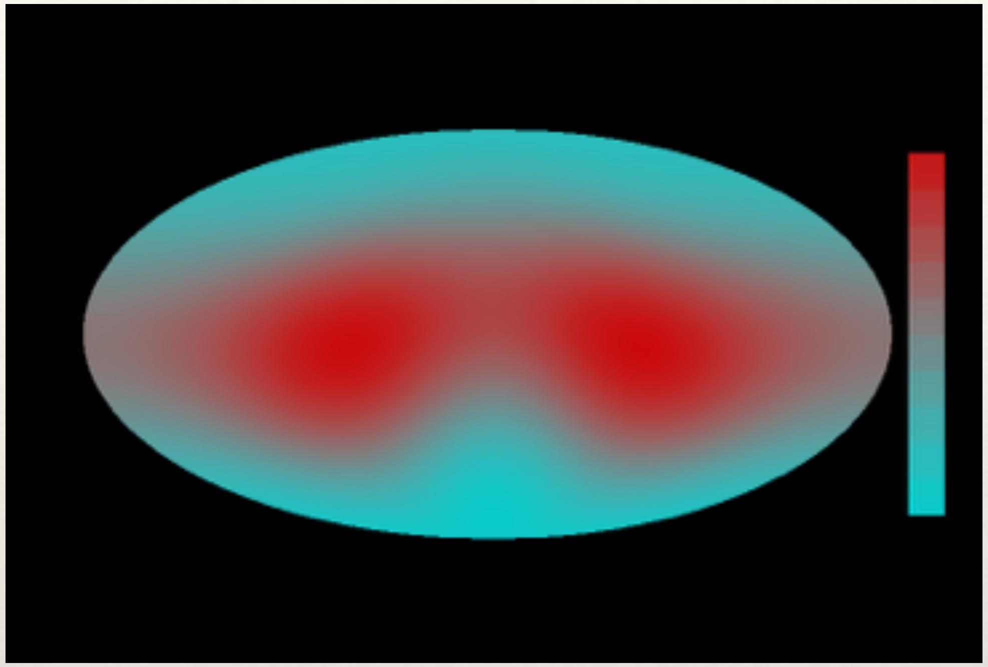
θ_2 vs. ϕ



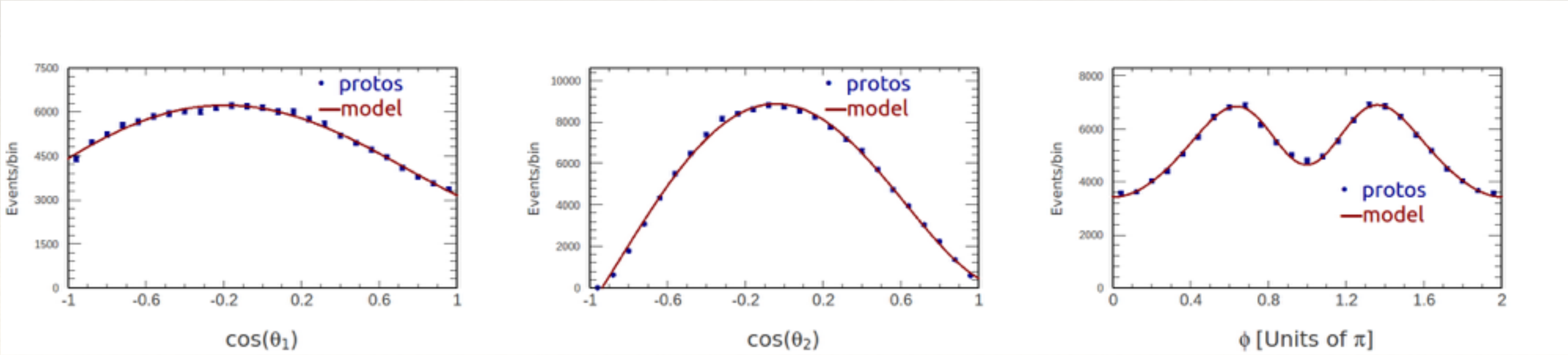
Single top t-channel (reconstruction level):



θ_1 vs. ϕ

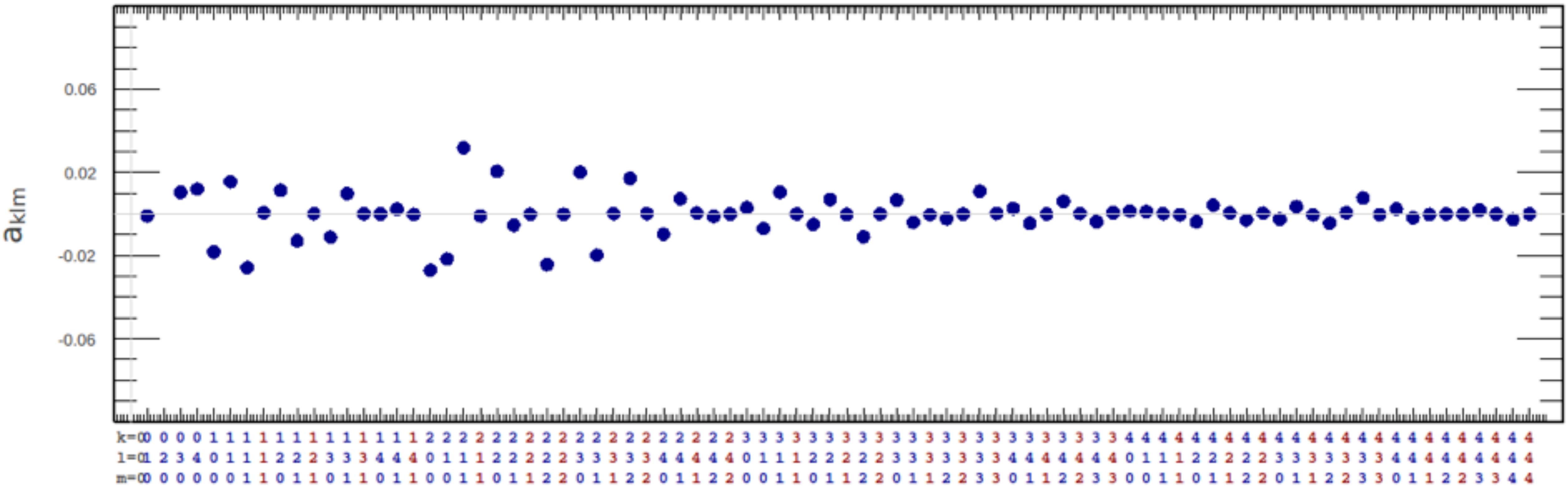


θ_2 vs. ϕ



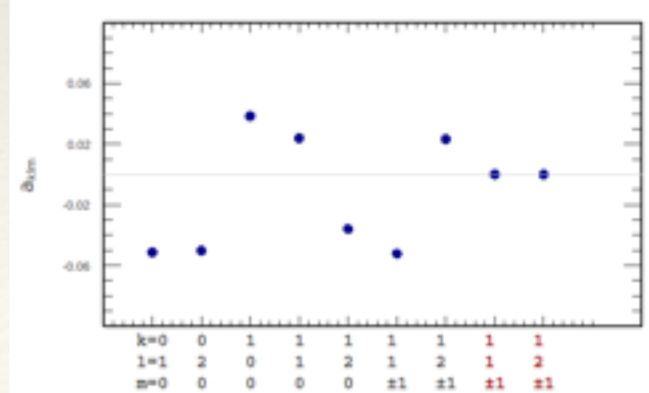
But detector effects have a very big impact!

Single top t-channel, reconstruction level coefficients.



....

Here in miniature is the original.



Our mission: recover the original coefficients from the reconstructed coefficients.

Detector effects

A particle produced at θ_{1T}, θ_{2T}, ϕ_T,

is either rejected by selection cuts,

or it is reconstructed at θ_{1R}, θ_{2R}, ϕ_R

The joint probability function is defined as

$$\mathcal{R}(\theta_{1T}, \theta_{2T}, \phi_T, \theta_{1R}, \theta_{2R}, \phi_R)$$

Manufacture a basis for this 6-D space out of M-functions:

$$r_{k,l,m,k',l',m'} M_{k,l}^{m}(\theta_{1T}, \theta_{2T}, \phi_T) M_{k',l'}^{m'}(\theta_{1R}, \theta_{2R}, \phi_R)$$

==> obtain the coefficients from Monte Carlo

Convert first:

You have found the coefficients of the joint pdf. $\mathcal{R}(\theta_{1R}, \theta_{2R}, \phi_R, \theta_{1T}, \theta_{2T}, \phi_T,)$

You need the coefficients of the conditional pdf $\mathcal{R}(\theta_{1R}, \theta_{2R}, \phi_R | \theta_{1T}, \theta_{2T}, \phi_T,)$

Convert by solving this matrix equation, which is obtained using Gaunt's theorem:

$$a^{\mu}_{\kappa,\lambda} \cdot W^{\mu',\mu,M}_{\kappa',\lambda',\kappa,\lambda,K,L} \cdot g_{\kappa',\lambda',\mu',K',L',M'} = r_{K,L,M,K',L',M'}$$

Physics coefficients in training sample

Gaunt coefficients

You need

You have

You can get what you need by inverting this system of equations

$$g_{\kappa',\lambda',\mu',K',L',M'}$$

Construct a *convolution theorem*

Multiply the physics distribution with the conditional probability and integrate over the truth angles. .

Theorem accommodates non isotropic smearing, which need not be independent of the decay angles:

$$g_{K,L,-M,k,l,m} \cdot a_{K,L,M} = \mathcal{A}_{k,l,m}$$

Which can be written in a matrix form:

$$\mathbf{G} \cdot \vec{a} = \vec{\mathcal{A}}$$

From MC

Reco coefficients measured in data

Physics coefficients. You want.

Deconvolve. Conceptually:

$$\mathbf{G} \cdot \vec{a} = \vec{A}$$

From MC

Reco coefficients measured in data

Physics coefficients. You want. $\vec{a} = \mathbf{G}^{-1} \cdot \vec{A}$

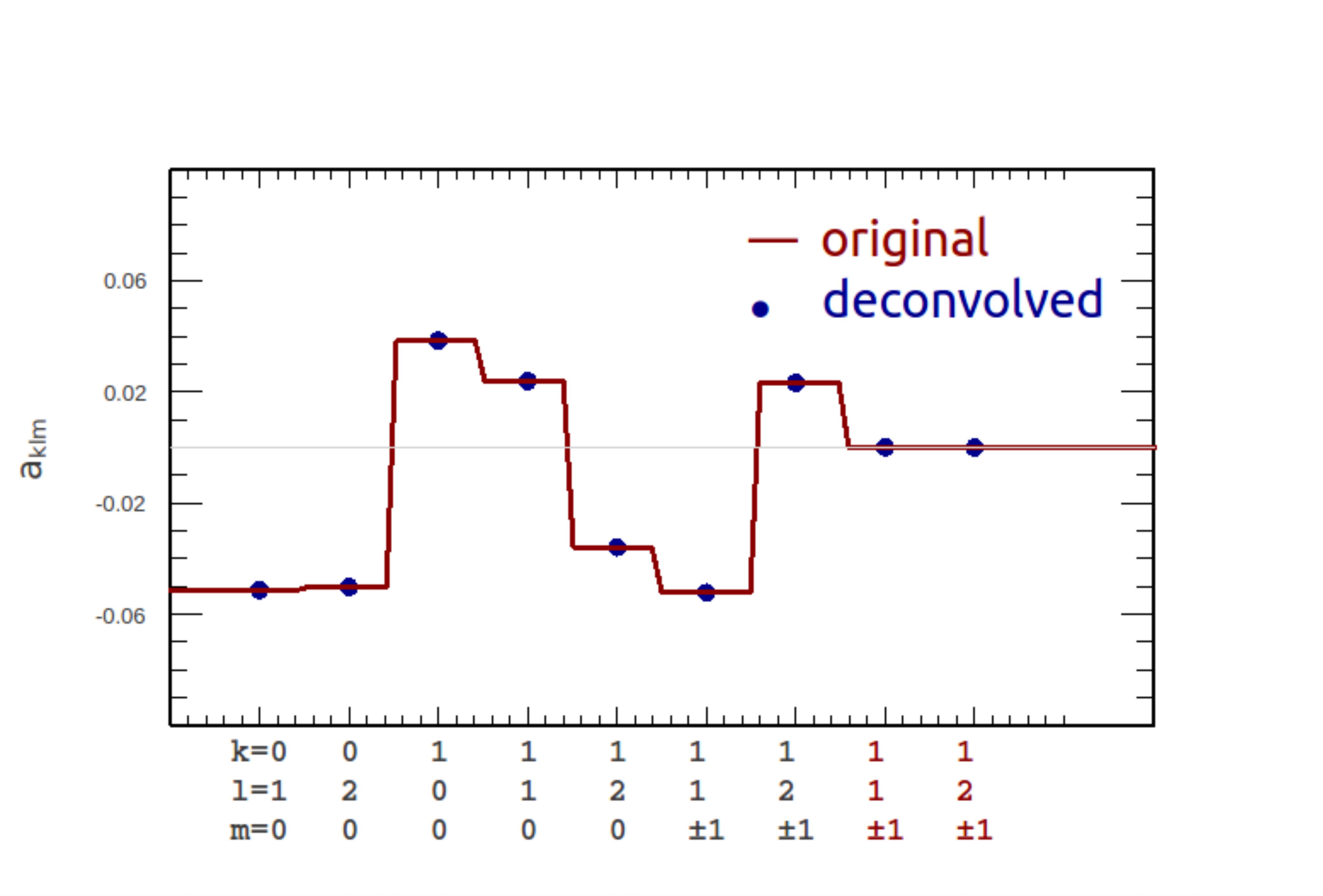
But given that you G is a rectangular matrix in general, you will have to minimize a χ^2:

$$\chi^2(\vec{a}) = (\vec{A} - \mathbf{G} \cdot \vec{a})^T \cdot \mathbf{C}^{-1} \cdot (\vec{A} - \mathbf{G} \cdot \vec{a})$$

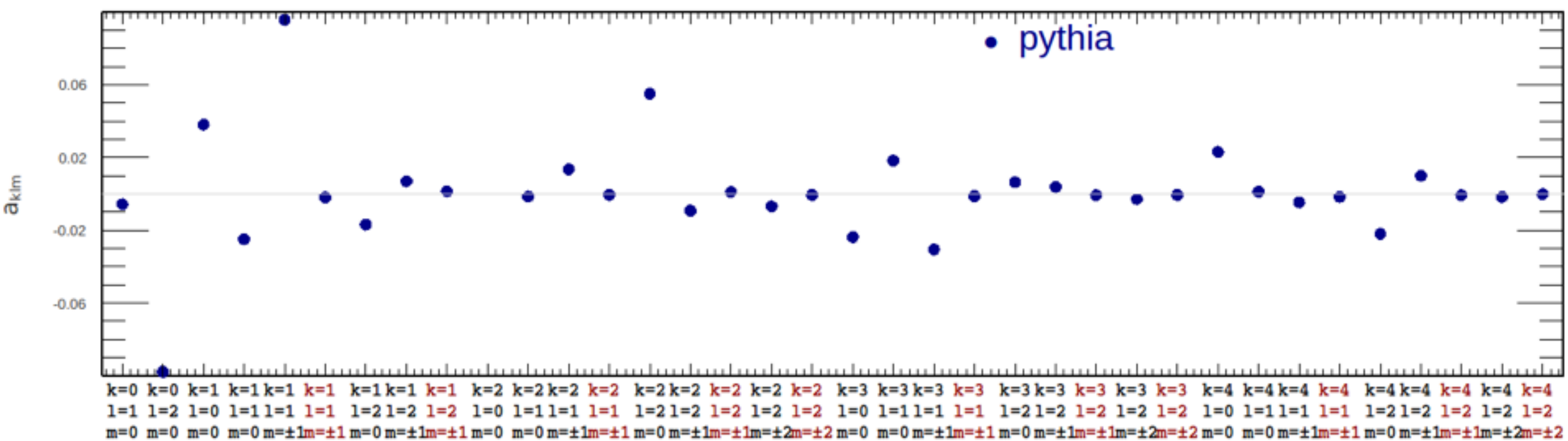
With analytic solution and error matrix:

$$\vec{a} = (\mathbf{G}^T\mathbf{C}^{-1}\mathbf{G})^{-1}\mathbf{G}^T\mathbf{C}^{-1}\vec{A} \qquad \mathbf{V} \equiv \mathrm{Cov}(\vec{a}) = (\mathbf{G}^T\mathbf{C}^{-1}\mathbf{G})^{-1}$$

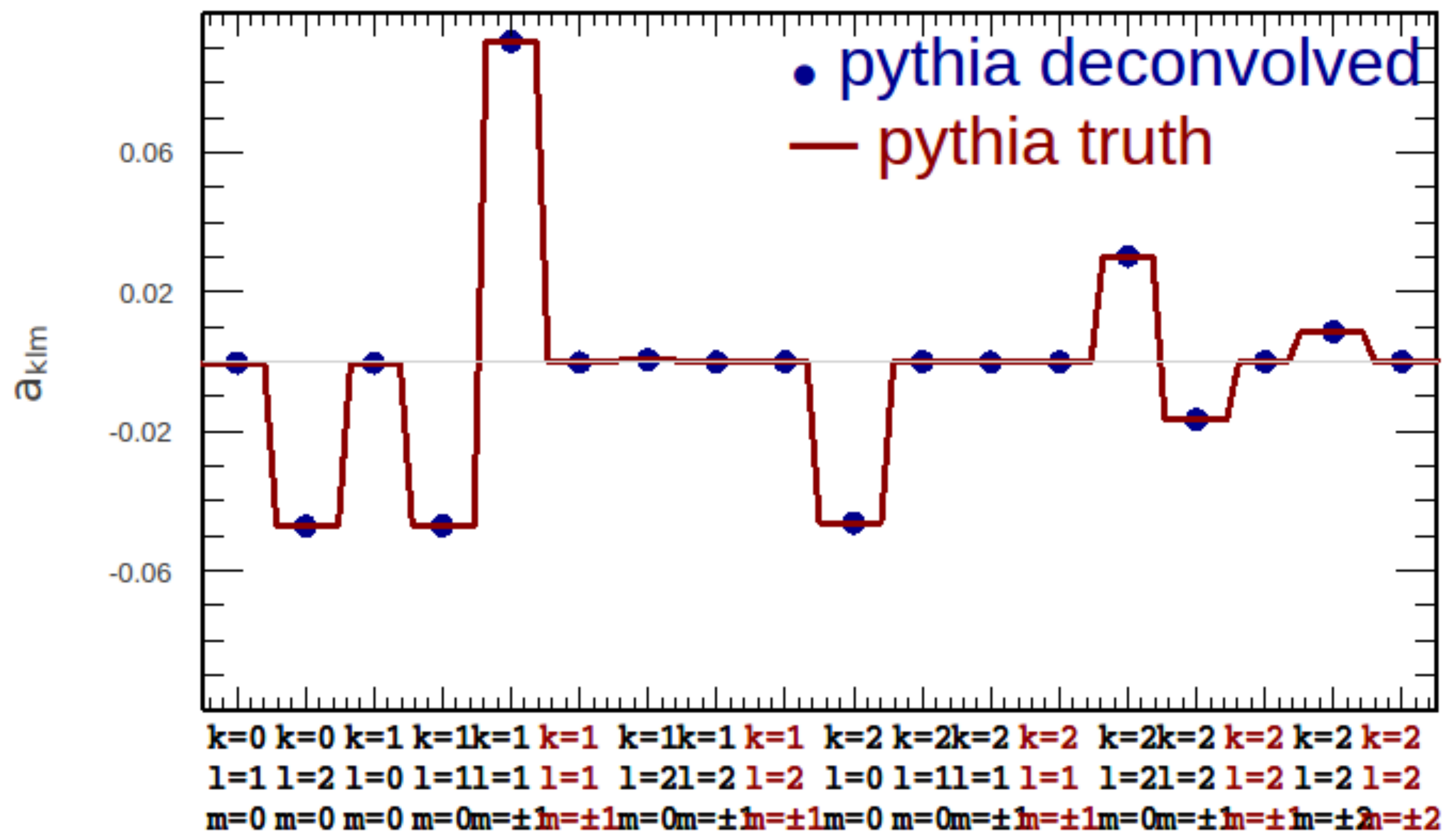
Single top t-channel



$H \rightarrow W^+W^-$ (reconstruction level)



$H \to W^+W^-$ (after deconvolution)



Deconvolving the detector from an observed signal in Fourier space: the recipe

1. Do an orthogonal series analysis of the Monte Carlo in the space of true and reconstructed angles.
2. Coefficient Conversion: Joint PDF-> Conditional PDF & determination of **G**. Procedure is sketched in the previous slides, will be fully described in proceedings.

3. Do an orthogonal series analysis of data sample to determine coefficients $\mathcal{A}$ of reconstructed angular distributions and their covariance matrix **C**.

4. Apply this equation:

$$\vec{a} = (\mathbf{G}^T\mathbf{C}^{-1}\mathbf{G})^{-1}\mathbf{G}^T\mathbf{C}^{-1}\vec{\mathcal{A}} \qquad \mathbf{V} \equiv \mathrm{Cov}(\vec{a}) = (\mathbf{G}^T\mathbf{C}^{-1}\mathbf{G})^{-1}$$

to obtain the physics coefficients & the full covariance thereof.

Then propagate the measurement to fundamental parameters: coupling constants in the interaction Lagrangian

Conclusions

- We showed a set of techniques for describing data using orthogonal functions.
- Introduced a set of functions useful for a certain types of processes with two polar and one azimuthal angle.
- The techniques benefit from an impressive mathematical toolkit, but one which is largely unfamiliar to physicists.
- This includes Gaunt's theorem and a variant of the convolution theorem.
- The latter is extremely useful for handling ferocious detector effects and the benefit is simultaneous determination of shape variables and / or physics parameters.
- The purpose of this talk and the accompanying proceedings is to make these techniques more familiar to the physics community.
- Thank you!