FormCalc 9 and Extensions

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Need not skip Version 9 since there's no FormCalc 95 or 97.

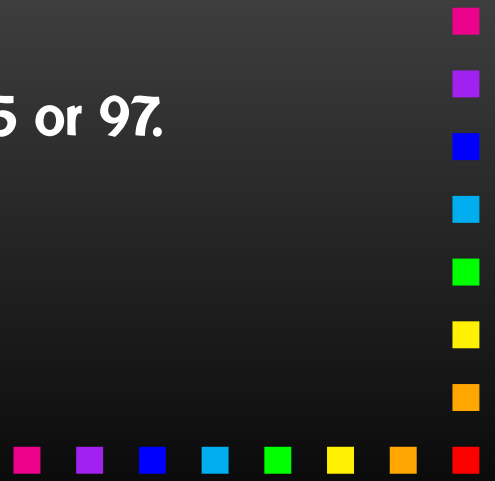


Diagram Evaluation in FeynArts, FormCalc, LoopTools

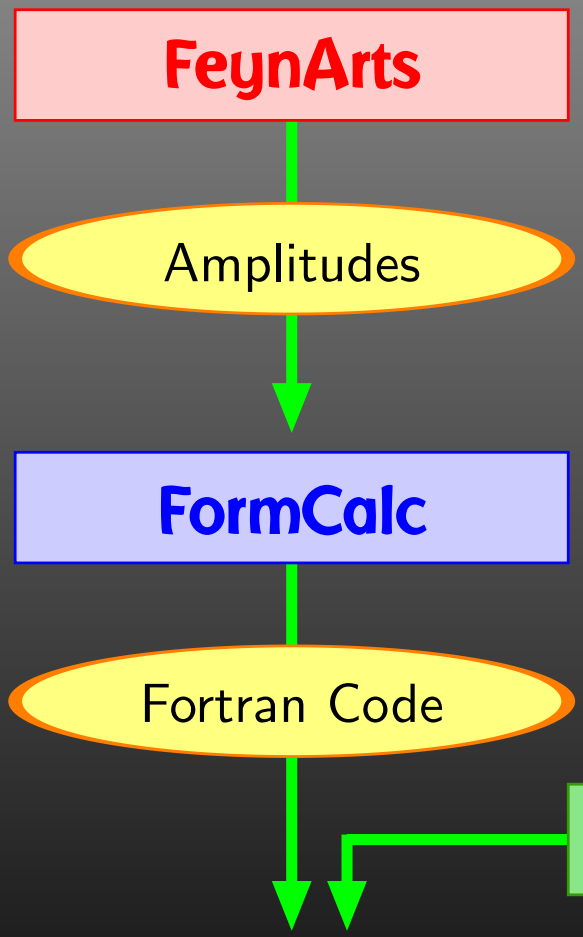


Diagram Generation:

- Create the topologies
- Insert fields
- Apply the Feynman rules
- Paint the diagrams

Algebraic Simplification:

- Contract indices
- Calculate traces
- Reduce tensor integrals
- Introduce abbreviations

Numerical Evaluation:

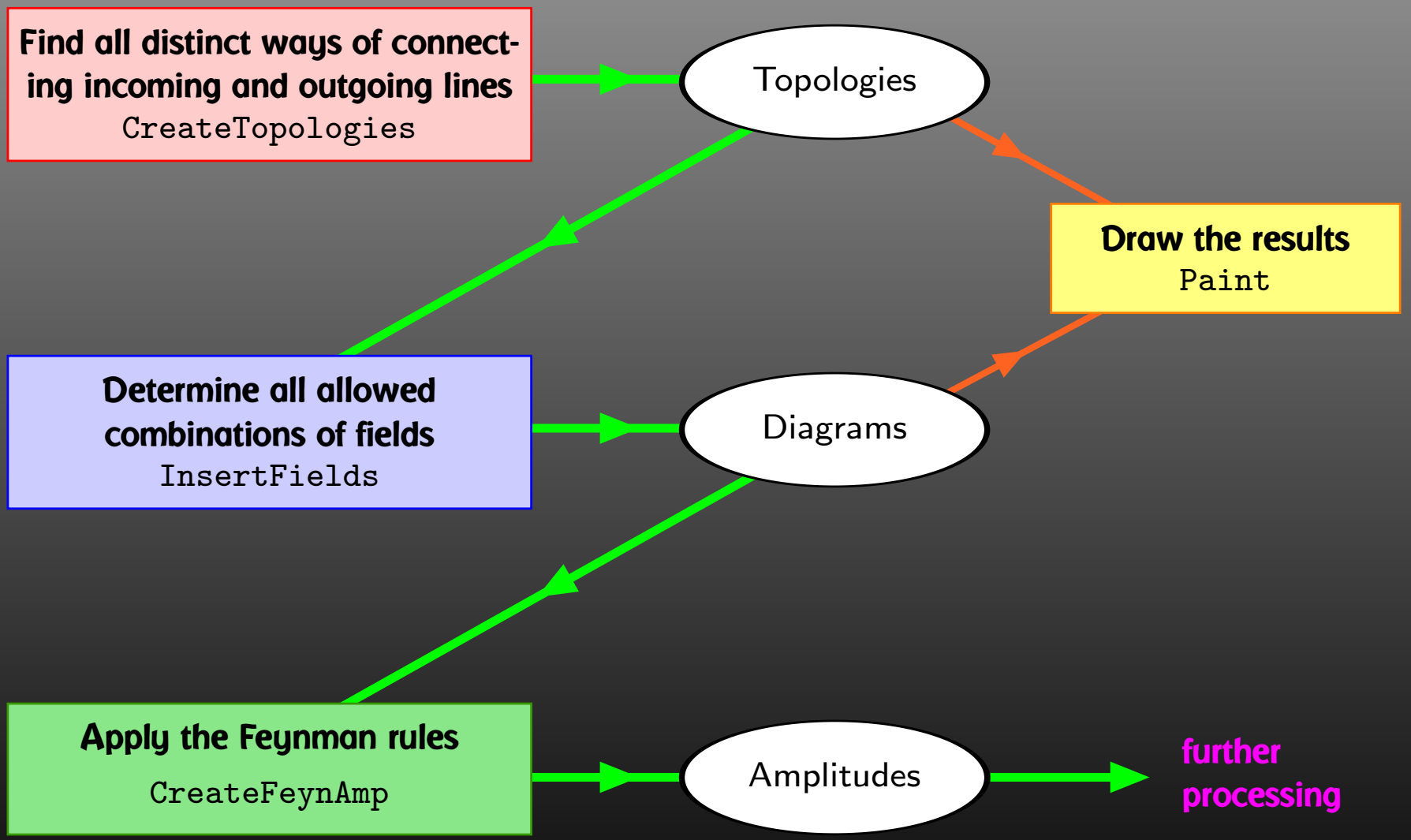
- Convert Mathematica output to Fortran code
- Supply a driver program
- Implementation of the integrals

Symbolic manipulation (Computer Algebra) for the structural and algebraic operations.

Compiled high-level language (Fortran) for the numerical evaluation.

$|\mathcal{M}|^2 \longrightarrow$ **Cross-sections, Decay rates, ...**

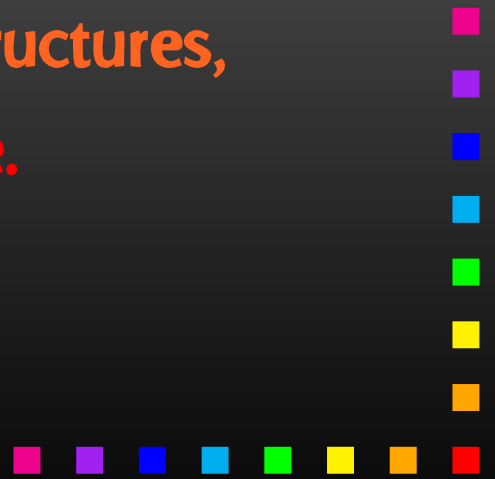
FeynArts


Find all distinct ways of connecting incoming and outgoing lines
CreateTopologies
Topologies
Draw the results
Paint
Determine all allowed combinations of fields
InsertFields
Diagrams
Apply the Feynman rules
CreateFeynAmp
Amplitudes
further processing

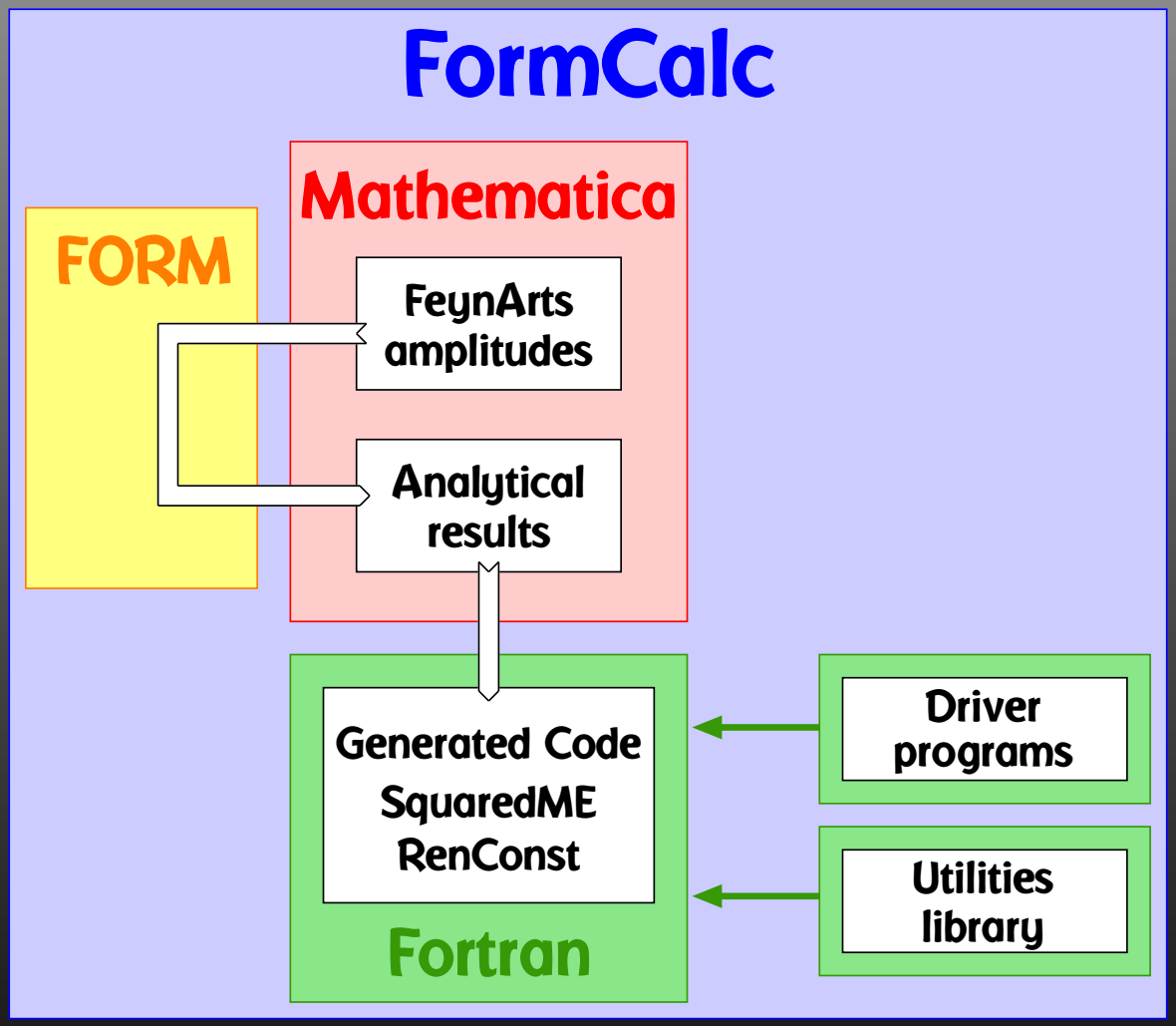
Algebraic Simplification

A number of steps have to be done analytically:

- contract indices as far as possible,
- evaluate fermion traces,
- perform the tensor reduction,
- add local terms arising from D·(divergent integral) (dim reg + dim red),
- simplify open fermion chains,
- simplify and compute the square of SU(N) structures,
- "compactify" the results as much as possible.



FormCalc Internals



FormCalc 9

Work in collaboration with Christian Schappacher

'Collected Wisdom' from several nontrivial SUSY calculations:
arXiv:1511.06002, 1410.2787, 1112.2830, 1111.7289
'Extended support' for MSSMCT model file.

New Features:

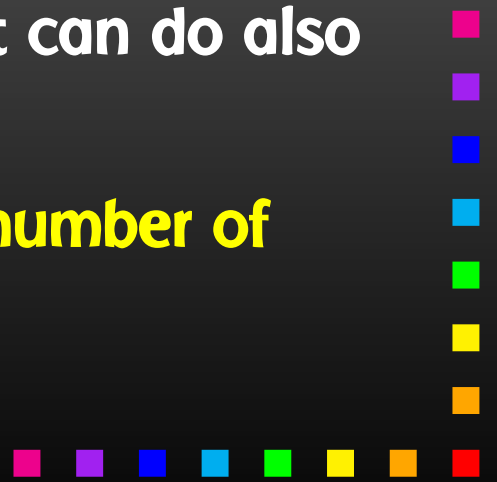
- Combining processes with (almost) arbitrary kinematics.
- Improved code generation + drivers.
- Optional output of non-vectorized code.
- Added various QCD functions (running, $\overline{\text{MS}} \to \overline{\text{DR}}$, etc.).
- Improvements in makefiles, build.

Combining Processes

- Generate each (partonic) process.
- Put description for each in `partonic.h`:

```
#define PID 1
#define PARTON1 i
#define PARTON2 j
#include "process/specs.h"
#include "parton.h"
```

- configure, compile.
- Typically used in hadronic computations, but can do also e.g. $e^+e^- \to \{e^+e^-, \mu^+\mu^-, \tau^+\tau^-\}$.
- Can combine only processes with the same number of external legs (same phase-space dim).



Veto Cuts

- Formerly only selected one-particle cuts available, applied by restricting the integration bounds.
- Now available: 'Veto' cuts which set the cross-section to zero if the cut condition applies:

`CUT_E`(i)
`CUT_k`(i)
`CUT_ET`(i)
`CUT_kT`(i)
`CUT_y`(i)
`CUT_eta`(i)
`CUT_deltatheta`(i)
`CUT_cosdeltatheta`(i)

`CUT_R`(i,j)
`CUT_rho`(i,j)
`CUT_deltay`(i,j)
`CUT_deltaeta`(i,j)
`CUT_yprod`(i,j)
`CUT_etaprod`(i,j)
`CUT_deltaalpha`(i,j)
`CUT_cosdeltaalpha`(i,j)
`CUT_invmass`(i,j)

straightforward to add more

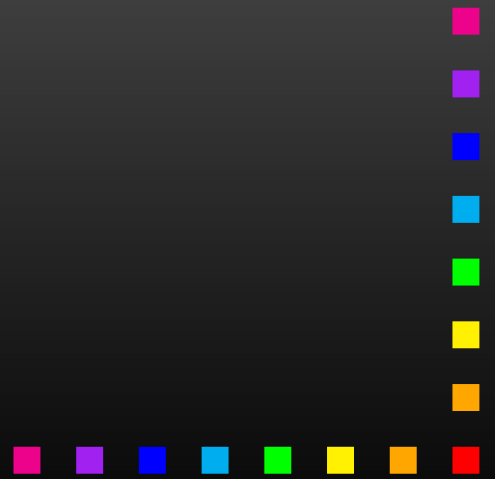
- Construct cuts in `run.F`, e.g.

```
#define CUT1 CUT_kT(3) .gt. 10
```

Non-vectorized Code

- Generated code is by default vectorized in the helicities of the external particles.
- Not always useful, e.g. if one wishes to include the generated code in own code (adds complexity).
- Can now remove all vector instructions/macros with

```
SetLanguage["Fortran","novec"]
SetLanguage["C","novec"]
```



Debugging and Checking Instructions

- **Debugging statements** already available for long:

```
var = expr
DEB("var", var)
```

- New: **Checking statements** of the form

```
CHK_PRE(var)
var = expr
CHK_POST("var", var)
```

- Implement e.g. as

```
#define DEB(tag,var) print *, tag, var
#define CHK_PRE(var) tmp = var
#define CHK_POST(tag,var) if( abs(var - tmp) .gt. 1D7 ) stop tag
```

- Very efficient for finding **problems with a particular parameter point** (in a ton of generated code).

New/improved QCD Functions

Added various QCD utility functions:

- $\alpha_s^{\overline{\text{MS}}}(Q)$ up to 4L, from RunDec [hep-ph/0004189],
- $m_q^{\overline{\text{MS}}}(Q_1) \to m_q^{\overline{\text{MS}}}(Q_2)$ up to 3L, from RunDec,
- $m_q^{\overline{\text{MS}}}(Q) \to m_q^{\text{OS}}$ up to 3L, from RunDec,
- $\alpha_s^{\overline{\text{MS}}} \to \alpha_s^{\overline{\text{DR}}}$ up to 2L, from arXiv:0706.2953.

More Renormalization Constants

- Can now define Renormalization constants **with arbitrary heads,** e.g. mass shifts. Each head goes in separate file:

  `RenConst[.] := ...` → `RenConst.F, .h` *(normal)*
  `MassShift[.] := ...` → `MassShift.F, .h` *(new)*

- Creates **subsets of RC-like objects,** i.e. results of loop calculation but constant (external kinematics fixed).

  Example: shifts used in the iterative determination of Δb:

  RCs need not all be recomputed in each iteration = faster.

- Computation:

  `CalcRenConst[`*expr*`, MassShift]` *(manual)*
  `$RenConst = {RenConst, MassShift}` *(automatic)*

  i.e. the Model file can request that `MassShift` be computed by adding it to `$RenConst`.

Extensions

Often special requirements:

- Resummations (e.g. hbb in MSSM),
- Approximations (e.g. gaugeless limit),
- K-factors,
- Nontrivial renormalization.

Software design so far:

- Mostly 'monolithic' (one package does everything).
- Often controlled by parameter cards, not easy to use beyond intended purpose.
- May want to/must use other packages.

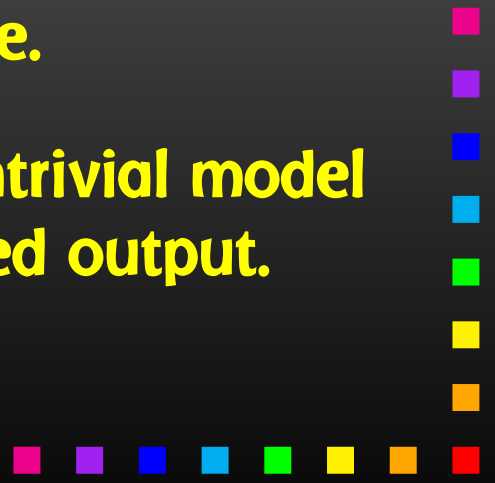
Example: $\mathcal{O}(\alpha_t^2)$ MSSM Higgs-mass corrections

Work in collaboration with Sebastian Paßehr

- **FeynHiggs computes MSSM Higgs mass corrections, including leading 2L contributions: $\mathcal{O}(\alpha_s\alpha_t)$, $\mathcal{O}(\alpha_t^2)$.**
- **$\mathcal{O}(\alpha_t^2)$: Diagrammatic 2L calculation in full cMSSM.**
  Hollik, Paßehr 2014
  1. **Unrenormalized 2L self-energies**
     $\Sigma^{(2)}_{hh}$, $\Sigma^{(2)}_{hH}$, $\Sigma^{(2)}_{hA}$, $\Sigma^{(2)}_{HH}$, $\Sigma^{(2)}_{HA}$, $\Sigma^{(2)}_{AA}$, $\Sigma^{(2)}_{H^+H^-}$
     in gaugeless approximation at $p^2 = 0$ at $\mathcal{O}(\alpha_t^2)$.
  2. **1L diagrams with insertions of 1L counterterms.**
  3. **2L counterterms for ①.**
  4. **2L tadpoles $T_h^{(2)}$, $T_H^{(2)}$, $T_A^{(2)}$ at $\mathcal{O}(\alpha_t^2)$ appearing in ③.**

Template for Calculations

- Starting point: several Mathematica 8 *(sic)* notebooks with parallel instructions poured all over, duplicate code (e.g. gaugeless limit implemented multiply), etc.
- Reorganization of entire procedure:
  - Break calculation into several steps.
  - Implement each step as independent program (invoked from command line).
  - In lieu of 'in vivo' debugging keep detailed logs.
  - Coordinate everything through a makefile.
- Outcome: Template for 2L calculation in nontrivial model with nontrivial renormalization with optimized output.



Wheels we don't reinvent

Will use external packages

- **FeynArts** (for diagram generation),

  Hahn 2001–2015

- **MSSMCT.mod** (MSSM model file with 1L counterterms),

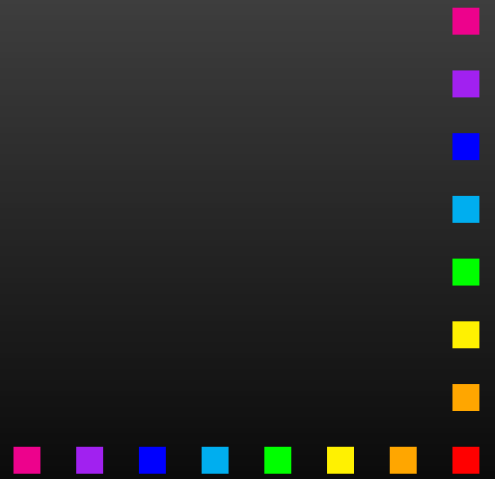
  Schappacher et al. 2014

- **FormCalc** (for 1L tensor reduction, code generation),

  Hahn 1996–2015

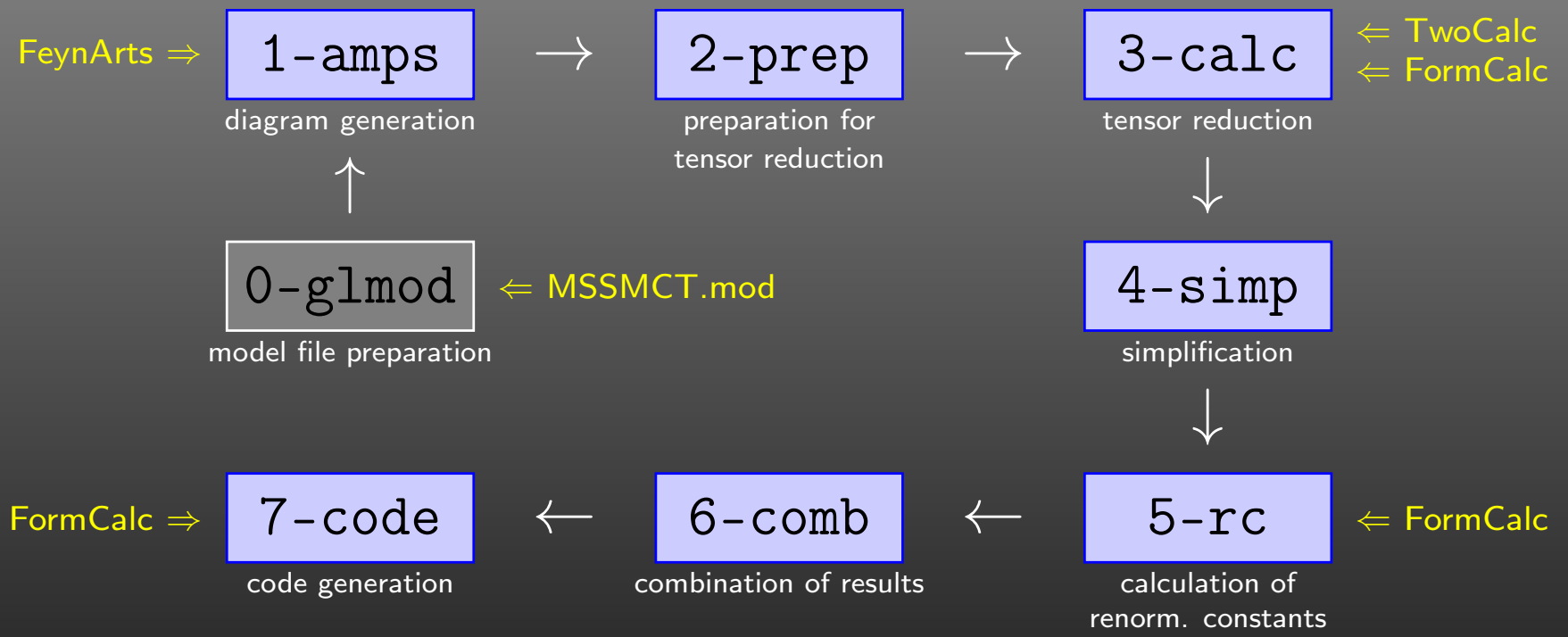
- **TwoCalc** (for 2L tensor reduction).

  Weiglein et al. 1992, 1994



Steps of the Calculation

Calculation split into 7 (8) steps:



Script Structure

- **Shell scripts (`/bin/sh`), run from command line as e.g.**
  `./1-amps arg1 arg2`
- `arg1` = `h0h0`, `h0HH`, `h0A0`, `HHHH`, `HHA0`, `A0A0`, `HmHp` **(self-energies),**
  `h0`, `HH`, `A0` **(tadpoles).**
- `arg2` = `0` **for virtual 2L diagrams,**
  `1` **for 1L diagrams with 1L counterterms.**
- **Inputs/outputs defined in first few lines, e.g.**

```
in=m/$1/2-prep.$2
out=m/$1/3-calc.$2
```

- **Symbolic output + log files go to '`m`' subdirectory.**
  Log file = Output file + `.log.gz`
- **Fortran code goes to '`f`' subdirectory.**

Scripting Mathematica

Efficient batch processing with Mathematica:

Put everything into a script, using sh's Here documents:

```
#! /bin/sh ................ Shell Magic
math << \_EOF_ ............ start Here document (note the \)
  << FeynArts`
  << FormCalc`
  top = CreateTopologies[...];
  ...
_EOF_ ..................... end Here document
```

Everything between "<< *tag*" and "*tag*" goes to Mathematica as if it were typed from the keyboard.

Note the "\" before *tag*, it makes the shell pass everything literally to Mathematica, without shell substitutions.

Scripting Mathematica

- Everything contained in one compact shell script, even if it involves several Mathematica sessions.
- Can combine with arbitrary shell programming, e.g. can use command-line arguments efficiently:

```
#! /bin/sh
math -run "arg1=$1" -run "arg2=$2" ... << \END
  ...
END
```

- Can easily be run in the background, or combined with utilities such as make.

Debugging hint: `-x` flag makes shell echo every statement,

```
#! /bin/sh -x
```

Step 0: Gaugeless Limit

Gaugeless approximation:

1. **Set gauge couplings** $g, g' = 0 \Rightarrow M_W, M_Z = 0$.
2. **Keep finite weak mixing angle.**
3. **Keep** $\frac{\delta M_W^2}{M_W^2}$ **and** $\frac{\delta M_Z^2}{M_Z^2}$ **finite.**

Must set $m_b = 0$ **so that** $\mathcal{O}(\alpha_t^2)$ **corrections form supersymmetric and gauge-invariant subset.**

Most efficient to modify Feynman rules (not ③, though):

- **Load** `MSSMCT.mod` **model file.**
- **Modify couplings, remove zero ones.**
- **Write out** `MSSMCTgl.mod` **model file.**

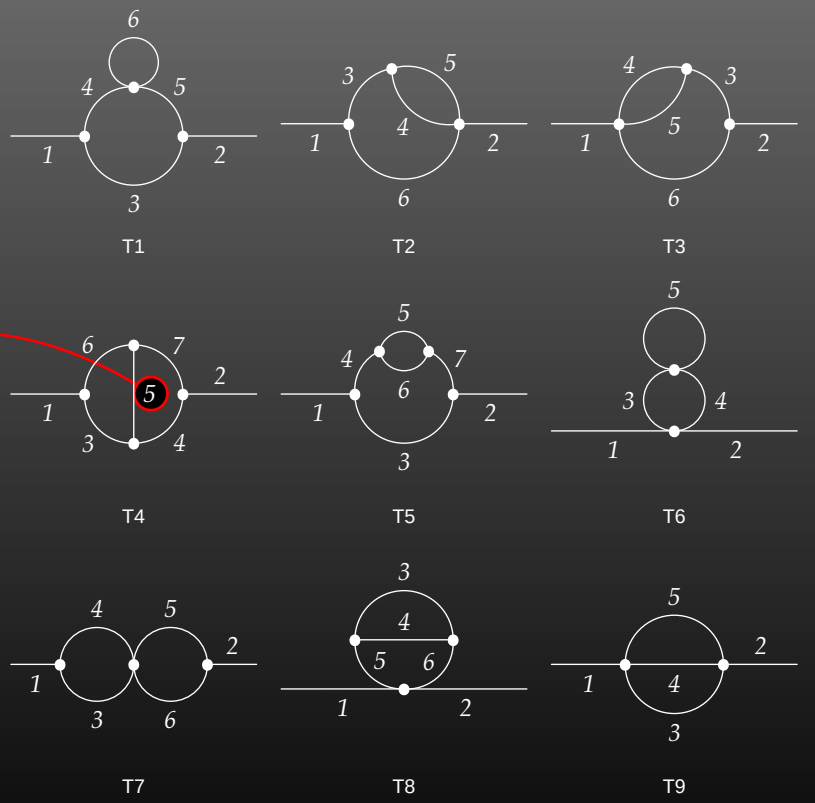
Step 1: Diagram Generation

- Generate 2L virtual and 1L+counterterm diagrams using wrappers for FeynArts functions.

Simple diagram selection functions, e.g.

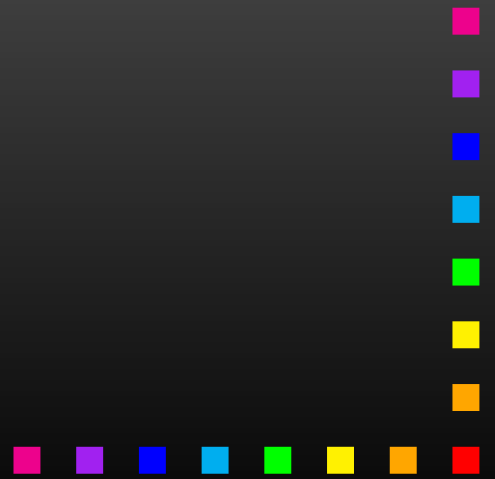
```
sel[0][S[_] -> S[_]] = {
  t[3] && htb[6],
  t[3] && tb[6],
  t[3] && tb[6],
  t[3] && t[4] && htb[5],
  t[3] && htb[5|6],
  t[3] && htb[5],
  t[3] && t[5],
  t[5] && ht[3|4],
  t[3|4|5] && ht[3|4|5] }
```

one of $h_i, \tilde{\chi}, t, \tilde{t}, b, \tilde{b}$



Step 2: Preparation for Tensor Reduction

- Take $p^2 \to 0$ limit.
- Simplify ubiquitous sfermion mixing matrices U_{ij}, mostly by exploiting unitarity ($\sim$ 50% size reduction).



Efficiently Exploit Unitarity in Mathematica

Unitarity of 2 x 2 matrix: $UU^\dagger = U^\dagger U = \mathbb{1}$, **i.e.**

$$
\begin{aligned}
U_{11}U_{11}^* + U_{12}U_{12}^* &= 1, & U_{11}U_{21}^* + U_{12}U_{22}^* &= 0,\\
U_{21}U_{21}^* + U_{22}U_{22}^* &= 1, & U_{21}U_{11}^* + U_{22}U_{12}^* &= 0,\\
U_{11}U_{11}^* + U_{21}U_{21}^* &= 1, & U_{11}U_{12}^* + U_{21}U_{22}^* &= 0,\\
U_{12}U_{12}^* + U_{22}U_{22}^* &= 1, & U_{12}U_{11}^* + U_{22}U_{21}^* &= 0.
\end{aligned}
$$

Problem: `Simplify` **will rarely arrange the** U**'s in just the way that these rules can be applied directly.**

Solution: Introduce auxiliary symbols which immediately deliver the r.h.s. once `Simplify` **considers the l.h.s., i.e. increase the 'incentive' for** `Simplify` **to use the r.h.s.**

But: Upvalues work only one level deep.

Efficiently Exploit Unitarity in Mathematica

Introduce

$$\texttt{USf}[1, j]\ \texttt{USfC}[1, j] \to \texttt{UCSf}[1, j],$$

$$\texttt{USf}[2, j]\ \texttt{USfC}[2, j] \to \texttt{UCSf}[2, j],$$

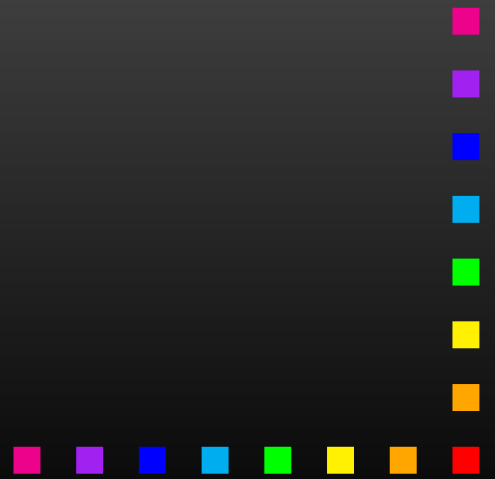
$$\texttt{USf}[1, j]\ \texttt{USfC}[2, j] \to \texttt{UCSf}[3, j],$$ **+ ditto for 1st index**

and formulate unitarity for the `UCSf`**:**

```
UCSf[2,1] = UCSf[1,2];    UCSf[3,2] = -UCSf[3,1];
UCSf[2,2] = UCSf[1,1];    UCSfC[3,2] = -UCSfC[3,1];
                          UCSf[2,3] = -UCSf[1,3];
...                       UCSfC[2,3] = -UCSfC[1,3];
```

Step 3: Tensor Reduction

- Relatively straightforward application of TwoCalc and FormCalc for tensor reduction.
- Observe: Need two Mathematica sessions since TwoCalc and FormCalc cannot be loaded into one session, easily accomodated in shell script.



Step 4: Simplification

- Tensor reduction traditionally increases # of terms most.
- Step 4 reduces size before combination of results.
- Empirical simplification recipe.
- 'DiagMark' trick (D. Stöckinger):
  - Introduce `DiagMark[`m_i`]` where m_i = masses in loop in FeynArts output.
  - Few simplifications can be made between parts with different `DiagMark` $\Rightarrow$ Can apply simplification as

    ```
    Collect[amp, _DiagMark, simpfunc]
    ```

  - Much faster.

Step 5: Calculation of Renormalization Constants

- Compute 1L renormalization constants (RC) with FormCalc.
- Substitute explicit mass dependence in

  $\texttt{dMVsq1} \to \texttt{MV2}\ \texttt{dMVsq1MV2} \quad (V = W, Z)$

  such that gaugeless limit can be taken safely.
- Expand in ε, collect powers for easier handling later, e.g.

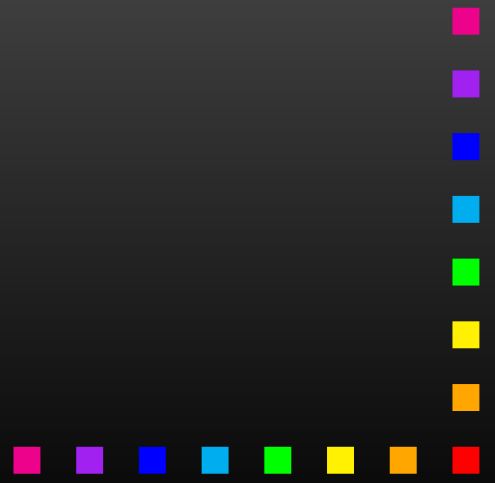
```
{ dMf1[3,3] -> RC[-1, dMf1[-1,3,3]] +
               RC[0, dMf1[0,3,3]],
  {dMf1[-1,3,3] -> ...,
   dMf1[0,3,3] -> ...} }
```

expansion

actual expressions for ε-coeffs

Step 6: Combination of Results

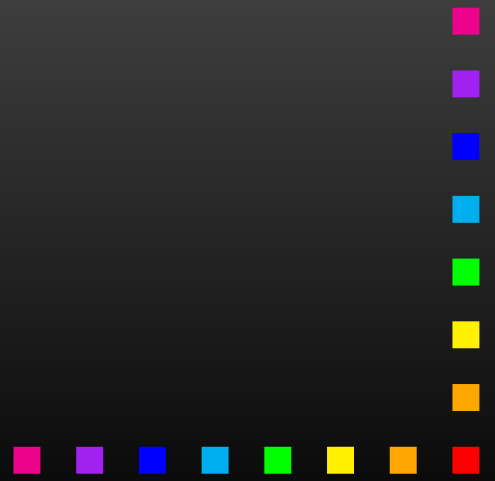
- Expand amplitude in ε (similar as RC).
- Insert RCs.
- Add genuine 2L counterterms (hand-coded).
- Pick only ε^0 term (unless debug flag set).
- Perform final simplification.



Step 7: Code Generation

- Introduce abbreviations to shorten code.
- Write out Fortran code using FormCalc's code-generation functions.
- Add static code which computes e.g. the necessary parameters for the generated code.
- Total final code size: 350 kBytes.

More details in arXiv:1508.00562.



Résumé

- **FormCalc 9 with many new features,** most adapted/ generalized from real-life SUSY calculations:
  - Combination of processes,
  - Improved code generation,
  - New/improved driver, util programs.

  feynarts.de/formcalc
- **Suite of scripts** as by-product of $\mathcal{O}(\alpha_t^2)$ M_h project, may be used as **Template for similar calculations:**
  - Two-loop,
  - Nontrivial model (MSSM) and renormalization,
  - Specific approximations (gaugeless, $p^2 = 0$),
  - Optimized output.

  feynhiggs.de (gen/tlsp directory)