Alternative method of Reduction of the Feynman Diagrams to a set of Master Integrals

Igor Kondrashuk

UBB, Chillan, Chile

Julio Borja, Igor Kondrashuk
Departamento de Ciencias Basicas, Universidad del Bio-Bio (Chile)

ACAT 2016, Chile, Valparaiso 2016

Abstract

We propose a new set of Master Integrals which can be used as a basis for multiloop calculation in any gauge massless field theory. In these theories we consider three-point Feynman diagrams with arbitrary number of loops. The corresponding multiloop integrals may be decomposed in terms of this set of the Master Integrals. We construct a new reduction procedure which can be applied to perform this decomposition.

Why do need this alternative method?

- It is most appropriate for determining the structure of the double ghost vertex in $\mathcal{N} = 4$ SYM
- The elements of the basis we propose may be represented in terms of triangle ladder diagrams in $d = 4 - 2\epsilon$
- Triangle ladders may be represented in terms of two-fold MB transforms
- The method is simple to program

The result is based on my previous papers. Co-authors

- Pedro Allendes (UdeC), Gorazd Cvetic (UTFSM), Ivan Gonzalez (UV), Bernd Kniehl (DESY), Anatoly Kotikov (JINR), Eduardo Notte-Cuello (ULS), Ivan Parra-Ferrada (UBB), Marko Rojas-Medar (UBB), Ivan Schmidt (UTFSM), Alvaro Vergara (UBB)

References

- G. Cvetič, I. Kondrashuk and I. Schmidt, "Effective action of dressed mean fields for $\mathcal{N}=4$ super-Yang-Mills theory," Mod. Phys. Lett. A **21** (2006) 1127 [hep-th/0407251].
- G. Cvetič, I. Kondrashuk, A. Kotikov and I. Schmidt, "Towards the two-loop Lcc vertex in Landau gauge," Int. J. Mod. Phys. A **22** (2007) 1905 [hep-th/0604112].
- G. Cvetič and I. Kondrashuk, "Further results for the two-loop Lcc vertex in the Landau gauge," JHEP **0802** (2008) 023 [hep-th/0703138].
- G. Cvetič and I. Kondrashuk, "Gluon self-interaction in the position space in Landau gauge," Int. J. Mod. Phys. A **23** (2008) 4145 [arXiv:0710.5762 [hep-th]].
- I. Kondrashuk and A. Kotikov, "Fourier transforms of UD integrals," in *Analysis and Mathematical Physics*, Birkhäuser Book Series Trends in Mathematics, edited by B. Gustafsson and A. Vasil'ev, (Birkhäuser, Basel, Switzerland, 2009), pp. 337 [arXiv:0802.3468 [hep-th]].
- I. Kondrashuk and A. Kotikov, "Triangle UD integrals in the position space," JHEP **0808** (2008) 106 [arXiv:0803.3420 [hep-th]].
- P. Allendes, N. Guerrero, I. Kondrashuk and E. A. Notte Cuello, "New four-dimensional integrals by Mellin-Barnes transform," J. Math. Phys. **51** (2010) 052304 [arXiv:0910.4805 [hep-th]].
- I. Kondrashuk and A. Vergara, "Transformations of triangle ladder diagrams," JHEP **1003** (2010) 051 [arXiv:0911.1979 [hep-th]].

References

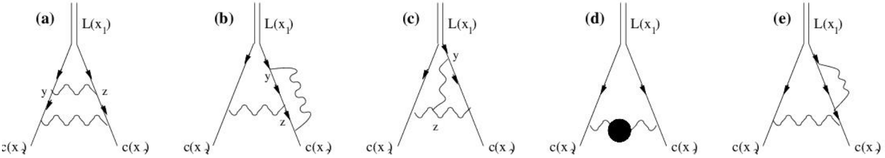
Ivan Gonzalez and Igor Kondrashuk, "Belokurov-Usyukina loop reduction in non-integer dimension", arXiv:1206.4763 [hep-th], Physics of Particles and Nuclei 44 (2013) 268-271

Pedro Allendes, Bernd Kniehl, Igor Kondrashuk, Eduardo A. Notte Cuello, Marko Rojas Medar, "Solution to Bethe-Salpeter equation via Mellin-Barnes transform", arXiv:1205.6257 [hep-th], Nuclear Physics B 870 (2013) 243-277

Ivan Gonzalez and Igor Kondrashuk, "Box ladders in non-integer dimension," arXiv:1210.2243 [hep-th], Theoretical and Mathematical Physics 177 (2013) 1515-1540

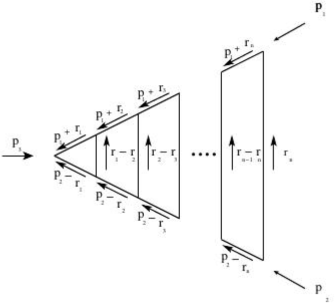
[Bernd Kniehl, Igor Kondrashuk, Eduardo A. Notte Cuello, Ivan Parra Ferrada, Marko Rojas Medar, "Two-fold Mellin-Barnes transforms of Usyukina-Davy-dychev functions", arXiv:1304.3004 [hep-th], Nuclear Physics B 876 (2013) 322-333

Double ghost vertex



- This vertex is finite in $\mathcal{N} = 4$ SYM in Landau gauge, it does not have poles in ϵ

Ladder diagrams



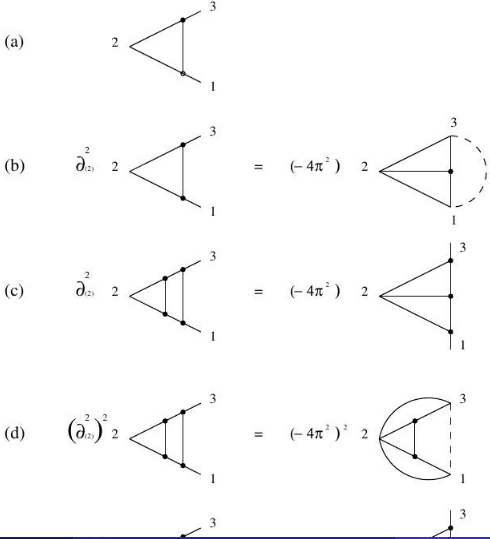
$$= \quad C^{(n)}(p_1^2, p_2^2, p_3^2) \equiv \frac{1}{(p_3^2)^n}\, \Phi^{(n)}\left(\frac{p_1^2}{p_3^2}, \frac{p_2^2}{p_3^2}\right)$$

V. V. Belokurov and N. I. Usyukina, "Calculation Of Ladder Diagrams In Arbitrary Order," J. Phys. A **16** (1983) 2811.

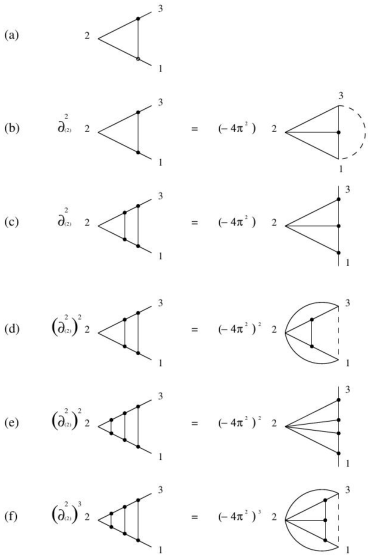
N. I. Usyukina and A. I. Davydychev, "An Approach to the evaluation of three and four point ladder diagrams," Phys. Lett. B **298** (1993) 363.

N. I. Usyukina and A. I. Davydychev, "Exact results for three and four point ladder diagrams with an arbitrary number of rungs," Phys. Lett. B **305** (1993) 136.

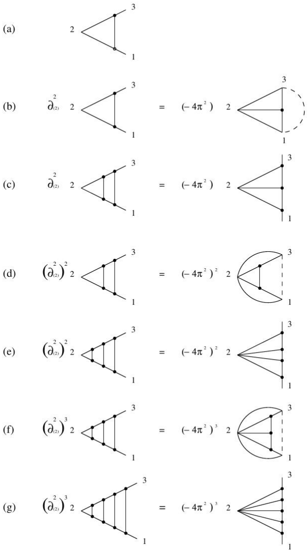
Fourier-invariance of UD functions



Fourier-invariance of UD functions



Fourier-invariance of UD functions



Fourier-invariance of UD functions via MB transform

$$\frac{1}{[31]^2}\Phi^{(n)}\left(\frac{[12]}{[31]},\frac{[23]}{[31]}\right) = \frac{1}{(2\pi)^4}\int \; d^4p_1 d^4p_2 d^4p_3 \;\delta(p_1+p_2+p_3)\times$$
$$\times e^{ip_2x_2}e^{ip_1x_1}e^{ip_3x_3}\frac{1}{(p_2^2)^2}\Phi^{(n)}\left(\frac{p_1^2}{p_2^2},\frac{p_3^2}{p_2^2}\right).$$

The explicit form of the function is given in Davydychev and Usyukina papers:

$$\Phi^{(n)}(x,y) = -\frac{1}{n!\lambda}\sum_{j=n}^{2n}\frac{(-1)^j j!\ln^{2n-j}(y/x)}{(j-n)!(2n-j)!}\left[\mathrm{Li_j}\left(-\frac{1}{\rho x}\right) - \mathrm{Li}_j(-\rho y)\right],$$

$$\rho = \frac{2}{1-x-y+\lambda}, \qquad \lambda = \sqrt{(1-x-y)^2-4xy}.$$

Mellin-Barnes transform for the ladder functions:

$$\Phi^{(n)}(x,y) = \oint dz_2 dz_3 x^{z_2} y^{z_3} \mathcal{M}^{(n)}(z_2,z_3)$$

$$\frac{1}{[31]^2}\Phi^{(n)}\left(\frac{[12]}{[31]},\frac{[23]}{[31]}\right) = \frac{1}{(2\pi)^4}\int\ d^4p_1 d^4p_2 d^4p_3\ \delta(p_1+p_2+p_3)\times$$

$$\times e^{ip_2x_2} e^{ip_1x_1} e^{ip_3x_3} \frac{1}{(p_2^2)^2}\Phi^{(n)}\left(\frac{p_1^2}{p_2^2},\frac{p_3^2}{p_2^2}\right) =$$

$$= \frac{1}{(2\pi)^8}\int\ d^4p_1 d^4p_2 d^4p_3 d^4x_5 e^{ip_2(x_2-x_5)} e^{ip_1(x_1-x_5)} e^{ip_3(x_3-x_5)}\times$$

$$\times \frac{1}{(p_2^2)^2}\Phi^{(n)}\left(\frac{p_1^2}{p_2^2},\frac{p_3^2}{p_2^2}\right) =$$

Fourier-invariance of UD functions via MB transform

$$= \frac{1}{(2\pi)^8}\int d^4p_1 d^4p_2 d^4p_3 d^4x_5 \oint dz_2 dz_3 \frac{e^{ip_2(x_2-x_5)}e^{ip_1(x_1-x_5)}e^{ip_3(x_3-x_5)}}{(p_2^2)^{2+z_2+z_3}(p_1^2)^{-z_2}(p_3^2)^{-z_3}} \times$$

$$\times \mathcal{M}^{(n)}(z_2,z_3) =$$

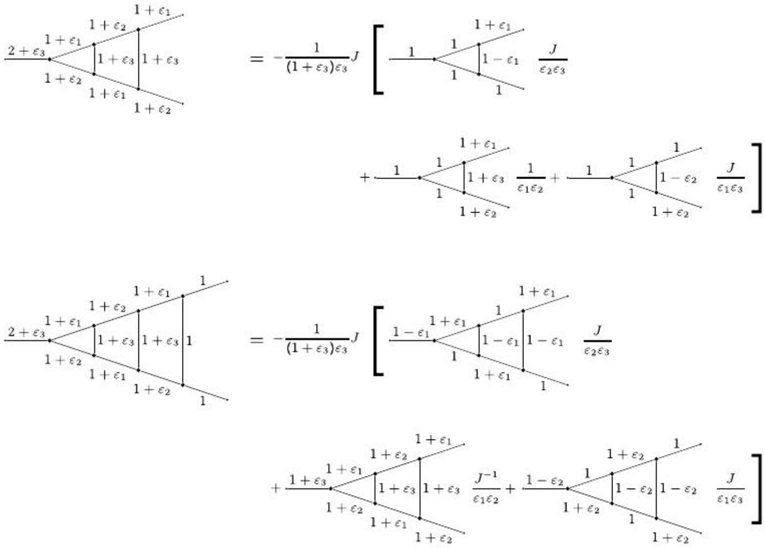
$$= \frac{(4\pi)^6}{(2\pi)^8}\int d^4x_5 \oint dz_2 dz_3 \frac{\Gamma(-z_2-z_3)}{\Gamma(2+z_2+z_3)}\frac{\Gamma(2+z_2)}{\Gamma(-z_2)}\frac{\Gamma(2+z_3)}{\Gamma(-z_3)}$$

$$\times \frac{2^{2z_2+2z_3-2(2+z_2+z_3)}\mathcal{M}^{(n)}(z_2,z_3)}{(x_2-x_5)^{-z_2-z_3}(x_1-x_5)^{2+z_2}(x_3-x_5)^{2+z_3}} =$$

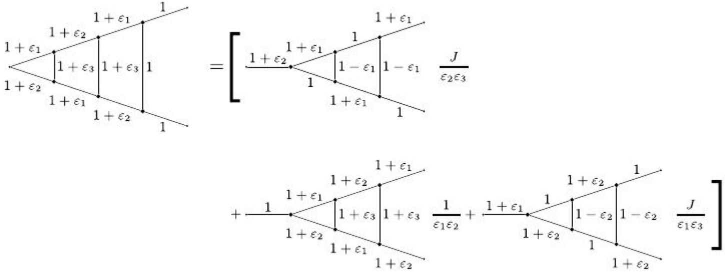
$$= \oint dz_2 dz_3 \frac{\mathcal{M}^{(n)}(z_2,z_3)}{[12]^{-z_3}[23]^{-z_2}[31]^{2+z_2+z_3}}$$

P. Allendes, N. Guerrero, I. Kondrashuk and E. A. Notte Cuello, "New four-dimensional integrals by Mellin-Barnes transform," J. Math. Phys. **51** (2010) 052304 [arXiv:0910.4805 [hep-th]].

Loop reduction in d=4 dimensions



Another form of the loop reduction in d=4 dimensions



V. V. Belokurov and N. I. Usyukina, "Calculation Of Ladder Diagrams In Arbitrary Order," J. Phys. A **16** (1983) 2811.

Pedro Allendes, Bernd Kniehl, Igor Kondrashuk, Eduardo A. Notte Cuello, Marko Rojas Medar, "Solution to Bethe-Salpeter equation via Mellin-Barnes transform", arXiv:1205.6257 [hep-th], Nuclear Physics B 870 (2013) 243-277

Ivan Gonzalez and Igor Kondrashuk, "Box ladders in non-integer dimension," arXiv:1210.2243 [hep-th], Theoretical and Mathematical Physics 177 (2013) 1515-1540

[Bernd Kniehl, Igor Kondrashuk, Eduardo A. Notte Cuello, Ivan Parra Ferrada, Marko Rojas Medar, "Two-fold Mellin-Barnes transforms of Usyukina-Davy-dychev functions", arXiv:1304.3004 [hep-th], Nuclear Physics B 876 (2013) 322-333

Formulas to prove, IK's talks at ICMP 2015, Santiago

$$\oint_C dz_2 dz_3 \; D^{(u,v)}[1+\varepsilon_1 - z_3, 1+\varepsilon_2 - z_2, 1+\varepsilon_3] \times$$
$$\times D^{(z_2,z_3)}[1+\varepsilon_2, 1+\varepsilon_1, 1+\varepsilon_3] =$$
$$J\left[\frac{D^{(u,v-\varepsilon_2)}[1-\varepsilon_1]}{\varepsilon_2\varepsilon_3} + \frac{D^{(u,v)}[1+\varepsilon_3]}{\varepsilon_1\varepsilon_2} + \frac{D^{(u-\varepsilon_1,v)}[1-\varepsilon_2]}{\varepsilon_1\varepsilon_3}\right],$$

$$D^{(z_2,z_3)}[\nu_1,\nu_2,\nu_3] = \frac{\Gamma(-z_2)\,\Gamma(-z_3)\,\Gamma(-z_2-\nu_2-\nu_3+d/2)}{\Pi_i\Gamma(\nu_i)}$$
$$\times\frac{\Gamma(-z_3-\nu_1-\nu_3+d/2)\,\Gamma(z_2+z_3+\nu_3)\,\Gamma(\Sigma\nu_i - d/2 + z_3 + z_2)}{\Gamma(d-\Sigma_i\nu_i)},$$

$$\varepsilon_1+\varepsilon_2+\varepsilon_3 = 0 \quad \text{and} \quad D^{(u,v)}[1+\nu] \equiv D^{(u,v)}[1,1,1+\nu].$$

Barnes Lemmas, Smirnov's textbook

$$\oint_C dz\, \Gamma(\lambda_1 + z)\Gamma(\lambda_2 + z)\Gamma(\lambda_3 - z)\Gamma(\lambda_4 - z) = \frac{\Gamma(\lambda_1+\lambda_3)\Gamma(\lambda_1+\lambda_4)\Gamma(\lambda_2+\lambda_3)\Gamma(\lambda_2+\lambda_4)}{\Gamma(\lambda_1+\lambda_2+\lambda_3+\lambda_4)},$$

$$\oint_C dz\, \frac{\Gamma(\lambda_1 + z)\Gamma(\lambda_2 + z)\Gamma(\lambda_3 + z)\Gamma(\lambda_4 - z)\Gamma(\lambda_5 - z)}{\Gamma(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5+z)} = \frac{\Gamma(\lambda_1+\lambda_4)\Gamma(\lambda_2+\lambda_4)\Gamma(\lambda_3+\lambda_4)\Gamma(\lambda_1+\lambda_5)\Gamma(\lambda_2+\lambda_5)\Gamma(\lambda_3+\lambda_5)}{\Gamma(\lambda_1+\lambda_2+\lambda_4+\lambda_5)\Gamma(\lambda_1+\lambda_3+\lambda_4+\lambda_5)\Gamma(\lambda_2+\lambda_3+\lambda_4+\lambda_5)},$$

$$\oint_C dz\, \frac{\Gamma(\lambda_1 + z)\Gamma(\lambda_2 + z)\Gamma(\lambda_3 - z)\Gamma(\lambda_4 - z)}{z} = \frac{\Gamma(\lambda_1+\lambda_3-1)\Gamma(\lambda_2+\lambda_3)\Gamma(\lambda_2+\lambda_4)\Gamma(\lambda_1+\lambda_4-1)}{\Gamma(1-\lambda_1)\Gamma(1-\lambda_2)} \times \left[\Gamma(1-\lambda_1)\Gamma(1-\lambda_2) - \Gamma(\lambda_3)\Gamma(\lambda_4)\right], \quad \lambda_1+\lambda_2+\lambda_3+\lambda_4 = 2$$

The Integrant

$$D^{(z_2,z_3)}[1+\varepsilon_2, 1+\varepsilon_1, 1+\varepsilon_3] = \frac{\Gamma(-z_2)\Gamma(-z_3)\Gamma(-z_2+\varepsilon_2)\Gamma(-z_3+\varepsilon_1)\Gamma(1+z_2+z_3)}{\Gamma(1+\varepsilon_1)\Gamma(1+\varepsilon_2)\Gamma(1+\varepsilon_3)} \times \times\Gamma(1+z_2+z_3+\varepsilon_3),$$

$$D^{(u,v)}[1+\varepsilon_1-z_3, 1+\varepsilon_2-z_2, 1+\varepsilon_3] = \frac{\Gamma(-u)\Gamma(-v)\Gamma(-u+\varepsilon_1+z_2)\Gamma(-v+\varepsilon_2+z_3)}{\Gamma(1+\varepsilon_1-z_3)\Gamma(1+\varepsilon_2-z_2)\Gamma(1+\varepsilon_3)\Gamma(1+z_2+z_3)} \times \Gamma(1+u+v+\varepsilon_3)\Gamma(1-z_2-z_3+u+v),$$

$$D^{(u,v)}[1+\varepsilon_1-z_3, 1+\varepsilon_2-z_2, 1+\varepsilon_3] D^{(z_2,z_3)}[1+\varepsilon_2, 1+\varepsilon_1, 1+\varepsilon_3]$$
$$= \frac{\Gamma(-u)\Gamma(-v)\Gamma(1+u+v+\varepsilon_3)}{\Gamma(1+\varepsilon_1)\Gamma(1+\varepsilon_2)\Gamma^2(1+\varepsilon_3)} \frac{1}{\varepsilon_1 - z_3} \frac{1}{\varepsilon_2 - z_2} \Gamma(-z_2)\Gamma(-z_3) \times \times\Gamma(1+z_2+z_3+\varepsilon_3)\Gamma(-u+\varepsilon_1+z_2)\Gamma(-v+\varepsilon_2+z_3) \times \times\Gamma(1-z_2-z_3+u+v).$$

Simple trick, IK's talks at ICMP 2015, Santiago

$$\frac{1}{z_3-\varepsilon_1}\frac{1}{z_2-\varepsilon_2}\Gamma\left(-z_2\right)\Gamma\left(-z_3\right)\Gamma\left(1+z_2+z_3+\varepsilon_3\right)\Gamma\left(-u+\varepsilon_1+z_2\right)\times$$
$$\times\Gamma\left(-v+\varepsilon_2+z_3\right)\Gamma\left(1-z_2-z_3+u+v\right)=$$
$$=\frac{z_2+z_2+\varepsilon_3}{(z_3-\varepsilon_1)(z_2-\varepsilon_2)}\Gamma\left(-z_2\right)\Gamma\left(-z_3\right)\Gamma\left(z_2+z_3+\varepsilon_3\right)\Gamma\left(-u+\varepsilon_1+z_2\right)\times$$
$$\times\Gamma\left(-v+\varepsilon_2+z_3\right)\Gamma\left(1-z_2-z_3+u+v\right)=$$
$$=\left(\frac{1}{z_3-\varepsilon_1}+\frac{1}{z_2-\varepsilon_2}\right)\Gamma\left(-z_2\right)\Gamma\left(-z_3\right)\Gamma\left(z_2+z_3+\varepsilon_3\right)\Gamma\left(-u+\varepsilon_1+z_2\right)\times$$
$$\times\Gamma\left(-v+\varepsilon_2+z_3\right)\Gamma\left(1-z_2-z_3+u+v\right).$$

$$\oint_C dz_2\frac{1}{z_2-\varepsilon_2}\Gamma\left(-z_2\right)\Gamma\left(-u+\varepsilon_1+z_2\right)\oint_C dz_3\Gamma\left(-z_3\right)\Gamma\left(z_2+z_3+\varepsilon_3\right)\times$$
$$\Gamma\left(-v+\varepsilon_2+z_3\right)\Gamma\left(1-z_2-z_3+u+v\right).$$

Application of the Barnes lemmas, IK's talks at ICMP 2015, Santiago

$$\oint_C dz_2 \frac{1}{z_2-\varepsilon_2}\Gamma(-z_2)\Gamma(-u+\varepsilon_1+z_2)\oint_C dz_3\Gamma(-z_3)\Gamma(z_2+z_3+\varepsilon_3)\times$$
$$\Gamma(-v+\varepsilon_2+z_3)\Gamma(1-z_2-z_3+u+v) =$$
$$\oint_C dz_2 \frac{1}{z_2-\varepsilon_2}\Gamma(-z_2)\Gamma(-u+\varepsilon_1+z_2)\times$$
$$\times\frac{\Gamma(z_2+\varepsilon_3)\Gamma(-v+\varepsilon_2)\Gamma(1+\varepsilon_3+u+v)\Gamma(1+\varepsilon_2+u-z_2)}{\Gamma(1+u-\varepsilon_1)} =$$
$$= \frac{\Gamma(-v+\varepsilon_2)\Gamma(1+\varepsilon_3+u+v)}{\Gamma(1+u-\varepsilon_1)}\oint_C dz_2 \frac{1}{z_2-\varepsilon_2}\Gamma(z_2+\varepsilon_3)\Gamma(-z_2)\times$$
$$\times\Gamma(-u+\varepsilon_1+z_2)\Gamma(1+\varepsilon_2+u-z_2).$$

Another simple trick,IK's talks at ICMP 2015, Santiago

$$\oint_C dz_2 \frac{1}{z_2-\varepsilon_2}\Gamma(z_2+\varepsilon_3)\Gamma(-z_2)\Gamma(-u+\varepsilon_1+z_2)\Gamma(1+\varepsilon_2+u-z_2) =$$

$$\oint_C dz_2 \frac{1}{(z_2-\varepsilon_2)(z_2-u+\varepsilon_1)}\Gamma(z_2+\varepsilon_3)\Gamma(-z_2)\Gamma(1-u+\varepsilon_1+z_2)\times$$

$$\times\Gamma(1+\varepsilon_2+u-z_2) =$$

$$-\frac{1}{u+\varepsilon_3}\oint_C dz_2\left[\frac{1}{z_2-\varepsilon_2}-\frac{1}{z_2-u+\varepsilon_1}\right]\Gamma(z_2+\varepsilon_3)\Gamma(-z_2)\times$$

$$\times\Gamma(1-u+\varepsilon_1+z_2)\Gamma(1+\varepsilon_2+u-z_2) =$$

$$-\frac{1}{u+\varepsilon_3}\left[\oint_C dz_2\frac{\Gamma(z_2+\varepsilon_3)\Gamma(-z_2)\Gamma(1-u+\varepsilon_1+z_2)\Gamma(1+\varepsilon_2+u-z_2)}{z_2-\varepsilon_2}\right.$$

$$\left.-\oint_C dz_2\frac{\Gamma(z_2+\varepsilon_3)\Gamma(-z_2)\Gamma(1-u+\varepsilon_1+z_2)\Gamma(1+\varepsilon_2+u-z_2)}{z_2-u+\varepsilon_1}\right].$$

Shift of the contour,IK's talks at ICMP 2015, Santiago

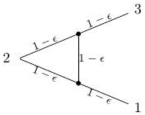
$$\oint_C dz_2 \frac{\Gamma(z_2+\varepsilon_3)\Gamma(-z_2)\Gamma(1-u+\varepsilon_1+z_2)\Gamma(1+\varepsilon_2+u-z_2)}{z_2-\varepsilon_2} -$$

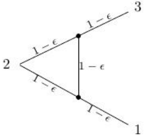
$$-\oint_C dz_2 \frac{\Gamma(z_2+\varepsilon_3)\Gamma(-z_2)\Gamma(1-u+\varepsilon_1+z_2)\Gamma(1+\varepsilon_2+u-z_2)}{z_2-u+\varepsilon_1} =$$

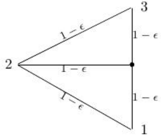
$$\oint_C dz_2 \frac{\Gamma(z_2-\varepsilon_1)\Gamma(-z_2-\varepsilon_2)\Gamma(1-u-\varepsilon_3+z_2)\Gamma(1+u-z_2)}{z_2} -$$

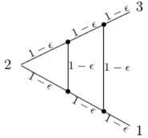
$$-\oint_C dz_2 \frac{\Gamma(z_2+u+\varepsilon_3-\varepsilon_1)\Gamma(-z_2-u+\varepsilon_1)\Gamma(1+z_2)\Gamma(1-\varepsilon_3-z_2)}{z_2}.$$

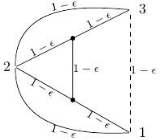
Conjecture in $d = 4 - 2\epsilon$

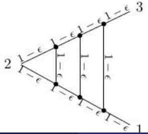


$\partial^2_{(2)}$ 

$= (-4\pi^2)$ 

$(\partial^2_{(2)})^2$ 

$= (-4\pi^2)^2$ 

$(\partial^2_{(2)})^2$ 

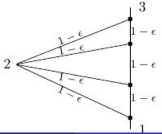
$= (-4\pi^2)^2$ 

Diagram with gluonic self-interacion in the limit $\epsilon \to 0$

$$
\begin{aligned}
&\left[\frac{13/2}{[12]^2[23]^2} + \frac{-33/2}{[12]^2[31]^2} + \frac{15/2}{[23]^2[31]^2} + \frac{11}{[12][23][31]^2} + \frac{-14}{[12][23]^2[31]} + \frac{10}{[12]^2[23][31]}\right] \\
&+ \left[\frac{6}{[12][23]^2} + \frac{-6}{[12][31]^2} + \frac{[12]}{[23]^2[31]^2} + \frac{4[23]}{[12]^2[31]^2} + \frac{-2}{[12]^2[31]} + \frac{-5}{[23]^2[31]} + \frac{-2[31]}{[12]^2[23]^2}\right] J(1,1,1) \\
&+ \left[\frac{-2}{[12]^2[23]^2} + \frac{6}{[12]^2[31]^2} + \frac{4}{[23]^2[31]^2} + \frac{1}{[12][23][31]^2} + \frac{-1}{[12][23]^2[31]} + \frac{-4}{[12]^2[23][31]}\right] \ln[12] \\
&+ \left[\frac{-1/2}{[12]^2[23]^2} + \frac{-5}{[12]^2[31]^2} + \frac{-3/2}{[23]^2[31]^2} + \frac{-7/2}{[12][23][31]^2} + \frac{2}{[12][23]^2[31]} + \frac{7/2}{[12]^2[23][31]}\right] \ln[23] \\
&+ \left[\frac{5/2}{[12]^2[23]^2} + \frac{-1}{[12]^2[31]^2} + \frac{-5/2}{[23]^2[31]^2} + \frac{5/2}{[12][23][31]^2} + \frac{-1}{[12][23]^2[31]} + \frac{1/2}{[12]^2[23][31]}\right] \ln[31] \, .
\end{aligned}
$$

$$
[yz] = (y - z)^2, \qquad [y1] = (y - x_1)^2
$$

G. Cvetič and I. Kondrashuk, "Gluon self-interaction in the position space in Landau gauge," Int. J. Mod. Phys. A **23** (2008) 4145 [arXiv:0710.5762 [hep-th]].

Basis

Typical contribution is

$$\frac{(31)_\nu}{[31]^2}\int\ Dy\ \frac{(2y)_\sigma}{[2y]^2}\frac{(1y)_\lambda}{[1y]^2}\int\ Dz\left(\partial_\mu^{(z)}\Pi_{\rho\nu}(z3)\right)\Pi_{\rho\lambda}(zy)\Pi_{\mu\sigma}(z2),$$

the gluonic propagator is

$$\Pi_{\rho\lambda}(zy) = \frac{g_{\rho\lambda}}{[yz]^{1-\epsilon}} + 2(1-\epsilon)\frac{(yz)_\rho(yz)_\lambda}{[yz]^{2-\epsilon}}$$

Elements of the basis

$$\int\ Dy\ Dz\ \frac{1}{[31]^{1-\epsilon}}\frac{1}{[2y]^{1-\epsilon}}\frac{1}{[3z]^{1-\epsilon}}\frac{1}{[z2]^{1-\epsilon}}\partial_\mu^{(y)}\frac{1}{[1y]^{1-\epsilon}}\partial_\mu^{(y)}\frac{1}{[yz]^{1-\epsilon}}$$

Reduction of the basis to ladder diagrams

$$\int Dy\ Dz\ \frac{1}{[31]^{1-\epsilon}}\frac{1}{[2y]^{1-\epsilon}}\frac{1}{[3z]^{1-\epsilon}}\frac{1}{[z2]^{1-\epsilon}}\partial_\mu^{(y)}\frac{1}{[1y]^{1-\epsilon}}\partial_\mu^{(y)}\frac{1}{[yz]^{1-\epsilon}} =$$
$$\frac{1}{2}\int Dy\ Dz\ \frac{1}{[31]^{1-\epsilon}}\frac{1}{[2y]^{1-\epsilon}}\frac{1}{[3z]^{1-\epsilon}}\frac{1}{[z2]^{1-\epsilon}} \times$$
$$\times\left[\partial^2_{(y)}\left(\frac{1}{[1y]^{1-\epsilon}}\frac{1}{[yz]^{1-\epsilon}}\right) - \left(\partial^2_{(y)}\frac{1}{[1y]^{1-\epsilon}}\right)\frac{1}{[yz]^{1-\epsilon}}\right.$$
$$\left. - \frac{1}{[1y]^{1-\epsilon}}\left(\partial^2_{(y)}\frac{1}{[yz]^{1-\epsilon}}\right)\right]$$