Automating QCD amplitude computations

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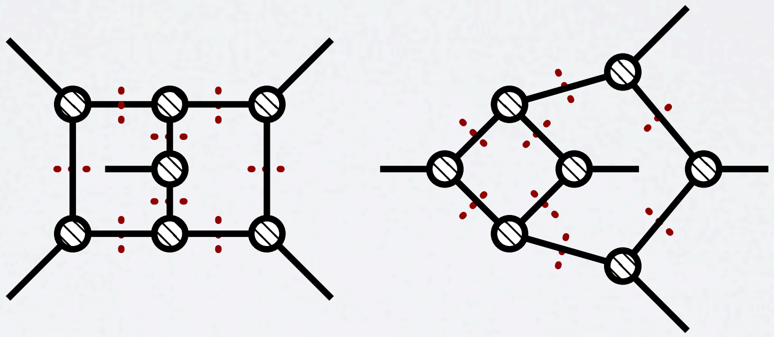
21st January 2016



Science & Technology Facilities Council

CENTRE FOR THEORETICAL PHYSICS · Higgs

ACAT 2016, Valaparaiso, Chile

Outline

high multiplicity

automating QCD amplitude computations

with on-shell methods

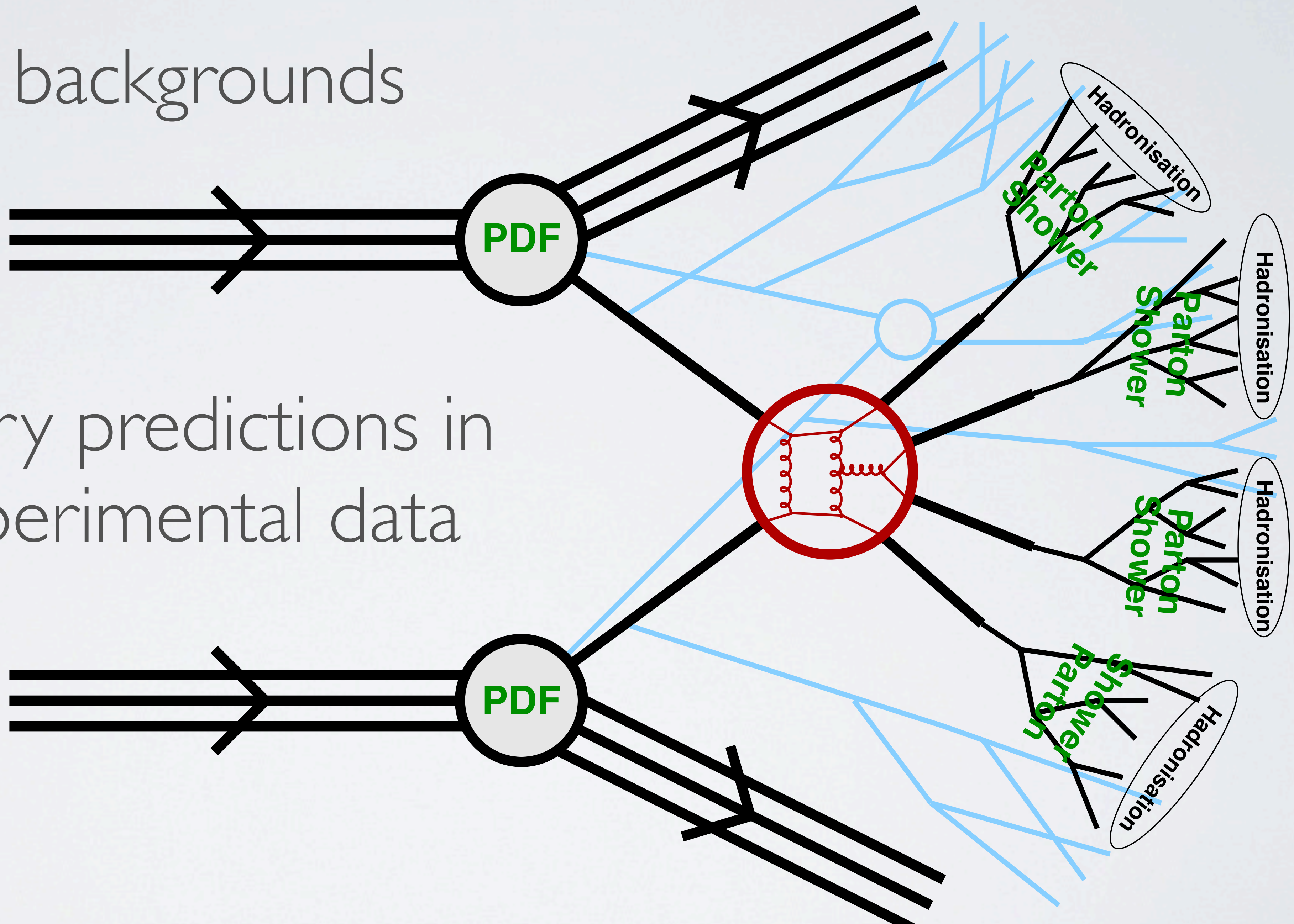
Big thanks to my collaborators! Hjalte Frellesvig, Albert Guffanti, Alex Ochriov, Gustav Mogull, Donal O'Connell, Benedikt Biedermann, Peter Uwer, Valery Yundin and Yang Zhang

Modelling hadron collisions

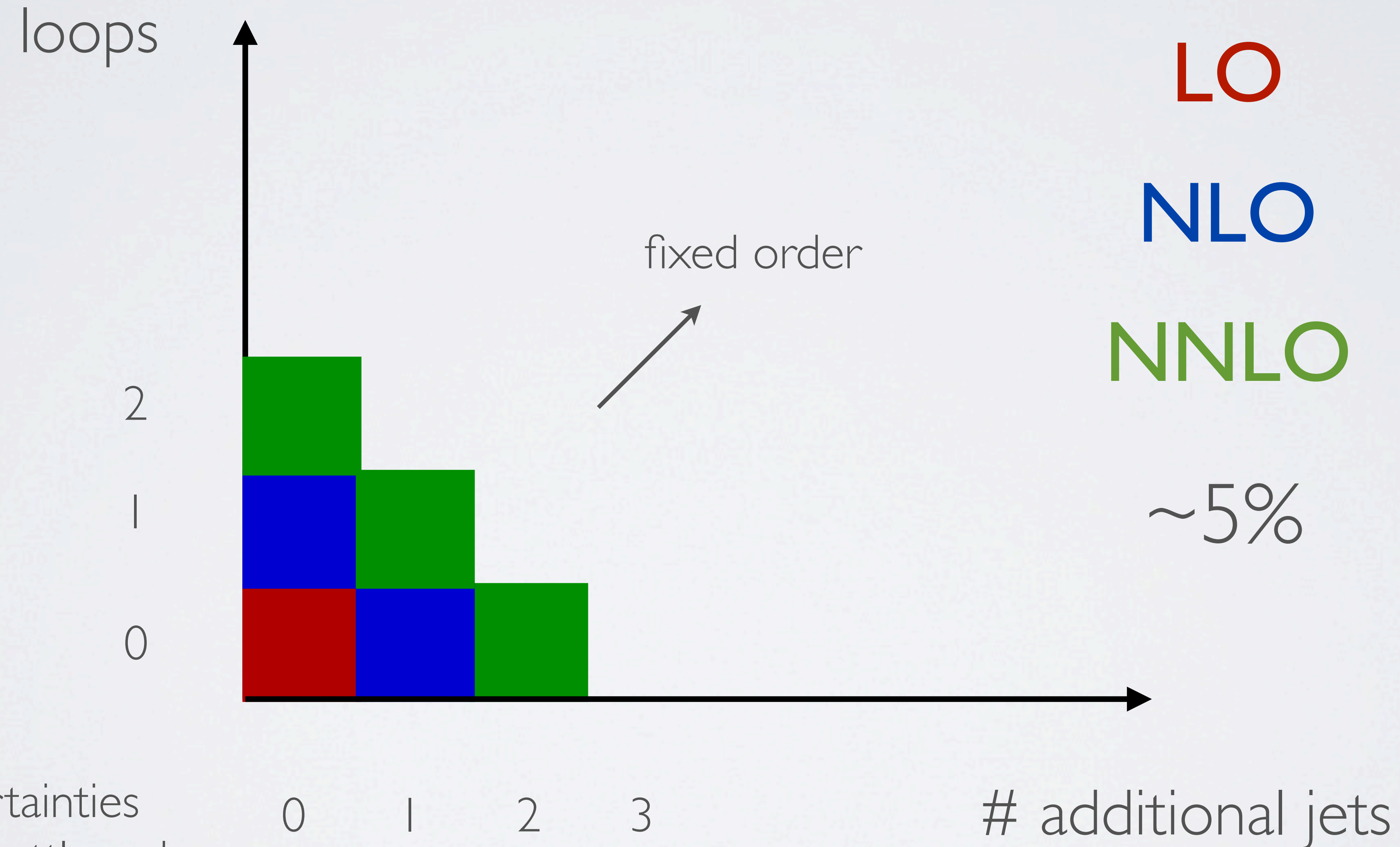
New physics backgrounds

Keeping theory predictions in line with experimental data

Determination of SM parameters



Ingredients for precision



LO

NLO

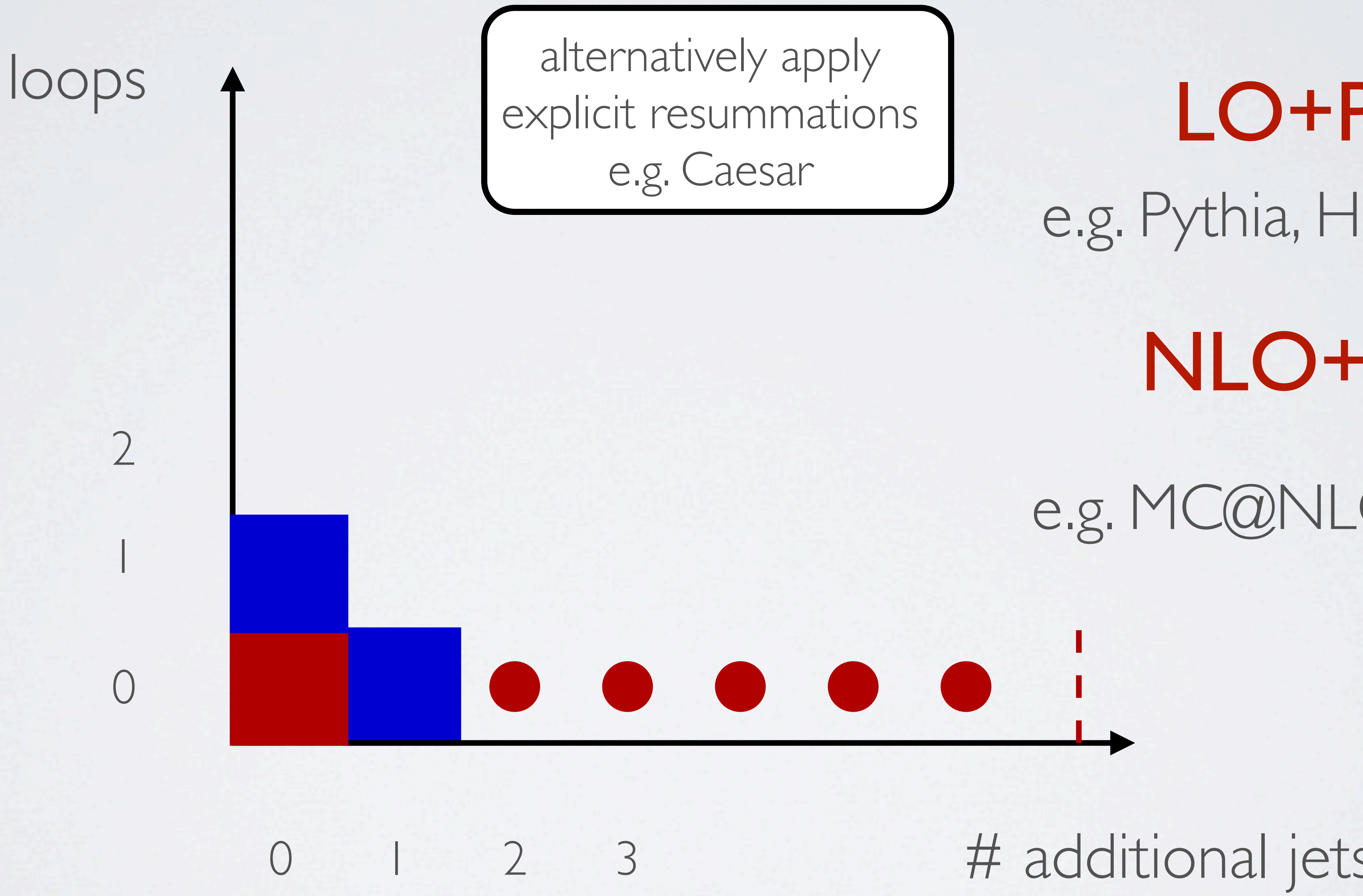
NNLO

~5%

pdf uncertainties
becoming bottleneck

Ingredients for precision

fixed order not good for all regions



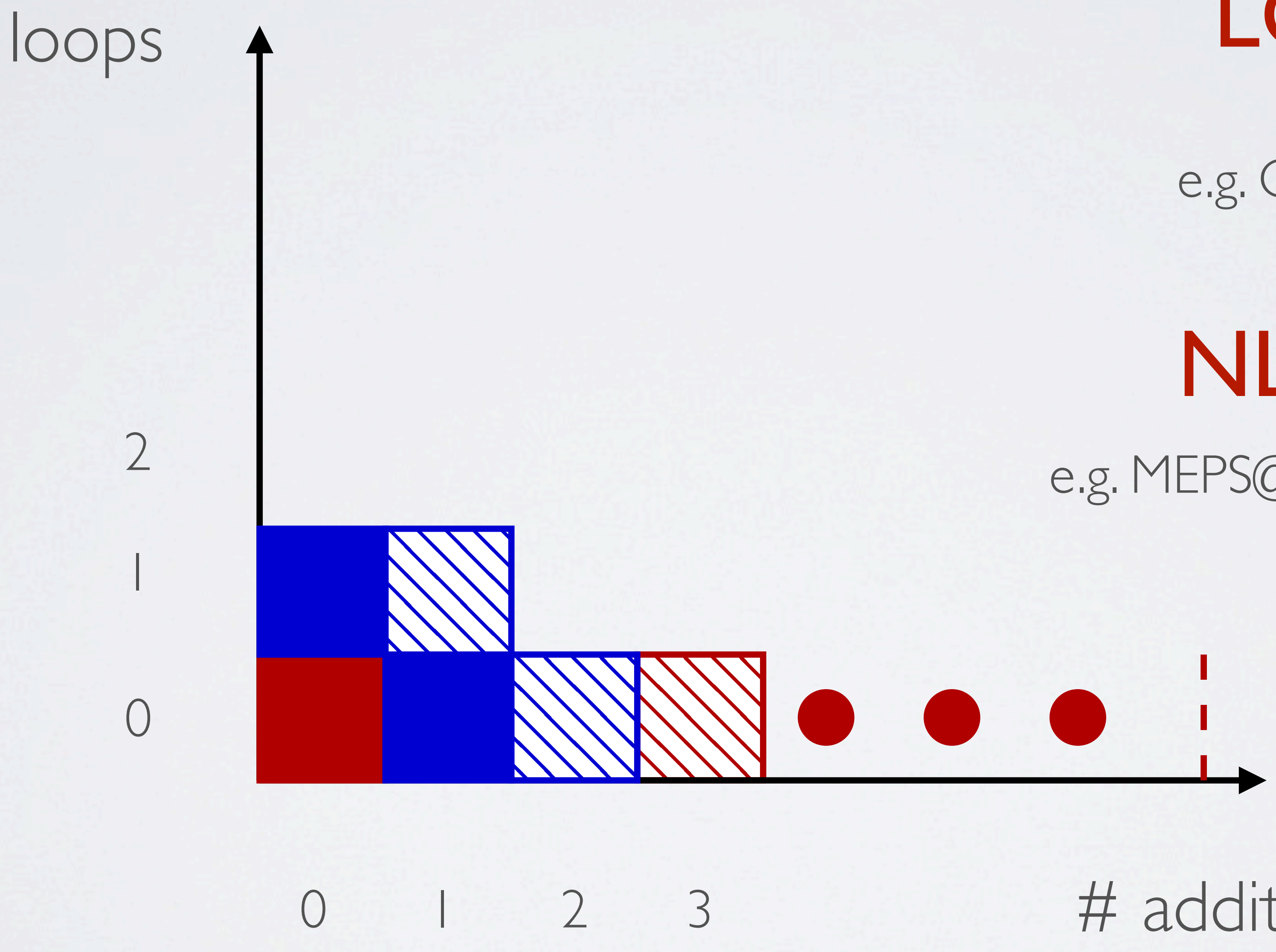
LO+PS

e.g. Pythia, Herwig, ...

NLO+PS

e.g. MC@NLO, POWHEG

Ingredients for precision



LO ME+PS

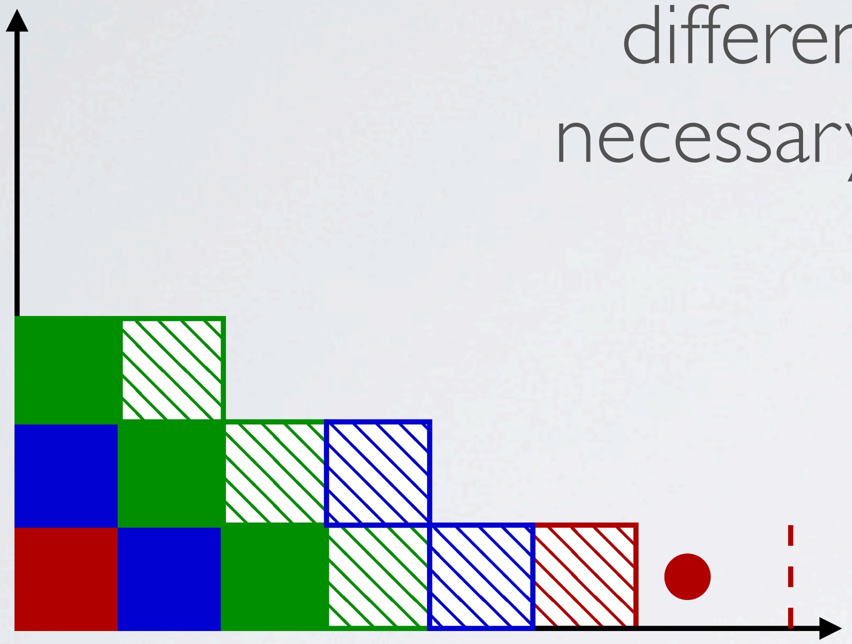
e.g. CKKW-L, MLM

NLO ME+PS

e.g. MEPS@NLO, UNLOPS, FxFx, ...

Ingredients for precision

differential predictions at ~5% level will be necessary to make the most of the Run II data



improving reliability of theoretical uncertainty estimates

requires availability of higher multiplicity matrix elements as well as higher loops

phase space becoming more complicated : high demands on amplitude efficiency

accurate matrix elements
(perturbative)

non-perturbative

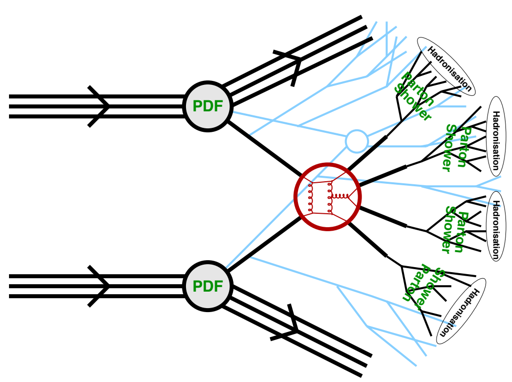
PDFs, hadronisation…

MC event generator

phase-space sampling

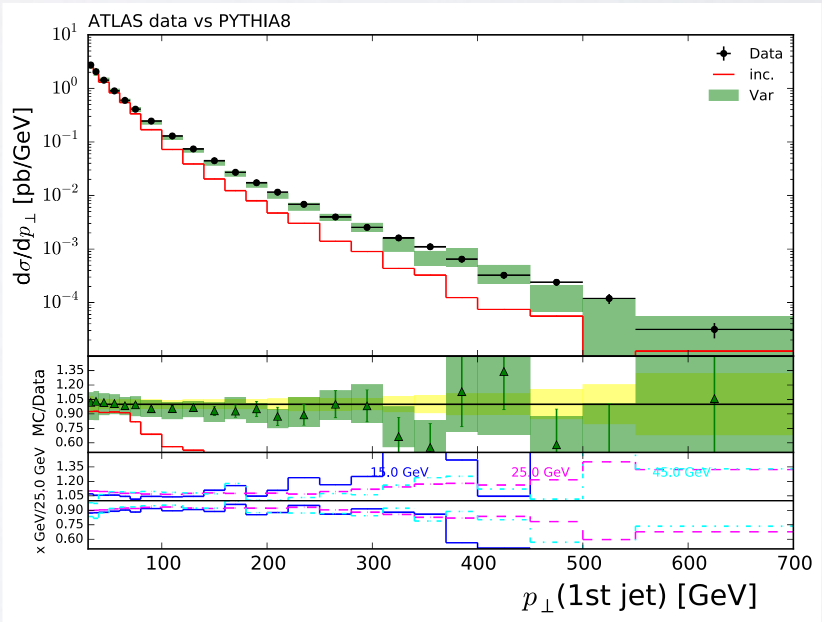
parton shower / resummation

underlying event



infra-red subtraction/
regularisation

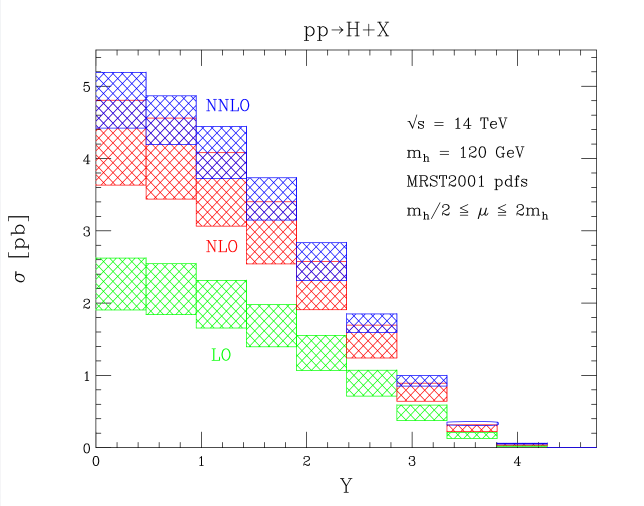
[Frederix et al. 1511.00847]



e.g. aMC@NLO + FxFx for W+0,1,2j

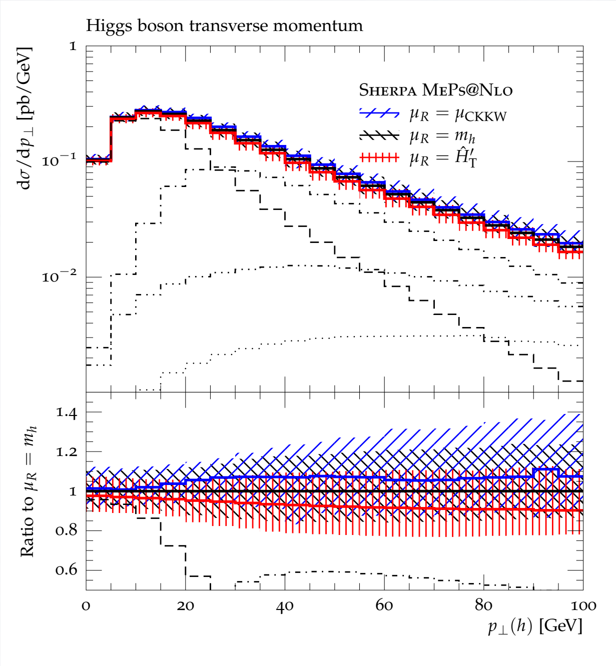
Reducing theoretical uncertainties

how reliable are scale variations?



[Anastasiou, Petriello, Melnikov (2005)]

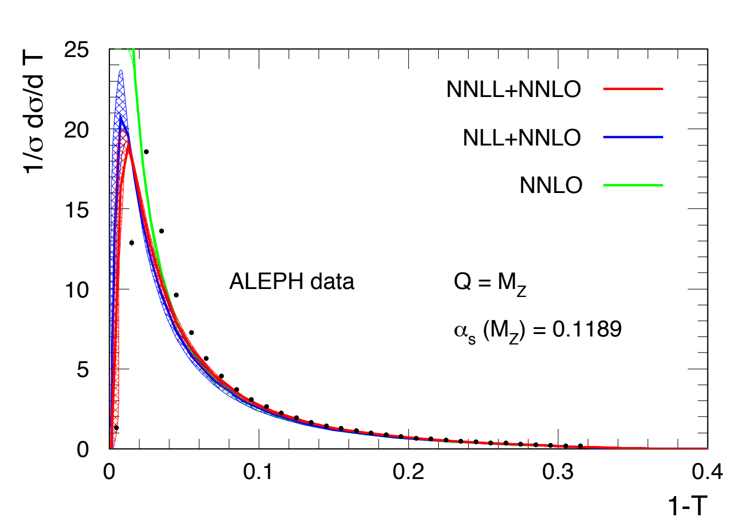
H+0,1,2j MEPS@NLO



[Hoeche, Krauss, Schoenherr (2014)]

multi-jet merging

resummation



e+e- event shapes

[Gehrmann, Lusioni, Monni(2014)]

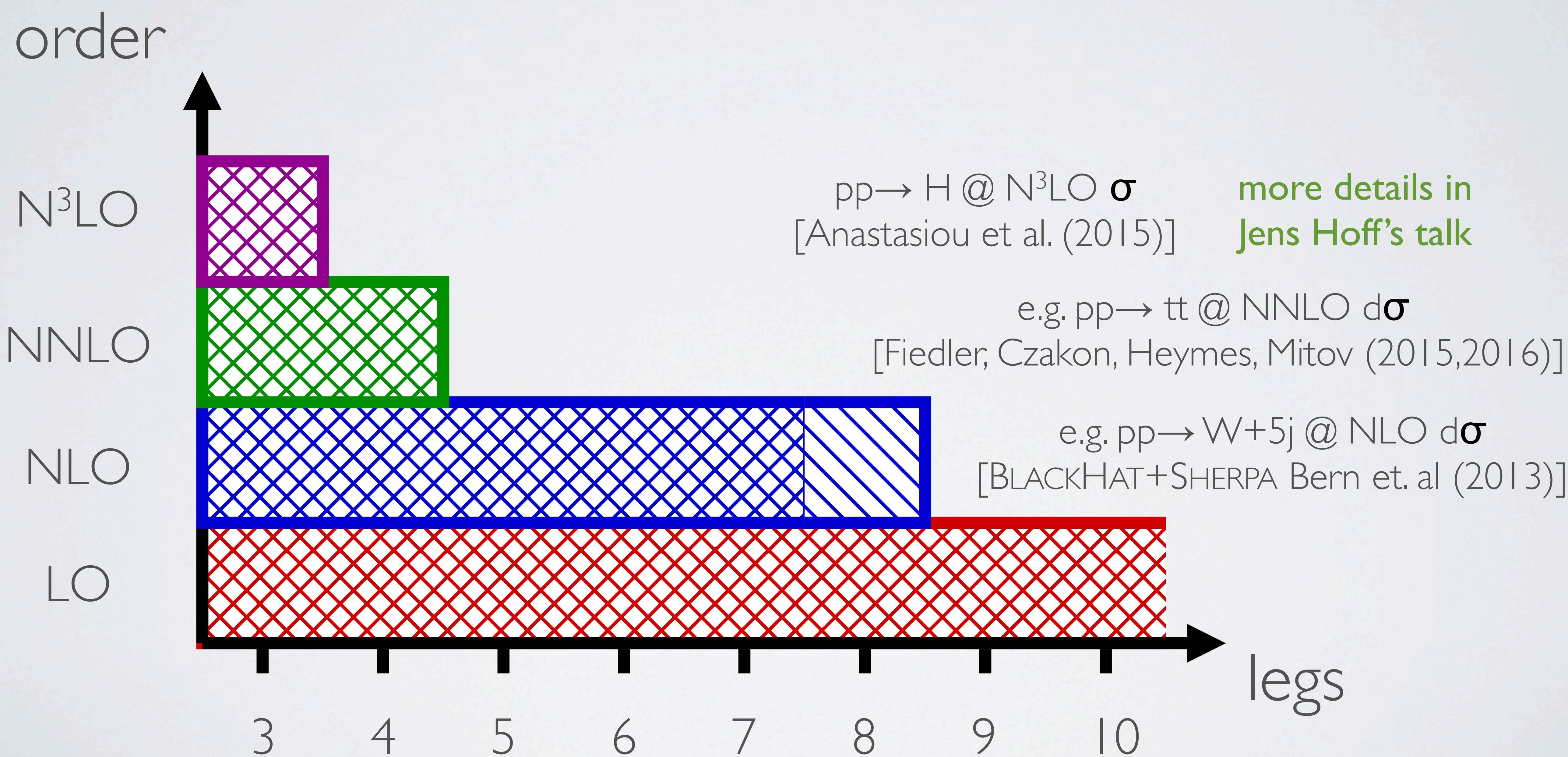
⇒more Loops

⇒more Legs

⇒matching

Complexity

~ #loops + #legs (+#scales)



Computational bottlenecks

- Large numbers of diagrams?
- Complicated basis of functions?
- Large cancellations due to redundant variables?
- Complicated kinematic algebra?

Computational bottlenecks

- Large nu
- Complica
- Large can
- Complica

maybe not such a problem - easy to automate
tree-level codes : MadGraph, CalcHEP, Alpgen,...

yes - multi-scale loop integrals are difficult.
evaluations methods are improving a lot...

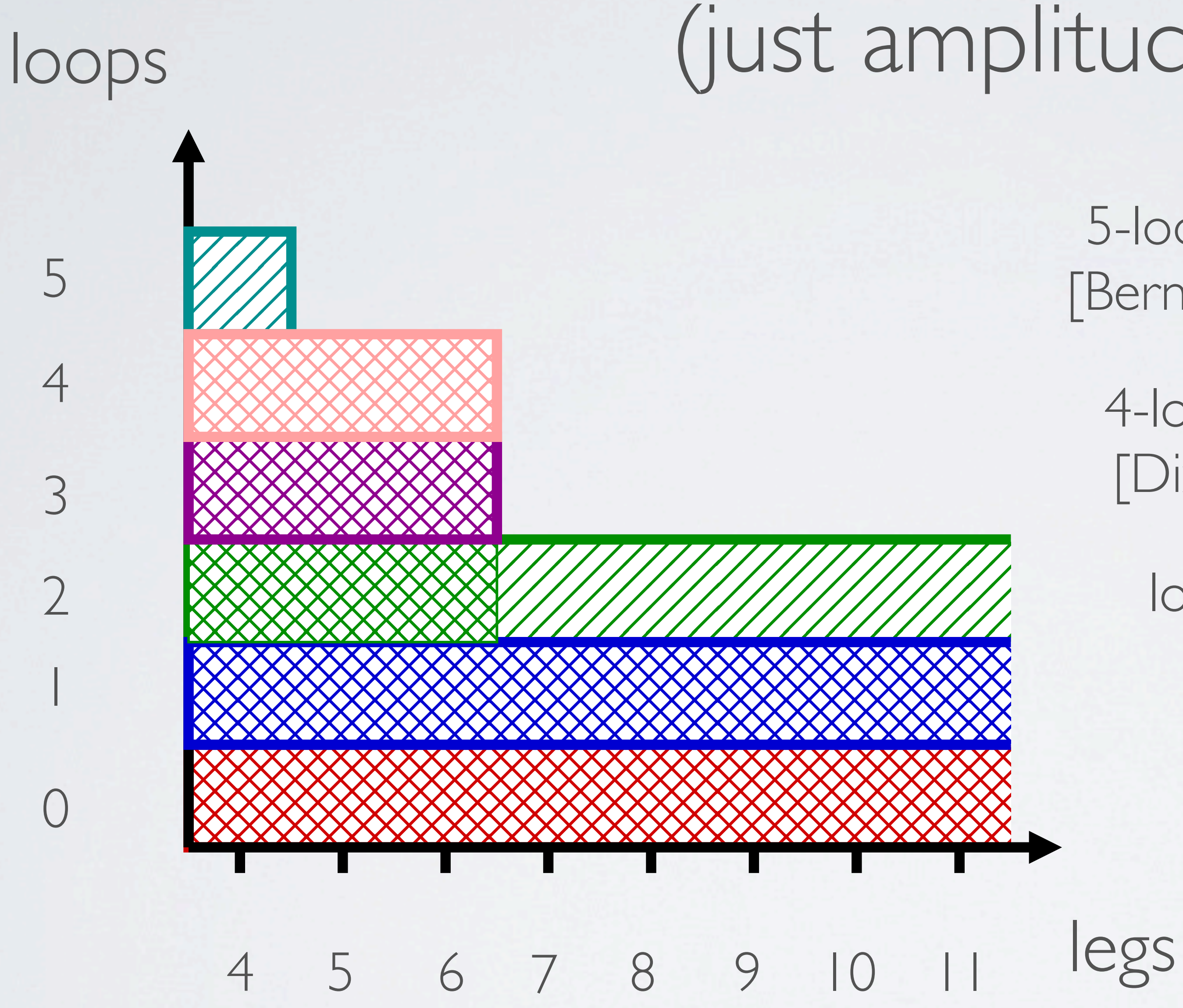
choosing the wrong basis of functions/variables can
compromise accuracy : try to work with physical
degrees of freedom as far as possible

on-shell methods, algebraic(numerical) methods,...

track 3 talks
Bogner,
Schroder,
Davydychev,
Kondrshuk,
Kompaniets,
Kataev,
Ueda,
Hoff,
Heinrich,
von Manteuffel

Complexity in $\mathcal{N} = 4$ SYM

(just amplitudes/integrands this time)



5-loop and planar 6-loop 4-point integrands
[Bern et al. 1210.7709][Bern et al. 1207.6666]

4-loop 6-point NMHV remainder function
[Dixon, von Hippel, McLeod 1509.08127]

local integrands for all two-loop amplitudes
[Trnka, Bourjailly 1505.05886]

analytic solution to BCFW
[Drummond, Henn 0808.2475]

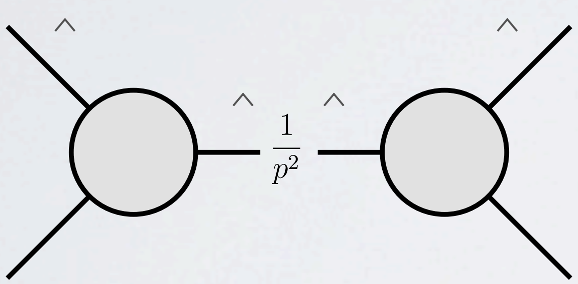
many other partial results, specific helicities, strong coupling etc.

Tree-level methods

recursion has played a key role in automated approaches

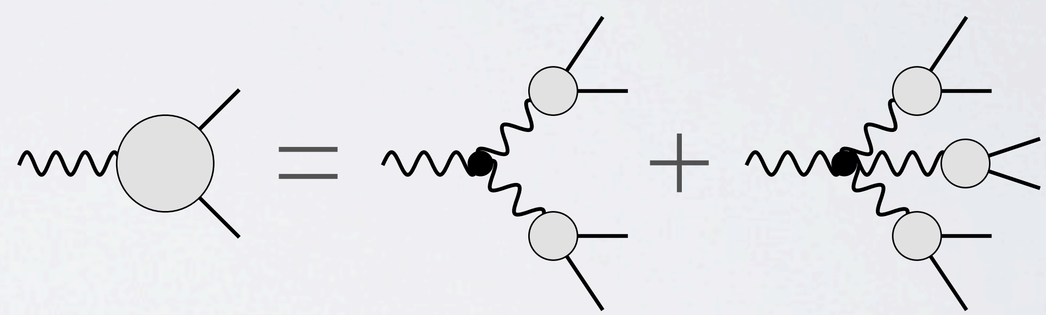
on-shell: BCFW

Britto, Cachazo, Feng, Witten (2005)



off-shell: BG

Berends-Giele (1988)



loop-level applications

rational terms in QCD

Berger, Bern, Dixon, Forde, Kosower (2005-2006)
BLACKHAT: Berger, Bern, Dixon, Febres-Cordero, Forde, Ita, Kosower, Matire (2008)

all-loop integrand for planar $\mathcal{N}$=4 SYM

Arkani-Hamed, Bourjailly, Cachazo, Caron-Huot, Trnka (2010)

tensor integrands

van Hameren (2009)

Becker, Reuschle, Weinzierl (2010)

OPENLOOPS: Cascoli, Pozzorini, Maierhöfer (2011)

Loop-level methods

diagrams $\xrightarrow{\text{reduction}}$ master integrals $\xrightarrow{\text{integration}}$ amplitude

integration-by-parts
[many Laporta style codes: FIRE5, Reduze2, Grinder, ...]

integrand reduction
[1-loop (CutTools,LoopTools), multi-loop: polyn. div.]

tensor reduction
[many implementations: LoopTools, Collier, FeynCalc, PJFry, ...]

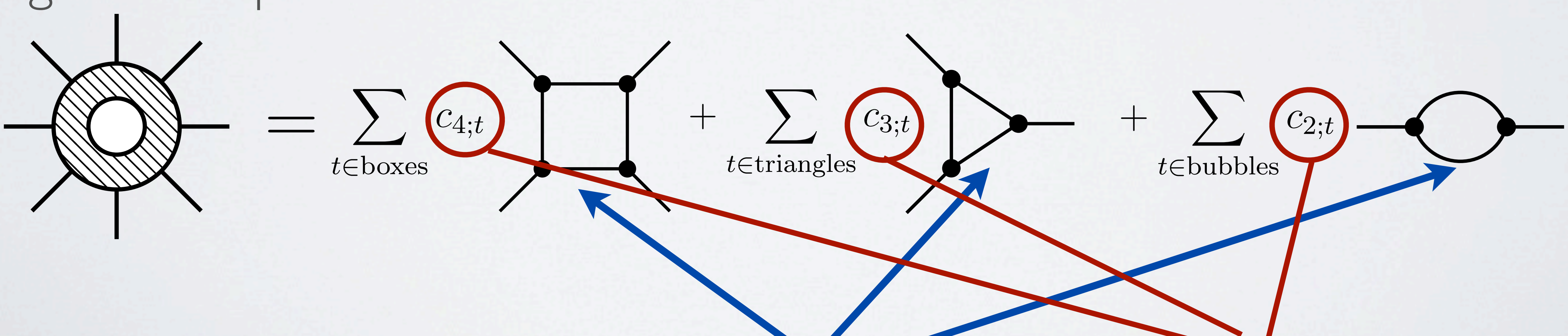
generalized unitarity
[BlackHat, NJet, Rocket,...]

sector decomposition
[numerical: FIESTA4, SecDec3]

differential equations
[a lot of progress with Henn's "canonical" approach]

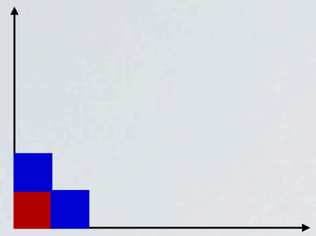
direct evaluation
[MPL (Bogner), HyperInt (Panzer)]

e.g. one-loop



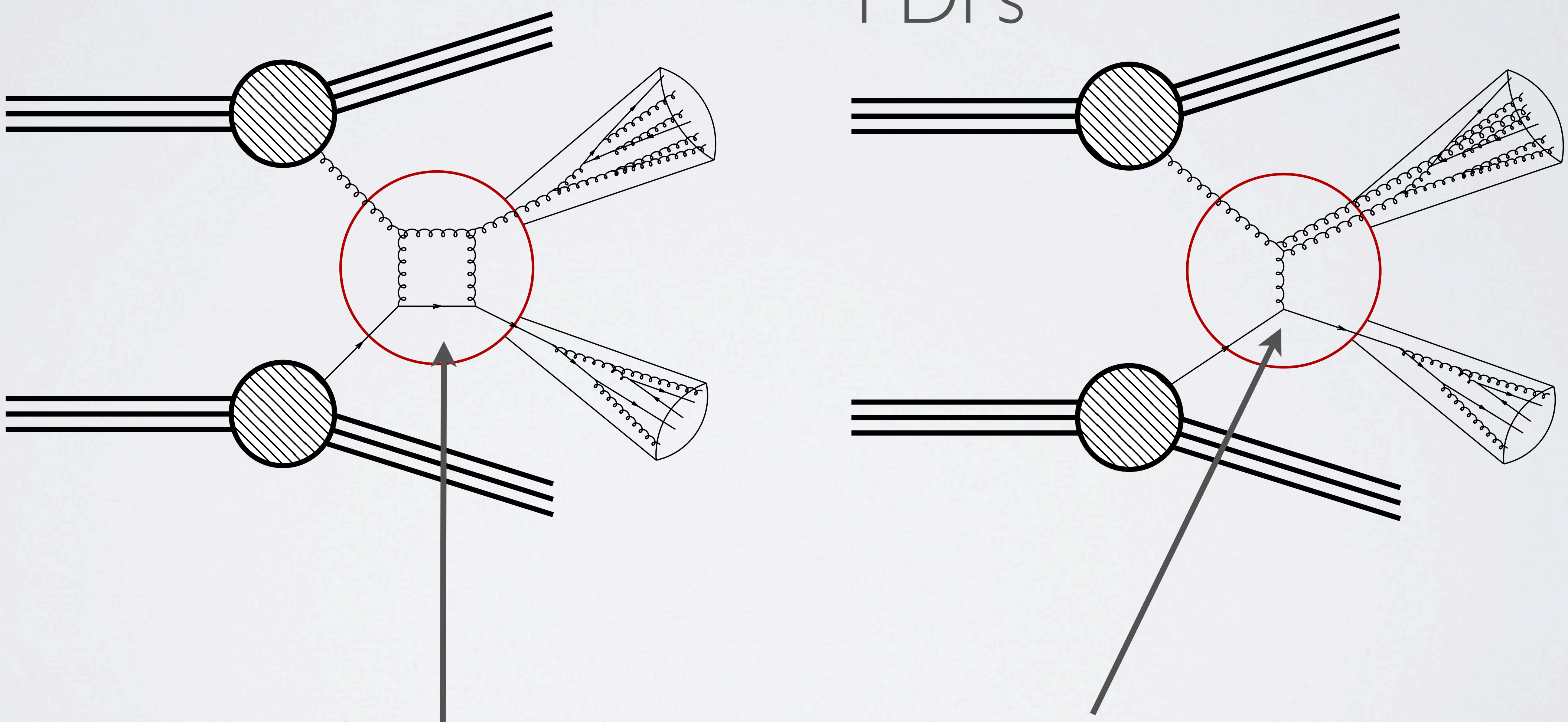
integral basis separates **analytic** and **algebraic** parts

QCD at NLO



$$\sigma_{pp\to X} = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, Q)\, f_j(x_2, Q)\, \widehat{\sigma}^{NLO}_{ij\to n}$$

PDFs

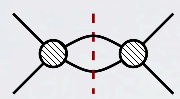


$$\widehat{\sigma}^{\delta NLO}_{ij\to n} = \int_n \left(d\sigma^V + \int_1 d\sigma^{S_1(R)} \right) + \int_{n+1} \left(d\sigma^R - d\sigma^{S_1(R)} \right)$$

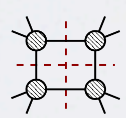
Automated one-loop amplitudes

solving on-shell conditions requires **complex** momenta
$\Rightarrow$ factorise residues into **tree amplitudes**

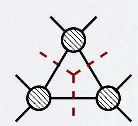
Unitarity: double cuts
[BDDK '94]
[triple cuts BDK '97]



Generalized unitarity:
quadruple cuts [BCF '04]

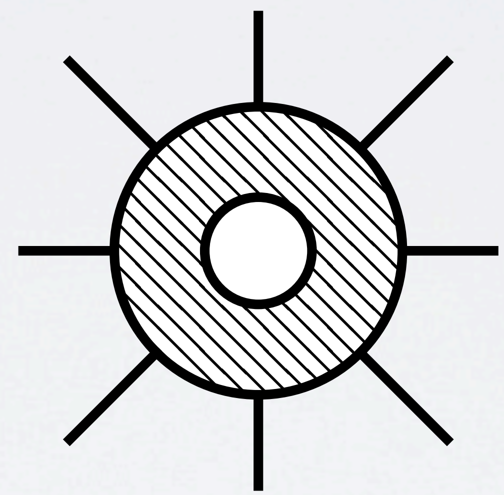


triple cuts [e.g. Forde '07]



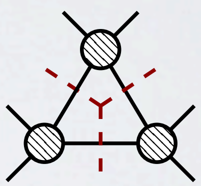
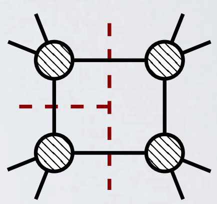
$$A = \sum_i (\text{rational})_i (\text{integral})_i$$

find complex contour to isolate
integral coefficient



multi-scale
kinematic algebra
performed
numerically

Integrand reduction [OPP '05]

$\Delta_3 =$  $-$ 

D-dim. generalized unitarity [GKM '08]

$$A = \int_k \sum_i \frac{\Delta_i(k,p)}{(\text{propagators})_i}$$

explicitly remove poles

Colour decompositions

for multi-jet final states colour permutation sums can be very large

minimise required kinematic information using
colour ordering and SU(N) symmetries

now scales
polynomially

n-gluon example:

tree-level

Kleiss-Kuijf relations (n-2)!

colour-kinematics relations (n-3)!
[Bern, Carrasco, Johansson (2010)]

multi-quark generalisations:
[Melia (2013, 2015)]
[Johansson, Ochirov (2015)]

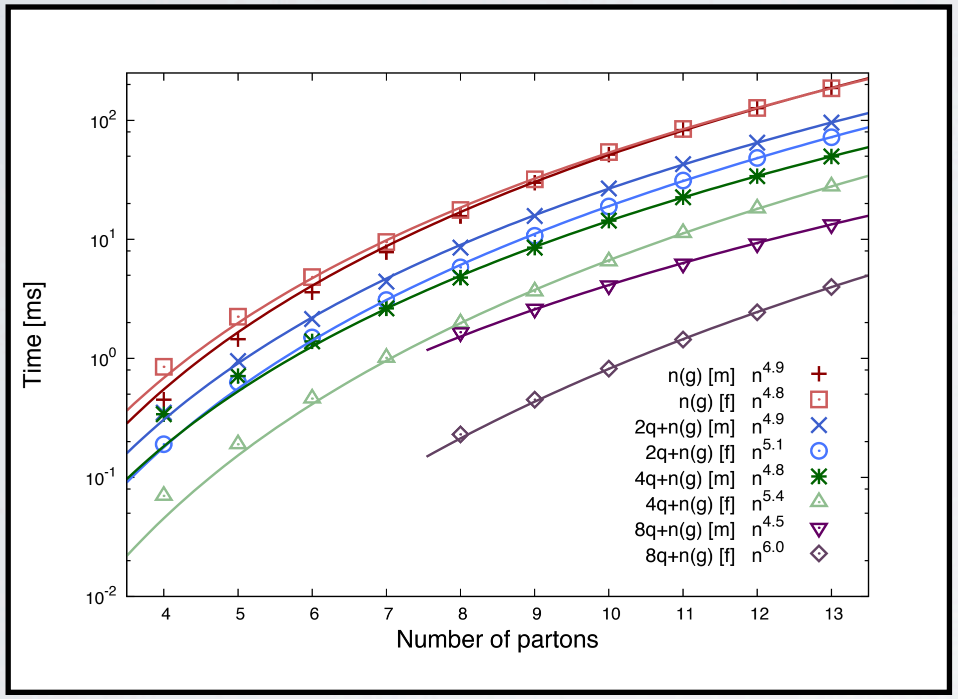
one loop primitive amplitudes

Del Duca, Dixon, Maltoni (n-1)/2!

general decompositions:
[Ita, Ozeren (2011)]
[SB,Biedermann, Uwer, Yundin (2012)]
[Reuschle, Wienzierl (2013)]
[Schuster (2013)]

Performance

NJET: SB, Biedermann, Uwer, Yundin https://bitbucket.org/njet/njet/



primitives scale ~n^6 for $n \lesssim 20$

process	T_{sd}[s]	$T_{4\text{ digits}}$[s]	(% fixed)
4g	0.030	0.030	(0.00)
2u2g	0.032	0.032	(0.00)
2u2d	0.011	0.011	(0.00)
4u	0.022	0.022	(0.00)

process	T_{sd}[s]	$T_{4\text{ digits}}$[s]	(% fixed)
5g	0.22	0.22	(0.22)
2u3g	0.34	0.35	(0.06)
2u2d1g	0.11	0.11	(0.00)
4u1g	0.22	0.22	(0.03)

process	T_{sd}[s]	$T_{4\text{ digits}}$[s]	(% fixed)
6g	6.19	6.81	(1.37)
2u4g	7.19	7.40	(0.38)
2u2d2g	2.05	2.06	(0.08)
4u2g	4.08	4.15	(0.21)
2u2d2s	0.38	0.38	(0.00)
2u4d	0.74	0.74	(0.00)
6u	2.16	2.17	(0.02)

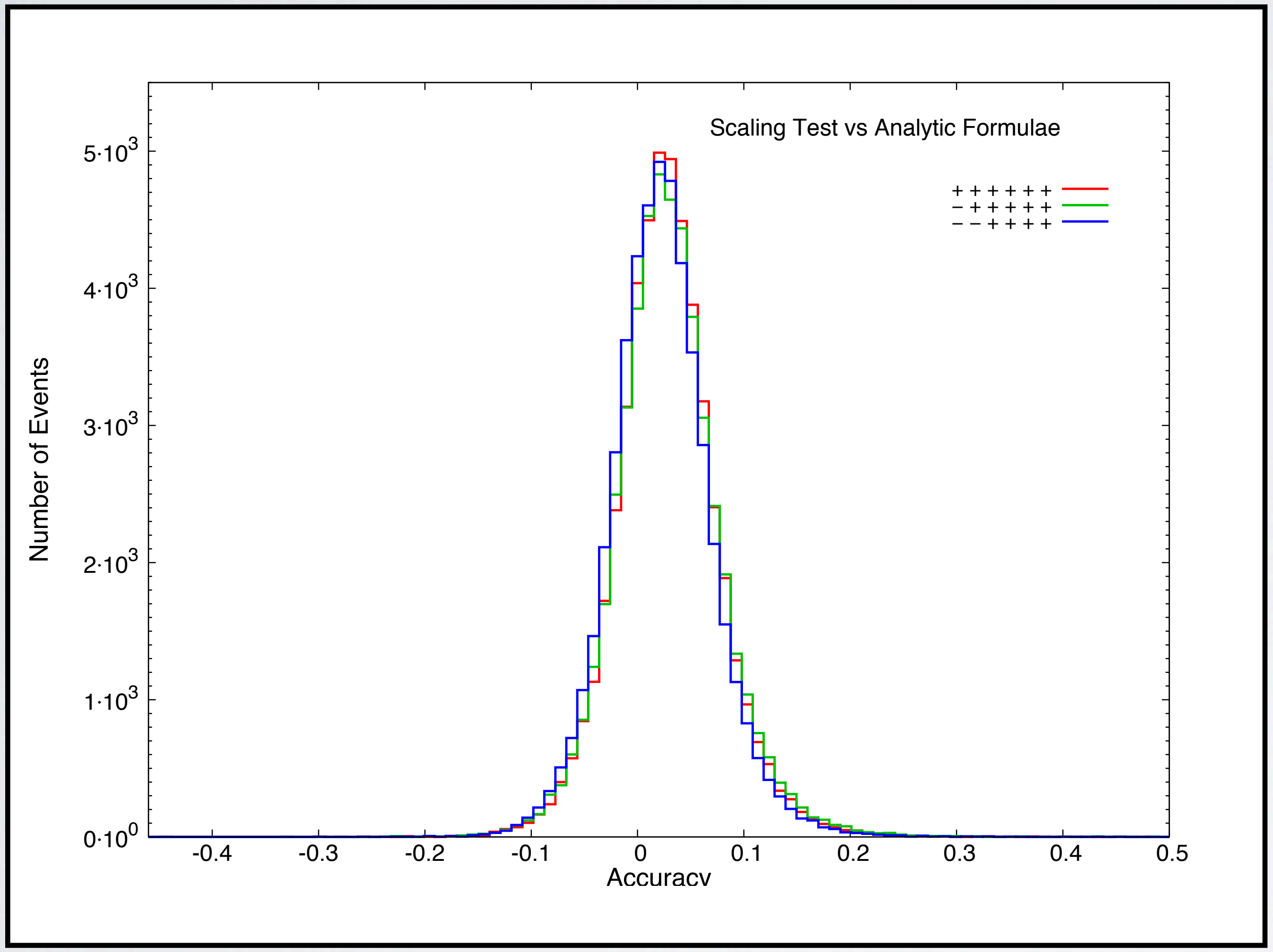
process	T_{sd}[s]	$T_{4\text{ digits}}$[s]	(% fixed)
7g	171.3	276.7	(8.63)
2u5g	195.1	241.2	(3.25)
2u2d3g	45.7	48.8	(0.88)
4u3g	92.5	101.5	(1.29)
2u2d2s1g	7.9	8.1	(0.23)
2u4d1g	15.8	16.2	(0.29)
6u1g	47.1	48.6	(0.41)

full colour sums a few seconds for 2⟶4

	$gg \to 2g$	$gg \to 3g$	$gg \to 4g$	$gg \to 5g$
standard sum	0.03	0.22	6.19	171.31
de-symmetrized	0.03	0.07	0.57	3.07

Accuracy

NJET: SB, Biedermann, Uwer, Yundin https://bitbucket.org/njet/njet/



$$\text{accuracy} = \log_{10}\left(\frac{A_{\text{NJet}}(s_1) + A_{\text{NJet}}(s_2)}{2(A_{\text{NJet}}(s_1) - A_{\text{NJet}}(s_2))}\right) - \log_{10}\left(\frac{A_{\text{NJet}}(1) + A_{\text{analytic}}}{2(A_{\text{NJet}}(1) + A_{\text{analytic}})}\right)$$

dimension scaling test

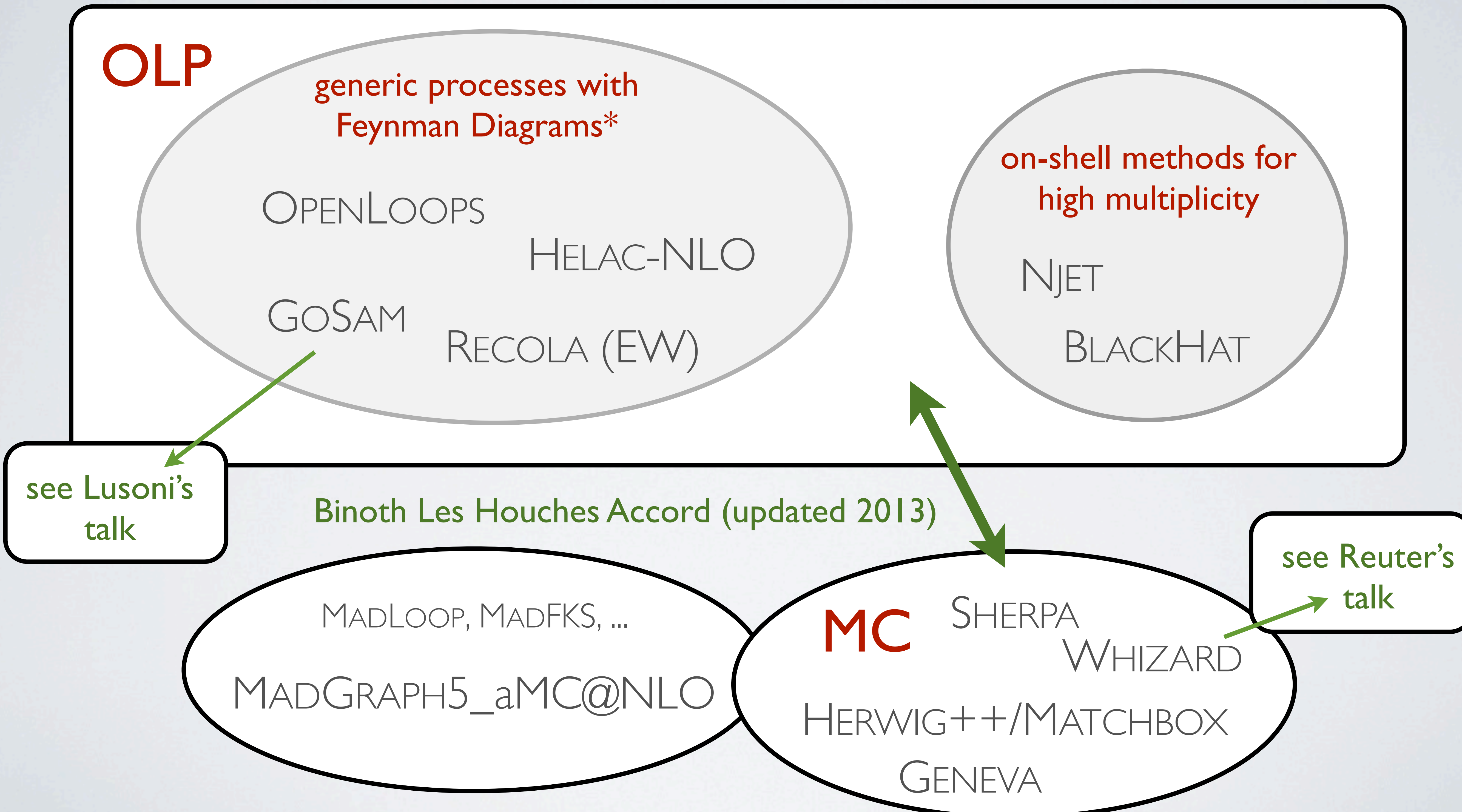
$$A(p_i, m_i, \mu_R) = x^{4-n} A(xp_i, xm_i, x\mu_R) := A_{\text{NJet}}(x)$$

$$\#\text{digits} = \log_{10}\left(\frac{A_{\text{NJet}}(s_1) + A_{\text{NJet}}(s_2)}{2(A_{\text{NJet}}(s_1) - A_{\text{NJet}}(s_2))}\right)$$

2 calls for the price of 1 using explicit vectorization with Vc

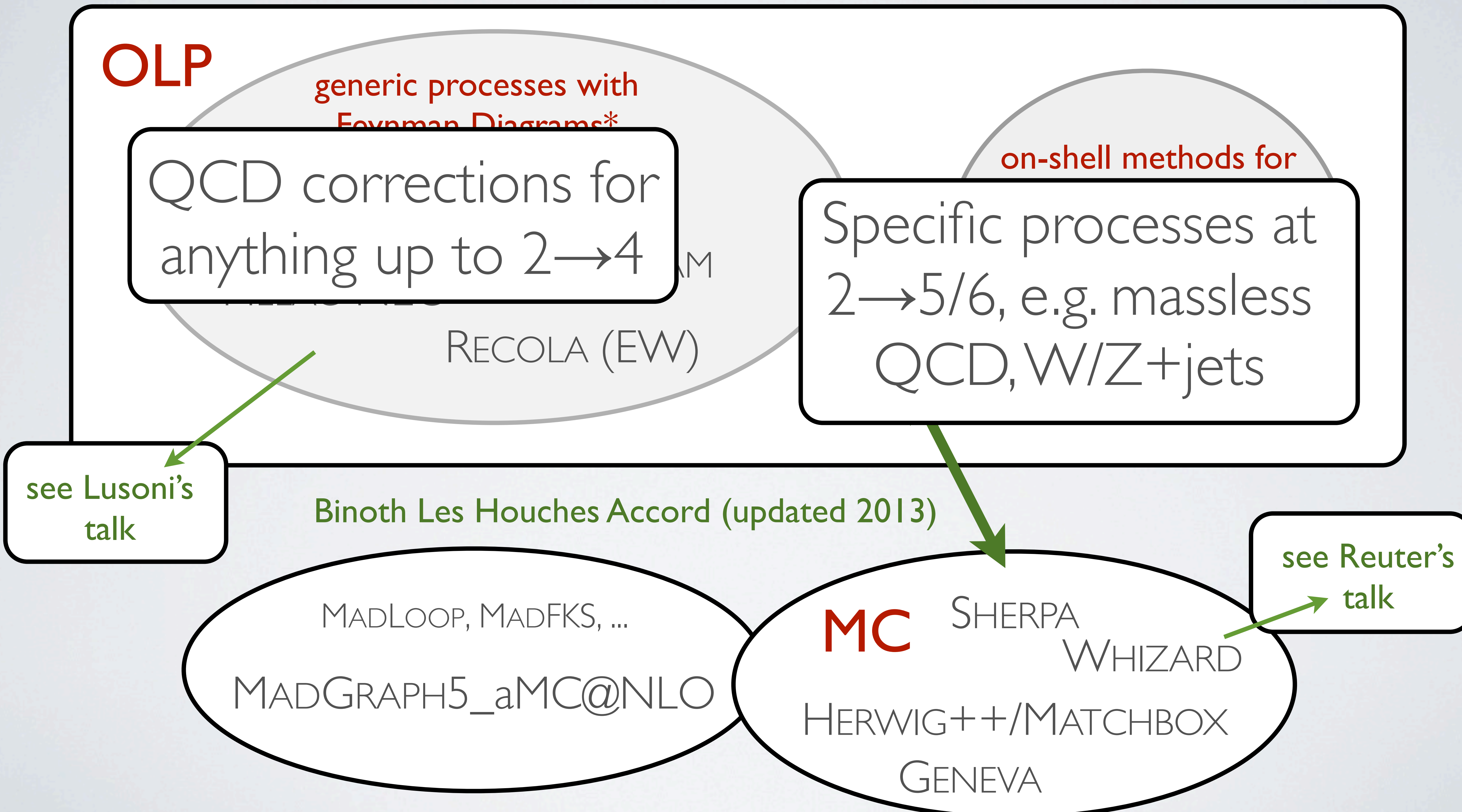
reliable but statistical: add ~2 digits on min. accuracy

Automated NLO



* efficient algorithms with off-shell recursion

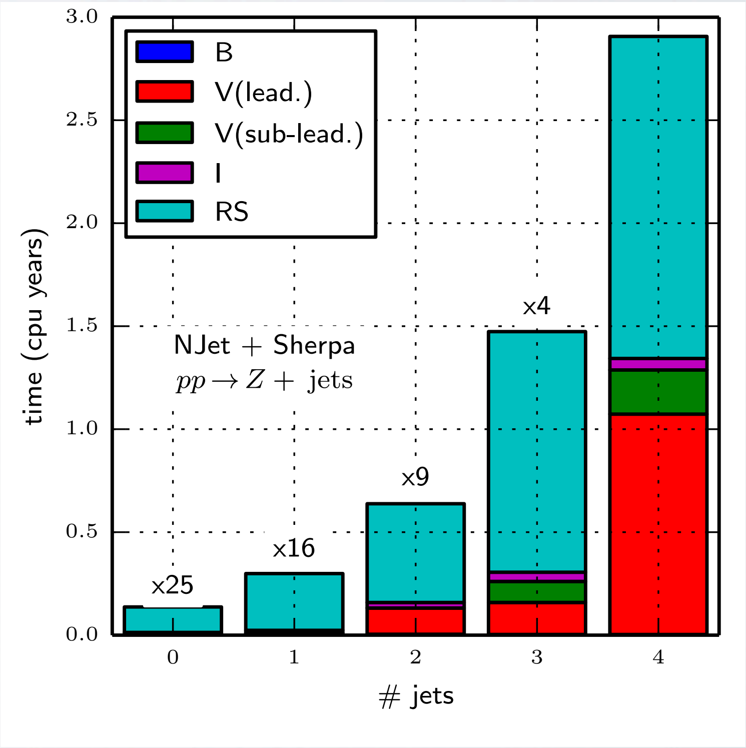
Automated NLO



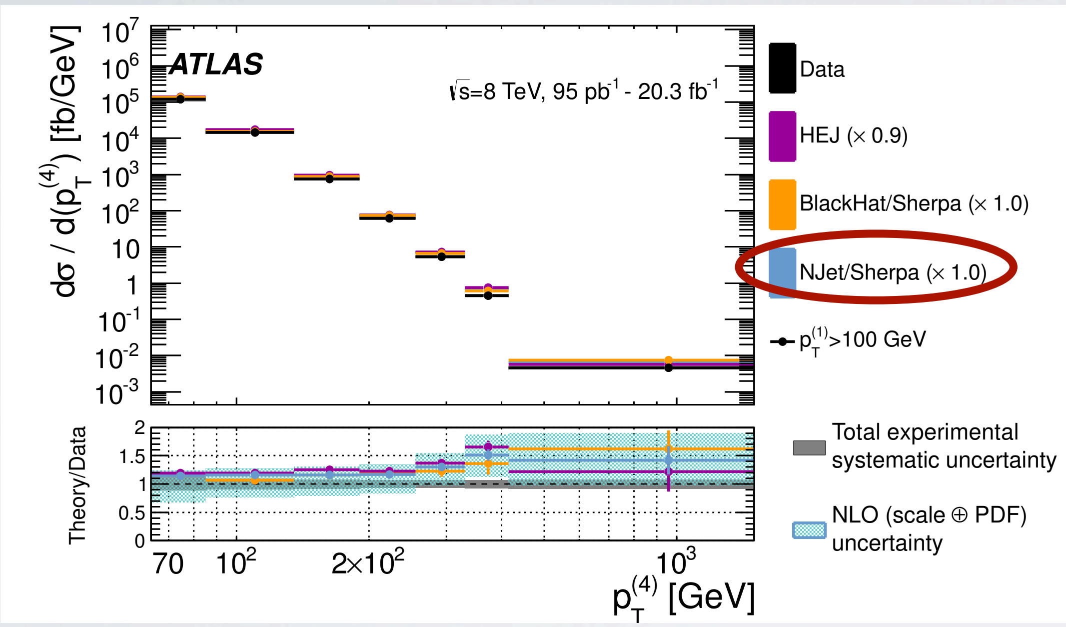
* efficient algorithms with off-shell recursion

Challenges at NLO?

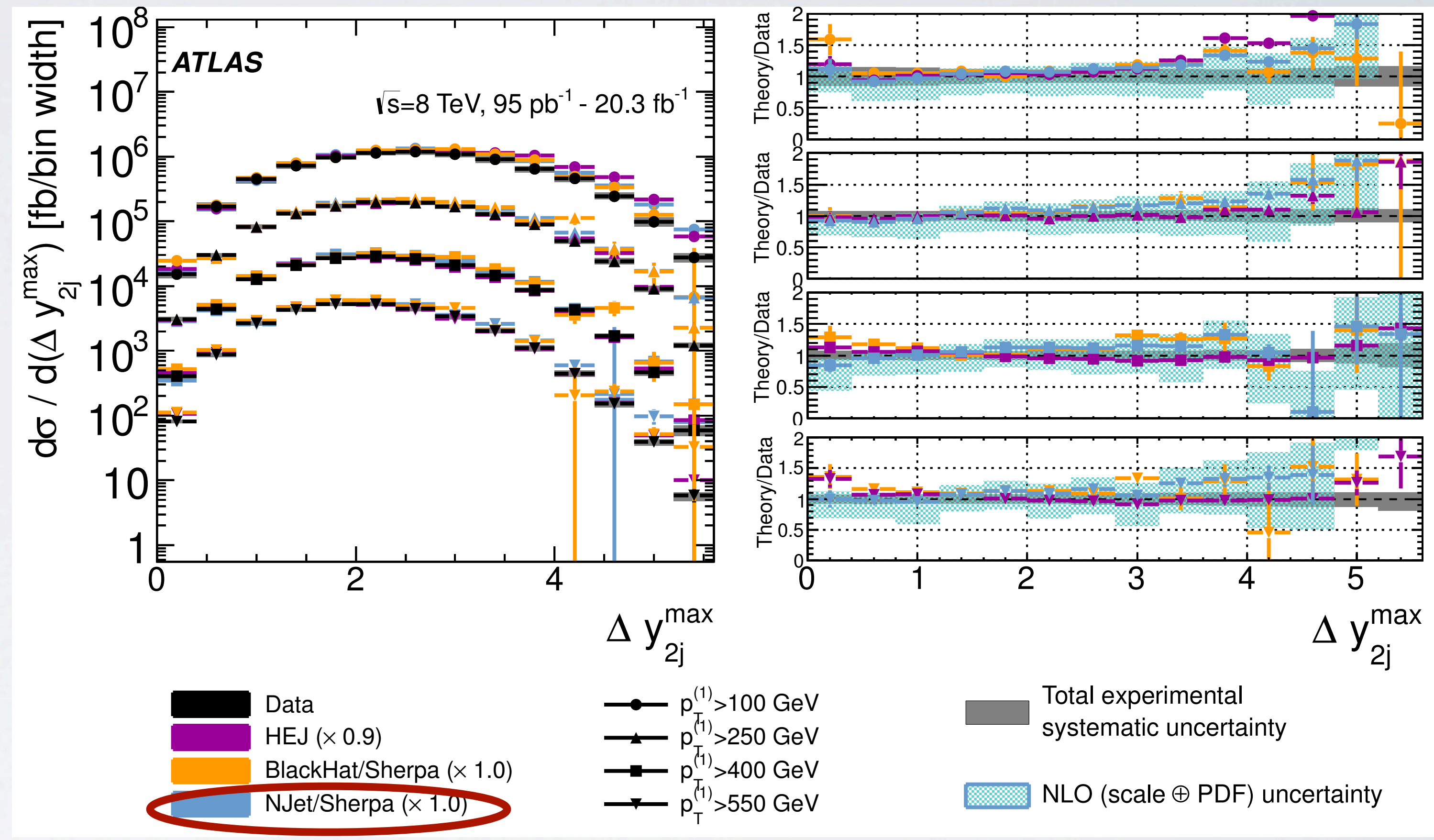
- NLO QCD is becoming standard precision in experiments
- automated NLO+PS and NLO ME+PS are available for many processes
- complete EW+QCD has been completed in a few cases
- FEYNRULES/LANHEP+UFO output can be used to implement arbitrary Langrangians (e.g. effective field theories)
- complicated final states still computationally intensive (mainly due to real radiation phase space)



high multiplicity NLO in practice



pp → 4j ATLAS 8 TeV 20.3fb^{-1} [1509.07335]



integration grids using MPI ~ 30 cores

event generation ~ 1000 cores

events stored on EOS file system at CERN

Root NTuples: (~2TB)

total XS ~ 0.5% MC error

towards automation at NNLO

$$\sigma_n^{NNLO} = \int_n (d\sigma^B + d\sigma^V + d\sigma^{VV}) + \int_{n+1} d\sigma^R + d\sigma^{RV} + \int_{n+2} d\sigma^{RR}$$

nearly all 2→2 processes
now computed at NNLO

talks by Bougezhal and von Manteuffel

extend integrand reduction and generalized unitarity techniques for higher multiplicity amplitudes

unitarity cuts commonly applied in super-symmetric and gravity computations

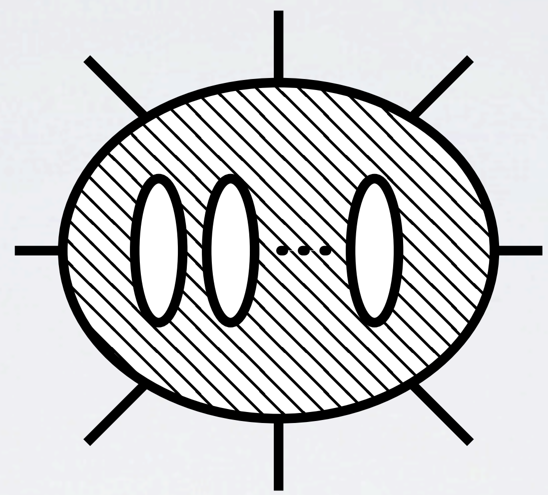
[Bern, Carrasco, Davies, Dennen, Huang, Dixon, Johansson, Kosower, Roiban]

e.g. 5-loop 4-point amplitude in N=4 sYM
[Bern, Carrasco, Johannaansson, Roiban 1207.6666]

multi-loop amplitudes from trees

Maximal unitarity

[Kosower, Larsen, Johansson, Caron-Huot, Zhang, Søgaard]



Integrand reduction via polynomial division

[Mastrolia, Ossola, SB, Frellesvig, Zhang, Mirabella, Peraro, Malamos, Kleiss, Papadopolous, Verheyen, Feng, Huang]

e.g. IBPs

$$A = \sum_i (\text{rational})_i (\text{integral})_i \longleftarrow A = \int_k \sum_i \frac{\Delta_i(k,p)}{(\text{propagators})_i}$$

IBPs must be free of doubled propagator MI

[Gluza, Kosower, Kajda 1009.0472]
[Schabinger 1111.4220][Ita 1510.05626]
[Larsen, Zhang 1511.01071]

integrand reduction

$$\Delta_{c;T}\Big|_{\mathrm{cut}} = \prod_i A_i^{(0)} - \sum_{T'} \frac{\Delta_{c;T'}}{\prod_{l\in T'/T} D_l}\Bigg|_{\mathrm{cut}}$$

on-shell the numerators can be written as products of tree-level amplitudes

fix basis of **monomials** in **irreducible scalar products** via **polynomial division** (Gröbner basis)

integrand parameterisations not unique - freedom in the choices of ISP monomials

5-gluon scattering in pure YM

D-dimensional
generalized unitarity cuts

integrand reduction via
polynomial division

[planar +++++ integrand
SB, Frellesvig, Zhang 1310.1051]

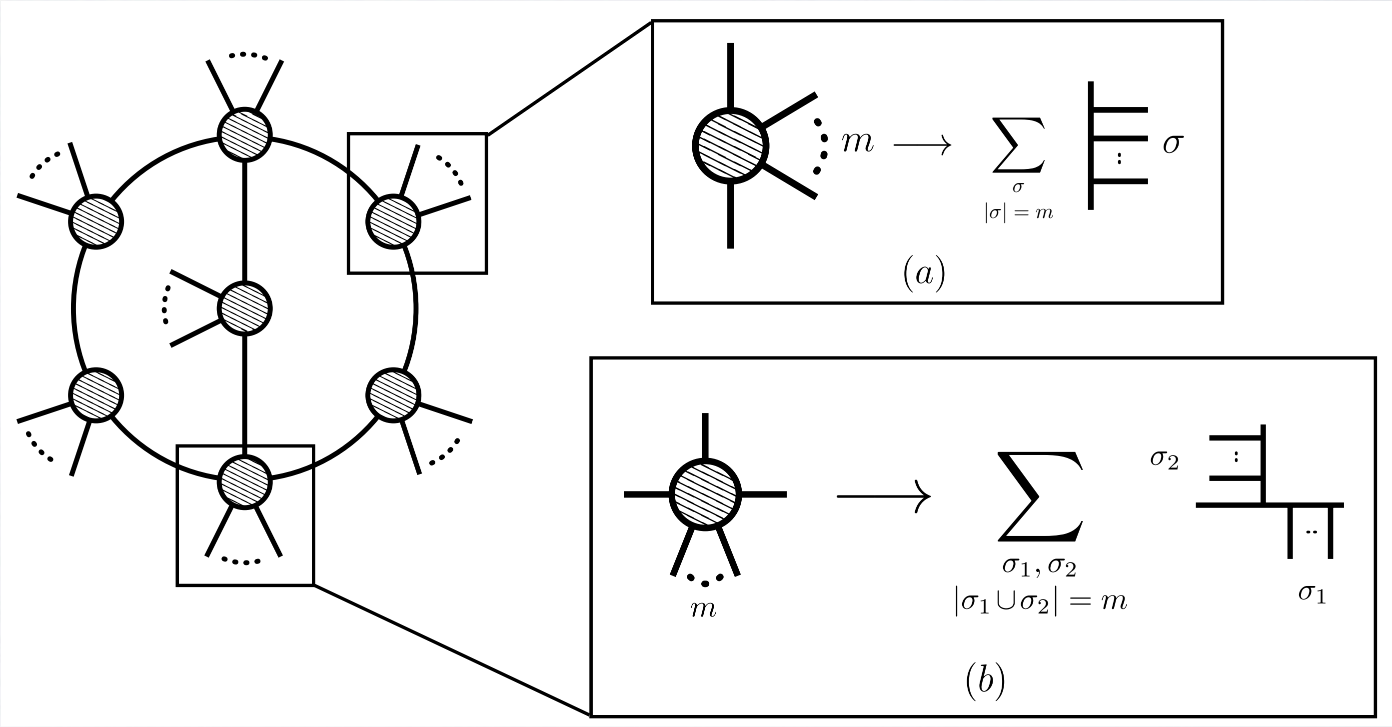
[complete +++++ integrand
SB, Mogull, Ochirov, O'Connell 1507.08797]

integration using diff. eqns.

[planar +++++ amplitude
Gehrmann, Henn, Lo Presti 1511.05409]

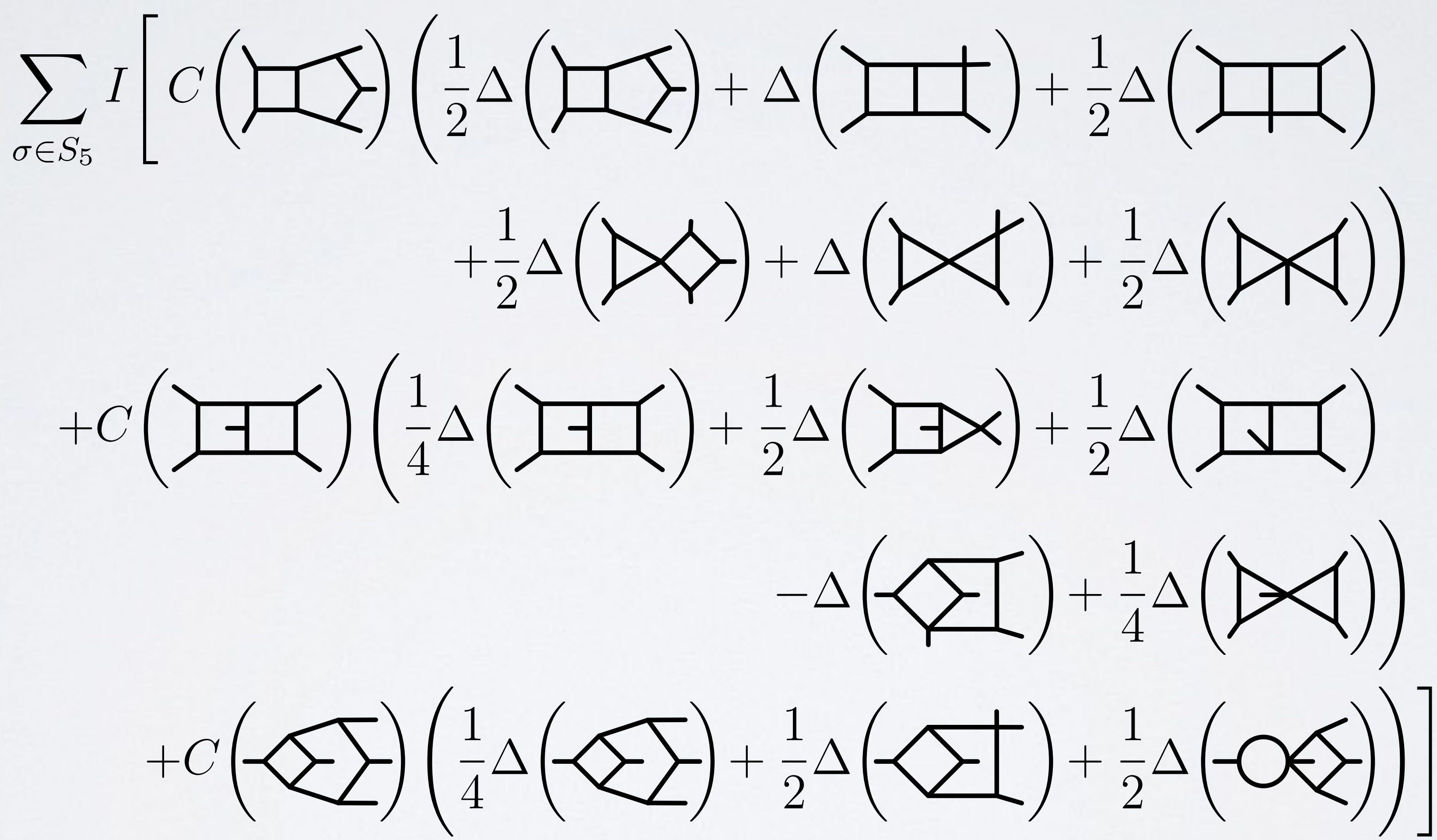
efficient colour
decompositions
incorporating
Kleiss-Kuijf relations

c.f. one-loop [Del Duca,
Dixon, Maltoni (1999)]



5-gluon scattering in pure YM

$$\mathcal{A}_5^{(2)}(1^+,2^+,3^+,4^+,5^+)=$$



Full colour result : non-planar from planar
using colour-kinematics duality

automating analytic computations

(a few of my thoughts on...)

momentum twistors

Hodges (2009)

rational parametrisation of the external kinematics

six-dimensional spinor-helicity

Cheung, O'Connell (2009)

Bern, Carrasco, Dennen, Huang, Ita (2010)

Davies (2011)

efficient **dimensionally regulated** cuts from gauge invariant trees

Outlook

- A high degree of automation is required in order to provide the required theoretical precision current collider experiments
- One-loop QCD computations now widely available and linked with Monte-Carlo tools
  - purely algebraic - numerical algorithms bypass complicated symbol manipulation
  - on-shell: unitarity and recursion relations $\Rightarrow$loops from trees
- Similar approaches to multi-loop amplitudes developing
  - First complete five-point amplitude in QCD
  - purely algebraic algorithms with rational momenta

Backup

Momentum twistors

[Hodges (2009)]

$$Z_{i,a} = (\lambda_{i,a}, \mu_{i,a})$$

momentum conservation automatically satisfied for any 4 x n matrix, Z

$$W_{i,\dot{a}} = (\tilde{\mu}_{\dot{a}}, \tilde{\lambda}_{\dot{a}}) = \frac{\epsilon_{\dot{a},b,c,d} Z_{i-1,b} Z_{i,c} Z_{i+1,d}}{\langle i-1 i\rangle \langle i i+1\rangle}$$

3n-10 independent variables

includes all Schouten identities

$$Z = \begin{pmatrix} 1 & 0 & -\frac{1}{s} & -\frac{1}{s}-\frac{1}{t} \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

complex phase should be evaluated separately

det.

$$\langle 12\rangle = 1 \qquad [12] = -s$$

Momentum twistors

$s_{12},\ s_{23},\ s_{34},\ s_{45},\ s_{15}$

5 point kinematics contains one Gram matrix relation

$$\mathrm{tr}_5(1,2,3,4)^2 = \left| G\begin{pmatrix} p_1 & p_2 & p_3 & p_4 \\ p_1 & p_2 & p_2 & p_4 \end{pmatrix} \right|$$

$$\begin{aligned}\mathrm{tr}_5^2 =& 2s_{12}\left(s_{15}^2(-s_{45}) + s_{15}(s_{23}(s_{34}+s_{45}) + s_{34}s_{45}) + s_{23}s_{34}(s_{45}-s_{23})\right) \\ &+ \left(s_{45}(s_{15}-s_{34}) + s_{23}s_{34}\right)^2 + s_{12}^2(s_{15}-s_{23})^2\end{aligned}$$

$$Z = \begin{pmatrix} 1 & 0 & \frac{1}{x_1} & \frac{1}{x_1}+\frac{1}{x_1x_2} & \frac{1}{x_1}+\frac{1}{x_1x_2}+\frac{1}{x_1x_2x_3} \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & \frac{x_4}{x_2} & 1 \\ 0 & 0 & 1 & 1 & 1-\frac{x_5}{x_4} \end{pmatrix}$$

$$x_1 = s_{12}$$
$$x_2 = \frac{\langle 23\rangle\langle 14\rangle}{\langle 12\rangle\langle 34\rangle}$$
$$x_3 = \frac{\langle 34\rangle\langle 15\rangle}{\langle 13\rangle\langle 45\rangle}$$
$$x_4 = \frac{s_{23}}{s_{12}}$$
$$x_5 = \frac{s_{45}}{s_{12}}$$

no square roots

$$\mathrm{tr}_5^2 = \frac{x_1^4}{x_2^2}\left(x_2^2x_3(x_5-1) + x_2(2x_3x_4+x_4) - (x_3+1)x_4(x_4-x_5)\right)^2$$

Momentum twistors at higher multiplicity

$$Z = \begin{pmatrix} 1 & 0 & f_1 & f_2 & f_3 & \dots & f_{n-3} & f_{n-2} \\ 0 & 1 & 1 & 1 & 1 & \dots & 1 & 1 \\ 0 & 0 & 0 & \frac{x_{n-1}}{x_2} & x_n & \dots & x_{2n-6} & 1 \\ 0 & 0 & 1 & 1 & x_{2n-5} & \dots & x_{3n-11} & 1 - \frac{x_{3n-10}}{x_{n-1}} \end{pmatrix}$$

$$f_i = \sum_{k=1}^{i} \frac{1}{\prod_{l=1}^{k} x_l}$$

$$x_i = \begin{cases} s_{12} & i = 1 \\ -\frac{\langle i\, i{+}1\rangle\langle i{+}2\, 1\rangle}{\langle 1\, i\rangle\langle i{+}1\, i{+}2\rangle} & i = 2, \dots, n-2 \\ \delta_{n,4} + (1 - \delta_{n,4})\frac{s_{23}}{s_{12}} & i = n-1 \\ -\frac{[2|P_{2,i-n+4}|i{-}n{+}5\rangle}{[21]\langle 1|\, i{-}n{+}5\rangle} & i = n, \dots, 2n-6 \\ \frac{\langle 1|P_{23}P_{2,i-2n+9}|i{-}2n{+}10\rangle}{s_{23}\langle 1|\, i{-}2n{+}10\rangle} & i = 2n-5, \dots, 3n-11 \\ \frac{s_{123}}{s_{12}} & i = 3n-10 \end{cases}$$

We can find an (**invertible**) representation for arbitrary number of massless particles