SIX LOOP BETA FUNCTION IN φ^4 *MODEL*

Tuesday 19th January, 2016

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ACAT2016, Valparaiso, Chile

OVERVIEW

1. Status of the multiloop calculations in φ^4 model
2. Two-point function
3. Four-point function
   - 3.1 IBP, R^*
   - 3.2 Calculating graphs with subdivergences using HyperInt
   - 3.3 Preliminary results
4. Predictions for 6 loop term based on Kazakov, Tarasov, Shirkov (1979) paper
5. Borel resummation
6. Summary

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$$S(\varphi) = -\int d\mathbf{x} \left(\frac{1}{2}\tau\varphi(\mathbf{x})^2 + \frac{1}{2}(\vec{\nabla}\varphi(\mathbf{x}))^2 + \frac{1}{24}g(\varphi(\mathbf{x})^2)^2 \right)$$

- critical exponents at 4-loop level:
  - **E. Brezin, J.C. LeGuillou and J. Zinn-Justin**, *Phys. Rev.*, D9 (1974) 1121.
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  - **K.G. Chetyrkin , S.G. Gorishny, S.A. Larin and F.V. Tkachov**, 1983 *Phys. Lett.* B **132** 351
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  - **K.G. Chetyrkin, S.G. Gorishny, S.A. Larin and F.V. Tkachov** 1986 *Preprint* INR P-0453, Moscow.
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- primitive[1] graphs up to 7-8 loops
  - **D. J. Broadhurst and D. Kreimer**, *Int. J. Mod. Phys. C* 6 (Aug., 1995) 519–524 (numerically, up to 7 loops)
  - **O. Schnetz**, *Commun. Number Theory Phys.* 4 (2010), no. 1 1–47 (analytically, up to 7 loops and almost all 8 loops)

[1] no subdivergences

TWO-POINT FUNCTION

TWO-POINT FUNCTION (ARXIV:1601.01960)

in collaboration with D.V. Batkovich and K.G. Chetyrkin

Number of graphs in Γ_2
(two-point 1PI Green function)

# of loops	1	2	3	4	5	**6**
# of graphs	0	1	1	4	11	**50**

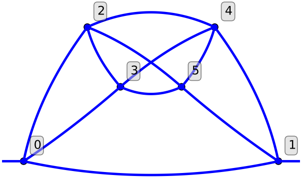
Theorem
(L)-loop counterterms can be calculated using R^* operation and (L-1)-loop IBP reduction

With 4-loop IBP reduction we can calculate only 48 out of 50 6-loop graphs

Graph equals to $G(1,5\varepsilon)F(\varepsilon)$, where $G(1,5\varepsilon) = -5/12 + \mathcal{O}(\varepsilon)$ and $F(\varepsilon)$ is pole part of 5-loop subgraph.

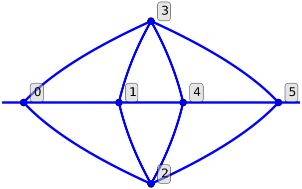
It is not possible to calculate $F(\varepsilon)$ using 4-loop IBP

Pole part of $F(\varepsilon)$ can be reconstructed from the 5-loop counterterm.



No IRR for this graph which allows to use 4-loop IBP

Calculated using transition to dual graph



TWO-POINT FUNCTION, ANOMALOUS DIMENSION OF THE FIELD (ARXIV:1601.01960)

$$
\begin{aligned}
\gamma_\varphi(u) =& \frac{u^2(n+2)}{36} - \left[8+n\right]\frac{u^3(n+2)}{432} + \left[500+90\,n-5\,n^2\right]\frac{u^4(n+2)}{5184} + \\
&+\Big[-77056+8832\,\zeta_3-25344\,\zeta_4+(-22752+3072\,\zeta_3-5760\,\zeta_4)\;n+ \\
&+(-296-288\,\zeta_3)\;n^2+(-39+48\,\zeta_3)\;n^3\Big]\frac{u^5(n+2)}{186624} + \\
&+\Big[\mathbf{1410544+297472\,\zeta_3+619776\,\zeta_4-833536\,\zeta_5-95232\,\zeta_3^2+1190400\,\zeta_6}+ \\
&+\left(\mathbf{549104+69888\,\zeta_3+215808\,\zeta_4-293632\,\zeta_5-28160\,\zeta_3^2+352000\,\zeta_6}\right)\;\mathbf{n}+ \\
&+\left(\mathbf{30184+14976\,\zeta_3+15744\,\zeta_4-23680\,\zeta_5-1024\,\zeta_3^2+12800\,\zeta_6}\right)\;\mathbf{n^2}+ \\
&+(\mathbf{-794+96\,\zeta_4})\;\mathbf{n^3}+(\mathbf{-29-16\,\zeta_3+48\,\zeta_4})\;\mathbf{n^4}\Big]\frac{\mathbf{u^6(n+2)}}{\mathbf{746496}} + O(u^7)
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$$

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$$+(\mathbf{-794+96\,\zeta_4})\;\mathbf{n^3}+(\mathbf{-29-16\,\zeta_3+48\,\zeta_4})\;\mathbf{n^4}\Big]\frac{\mathbf{u^6(n+2)}}{\mathbf{746496}}+\mathcal{O}(u^7)$$

No ζ_7 here

TWO-POINT FUNCTION, CRITICAL EXPONENT η (ARXIV:1601.01960)

$N = 1$, $D = 4 - 2\varepsilon$

$$\eta(\varepsilon) = 2\gamma_\varphi(u_*) = \frac{2}{27}\varepsilon^2 + \frac{109}{729}\varepsilon^3 + \left(\frac{7217}{39366} - \frac{64}{243}\zeta_3\right)\varepsilon^4 +$$
$$+ \left(\frac{321511}{2125764} - \frac{32}{81}\zeta_4 - \frac{1316}{2187}\zeta_3 + \frac{1280}{729}\zeta_5\right)\varepsilon^5 +$$
$$+ \left(\mathbf{\frac{3421613}{38263752}} - \mathbf{\frac{181462}{177147}}\zeta_3 - \mathbf{\frac{658}{729}}\zeta_4 + \mathbf{\frac{73232}{19683}}\zeta_5 + \mathbf{\frac{2432}{2187}}\zeta_3^2 + \mathbf{\frac{3200}{729}}\zeta_6 - \mathbf{\frac{3136}{243}}\zeta_7\right)\varepsilon^6 + \mathcal{O}(\varepsilon^7)$$
$$= 0.074074\,\varepsilon^2 + 0.149520\,\varepsilon^3 - 0.133260\,\varepsilon^4 + 0.821006\,\varepsilon^5 - \mathbf{5.201449}\,\varepsilon^6 + \mathcal{O}(\varepsilon^7)$$

ζ_7 originates from the fixed point value (u_*)

$1/N$-expansion up to $1/N^3$ using conformal bootstap technique

A.N. Vasiliev, Yu.M. Pis'mak, J. Honkonen, *Theor. Math. Phys.*, 50,N 2, p127 (1982) [2]:

$$\eta_N = \frac{\eta_1(\varepsilon)}{N} + \frac{\eta_2(\varepsilon)}{N^2} + \frac{\eta_3(\varepsilon)}{N^3} + \mathcal{O}\left(\frac{1}{N^4}\right) \quad \text{for any } \varepsilon,$$

We know η_N up to $1/N^3$ for any ε and we know η_ε up to ε^6 for any N.

$$\boxed{\eta_N \text{ expansion by } \varepsilon = \eta_\varepsilon \text{ expansion by } 1/N}$$

[2] This paper contains misprint in the second term of the r.h.s of eq. (22): the denominator must be $3(2-\mu)^3$ (for details see **Vasilev A. N.**, Quantum Field Renormalization Group in Critical Behavior Theory and Stochastic Dynamics, 2004)

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Three independent relations for 6 loop diagrams.

All of 6-loops self-energy diagrams give contribution to η_3.

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FOUR-POINT FUNCTION

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Number of graphs in Γ_4
(four-point 1PI Green function)

# of loops	1	2	3	4	5	**6**
# of graphs	1	2	8	26	124	**627**

- ***factorized*** – can be represented as product of two graphs $\gamma = \gamma_1 \cdot \gamma_2$,
  $KR'(\gamma) = -KR'(\gamma_1) \cdot KR'(\gamma_2)$
- ***4-loop reducible*** – graphs which contains 4-loop(or less) p-integral as subgraph and the rest part can be integrated using *G*-functions
  (two vertex reducible graphs)
- ***primitive*** – graphs with no subdivergences [3]
  (two vertex irreducible graphs)
- ***4-loop irreducible*** – graphs cannot be calculated using 4-loop IBP and R^* [4]
  (two vertex irreducible graphs with subdivergences)

factorized	124
4-loop reducible	481
primitive	10
4-loop irreducible	12

[3] **O. Schnetz**, *Commun. Number Theory Phys.* 4 (2010), no. 1 1–47, arXiv:0801.2856

[4] **E. Panzer**, *Comp. Phys. Comm.*, 188 (2015), pp. 148-166, arXiv:1403.3385

4-LOOP REDUCIBLE GRAPHS, IBP, R^*

- Infrared rearrangement(IRR) and R^*-operation implemented as *Python* library [5] on top of GraphState/Graphine library

- IBP reduction also implemented on *Python* with *mongodb* as cache. Reduction rules are generated by LiteRed[6]

- Master integral values are taken from Baikov/Chetyrkin paper[7] and Lee/Smirnov/Smirnov paper[8]

  Surprisingly that for 6-loop Γ_2 graphs it is enough master integral values from Baikov/Chetyrkin paper, but for 6-loop Γ_4 graphs one more term from Lee/Smirnov/Smirnov paper is required.

- Counterterms for graphs calculated with all possible rearrangements of external momenta (extensive consistency check for IBP and R^*)

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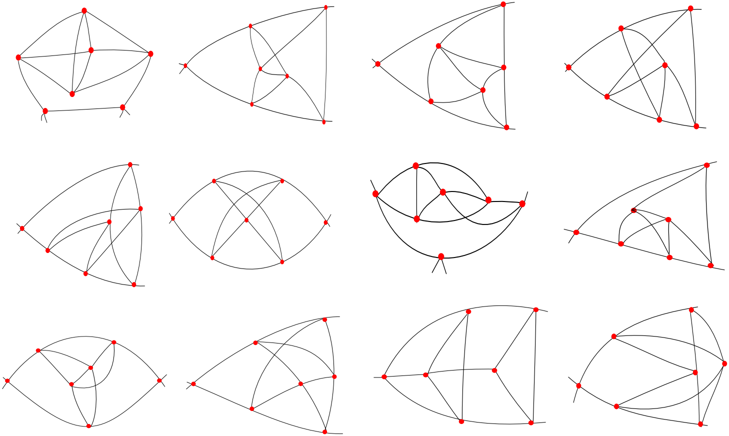
[5] **Batkovich D V, Kompaniets M V** 2015 *J. Phys.: Conf. Ser.* 608 012068

[6] **Lee R N**, 2012 Presenting LiteRed: a tool for the loop integrals reduction (Preprint hep-ph/1212.2685)

[7] **Baikov P A, Chetyrkin K G**, 2010 *Nucl. Phys. B* 837 186

[8] **Lee R N, Smirnov A V, Smirnov V A** 2012 *Nucl. Phys. B* 856 95

4-LOOP IRREDUCIBLE GRAPHS



All graphs are two vertex irreducible and contain subdivergences (up to 4 subgraphs)

HYPERINT (E.PANZER)

Many Feynman integrals are expressible via multiple polylograithms (MPL)

$$\mathtt{Li}_{n_1,\dots,n_d}(z_1,\dots,z_d) = \sum_{0<k_1<\dots<k_d} \frac{z_1^{k_1}\cdots z_d^{k_d}}{k_1^{n_1}\cdots k_d^{n_d}}$$

and there special values, like multiple zeta values (MZV) $\zeta_{n_1,\dots,n_d} = \mathtt{Li}_{n_1,\dots,n_d}(1,\dots,1)$

MPL can be rewritten in terms of hyperlogarithms

$$G(\sigma_1,\dots,\sigma_w;z) = \int_0^z \frac{dz_1}{z_1-\sigma_1}\int_0^{z_1}\frac{dz_2}{z_2-\sigma_2}\cdots\int_0^{z_{w-1}}\frac{dz_w}{z_w-\sigma_w}$$

Main idea: consider graph in Feynman representation and integrate out one variable after other

$$f_n = \int_0^\infty f_{n-1}\,d\alpha_n = \int_{(0,\infty)^n} f_0\,d\alpha_1\cdots d\alpha_n, \qquad f_0 = \frac{\psi^{sdd-D/2}}{\phi^{sdd}}\prod_e \alpha_e^{a_e-1}, sdd = \sum_e a_e - DL/2$$

If on each step of integration function f_{n-1} can be represented in terms of hyperlogarithms in α_n with rational prefactors

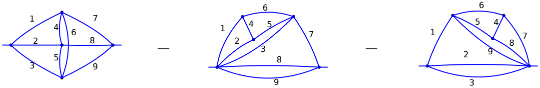
$$f_{n-1} = \sum_{\vec{\sigma},\tau,k}\frac{G(\vec{\sigma};\alpha_n)}{(\alpha_n-\tau)^k}\lambda_{\sigma,\tau,k}, \quad \texttt{with } \sigma \texttt{ and } \tau \texttt{ independent of } \alpha_n$$

graph is called *linearly reducible* and can be integrated analytically using HyperInt.

for details see **E.Panzer**, *Comp. Phys. Comm.*, 188 (2015), pp. 148-166,
E.Panzer, Feynman integrals and hyperlogarithms (PhD Thesis) arXiv:1506.07243 and references therein

HYPERINT (E.PANZER), GRAPHS WITH SUBDIVERGENCES

- algorithm does not limited by some particular loop number
- some limitations on the graph combinatorial type (graph must be linearly reducible)
  At this moment all graphs we need to calculate are linearly reducible
- Integrand passed to HyperInt must contain no subdivergences
- we need to find some combination of graphs which is free from subdivergences (on the integrand level), this combination except the target graph must contain graphs which are "easier" to calculate [9]



- Subtraction automation: heuristic vs R-operation approach
  - heuristic - less subtraction terms, but fail for complicated graphs
  - **R-operation approach - more subtraction terms, but work for any graph** (BPHZ-like subtractions)
  - dimensional shift - large number of terms

[9] **Brown F., Kreimer D.** *Letters in Mathematical Physics* September 2013, Volume 103, Issue 9, pp 933-1007, arXiv:1112.1180

PRELIMINARY RESULTS FOR BETA-FUNCTION

$N = 1$, $D = 4 - 2\varepsilon$, $\overline{MS}$-scheme

$$\beta(u) = u\left(-2\,\varepsilon + 3u - \frac{17}{3}u^2 + \Big[145 + 96\,\zeta_3\Big]\frac{u^3}{8} + \Big[-3499 - 3744\,\zeta_3 + 864\,\zeta_4 - 5760\,\zeta_5\Big]\frac{u^4}{48} + \right.$$

$$+\Big[764621 + 1146960\,\zeta_3 - 342432\,\zeta_4 + 2274048\,\zeta_5 +$$

$$+103680\,\zeta_3^2 - 777600\,\zeta_6 + 3048192\,\zeta_7\Big]\frac{u^5}{2304} +$$

$$+\Big[-\mathbf{94207135} - \mathbf{187104720}\,\zeta_3 + \mathbf{61160400}\,\zeta_4 - \mathbf{367044480}\,\zeta_5 - \mathbf{99970560}\,\zeta_3^2 +$$

$$+\mathbf{192700800}\,\zeta_6 - \mathbf{119771136}\,\zeta_{3,5} + \mathbf{9331200}\,\zeta_3\,\zeta_4 - \mathbf{732983040}\,\zeta_7 - \mathbf{270950400}\,\zeta_3\,\zeta_5 +$$

$$\left. +\mathbf{609507072}\,\zeta_8 - \mathbf{44236800}\,\zeta_3^3 - \mathbf{903782400}\,\zeta_9\Big]\frac{\mathbf{u^6}}{\mathbf{57600}} + \mathcal{O}(u^7)\right) =$$

$$= u\left(-2\varepsilon + 3u - \frac{17}{3}u^2 + 32.54968u^3 - 271.60578u^4 + \right.$$

$$\left. + 2848.56826u^5 - \mathbf{35096.77397u^6} + \mathcal{O}(u^7)\right)$$

$$\zeta_{3,5} = \sum_{0<n<m} \frac{1}{n^3 m^5} \approx 0.03770767298$$

$\zeta_{3,5}$ in 6 loop term of β function was predicted by **D. J. Broadhurst** in 1985

PREDICTIONS FOR 6 LOOP TERM BASED ON KAZAKOV, TARASOV, SHIRKOV (1979) PAPER

PREDICTIONS FOR 6 LOOP TERM

$$\overline{\beta}(u) = \frac{\beta(u) + 2\varepsilon u}{2} = \frac{3}{2}u^2 - \frac{17}{6}u^3 + 16.27u^4 - 135.80u^5 + 1424.28u^6 - 17548.38u^7 + \mathcal{O}(u^8)$$

Borel transform:
$$\beta^B(u) = \int_0^\infty \frac{dx}{u} e^{-x/u} \left(x \frac{\partial}{\partial x} \right)^b B(x), \quad B_n = \frac{\overline{\beta}_n}{n! n^b}$$

with consequent conformal mapping

$$B(x) = \sum_{n=2}^{N} x^n B_n \to B(x) = \left(\frac{x}{\omega} \right)^\nu \left(a_0 + a_1 \omega + a_2 \omega^2 + \cdots + a_N \omega^N \right), \quad \omega(x) = \frac{\sqrt{1+x} - 1}{\sqrt{1+x} + 1}$$

coeffs a_i are fixed from the requirement to reproduce coefficients B_n known from loop expansion,
ν defines asymptotic of $\beta(g)$ when $g \to \infty$, if asymptotic is not known ν is defined from minimization of $\eta_{N+1} = 1 - \beta^B_{N+1}/\beta^B_N$.

$$1.7 < \nu < 2.2$$

Expanding $B(x)$ up to x^{N+1} one can get predicition for $N+1$ coefficient for beta function

for details see
D. I. Kazakov, O. V. Tarasov, D. V. Shirkov, (1979) "Analytic continuation of the results of perturbation theory for the model $g\varphi^4$ to the region $g \geq 1$", *Theoretical and Mathematical Physics*, Volume 38, Issue 1, pp 9-16
Kazakov, D. I., Shirkov, D. V. (1980), "Asymptotic Series of Quantum Field Theory and Their Summation." *Fortschr. Phys.*, 28: 465–499.

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predictions based on 4,5,6 loop beta function ($\nu = 2$)[10]

$$\overline{\beta}^{P4}(u) = \frac{3}{2}u^2 - \frac{17}{6}u^3 + 16.27u^4 - 135.80u^5 + \mathbf{1404.30u^6} - \mathbf{16537.8u^7} + \mathbf{213452.8u^8} + \mathcal{O}(u^9)$$

1.5% 5.7%

[10] **D. I. Kazakov, O. V. Tarasov, D. V. Shirkov**, (1979), *Theoretical and Mathematical Physics*, Volume 38, Issue 1, pp 9-16

PREDICTIONS FOR 6 LOOP TERM

$$\overline{\beta}(u) = \frac{3}{2}u^2 - \frac{17}{6}u^3 + 16.27u^4 - 135.80u^5 + 1424.28u^6 - 17548.38u^7 + \mathcal{O}(u^8)$$

predictions based on 4,5,6 loop beta function ($\nu = 2$)[11]

$$\overline{\beta}^{P4}(u) = \frac{3}{2}u^2 - \frac{17}{6}u^3 + 16.27u^4 - 135.80u^5 + \mathbf{1404.30u^6} - \mathbf{16537.8u^7} + \mathbf{213452.8u^8} + \mathcal{O}(u^9)$$

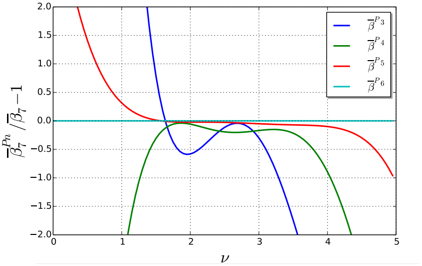
1.5% 5.7%

$$\overline{\beta}^{P5}(u) = \frac{3}{2}u^2 - \frac{17}{6}u^3 + 16.27u^4 - 135.80u^5 + 1424.28u^6 - \mathbf{17150.62u^7} + \mathbf{226595.68u^8} + \mathcal{O}(u^9)$$

2.3%!!

$$\overline{\beta}^{P6}(u) = \frac{3}{2}u^2 - \frac{17}{6}u^3 + 16.27u^4 - 135.80u^5 + 1424.28u^6 - 17548.38u^7 + \mathbf{242105.8u^8} + \mathcal{O}(u^9)$$

prediction for 7 loop term



ν dependence of the relative error of the prediction for 6 loop term based on 3-6 loop beta function

[11] **D. I. Kazakov, O. V. Tarasov, D. V. Shirkov**, (1979), *Theoretical and Mathematical Physics*, Volume 38, Issue 1, pp 9-16

PREDICTIONS FOR ANOMALOUS DIMENSION OF THE FIELD

$$\gamma_\varphi(u) = 0.0833u^2 - 0.0625u^3 + 0.3385u^4 - 1.9255u^5 + 14.383u^6 + \mathcal{O}(u^7)$$

Predictions for $\gamma_\varphi(u)$ based on 4,5,6 loop results ($\nu = 3$)

$$\gamma_\varphi^{P4}(u) = 0.0833u^2 - 0.0625u^3 + 0.3385u^4 - \underset{7\%}{\mathbf{2.0663u^5}} + \underset{11\%}{\mathbf{16.068u^6}} - \mathbf{145.86u^7} + \mathcal{O}(u^8)$$

PREDICTIONS FOR ANOMALOUS DIMENSION OF THE FIELD

$$\gamma_\varphi(u) = 0.0833u^2 - 0.0625u^3 + 0.3385u^4 - 1.9255u^5 + 14.383u^6 + \mathcal{O}(u^7)$$

Predictions for $\gamma_\varphi(u)$ based on 4,5,6 loop results ($\nu = 3$)

$$\gamma_\varphi^{P4}(u) = 0.0833u^2 - 0.0625u^3 + 0.3385u^4 \underbrace{-\mathbf{2.0663u^5}}_{7\%} + \underbrace{\mathbf{16.068u^6}}_{11\%} - \mathbf{145.86u^7} + \mathcal{O}(u^8)$$

$$\gamma_\varphi^{P5}(u) = 0.0833u^2 - 0.0625u^3 + 0.3385u^4 - 1.9255u^5 + \underbrace{\mathbf{14.316u^6}}_{0.5\%!!} - \mathbf{125.99u^7} + \mathcal{O}(u^8)$$

$$\gamma_\varphi^{P6}(u) = 0.0833u^2 - 0.0625u^3 + 0.3385u^4 - 1.9255u^5 + 14.383u^6 \underbrace{-\mathbf{127.29u^7}}_{\text{prediction for 7 loop term}} + \mathcal{O}(u^8)$$

BOREL RESUMMATION

RESUMMATION FOR ISING UNIVERSALITY CLASS ($N = 1$)

Critical exponents that can be measured in experiment

$$\alpha = \frac{2\Delta_\tau - d}{\Delta_\tau}, \quad \beta = \frac{\Delta_\phi}{\Delta_\tau}, \quad \gamma = \frac{d - 2\Delta_\phi}{\Delta_\tau}, \quad \delta = \frac{d - \Delta_\phi}{\Delta_\phi}, \quad \eta = 2\Delta_\phi - d + 2 = 2\gamma^*_\phi \tag{1}$$

where $\Delta_\tau = 2 + \gamma^*_\tau = 1/\nu$, $\Delta_\phi = d/2 - 1 + \gamma^*_\phi$

$D = 3$	α	β	γ	δ	ν	η
$\epsilon - exp^{(4)}$	0.1295	0.3217	1.2270	4.8139	0.6235	0.03201
$\epsilon - exp^{(5)}$	0.1119	0.3249	1.2382	4.8108	0.6294	0.03256
$\epsilon - exp^{(6)}$	0.1157	0.3255	1.2334	4.7899	0.6281	0.03629
HT & MC	0.110(1)	0.3265(3)	1.2372(5)	4.789(2)	0.6301	0.03601(4)
experiment	0.104-0.111	0.315-0.341	1.14-1.32		0.606-0.70	0.030-0.058

$D = 2$	α	β	γ	δ	ν	η
$\epsilon - exp^{(4)}$	0.2523	0.07496	1.598	22.315	0.8738	0.1716
$\epsilon - exp^{(5)}$	0.1279	0.0851	1.702	20.999	0.9361	0.1818
$\epsilon - exp^{(6)}$	0.1645	0.1092	1.617	15.816	0.9178	0.2379
exact		0.125	1.75	15	1	0.25

HT, MC and experimental data are taken from **Pelissetto A., Vicari E.** "Critical Phenomena and Renormalization-Group Theory", *Phys.Rept.* 368:549-727,2002; arXiv:cond-mat/0012164

SUMMARY

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- 6 loop calculation for φ^4 model are almost complete, beta function contains multiple zeta value $\zeta_{3,5}$
  (TODO: recalculate all integrals with HyperInt)
- Automation for calculation of the graphs with subdivergences using HyperInt
- Predictions of Kazakov/Tarasov/Shirkov for 6 loop terms gives only 2% error!
- Resumed critical exponents for 3D Ising model are in good agreement with both simulation and experimental data
- Resumed critical exponents for 2D Ising model are still far from exact solution