Numerical multi-loop calculations: tools and applications

Gudrun Heinrich

Max Planck Institute for Physics, Munich

ACAT 2016

Universidad Técnica Federico Santa María (UTFSM)

Valparaíso, Chile

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

The precision frontier

- LHC Run II:

  high precision predictions are of major importance !

- beyond NLO:

analytic calculation of integrals with two or more loops tough, in particular if several mass scales are involved

while the automation of NLO calculations has seen great progress in recent years, $\mathrm{N^xLO}$ with $x \geq 2$

still requires tedious process-specific calculations

- NLO automation:

- NLO automation:

pretty advanced

NLO matched to parton shower is new state of the art

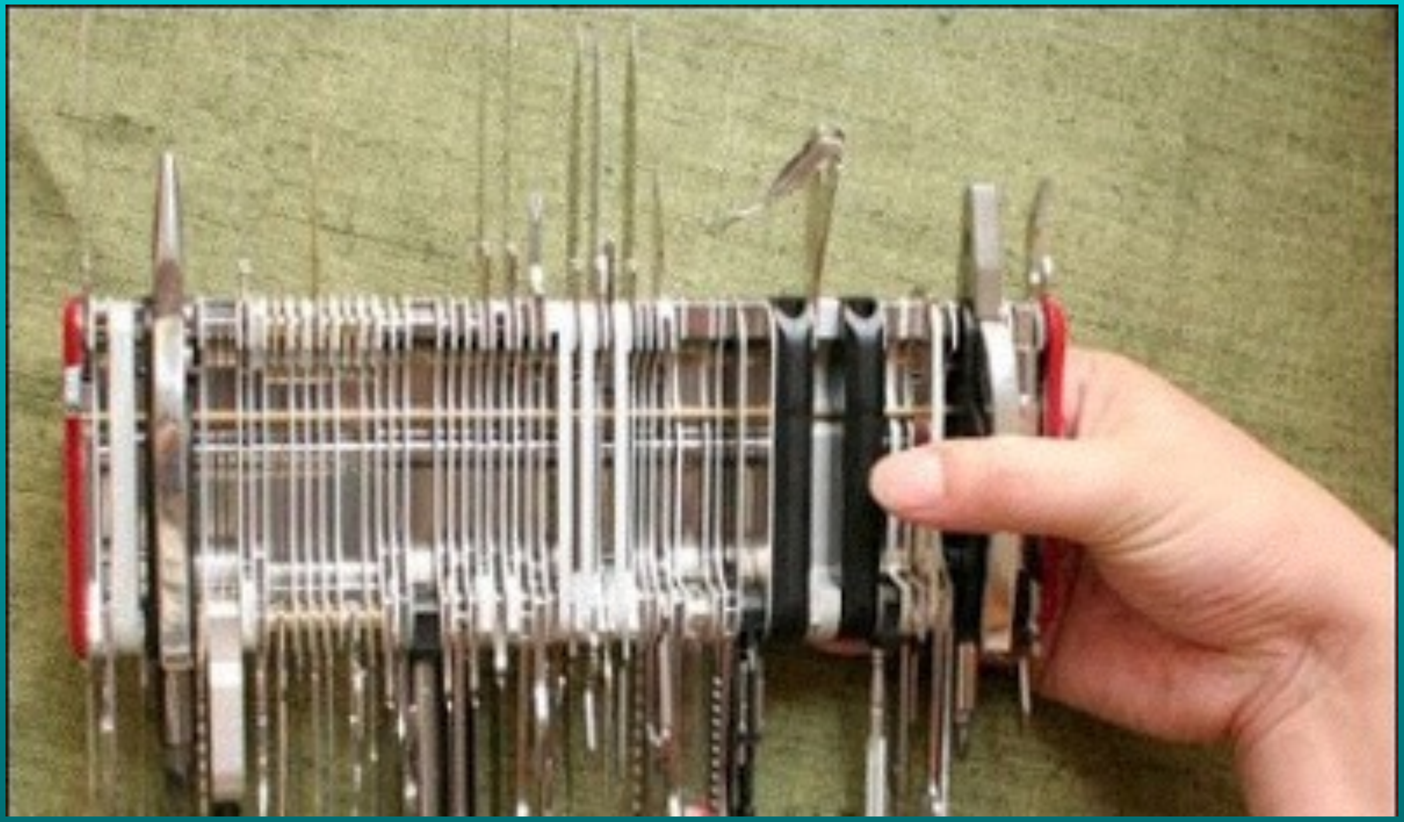
- NLO automation:

pretty advanced

NLO matched to parton shower is new state of the art

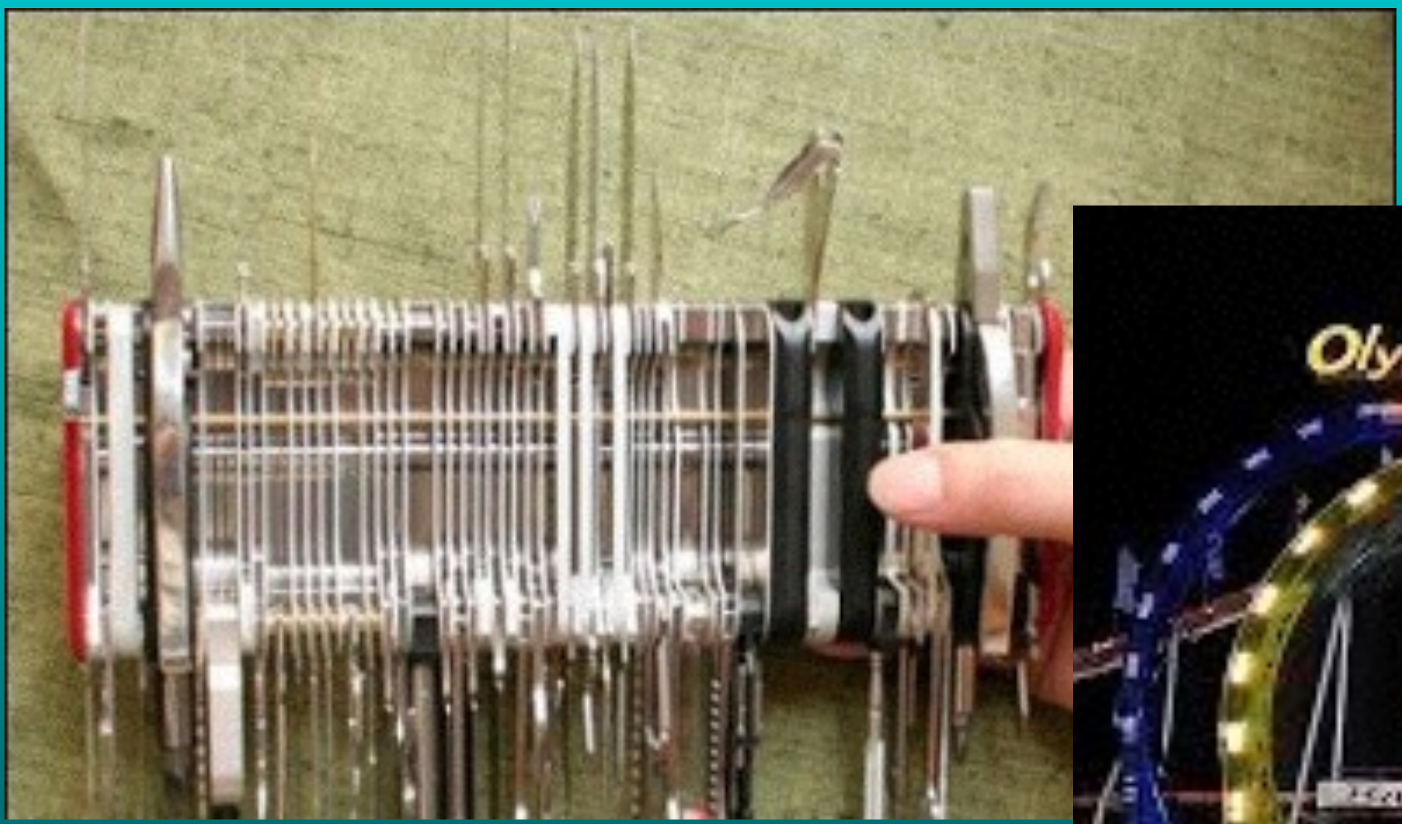
- NNLO: still a long way to automation

what we would like to have is a multi-functional tool which can calculate integrals with various mass configurations, number of external legs, etc.



what we would like to have is a multi-functional tool which can calculate integrals with various mass configurations, number of external legs, etc.

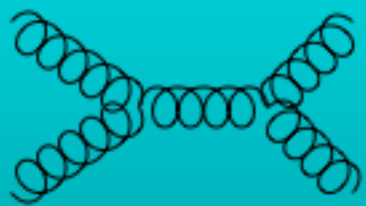
beyond one loop



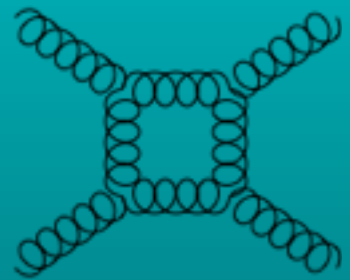
building blocks of higher order calculations

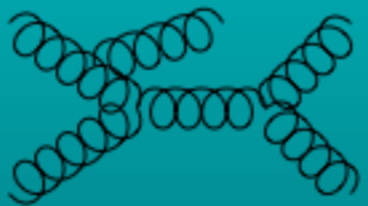
example 2 to 2 scattering

LO: usually tree level diagrams

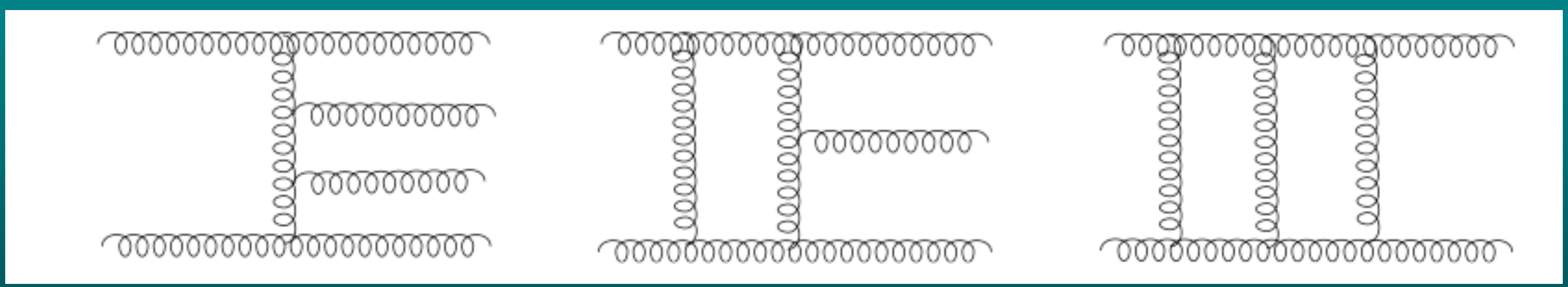


NLO: one loop (virtual) + extra real radiation + subtraction terms





NNLO:



double real

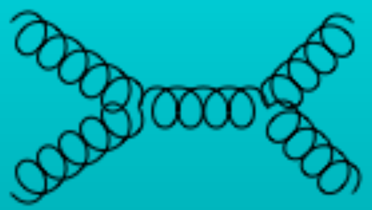
1-loop virtual
⊗ single real

2-loop virtual

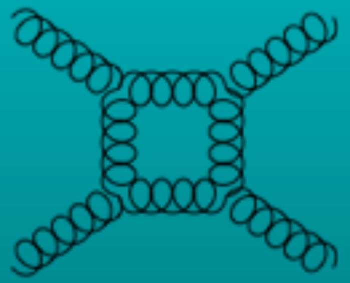
building blocks of higher order calculations

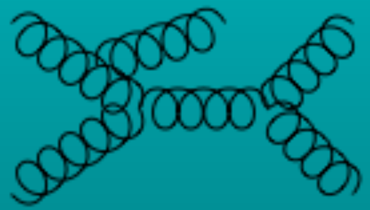
example 2 to 2 scattering

LO: usually tree level diagrams

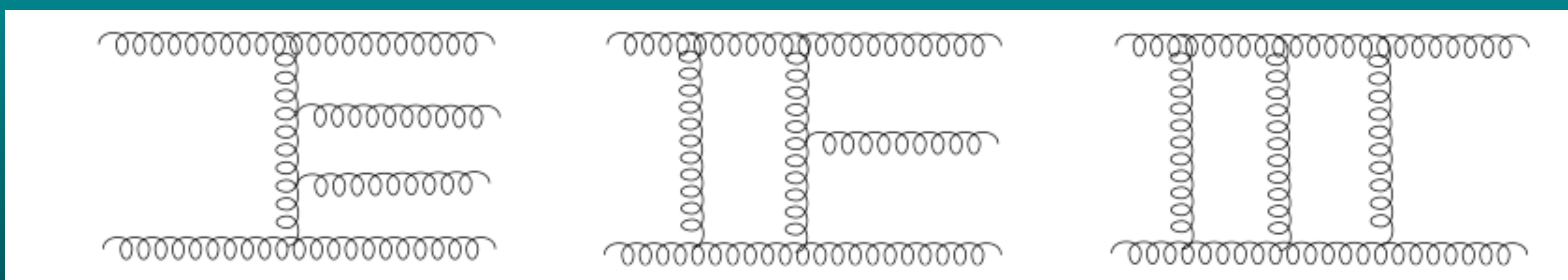


NLO: one loop (virtual) + extra real radiation + subtraction terms





NNLO:



double real

1-loop virtual ⊗ single real

2-loop virtual

see talk by R. Boughezal

need efficient methods to

- generate the amplitudes
- reduce the loop amplitudes to coefficients $\otimes$ master integrals

- calculate the master integrals

individual contributions to an amplitude (virtual/real) are usually ***divergent***

- requires the isolation of the singularities in epsilon

  (dimensional regularisation)

- need a good subtraction method for singularities of individual contributions

need efficient methods to

- generate the amplitudes
- reduce the loop amplitudes to coefficients $\otimes$ master integrals

- calculate the master integrals ← SecDec

individual contributions to an amplitude (virtual/real) are usually ***divergent***

- requires the isolation of the singularities in epsilon

(dimensional regularisation)

- need a good subtraction method for singularities of individual contributions

need efficient methods to

- generate the amplitudes
- reduce the loop amplitudes to coefficients $\otimes$ master integrals

- calculate the master integrals ← SecDec

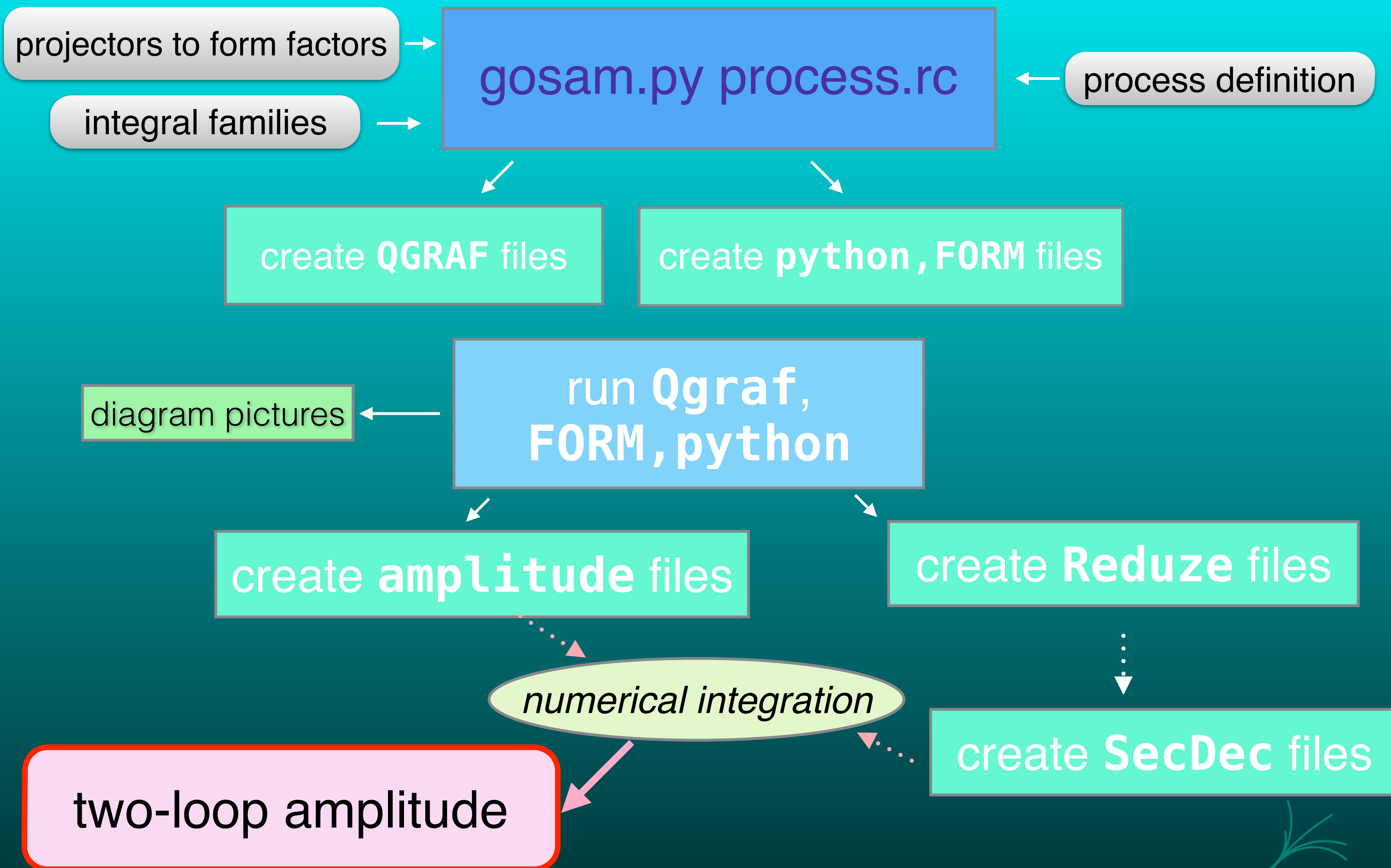
individual contributions to an amplitude (virtual/real) are usually *divergent*

- requires the isolation of the singularities in epsilon ← SecDec

(dimensional regularisation)

- need a good subtraction method for singularities of individual contributions

automated 2-loop amplitudes: GoSam @ 2 loops



MAX-PLANCK-GESELLSCHAFT

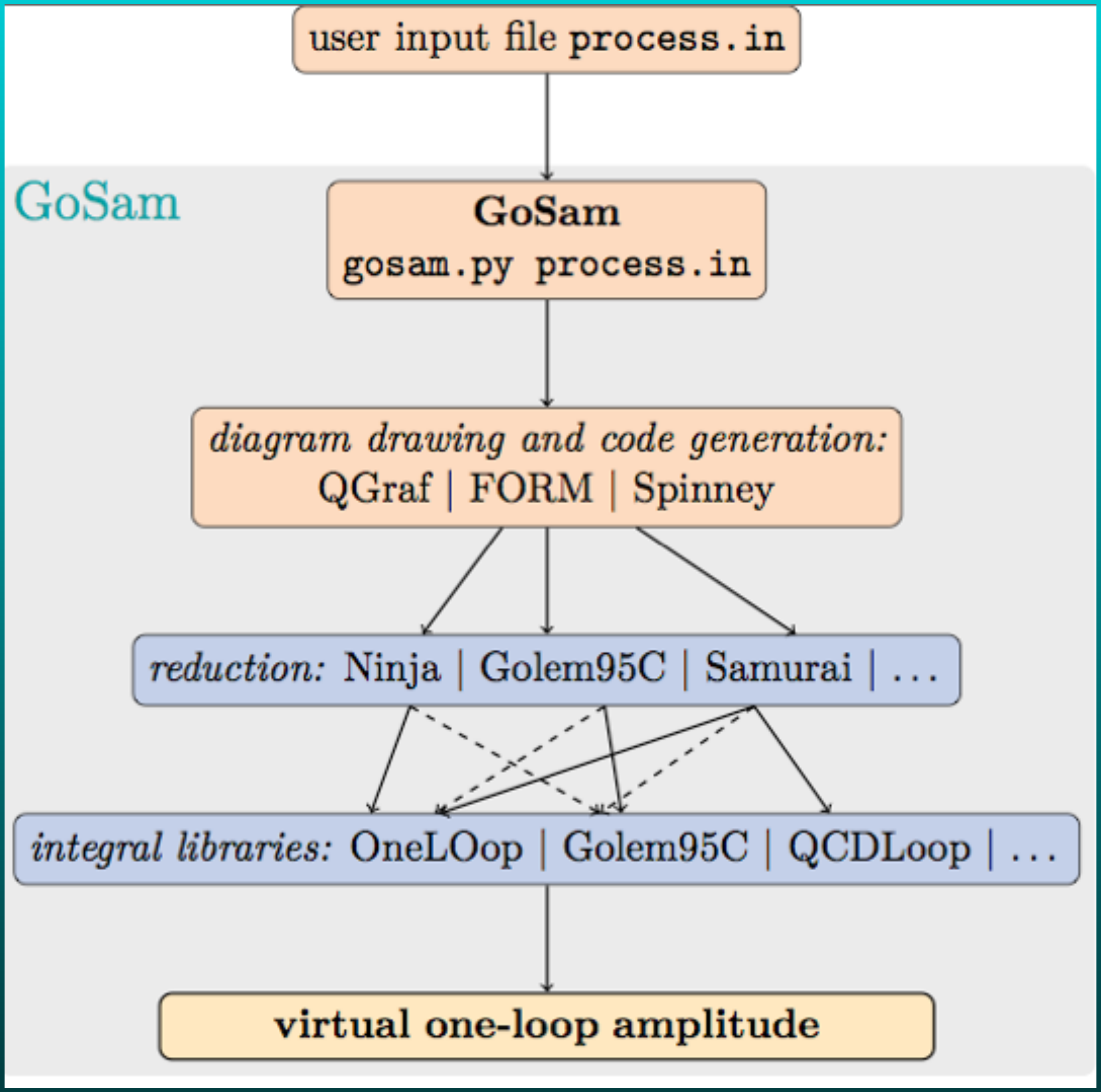
Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

credits

GoSam 2-loop	N.Greiner, GH, S.Jahn, S.Jones, M.Kerner, P. Mastrolia, J.Schlenk, T.Zirke
QGRAF	P. Nogueira
FORM	J. Vermaseren, J. Kuipers, T. Ueda, J. Vollinga
Reduze	C. Studerus, A. von Manteuffel
GoSam 1-loop	see later
SecDec	see later

GoSam @ 1-loop

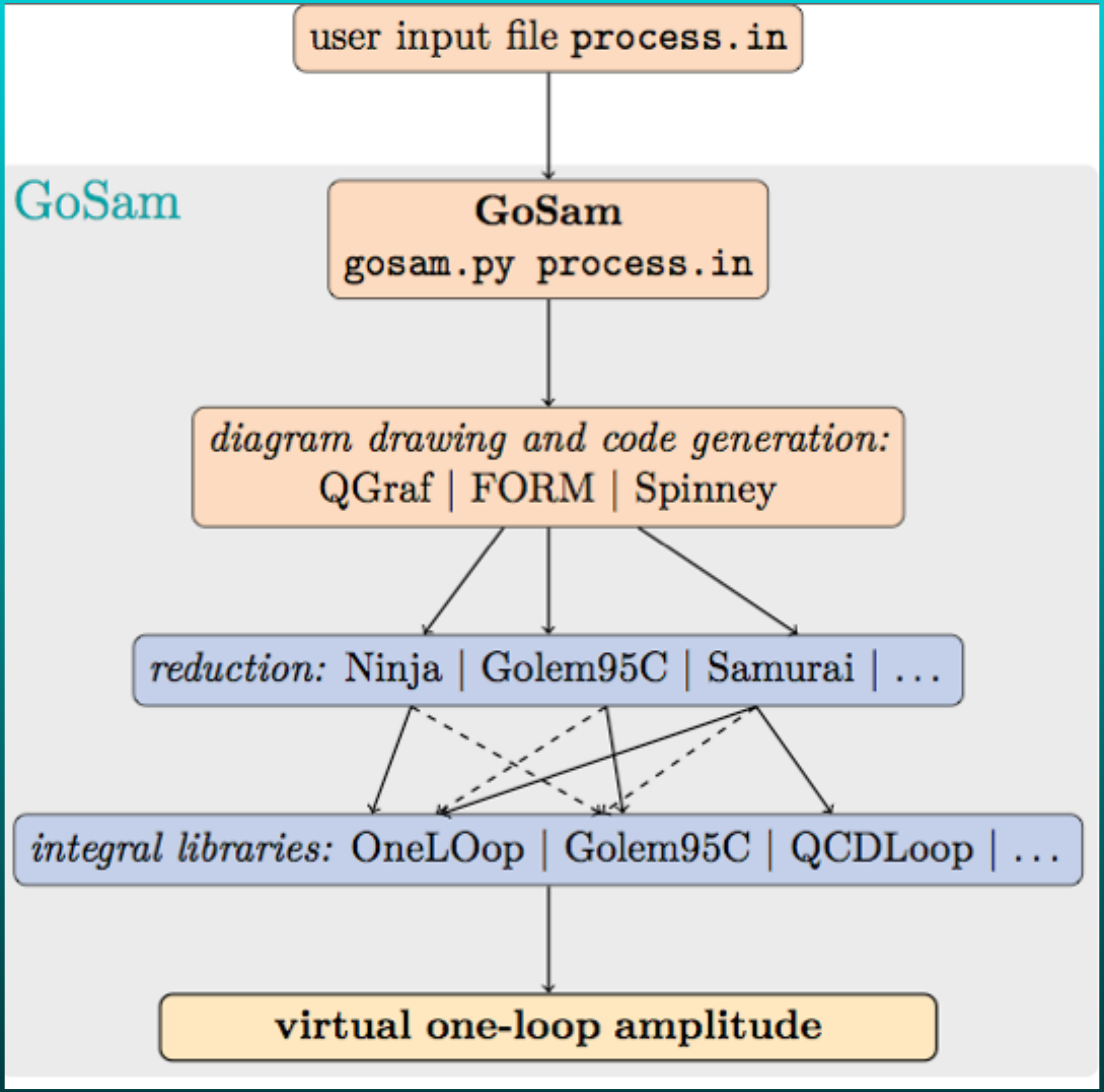
T. Binoth, G.Cullen, H.van Deurzen, N.Greiner, GH, S.Jahn, G.Luisoni, P. Mastrolia, E.Mirabella, G. Ossola, T. Peraro, T. Reiter, J. Reichel, J. Schlenk, J.F. von Soden-Fraunhofen, F. Tramontano



http://gosam.hepforge.org

GoSam @ 1-loop

T. Binoth, G.Cullen, H.van Deurzen, N.Greiner, GH, S.Jahn, G.Luisoni, P. Mastrolia, E.Mirabella, G. Ossola, T. Peraro, T. Reiter, J. Reichel, J. Schlenk, J.F. von Soden-Fraunhofen, F. Tramontano



see talk by G.Luisoni tomorrow

http://gosam.hepforge.org

SecDec

http://secdec.hepforge.org

algorithm: T.Binoth, GH '00

version 1.0: J. Carter, GH '10

version 2.0: S.Borowka, J. Carter, GH '12

version 3.0: S.Borowka, GH, S.Jones, M.Kerner, J.Schlenk, T.Zirke '15

arXiv:1502.06595 (CPC 2015)

other public programs based on sector decomposition:

- **sector_decomposition** (uses Ginac) (only Euclidean region)
  [Bogner, Weinzierl '07]

  supplemented with **CSectors**
  for construction of integrand in terms of Feynman parameters

  [Gluza, Kajda, Riemann, Yundin '10]

- FIESTA (versions 1,2,3,4) (use Mathematica, C++)

  [A.Smirnov, V.Smirnov, Tentyukov, '08,'09,'13,'15]

SecDec

based on method of sector decomposition

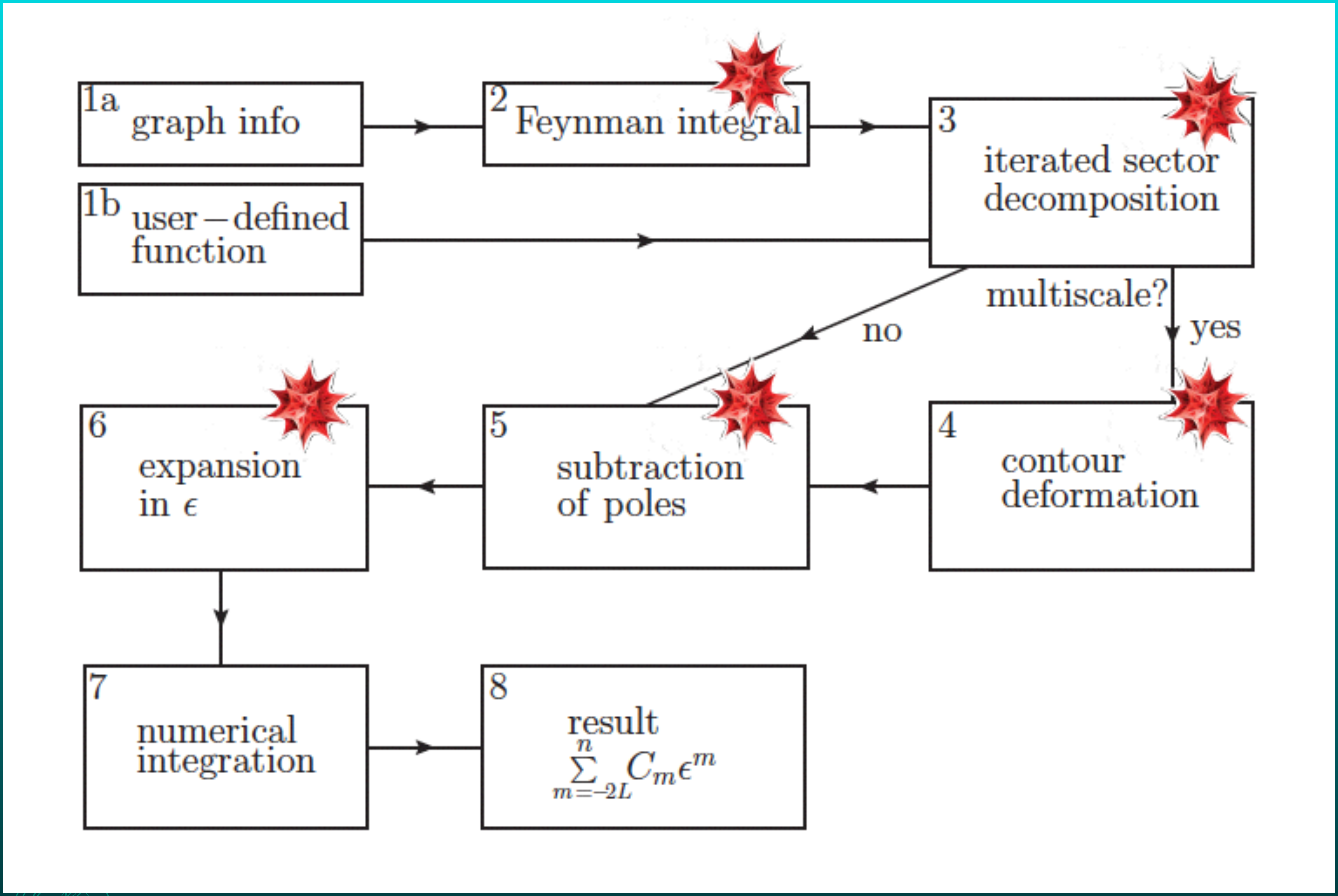
(Hepp 66; Denner & Roth 96; Binoth & GH 00)

- factorizes poles in dim. regulator epsilon from
  - multi-loop integrals
  - multi-dimensional parameter integrals
- produces Laurent series in epsilon, coefficients will be finite parametric integrals
- integrates coefficients numerically

  uses **Cuba** library (T.Hahn) or NIntegrate (Wolfram Research)

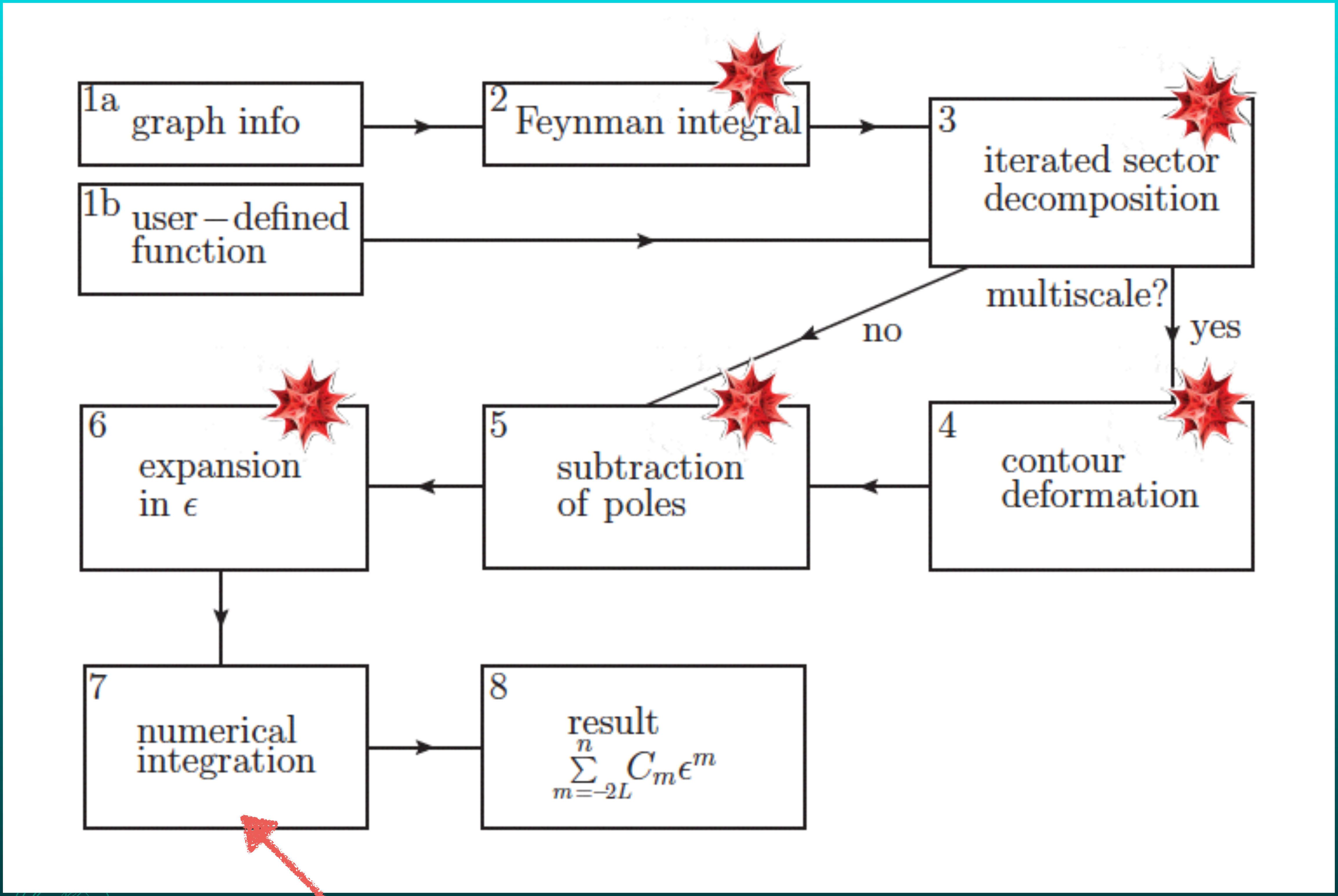
  1-dim: cquad (Gonnet)

SecDec basic workflow



graphics by S.Borowka

SecDec basic workflow



optionally on a cluster

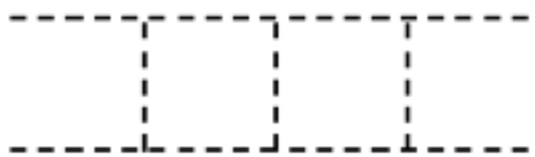
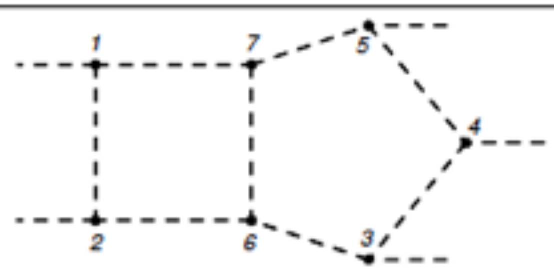
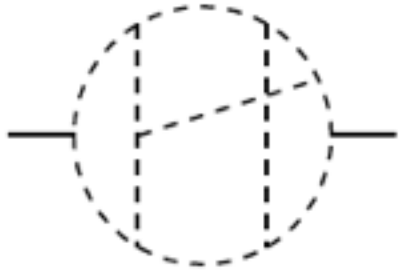
graphics by S.Borowka

new features in SecDec-3.0

- implementation of two new decompositions strategies G1, G2 based on a geometric algorithm (**J. Schlenk**, inspired by Kaneko/Ueda '10)

  uses Normaliz (for triangulation) (Bruns, Ichim, Römer, Söger)

⟶ guaranteed to stop, produces less sectors than original strategy X

Diagram	Strategy X	Strategy G1	Strategy G2
	282 sectors 1 s	266 sectors 8 s	166 sectors 4 s
	368 sectors 1 s	360 sectors 9 s	235 sectors 5 s
	548 sectors 3 s	506 sectors 15 s	304 sectors 4 s
	infinite recursion	72 sectors 5 s	76 sectors 1 s
	27336 sectrs 5510 s	32063 sectrs 11856 s	27137 sectrs 443 s

new features in SecDec-3.0

- improved user interface → easy input files, custom definition of kinematics
- propagators with zero or negative powers are possible
  → easy interface to reduction programs
- linear propagators can be treated
- usage on a cluster facilitated
- speed improvements
- option to use numerical integrators from Mathematica
- complex masses

coming soon:

algebraic part in python

new IBP method

numerical part on GPU

speedup by sampling adjustment for (sub-)dominant sectors

SecDec as a **library** to be linked to any amplitude calculation

S. Borowka, GH, S. Jahn, M. Kerner, S. Jones, J. Schlenk, T. Zirke

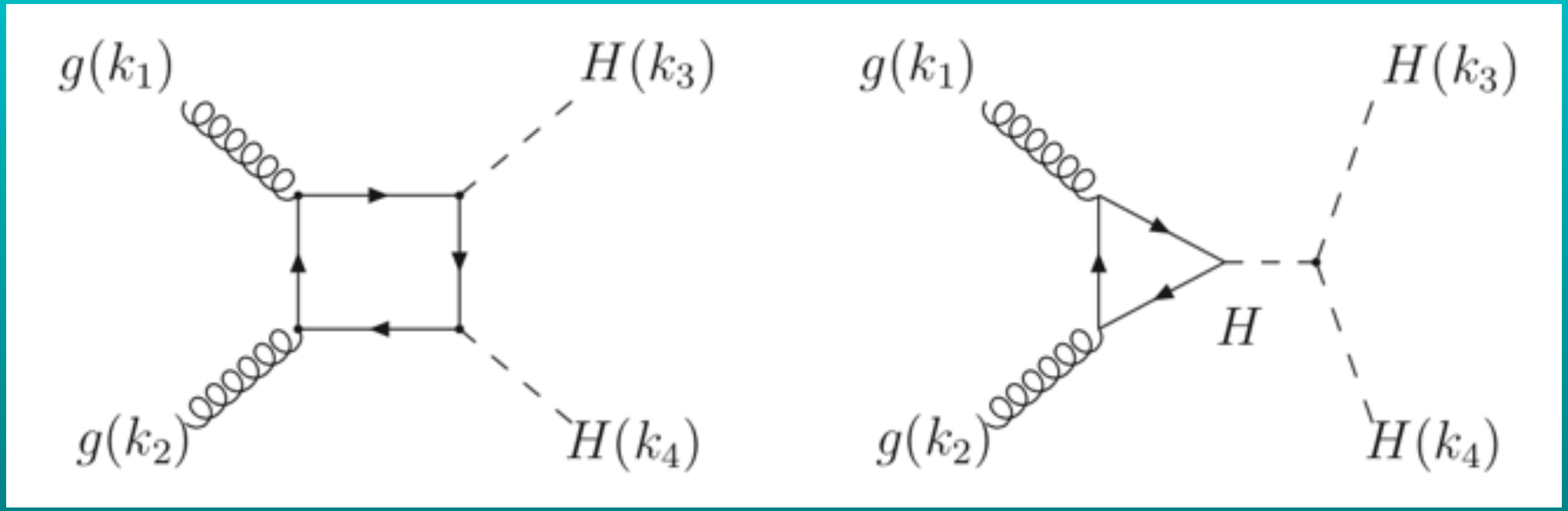
MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

application to loop integrals with several mass scales

example $gg \to HH$ **: 4 independent scales s12, s23, mH, mt**

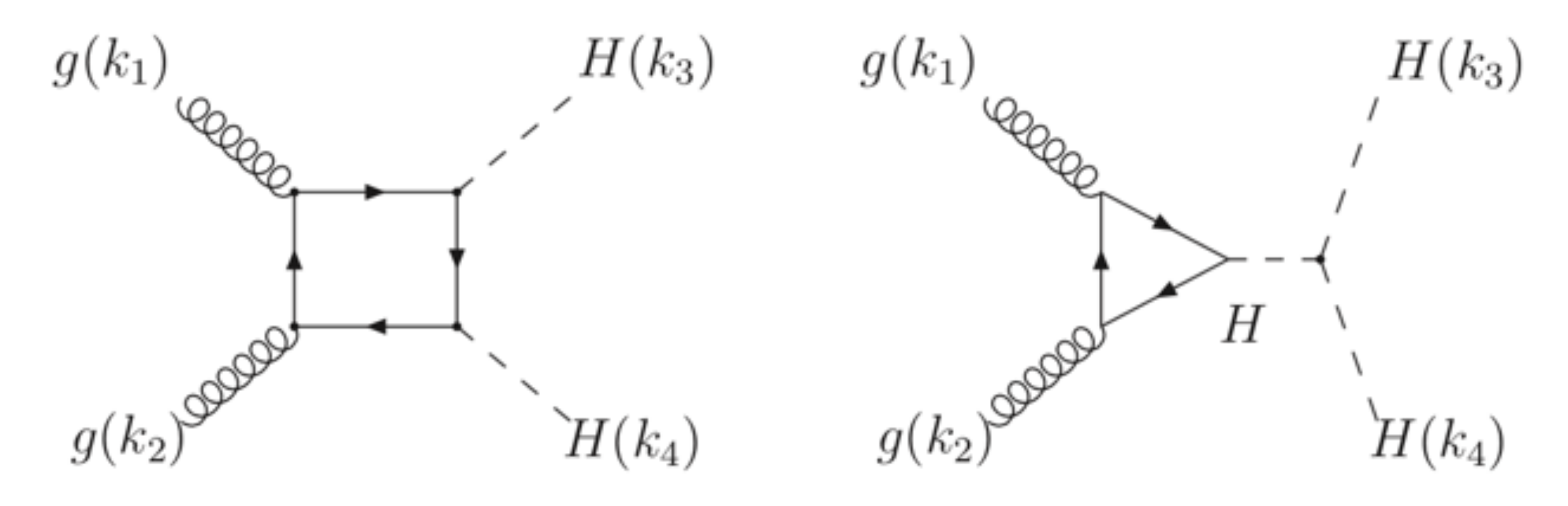
Leading Order already involves 1-loop diagrams



application to loop integrals with several mass scales

example $gg \to HH$: 4 independent scales s12, s23, mH, mt

Leading Order already involves 1-loop diagrams

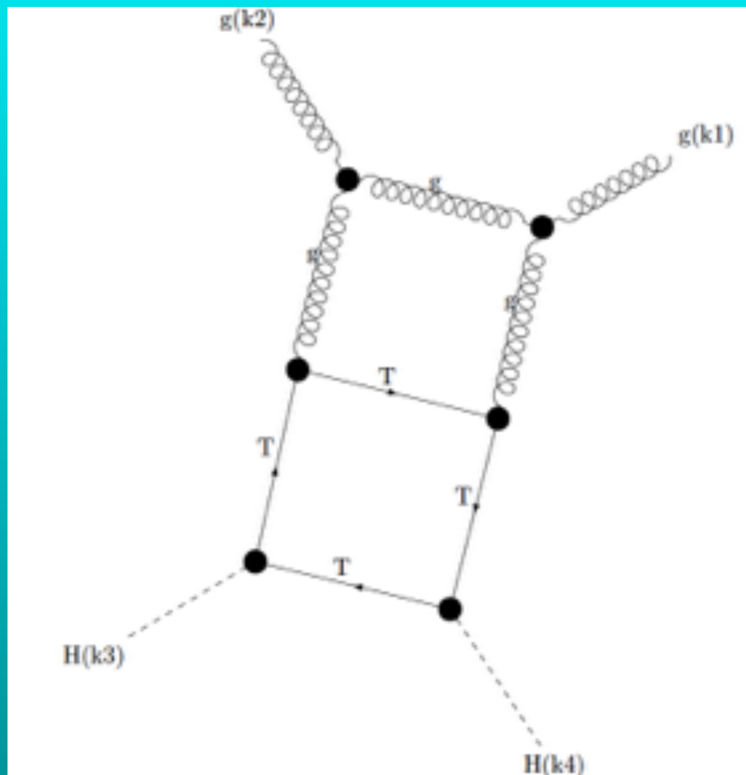


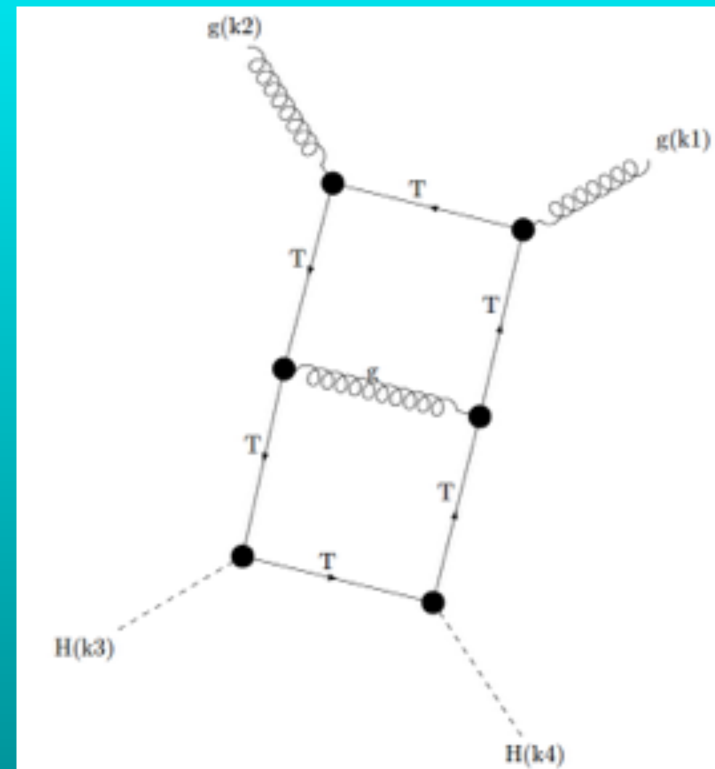
NLO (= 2 loops):

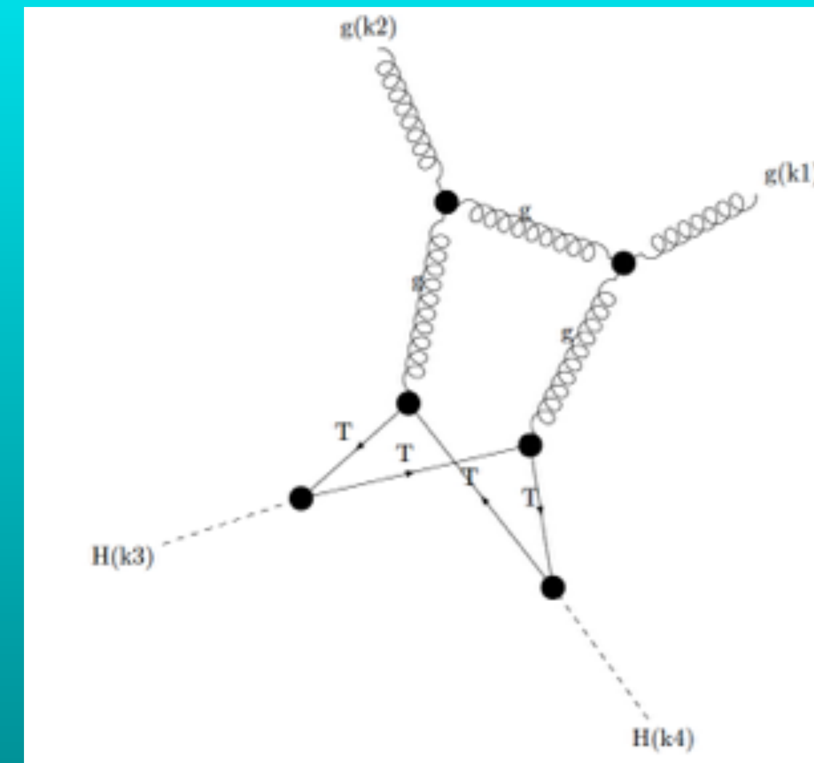
(most) 2-loop diagrams not known analytically

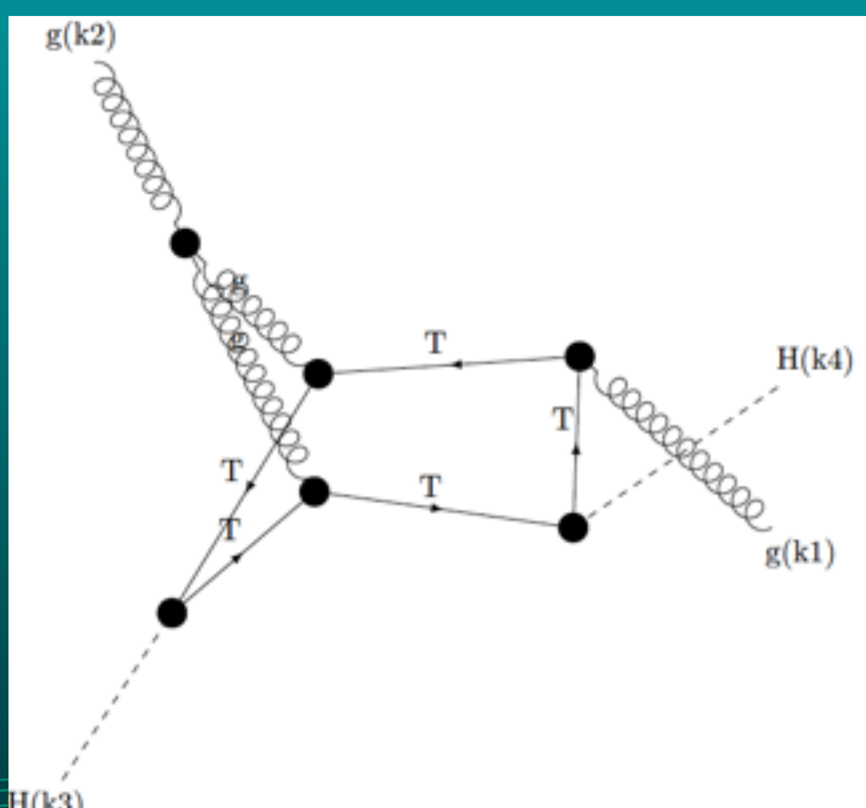
with full mass dependence

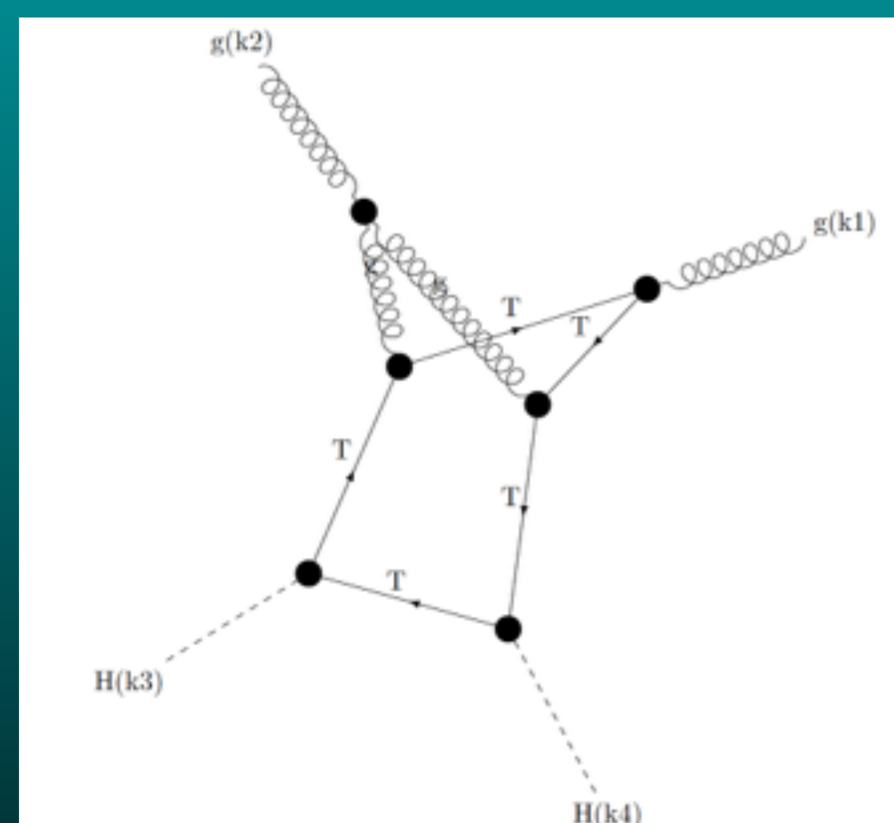
examples of 2-loop box diagrams







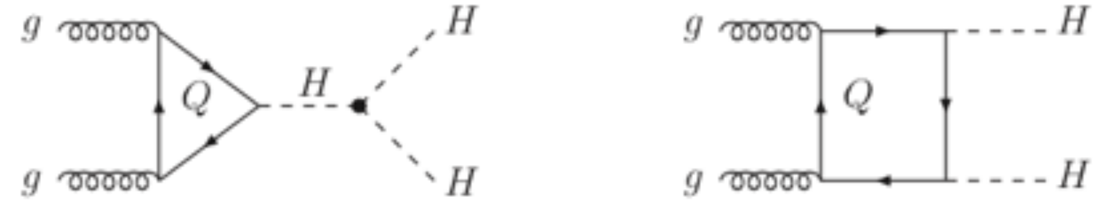




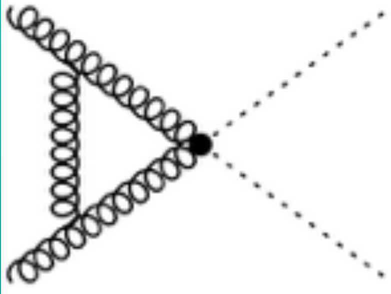
results in the literature so far

LO with full heavy quark mass dependence

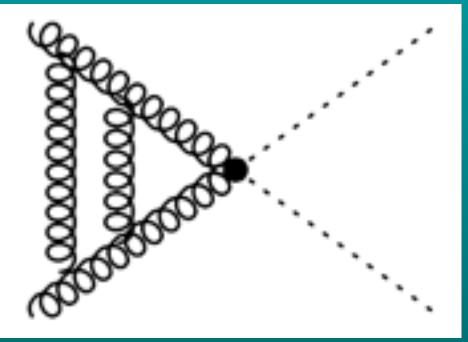
Glover, van der Bij '88, Plehn, Spira, Zerwas '96



NLO in $m_t \to \infty$ limit (EFT): Dawson, Dittmaier, Spira '98 (**HPAIR**)



- **supplemented with $1/m_t$ expansion:** $(\pm 10\%)$
  Grigo, Hoff, Melnikov, Steinhauser '13, '15
- **full mass dependence in NLO real radiation part and matching to parton shower**
  Frederix, Hirschi, Mattelaer, Maltoni, Torrielli, Vryonidou, Zaro '14;
  Maltoni, Vryonidou, Zaro '14



NNLO in $m_t \to \infty$ limit:

De Florian, Mazzitelli '13

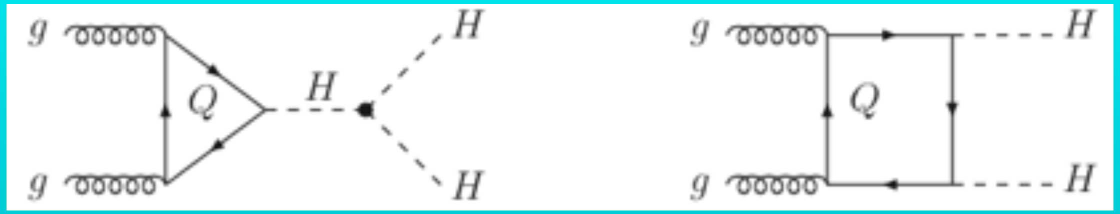
- **including all matching coefficients** Grigo, Melnikov, Steinhauser '14
- **supplemented with $1/m_t$ expansion:** Grigo, Hoff, Steinhauser '15
- **soft gluon resummation NNLL matched to NNLO** De Florian, Mazzitelli '15

+ lots of phenomenological studies

Baglio, Barr, Dolan, Englert, Ferreira de Lima, Goncalves-Netto, Greiner, Gröber, Krauss, Maierhöfer, Maltoni, Mühlleitner, Papaefstathiou, Spannowsky, Spira,Thompson, Vryonidou, Zaro, Zurita, ... '12,'13,'14,'15

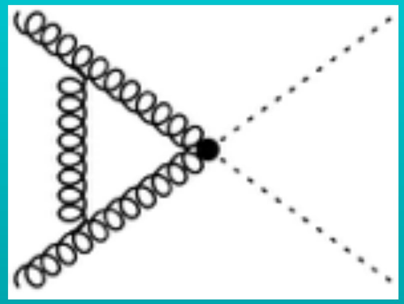
results in the literature so far

LO with full heavy quark mass dependence



Glover, van der Bij '88, Plehn, Spira, Zerwas '96

NLO in $m_t \to \infty$ limit (EFT): Dawson, Dittmaier, Spira '98 (**HPAIR**)

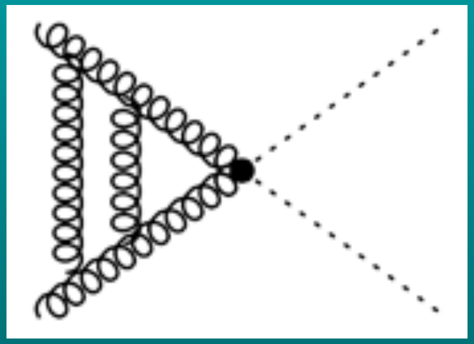


- **supplemented with $1/m_t$ expansion:** $(\pm 10\%)$

  Grigo, Hoff, Melnikov, Steinhauser '13, '15

- **full mass dependence in NLO real radiation part and matching to parton shower** **-10%**

  Frederix, Hirschi, Mattelaer, Maltoni, Torrielli, Vryonidou, Zaro '14;
  Maltoni, Vryonidou, Zaro '14



NNLO in $m_t \to \infty$ limit:

De Florian, Mazzitelli '13

- **including all matching coefficients** Grigo, Melnikov, Steinhauser '14
- **supplemented with $1/m_t$ expansion:** Grigo, Hoff, Steinhauser '15
- **soft gluon resummation NNLL matched to NNLO** De Florian, Mazzitelli '15

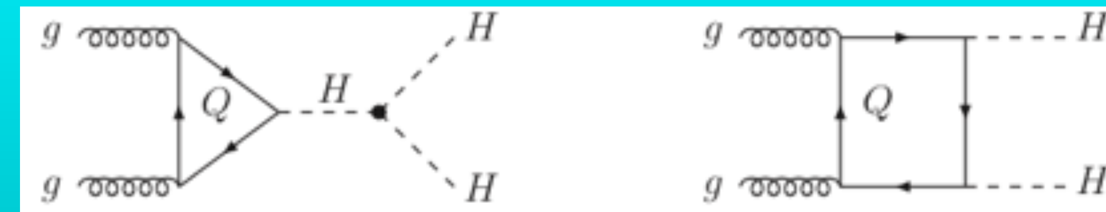
+ lots of phenomenological studies

Baglio, Barr, Dolan, Englert, Ferreira de Lima, Goncalves-Netto, Greiner, Gröber, Krauss, Maierhöfer, Maltoni, Mühlleitner, Papaefstathiou, Spannowsky, Spira,Thompson, Vryonidou, Zaro, Zurita, ... '12,'13,'14,'15

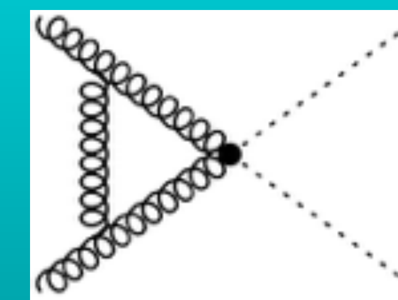
results in the literature so far

LO with full heavy quark mass dependence

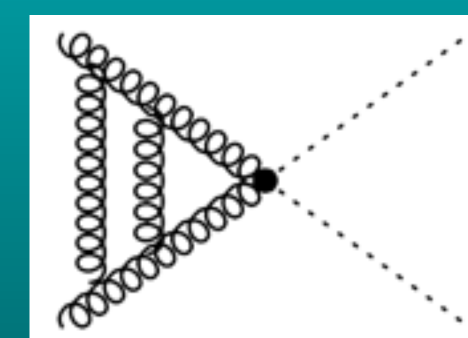
Glover, van der Bij '88, Plehn, Spira, Zerwas '96



NLO in $m_t \to \infty$ limit (EFT): Dawson, Dittmaier, Spira '98 (**HPAIR**)



- **supplemented with $1/m_t$ expansion:** $(\pm 10\%)$
  Grigo, Hoff, Melnikov, Steinhauser '13, '15
- **full mass dependence in NLO real radiation part and matching to parton shower** **-10%**
  Frederix, Hirschi, Mattelaer, Maltoni, Torrielli, Vryonidou, Zaro '14;
  Maltoni, Vryonidou, Zaro '14



NNLO in $m_t \to \infty$ limit: **+20%**

De Florian, Mazzitelli '13

- **including all matching coefficients** Grigo, Melnikov, Steinhauser '14
- **supplemented with $1/m_t$ expansion:** Grigo, Hoff, Steinhauser '15
- **soft gluon resummation NNLL matched to NNLO** De Florian, Mazzitelli '15

+ lots of phenomenological studies

Baglio, Barr, Dolan, Englert, Ferreira de Lima, Goncalves-Netto, Greiner, Gröber, Krauss, Maierhöfer, Maltoni, Mühlleitner, Papaefstathiou, Spannowsky, Spira,Thompson, Vryonidou, Zaro, Zurita, ... '12,'13,'14,'15

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)

calculation of the 2-loop amplitude

S.Borowka, N.Greiner, GH, S.Jones, M.Kerner, J.Schlenk, U.Schubert, T.Zirke

- use GoSam-2loop to generate the amplitude
- reduction with Reduze2 [C. Studerus, A. von Manteuffel]

  (Fire5 [A.V. Smirnov] , LiteRed [R.N. Lee])

  ➡ 8 integral families with 9 propagators each

  ➡ partly finite basis

- produce input files for SecDec with GoSam-2loop
- independent implementation with Qgraf, Reduze2, Mathematica [M. Kerner]
- evaluate integrals (SecDec) & coefficients

2-loop amplitude

integrals	1-loop	2-loop
direct	63	~10000
use symmetries	21	~1600
use IBP's	8	~300

# of sampling points determined by

- contribution to amplitude
- time per sampling point

target accuracy set at amplitude level

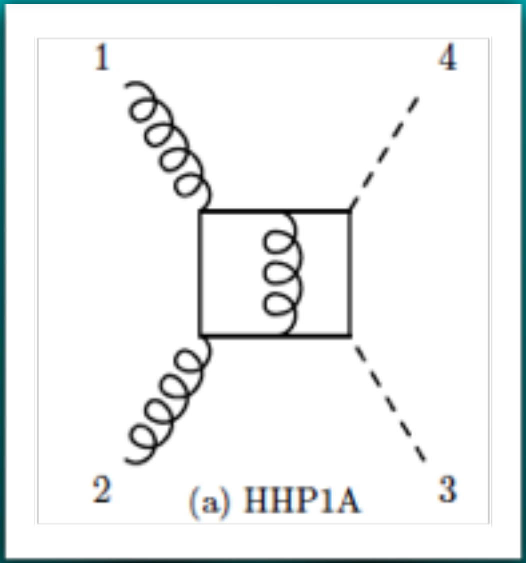
MAX-PLANCK-GESELLSCHAFT

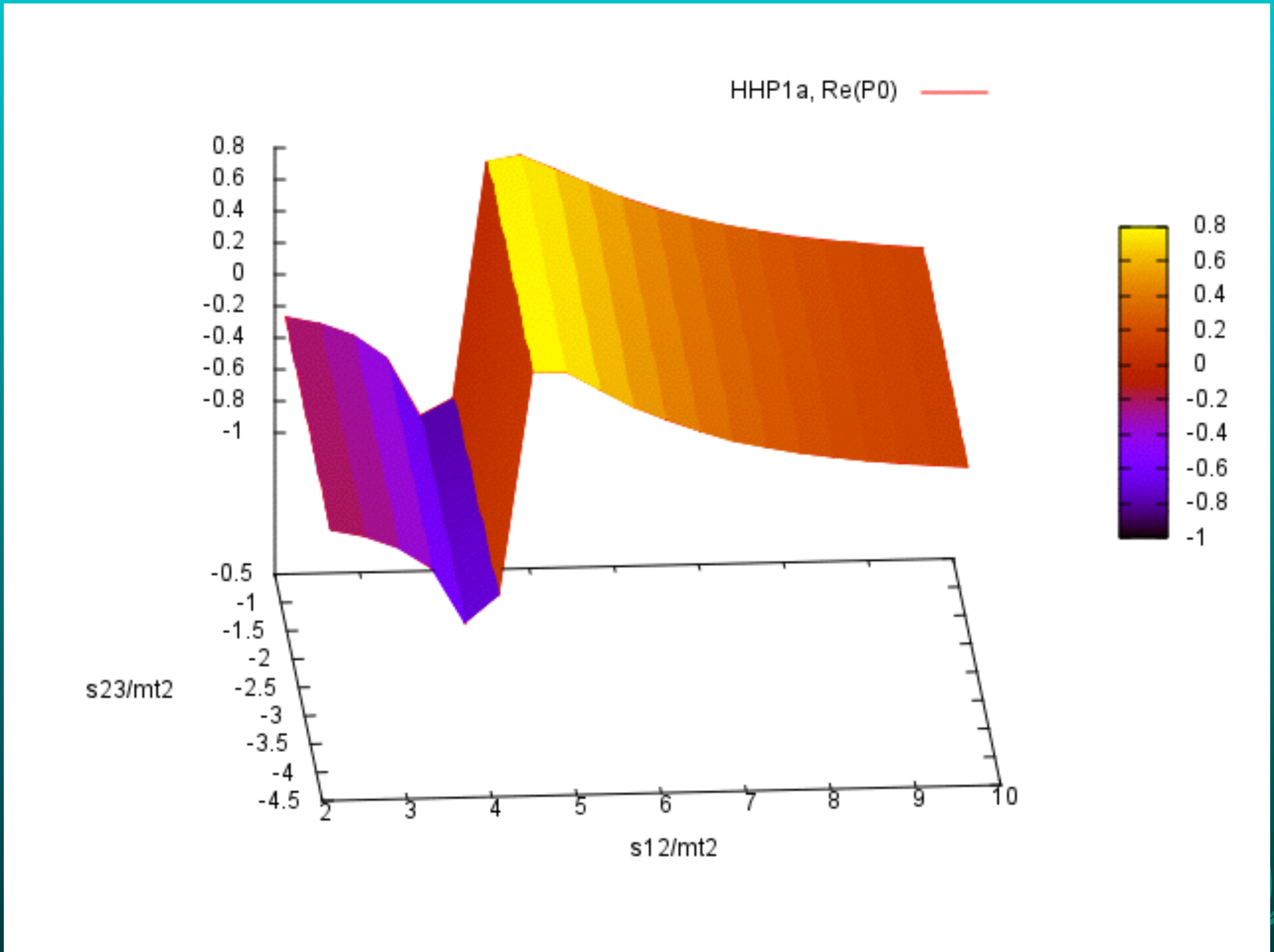
Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

examples of master integrals

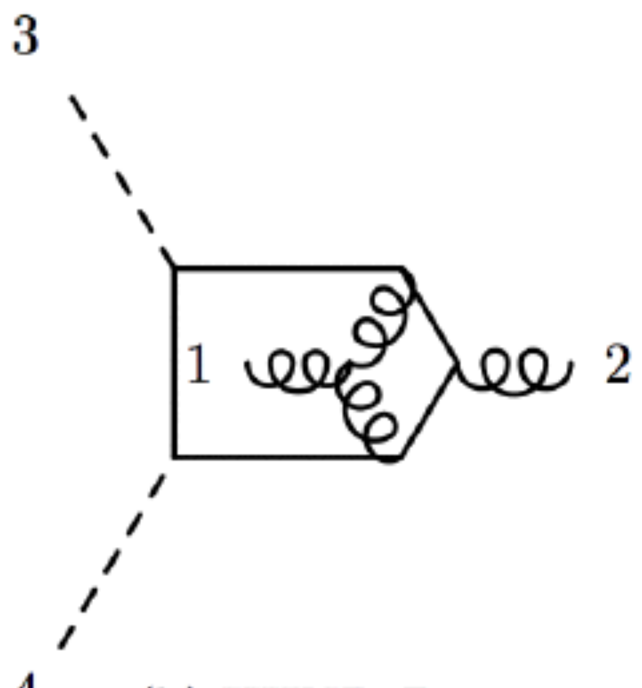
a planar 7 propagator integral:

$$m_H = 125\, GeV$$
$$m_t = 173\, GeV$$

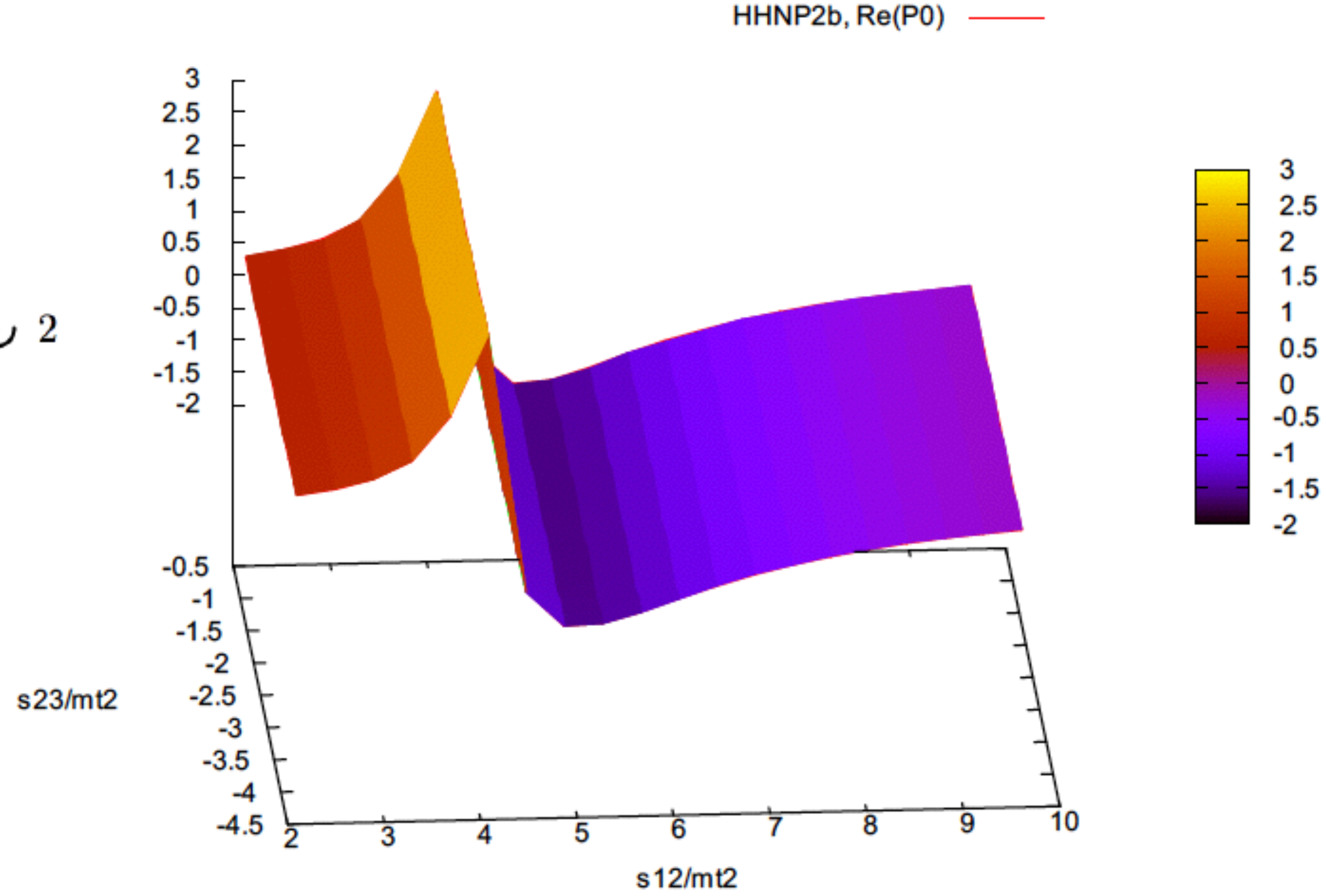




a non-planar 7 propagator integral:



(b) HHNP2B



$$I = \frac{P_{-1}}{\varepsilon} + P_0$$

$$m_H = 125\ \text{GeV}$$
$$m_t = 173\ \text{GeV}$$

Summary and Outlook

- LHC Run II is a precision game!
- NNLO techniques are advancing rapidly
  ⟶ automation feasible !
- tools towards this aim presented here:
  ➡ **GoSam-2loop** based on GoSam-1loop (public)
  ➡ **SecDec**
  ✓ can do integrals with several mass scales numerically
  ✓ is being made ready for large scale phenomenological applications

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

try out the tools!

http://gosam.hepforge.org

http://secdec.hepforge.org

tensor decomposition and form factors

Expose tensor structure: $\mathcal{M} = \epsilon^1_\mu \epsilon^2_\nu \mathcal{M}^{\mu\nu}$

Decomposition:

Form Factors (Contain integrals)

$$\mathcal{M}^{\mu\nu} \propto A_1(s,t,m_H^2,m_T^2,d)T_1^{\mu\nu} + A_2(s,t,m_H^2,m_T^2,d)T_2^{\mu\nu}$$

(Tensor) Basis

Choose: $\mathcal{M}^{++} = \mathcal{M}^{--} = -A_1$

$\mathcal{M}^{+-} = \mathcal{M}^{-+} = -A_2$

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_2^\mu p_1^\nu}{p_1 \cdot p_2} \qquad\qquad p_T^2 = \frac{ut - m_H^4}{s}$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{m_H^2 p_2^\mu p_1^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2p_1 \cdot p_3 p_2^\mu p_3^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2p_2 \cdot p_3 p_3^\mu p_1^\nu}{p_T^2 p_1 \cdot p_2} + \frac{2p_3^\mu p_3^\nu}{p_T^2}$$

Glover, van der Bij 88

thanks: Stephen Jones

projectors

Construct Projectors:

No Integrals

$$P_j^{\mu\nu} = \sum_{i=1}^{2} B_{ji}(s,t,m_H^2,d) T_i^{\mu\nu}$$

Same Basis as amplitude

Such that:

$$P_{1\mu\nu}\mathcal{M}^{\mu\nu} = A_1$$

$$P_{2\mu\nu}\mathcal{M}^{\mu\nu} = A_2$$

Explicitly; separately calculate the contraction of each projector with $\mathcal{M}^{\mu\nu}$

Recall:

$$\mathcal{M}^{++} = \mathcal{M}^{--} = -A_1$$

$$\mathcal{M}^{+-} = \mathcal{M}^{-+} = -A_2$$

- Self-coupling diagrams are 1PR by cutting a scalar propagator
- By angular momentum conservation they contribute only to A_1

thanks: Stephen Jones

SecDec can also do

- integrals with inverse propagators (numerators), e.g.

$$I^{-1,0}_{NP2B} = \frac{\int d^D p_1 \int d^D p_2 \quad (p_1+k_1)^2}{(p_2^2-m_t^2)((p_2+k_1+k_2)^2-m_t^2)((p_2+k_1+k_2+k_3)^2-m_t^2)(p_2-p_1)^2(p_2-p_1+k_1)^2(p_1^2-m_t^2)((p_1+k_2)^2-m_t^2)}$$

- integrals with contracted tensor numerators, e.g.

$$I^{t2}_{NP2B} = \frac{\int d^D p_1 \int d^D p_2 \quad (p_1\cdot k_1)\,(p_2\cdot k_3)}{(p_2^2-m_t^2)((p_2+k_1+k_2)^2-m_t^2)((p_2+k_1+k_2+k_3)^2-m_t^2)(p_2-p_1)^2(p_2-p_1+k_1)^2(p_1^2-m_t^2)((p_1+k_2)^2-m_t^2)}$$

⟶ no need for a scalar integral basis