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The Matrix Element Method at the LHC: Status and Prospects for Run 2

Sébastien Wertz

In collaboration with Alexandre Mertens and Sébastien Brochet

On behalf of the CP3 Matrix Element group

CP3 - Center for Cosmology, Particle Physics and Phenomenology
Université catholique de Louvain - Belgium

Outline

1. The Matrix Element Method
   - Introduction
   - Description of the method
2. Use of the MEM at the LHC
3. Implementation of the MEM
   - Introduction
   - The automated way: MadWeight
   - A new project: MoMEMta
   - Example applications
4. Conclusions
5. References & Backup

The Matrix Element Method

Introduction

- LHC experiments: recorded *huge* amounts of data
- Processes, signals, …, can be very hard to extract:

  Large backgrounds, limited discrimination…

  → **multivariate** techniques (MVA) → make the most of the data!
- Machine Learning (BDTs, ANNs) widely available & used
- Alternative technique: **Matrix Element Method (MEM)**
  - Pioneered at Tevatron (Top mass measurement) [1-3 and many others]
  - <u>No training</u> → good discrimination even if limited Monte Carlo statistics
  - Main difficulty: computationally intensive

The Matrix Element Method

Description of the method

Probability density („weight“) for event **x** given hypothesis α?

Possible uses:

Sample likelihood	Neyman-Pearson discriminant [4]
→ M.L. parameter fit	→ Hypothesis testing/search for rare process
$\prod_{i \in events} P(\mathbf{x}_i \vert \alpha)$	$P(\mathbf{x} \vert S) / \sum_i r_i P(\mathbf{x} \vert B_i)$

… Can be computed!

$$P(\mathbf{x}|\alpha) = \frac{1}{A_\alpha \sigma_\alpha} \int \mathrm{d}\Phi(y) \frac{\mathrm{d}x_1 \mathrm{d}x_2}{x_1 x_2 s} f(x_1) f(x_2) |\mathcal{M}_\alpha(y, x_1, x_2)|^2 W(\mathbf{x}|y) \epsilon_\alpha(y)$$

Theoretical hypothesis (Matrix Element) + Parton shower + Detector (transfer functions, efficiencies) + Experimental information (whole event **x**)

Outline

Use of the MEM at the LHC

- **CMS & ATLAS**: several searches for **ttH process** [5-9]
- MEM: h→bb, tt→semi-/dileptonic [6,9]

Dedicated implementations

- **ATLAS**: search for **s-channel single Top** [11]
- Most sensitive analysis at LHC so far [12,13]

More details in Backup!

MadWeight [17]

- **CMS**: measurement of **spin correlations in tt** production (μ+jets) [10]
- Hypothesis test (cor./uncor.)
- Template fit of ME discriminant →fraction of correlated events

No integration (ME only)

- **ATLAS & CMS**: **h→ZZ*→4l** [14,15,16]
- Search for the SM Higgs boson
- Spin-parity measurements

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Implementation of the MEM

Introduction

Many implementations…
Mostly process-dedicated

↔

General code: MadWeight (next slide)…

Phase-space parametrisation
(integrate out delta from momentum conservation)

Matrix element evaluation
(exported from MadGraph/MCFM/…)

Numerical integration
(VEGAS in ROOT or Cuba)

PDFs (LHAPDF)

Transfer functions

Input/Output

…

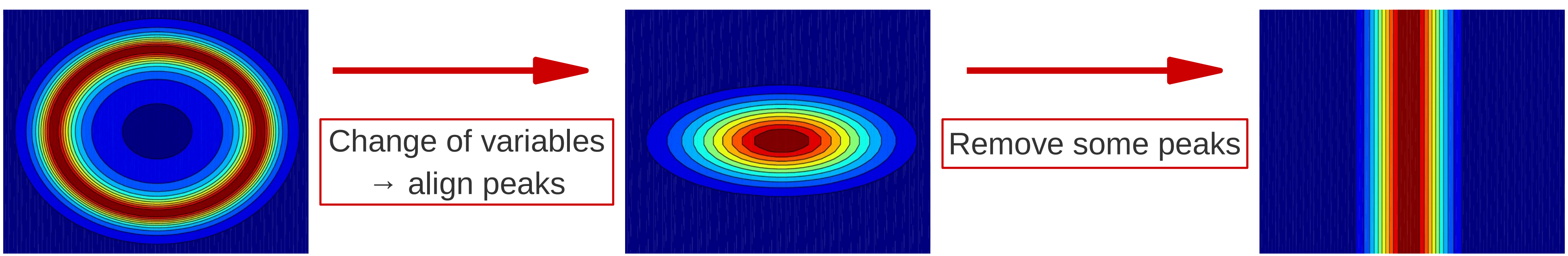
- **Speed:** multi-dimensional (typically 4-8D) integrals of **highly peaked functions = difficult!**
  - Propagators (ME), transfer functions
  - Adaptive MC algorithms fail if peak structure too complicated

Implementation of the MEM

The automated way: MadWeight [17]

- ➢ Part of MadGraph5_aMC@NLO [18]
- ✔ Can **automatically** compute MEM weights for (almost) any user-specified process
- ✔ Uses smart phase-space mappings to **align peaks** of integrand with coordinate axes
  - Integration using VEGAS [19]
  - Some peaks (Breit-Wigners) can be removed

Much improved convergence speed!



… But not ideally suited for real-life analysis:

- ✗ Not in development anymore; hard to maintain – written in Fortran*, rigid structure
- ✗ I/O unadapted (text files)

* Not common knowledge in experimental HEP

Implementation of the MEM

A new project: MoMEMta

- Full automation not desirable for dedicated analysis (adapt I/O, use approximations, …)
- MadWeight: changes of variable highly efficient → should be used!

→ New project: **Mo**dular **M**atrix **E**lement **M**ethod implemen**ta**tion

Goal:

- **Modular** framework written in **C++**
- Changes of variables, transfer functions, interface to matrix elements, I/O, …: modules
- Provide default modules + easily add user-tailored modules

Status:

- Non-modular prototype validated ↔ MadWeight (next slides)
- Based on: **Cuba** (integration) [20], ROOT, **LHAPDF** [21]
- Modular skeleton in place & in active development – available on github
- New MadGraph C++ matrix element exporter → faster, easier use: under review

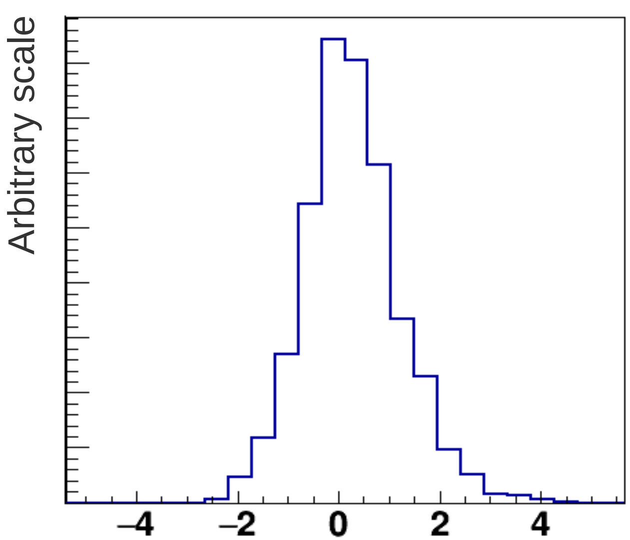
Implementation of the MEM

A new project: MoMEMta

Non-modular prototype → Modular skeleton

Non-modular prototype

- Dileptonic tt process
- MadWeight change of variables
- Matrix element from MadGraph
- Cross-check w.r.t. MadWeight:



$$(P_{\text{C++}} - P_{\text{MW}})/\sqrt{\sigma^2_{P_{\text{C++}}} + \sigma^2_{P_{\text{MW}}}}$$

Modular skeleton

- Reads Lua scripting file
- Modular structure resolved when loading script: modules' inputs & outputs linked in memory pool → negligible overhead during computation!
- Modules: dynamically loaded libraries

Module type (class deriving from „Module“ abstract class)

Module name

```
GaussianTransferFunction.tf_p1 = {
    ps_point = 'cuba::ps_points/0',
    reco_particle = 'input::particles/0',
    sigma = 0.05,
}
```

Define parameters

```
BlockD.blockd = {
    inputs = {
        'tf_p1::output',
    },
    s13 = 'flatter_s13::s',
}
```

Define input to module: „given“ or =output of other modules

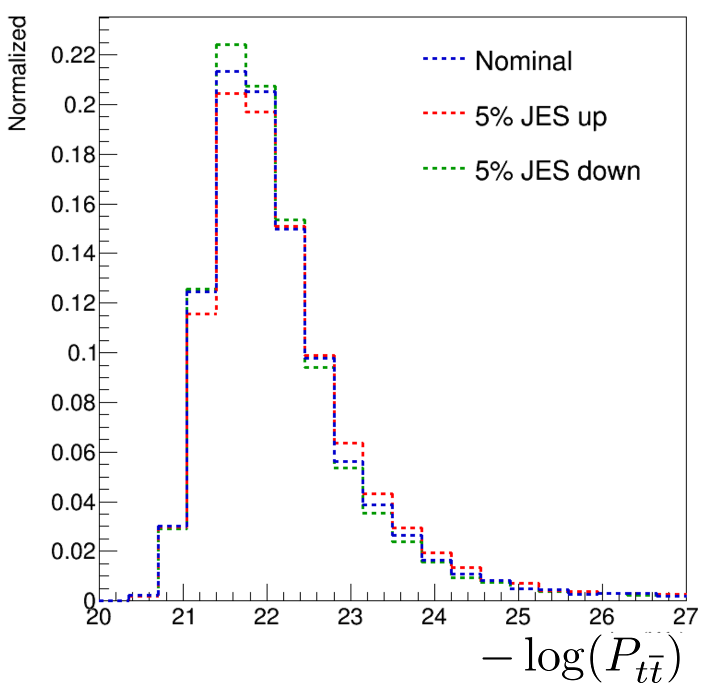
Implementation of the MEM

Example application: Multiple weights & systematics

- Many systematic uncertainties: Monte Carlo event reweighting
  → Propagate to final distributions *without modifying event content*
  → Matrix Element weights: no re-computation
- **Jet Energy Scale & Resolution** uncertainties: modify events' jet energies/content
  → Recompute ME for each variation: heavy!
- Most events: same object content → only slight variation of energies
  - Compute **all variations simultaneously** to nominal
  - *Vector* integrand (Cuba): VEGAS grid ~ adapted to all components (small variations in integrand)
  - Transfer function: different E_{rec}, but:
    ME evaluation (slow!) common → gain!

$$\int \mathrm{d}E_{gen}\, |\mathcal{M}(E_{gen})|^2 \times \begin{bmatrix} W(E^+_{rec}|E_{gen}) \\ W(E_{rec}|E_{gen}) \\ W(E^-_{rec}|E_{gen}) \end{bmatrix} \times \dots$$

- Variations of theoretical parameters in ME



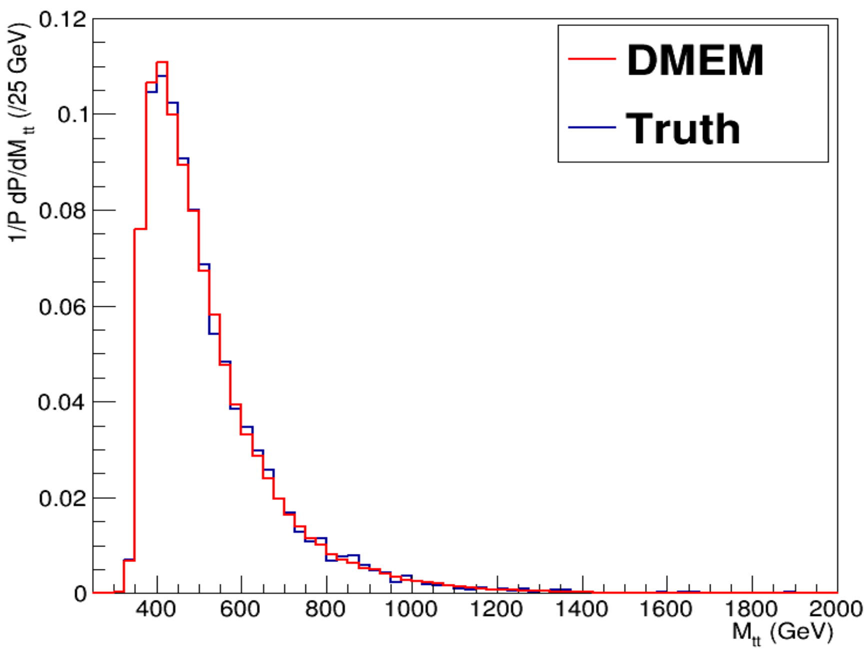
Implementation of the MEM

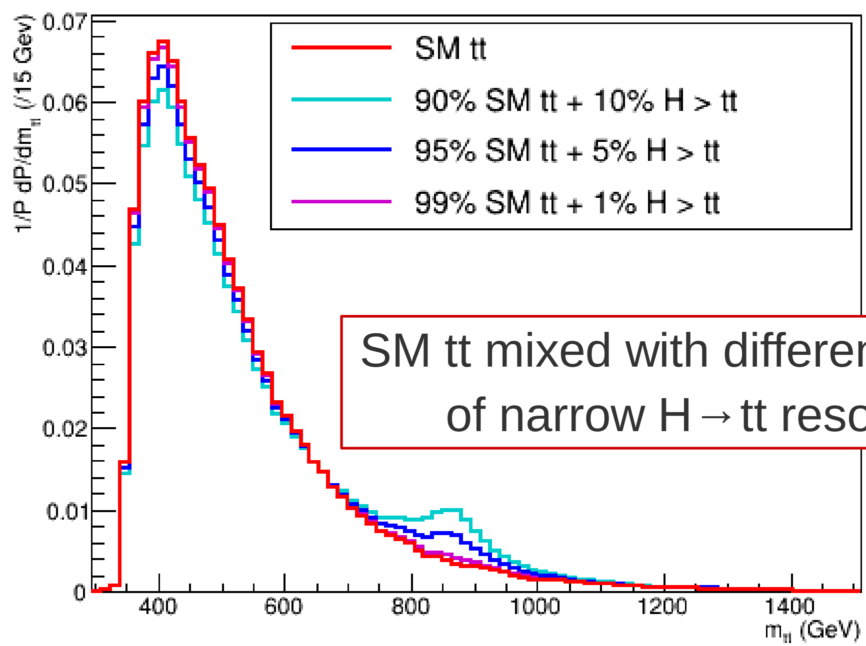
Example application: differential MEM [22]

- Per-event likelihood → per-event *differential* likelihood?

$$\frac{1}{P}\frac{\partial P(\mathbf{x})}{\partial Z}\bigg|_{Z_0} = \int \mathrm{d}\Phi(\mathbf{x})\,\frac{\mathrm{d}x_1 \mathrm{d}x_2}{x_1 x_2 s}\, f(x_1) f(x_2)\,|\mathcal{M}(\mathbf{y})|^2\, W(\mathbf{x}|\mathbf{y})\,\delta(Z(\mathbf{y}) - Z_0)$$

- Each event → histogram → stack / get max. likelihood values / …
- MoMEMta: write a module, define how to fill histogram in config script
- Non-modular code: test on dileptonic tt sample → reconstruct m_{tt} (! neutrino ambiguities):



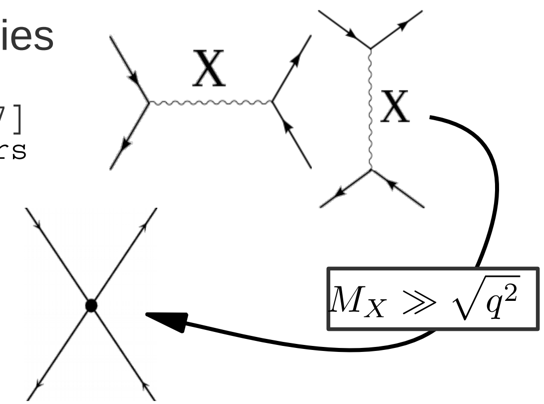


SM tt mixed with different fractions of narrow H → tt resonance

Implementation of the MEM

Work in progress!

Example application: search for higher-dimensional operators in tt production



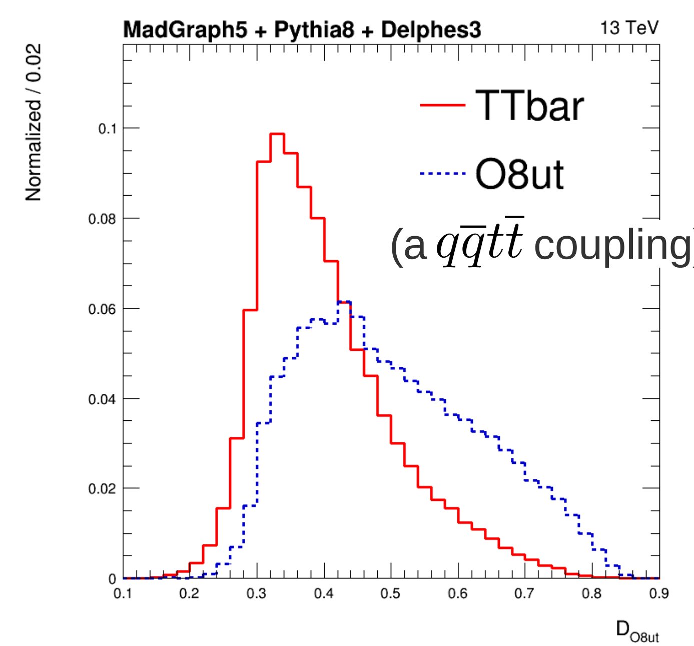
- New Physics too heavy → *indirect effects on SM observables* at lower energies
- Parametrise effects in *model-independent way*: **effective operators** [23-27] and many others
- This work: tt production (Top ↔ BSM?), dileptonic channel
  - Dominant effect: *interference* of Dim6 operators with SM amplitude
  - Goal: global fit of operators' couplings
- Problem: some operators hardly distinguishable from the SM & from each other
- Use **MEM**: compute weights, two hypotheses:

  SM tt (P_{tt}) + interference SM ↔ operator i (P_i)

  Not well-defined likelihood, but discriminates operator's effects:

$$D_i = (\arctan(\log(|\tilde{P}_i|/P_{t\bar{t}})) + \pi/2)/\pi$$

- Set of operators implemented in MG5_aMC [28,29]
- Events generated with MG5_aMC, Pythia8 [30,31], Delphes [32]
- Generate each interference SM ↔ 1 operator
  → „signals“ linear in operators' couplings



Outline

Conclusions

- **Matrix Element Method**: powerful physics-driven multivariate technique

  → measure SM parameters/observables, search for rare processes, …

- Successfully used at Tevatron & LHC

- Many analysis-dedicated implementations of MEM computation

- MadWeight: fully automated phase-space generator + efficient integration

  … but: hard to maintain, not adapted to large scale analyses

Conclusions

- **MoMEMta** → goal: modular, easy to extend, as efficient as MadWeight
  - Prototype validated for dileptonic tt topology
  - Modular framework under development
- Applications: differential MEM, systematics, constraining effective operators, …

Stay tuned: follow updates on GitHub!

References

[1] K. Kondo, *J. Phys. Soc. Jap.* **57** (1988) 4126

[2] K. Kondo, *J. Phys. Soc. Jap.* **60** (1991) 836

[3] D0 Collaboration, *Nature* **429** (2004) 638

[4] J. Neyman and E. Pearson, *Phil. Trans. R. Soc. Lond. A* **231** (1933) 289

[5] CMS Collaboration, *JHEP* **09** (2014) 087. hep-ex/1408.1682

[6] CMS Collaboration, *Eur. Phys. J. C* **75** (2015) 251. hep-ex/1502.02485

[7] ATLAS Collaboration, *Phys. Lett. B* **749** (2015) 519. hep-ex/1506.05988

[8] ATLAS Collaboration, *Phys. Lett. B* **740** (2015) 222. hep-ex/1409.3122

[9] ATLAS Collaboration, *Eur. Phys. J. C* **75** (2015) 349. hep-ex/1503.05066

[10] CMS Collaboration, CMS-TOP-13-015, submitted to Phys. Lett. B. hep-ex/1511.06170

[11] ATLAS Collaboration, CERN-PH-EP-2015-286, submitted to Phys. Lett. B. hep-ex/1511.05980

[12] ATLAS Collaboration, *Phys. Lett. B* **740** (2015) 118. hep-ex/1410.0647

[13] CMS Collaboration, CMS-PAS-TOP-13-009 (2013)

[14] ATLAS Collaboration, *Phys. Rev. D* **91** (2015) 012006. hep-ex/1408.5191

[15] CMS Collaboration, *Phys. Lett. B* **716** (2012) 30. hep-ex/1207.7235

[16] CMS Collaboration, *Phys. Rev. D* **89** (2014) 092007. hep-ex/1312.5353

[17] P. Artoisenet, V. Lemaître, F. Maltoni and O. Mattelaer, *JHEP* **1012** (2010) 068.hep-ph/1007.3300

[18] J. Alwall *et al.*, *JHEP* **1407** (2014) 079. hep-ph/1405.0301

References

[19] G. P. Lepage, *J. Comput. Phys.* **27** (1978) 192

[20] T. Hahn, *Comput. Phys. Commun.* **168** (2005) 78. hep-ph/0404043

[21] A. Buckley *et al.*, Report GLAS-PPE/2014-05, MCnet-14-29, IPPP/14/111, DCPT/14/222. hep-ph/1412.7420

[22] A. Pin and O. Mattelaer, Il Nuovo Cimento C, Top 2011 Conf. Proc. DOI: 10.1393/ncc/i2012-11254-1

[23] S. Weinberg, *Phys. Rev. Lett.* **43** (1979) 1566

[24] C. N. Leung, S. T. Love and S. Rao, *Z. Phys. C* **31** (1986) 433

[25] W. Buchmuller and D. Wyler, *Nucl. Phys. B* **268** (1986) 621

[26] J. A. Aguilar-Saavedra, *Nucl. Phys. B* **812** (2009) 181. hep-ph/0811.3842

[27] B. Grzadkowski *et al.*, *JHEP* ***1010*** (2010) 085. hep-ph/1008.4884

[28] C. Degrande *et al.*, *JHEP* **1103** (2011) 125. hep-ph/1010.6304

[29] C. Zhang and S. Willenbrock, *Phys. Rev. D* **83** (2011) 034006. hep-ph/1008.3869

[30] T. Sjöstrand, S. Mrenna and P. Skands, *JHEP* **0605** (2006) 026. hep-ph/0603175

[31] T. Sjöstrand, S. Mrenna and P. Skands, *Comput. Phys. Comm.* **178** (2008) 852. hep-ph/0710.3820

[32] J. de Favereau *et al.*, *JHEP* **1402** (2014) 057. hep-ex/1307.6346

[33] I. Volobouev (2011), data-an/1101.2259

[34] J. Alwall, A. Freitas and O. Mattelaer, *Phys. Rev. D* **83** (2011) 074010. hep-ph/1010.2263

Backup

The Matrix Element Method

Transfer function, likelihood normalisation

Transfer function: proba. to observe event **x** given partonic config. y

Encodes parton shower, hadronisation, detector & reconstruction effects

- In practice, one assumes $W(\mathbf{x}|y) = \prod_i W_i(E_i^{rec}|E_i^{gen})\,\delta(\theta_i^{rec} - \theta_i^{gen})\,\delta(\phi_i^{rec} - \phi_i^{gen})$
- $W(E^{rec}|E^{gen})$ can be
  - Fitted: single/double Gaussian, $\sigma, \mu \sim A + B \cdot \sqrt{E_{gen}} + C \cdot E_{gen}$
  - Binned: 2D histogram of E_{rec} vs. E_{gen}

Normalisation:

If $\int \mathrm{d}\mathbf{x}\, W(\mathbf{x}|y) = 1$ (integrating over *selected* phase-space),

and $A_\alpha = \langle \epsilon(y, \alpha) \rangle$, i.e. = #selected / #generated events for hypothesis α,

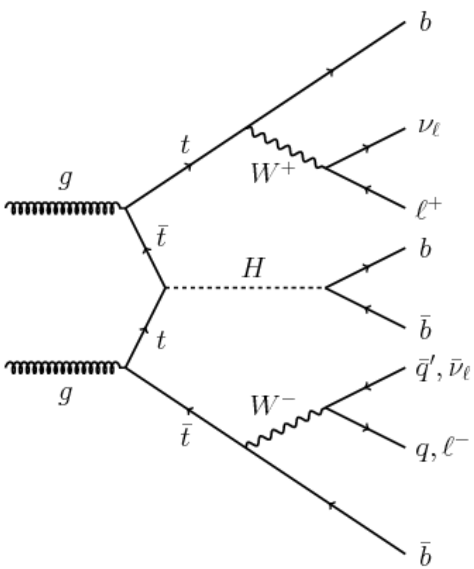
then $P(\mathbf{x}|\alpha)$ is correctly normalised as a likelihood [33]

- Normalisation & efficiencies crucial for M.L. fits
- S/B discrimination: worst case = small discrimination loss

Use of the MEM at the LHC

Searches for tth in ATLAS & CMS

- tth process: allows for *direct* measurement of Top-Higgs Yukawa coupling (~1 in SM…?)
- Small cross-section (~130fb @ 8 TeV), huge backgrounds & combinatorics
- RunI (25fb^{-1}): upper limits on $\mu = \sigma/\sigma_{SM}$; RunII ($\sigma_{SM} \rightarrow$ 500fb, 100fb^{-1}): expect discovery



CMS:

1. h→bb, h→γγ, h→ττ/WW/ZZ→leptons
   - BDTs used for h→bb & h→leptons
   - Final exp./obs. limit: μ<1.7/4.5 (3.5/4.1 for h→bb alone) [6]
2. h→bb; tt→1/2 leptons (8 TeV only)
   - Discrimination using **MEM**
   - Final exp./obs. limit; μ<2.9/3.3 [7]

ATLAS:

1. h→ττ/WW/ZZ→leptons (8 TeV only)
   - Final exp./obs. limit: μ<2.4/4.7 [8]
2. h→γγ; tt→0,1,2 leptons
   - Final exp./obs. limit: μ<4.9/6.7 [9]
3. h→bb; tt→1/2 leptons (8 TeV only)
   - Discrimination using **MEM**+ANN
   - Final exp./obs.limit: μ<2.2/3.4 [10]

ATLAS & CMS matrix element analyses used dedicated implementations for the integration

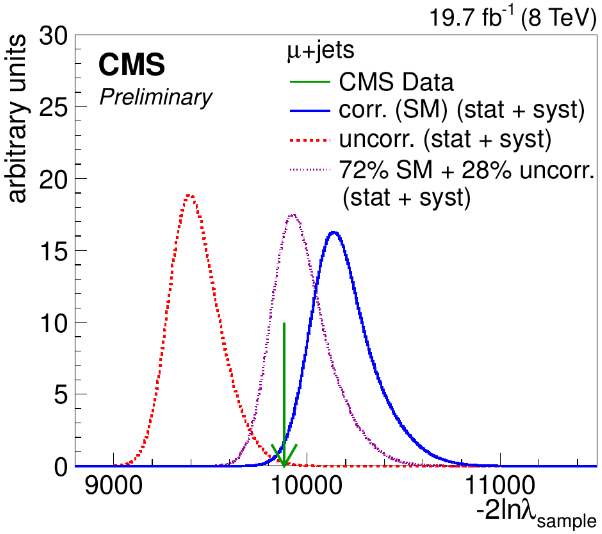
Use of the MEM at the LHC

Measurement of tt spin correlations in CMS [11]

- Top/anti-top pairs are produced spin-correlated in the SM
- Tops decay before they hadronize/decorrelate → correlation imprinted on decay products
- Measurement of the correlation: test of the SM, search for BSM deviations
- Many decay products, missing energy → expect improvement from **MEM**

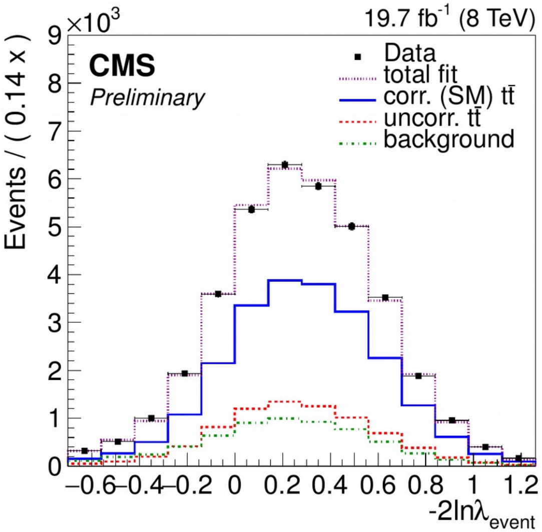
CMS analysis in the μ+jets channel: $\lambda_{event} = \frac{P(uncor)}{P(cor)}$

Hypothesis testing:



$$-2\ln(\lambda_{sample}) = -\sum 2\ln(\lambda_{event})$$

Template fit:



Results:

- Calibrated fit: fraction of spin-correlated events = 0.72 ± 0.09 (stat) $^{+0.15}_{-0.13}$ (syst) (most precise to date in this channel)
- Agreement under spin-correlated hypothesis: 2.2 σ
- Agreement under spin-uncorrelated hypothesis: 2.9 σ

MEM Integration using MadWeight

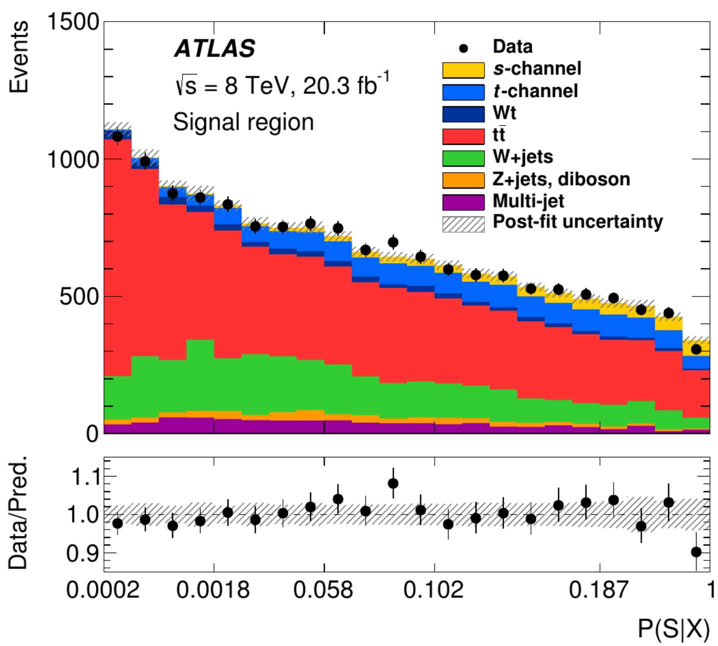
Use of the MEM at the LHC

Evidence for s-channel single Top quark production in ATLAS [12]

- S-channel: rarest Top quark production mode (~5pb @LHC8… compare with tt: ~250pb)
- First evidence at Tevatron (D0+CDF) using combination of MEM and ML techniques
- At LHC → **BDTs** →exp./obs. significance: ATLAS 1.4/1.3 σ [13], CMS 0.9/0.7 σ [14]
- ATLAS: re-analysis of 8 TeV dataset → updated calibrations, **MEM** instead of BDT

$$P(S|X) = \frac{\sum_i \alpha_{S_i} \mathcal{P}(X|S_i)}{\sum_i \alpha_{S_i} \mathcal{P}(X|S_i) + \sum_j \alpha_{B_j} \mathcal{P}(X|B_j)}$$

- α's: relative expected yields
- Signal hyp.: s-channel (2 & 3 parton final states)
- Bkg. hyp.: t-channel, semi- & di-leptonic tt, W+jj, W+cj, W+bb

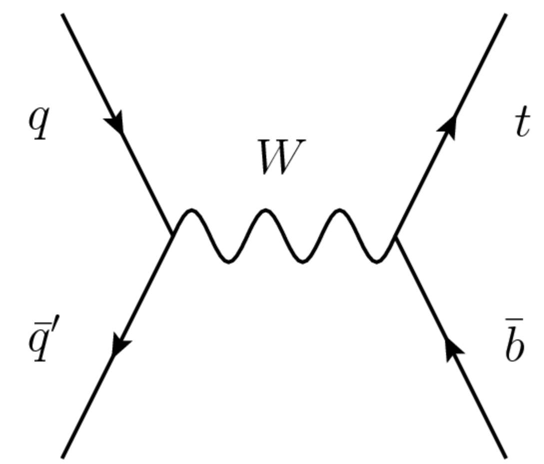


Results:

Exp./obs. significance of 3.9/3.3 σ

σ_s = 4.8 ± 1.1 (stat) $^{+2.2}_{-2.0}$ (syst) pb

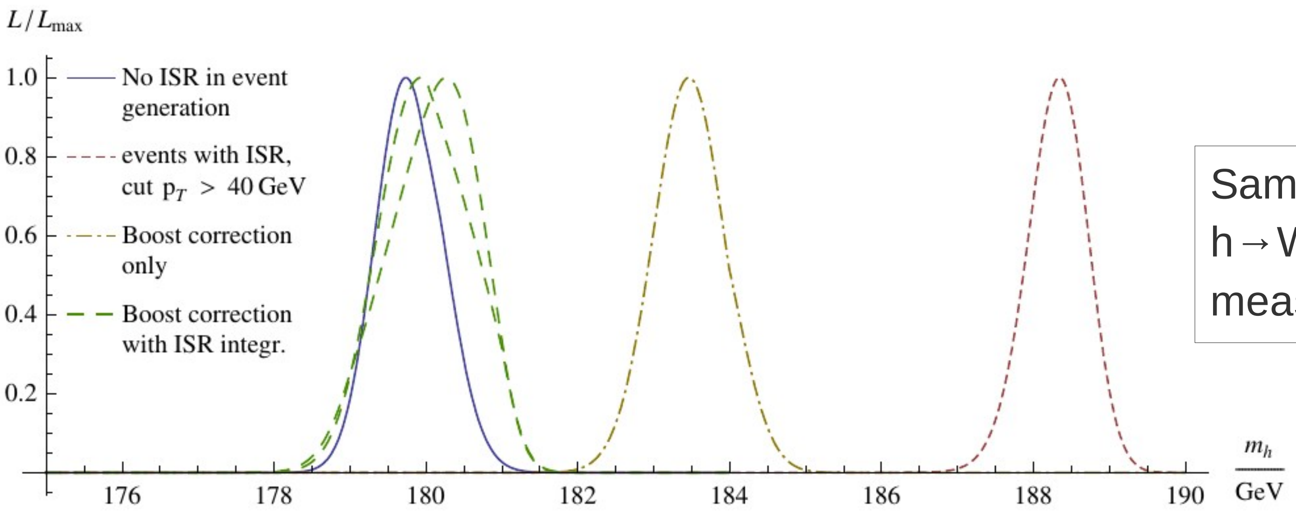
Used dedicated framework for MEM computation



„approximately half of the improvement can be attributed to the change in method from BDT to ME“

MEM & Initial State Radiation

- Hadron collider → extra jets from initial state radiation (ISR)
- Use of LO matrix elements (computing time prohibitive otherwise):
  - ✗ No description of extra radiation
  - ✗ Total transverse momentum imbalance
- Possible to treat ISR in an effective way: [34]
  - **Transverse boost** → system back into zero momentum frame
  - Integrate over p_T of initial partons, use dedicated **transfer functions**
- FSR less problematic: less abundant, somewhat modeled by transfer functions



Sample likelihood for $h \to WW \to 2l2\nu$ mass measurement (m_h=180GeV)