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Phase advance matching for chromaticity correction for FCC-ee

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What do we do?

Systematic investigation of chromaticity correction schemes for FCC-ee:

1. Interleaved sextupole scheme using Montague Formalism → LEP
2. Non-interleaved sextupole scheme
3. Independent sextupole pairs

4. Combination of local CCS and arc CC

Chromaticity

- Change of the tune with energy deviation

- Textbook: $\Delta Q = \xi \cdot \Delta p / p$

- In our case not precise enough: $\left(\delta = \Delta p / p\right)$

$$Q(\delta) = Q_0 + \frac{\partial Q}{\partial \delta}\delta + \frac{1}{2}\frac{\partial^2 Q}{\partial \delta^2}\delta^2 + \frac{1}{6}\frac{\partial^3 Q}{\partial \delta^3}\delta^3 + \ldots$$

Montague functions

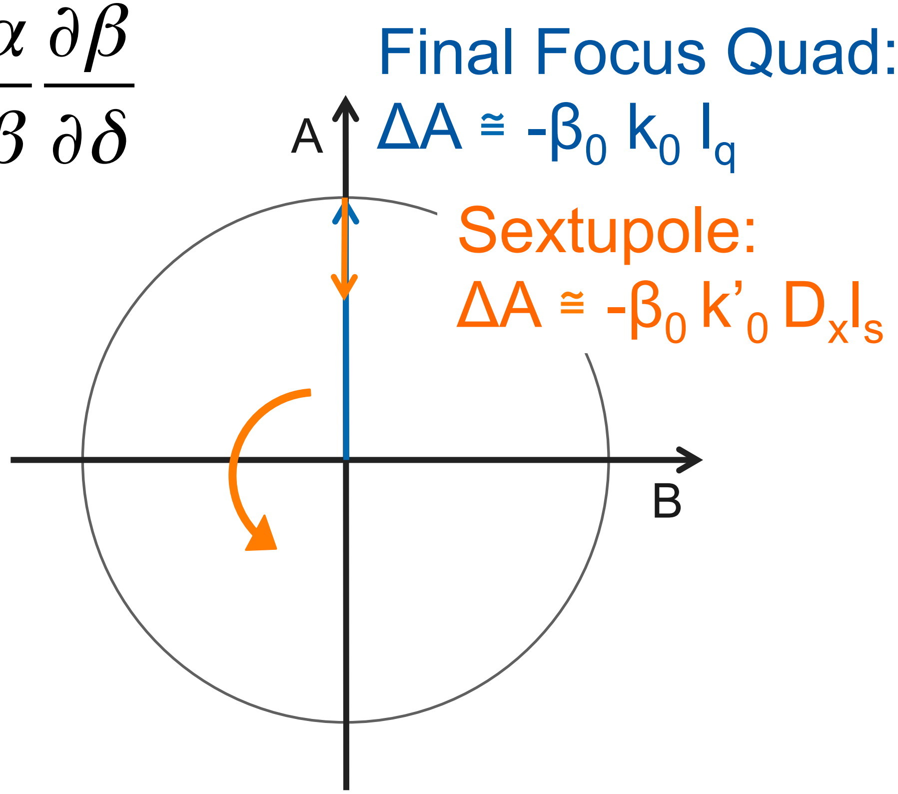
- Chromatic variables

$$B = \frac{1}{\beta}\frac{\partial \beta}{\partial \delta} \qquad A = \frac{\partial \alpha}{\partial \delta} - \frac{\alpha}{\beta}\frac{\partial \beta}{\partial \delta}$$

- W-vector

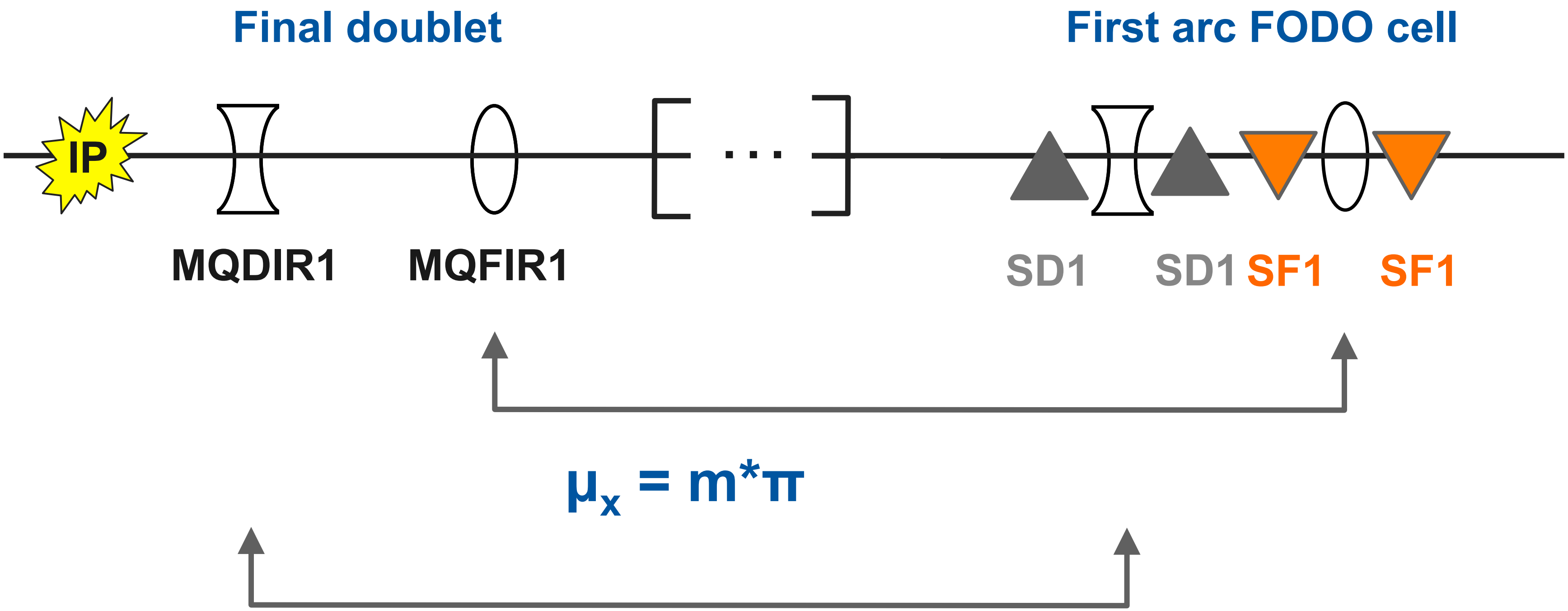
$$\vec{W} = \frac{1}{2}\left(B + iA\right)$$

$$= \frac{1}{2}\sqrt{A^2 + B^2}\, e^{i2\mu}$$



Rotates with twice the phase advance!

Phase advance FD – 1st Sext.

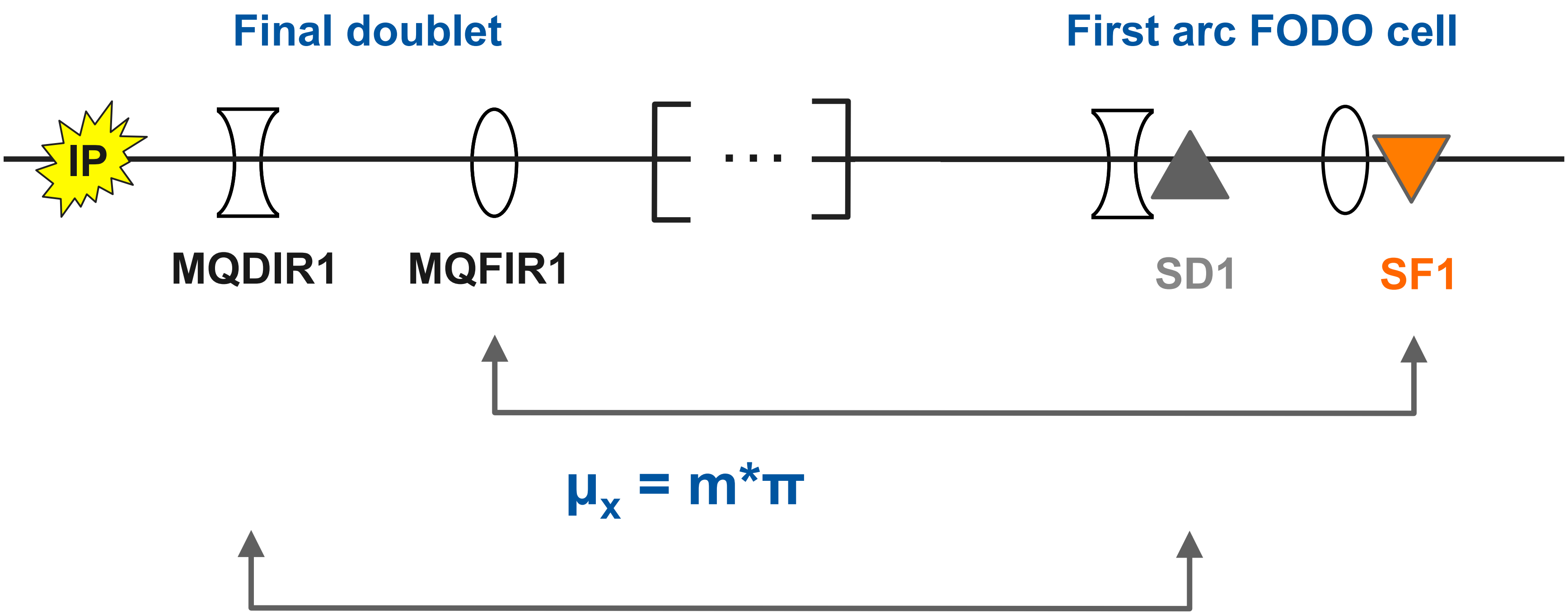


$\mu_x = m*\pi$

$\mu_y = n*\pi$

(m,n integers)

Phase advance FD – 1st Sext.



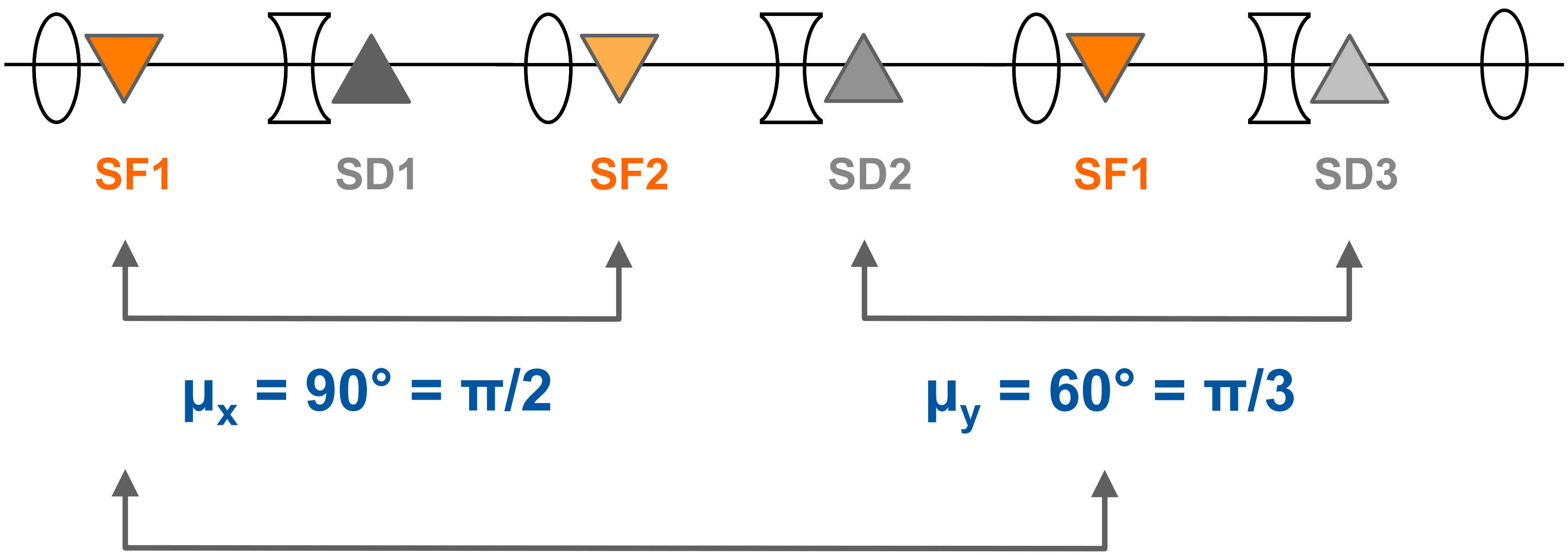
$$\mu_x = m*\pi$$

$$\mu_y = n*\pi$$

(m,n integers)

Bastian Haerer (bastian.harer@cern.ch)

FCC-ee sextupole scheme



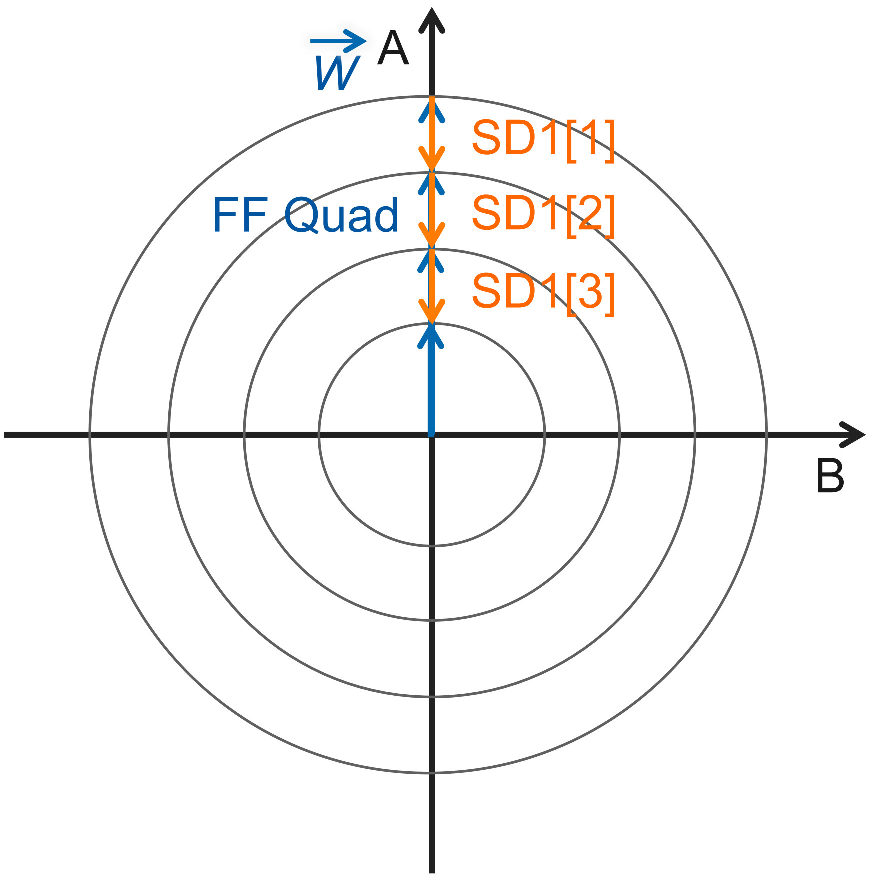
$\mu_x = 180° = \pi$ (→ -I transformation)

Even number of sextupoles per family!

-I transformation

- Sextupoles of each family are in phase

→ W-vector rotates by 2π



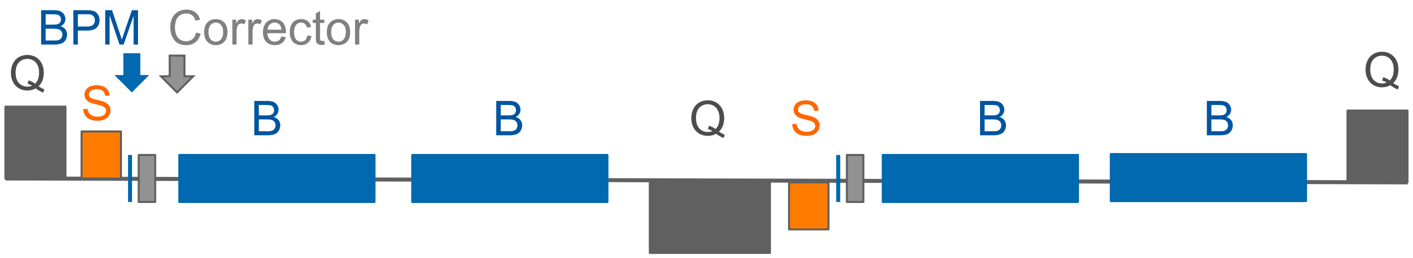
Next steps:

1. Compare non-symmetric FODO cell lattice with symmetric FODO cell lattice

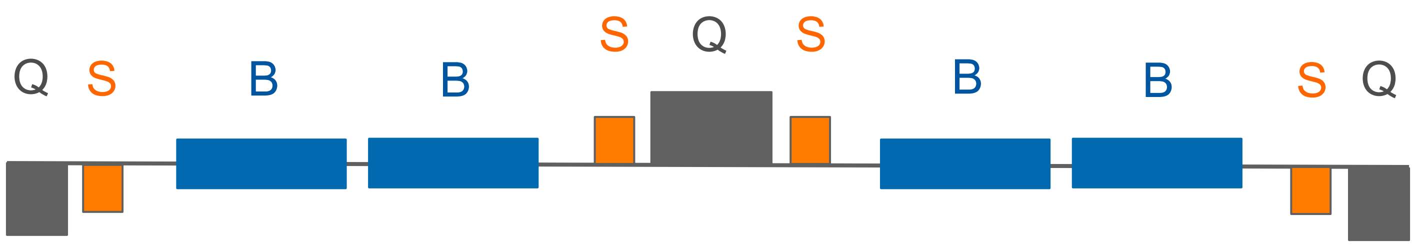
2. Compare 2 IR 12-fold layout with 2 IR baseline layout

1) FODO cells

Non-symmetric FODO cell (V15):



Symmetric FODO cell (V16):

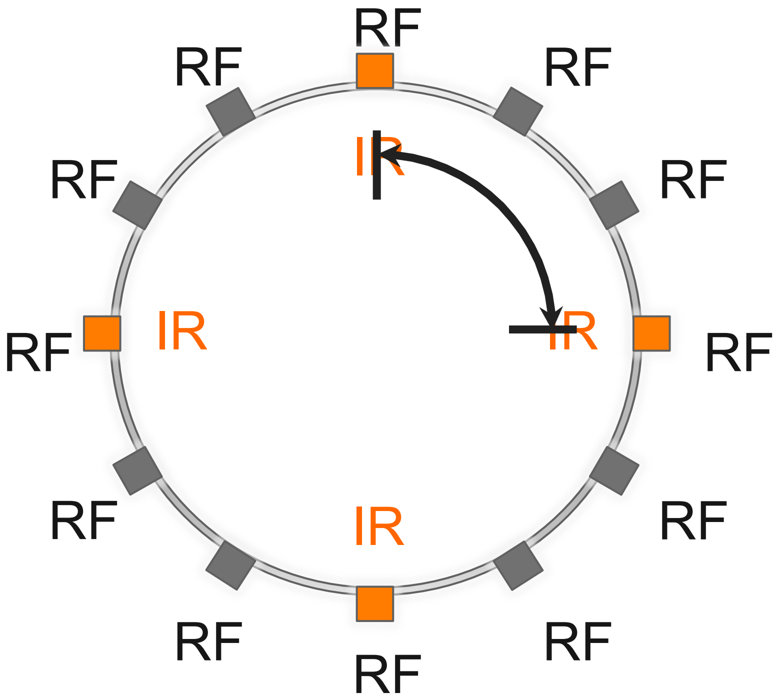


B = bending magnet, Q = quadrupole, S = sextupole

12-fold lattice

Circumference:	100 km
Arc length:	6.8 km
Straight section length:	1.5 km

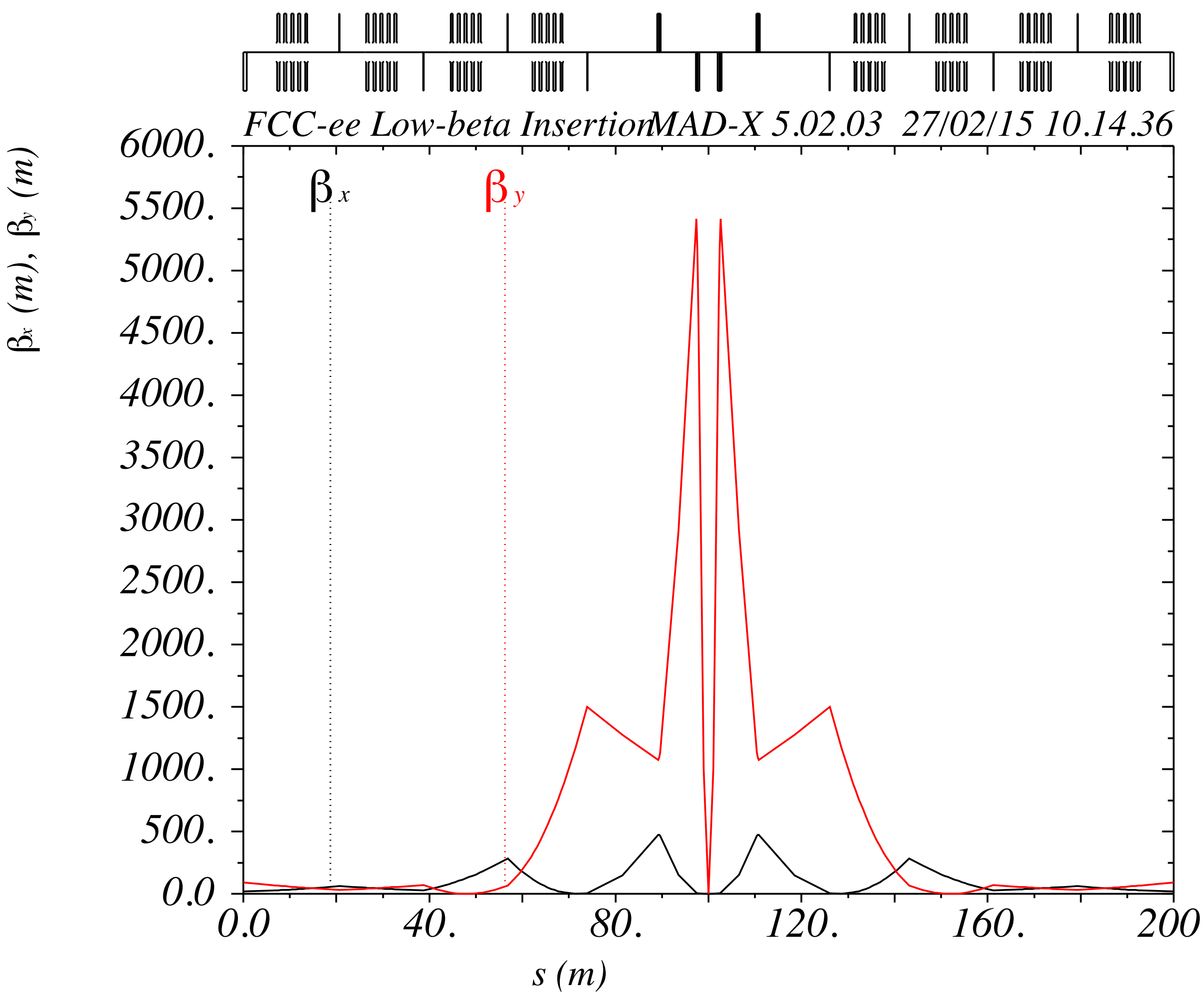
4 mini-beta insertions (IR)!



Objectives:

- One quarter of the ring
- Correct W function with the arcs next to the IPs

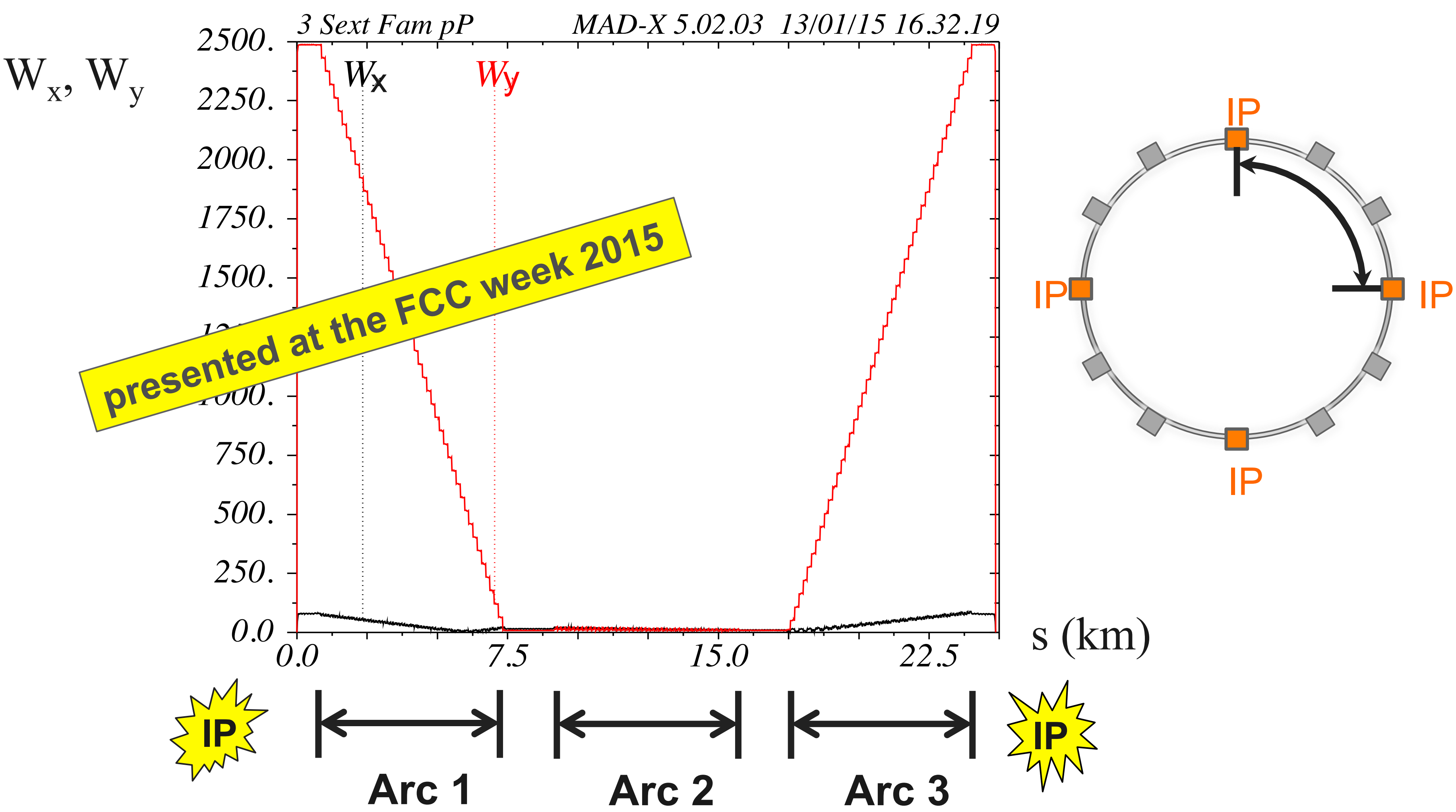
IR without local CCS



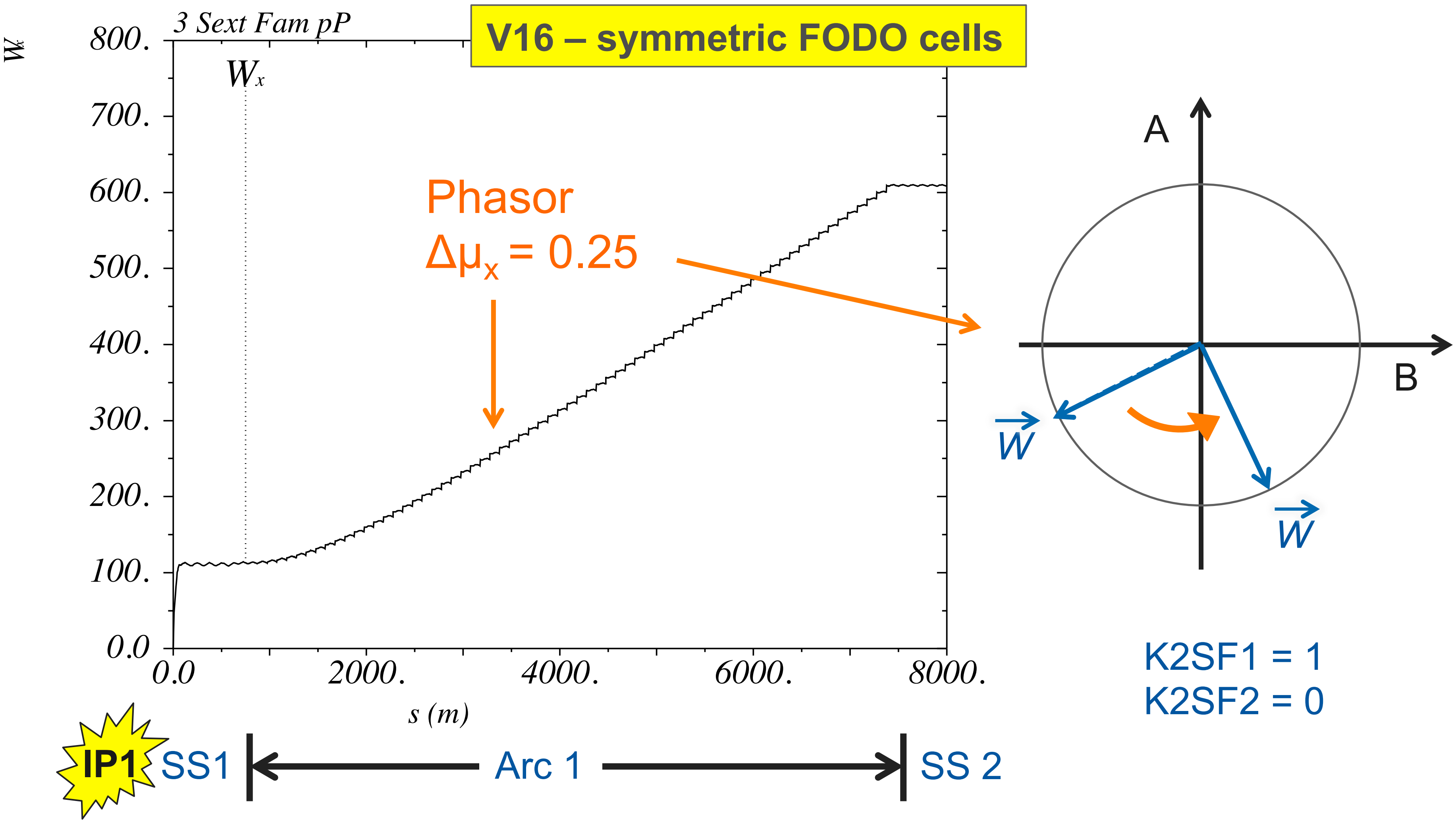
No local chromaticity correction scheme!

L*	= 2 m
β^*_x	= 1 m
β^*_y	= 0.001 m

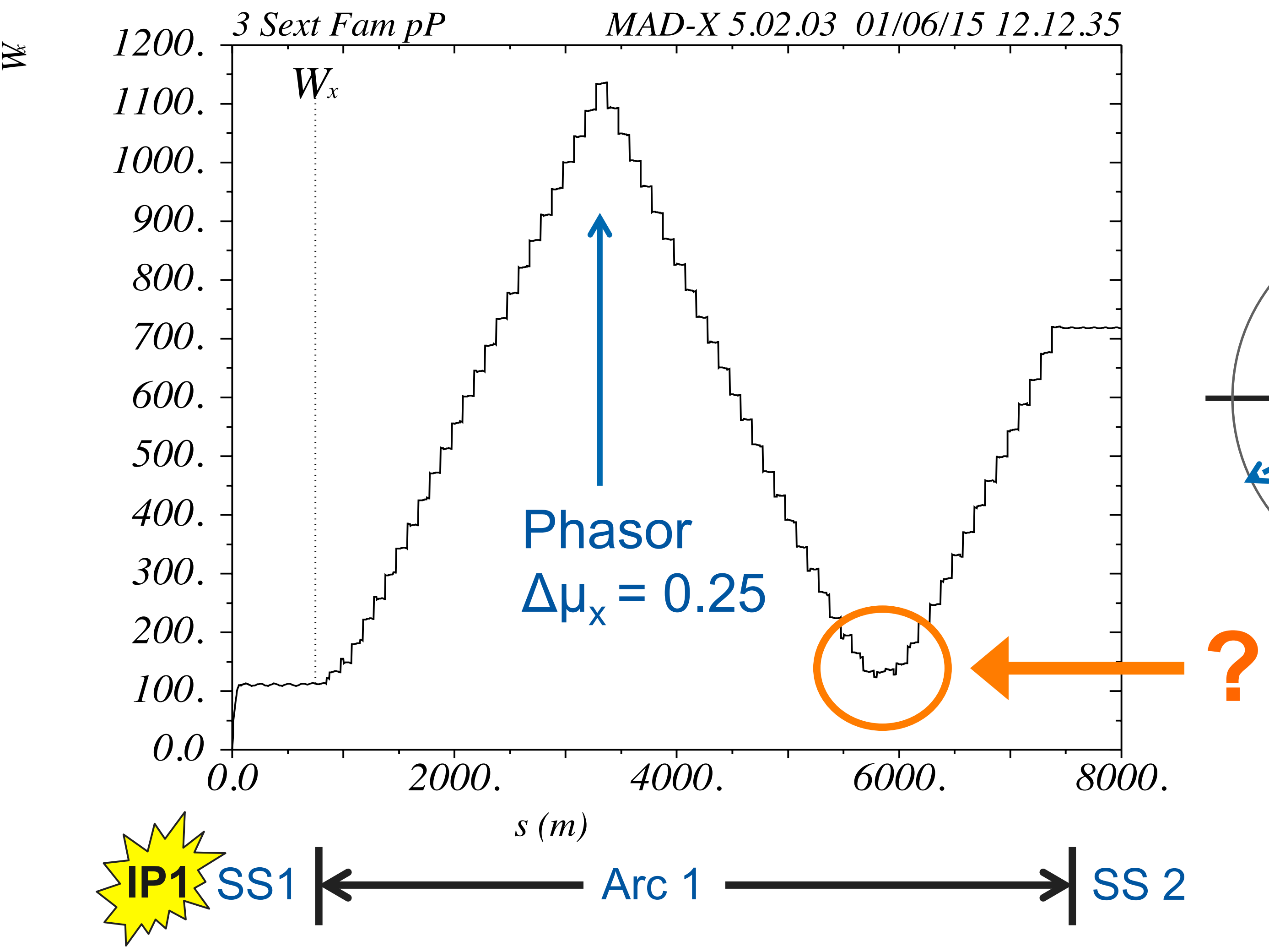
W functions non-symmetric FODO

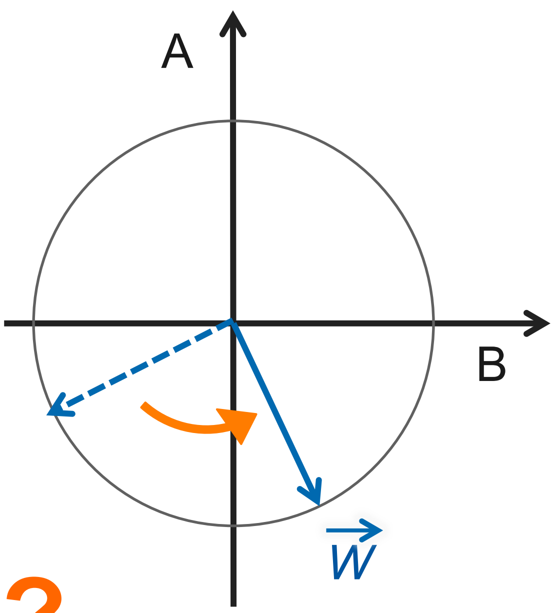


Hor. W function in first arc



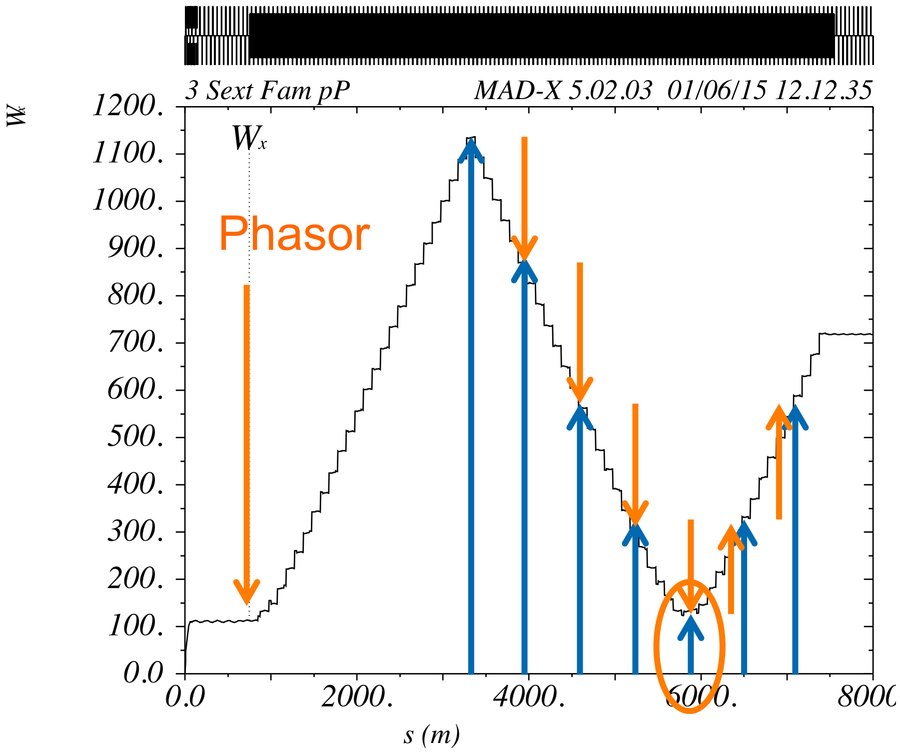
Hor. W function in first arc

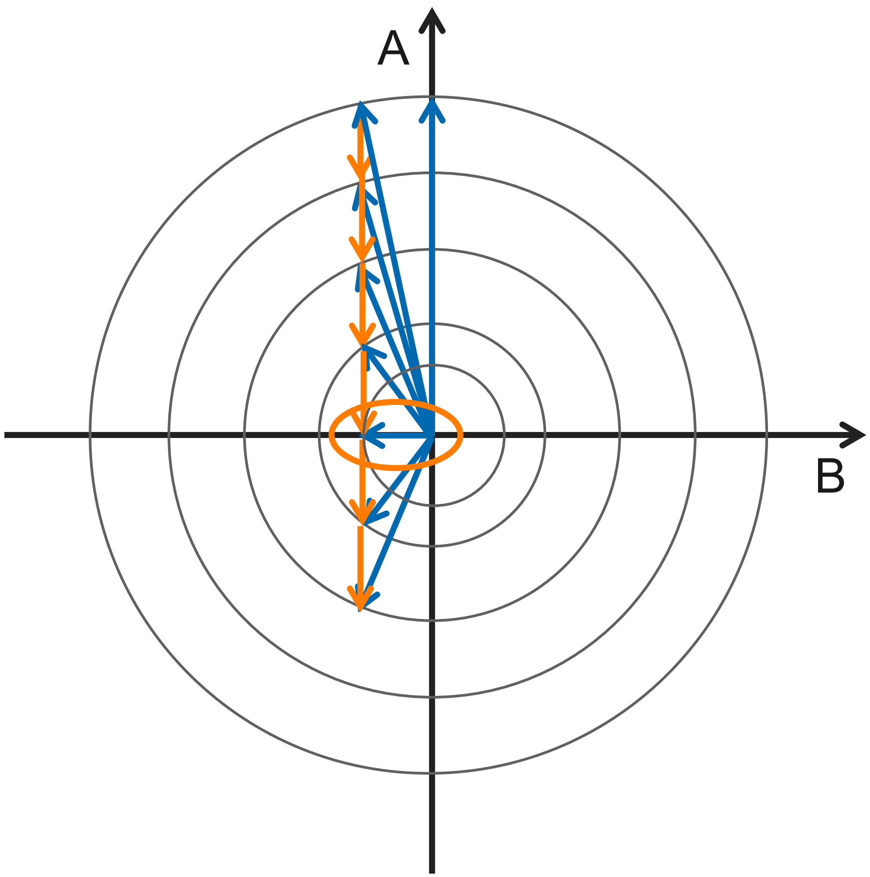




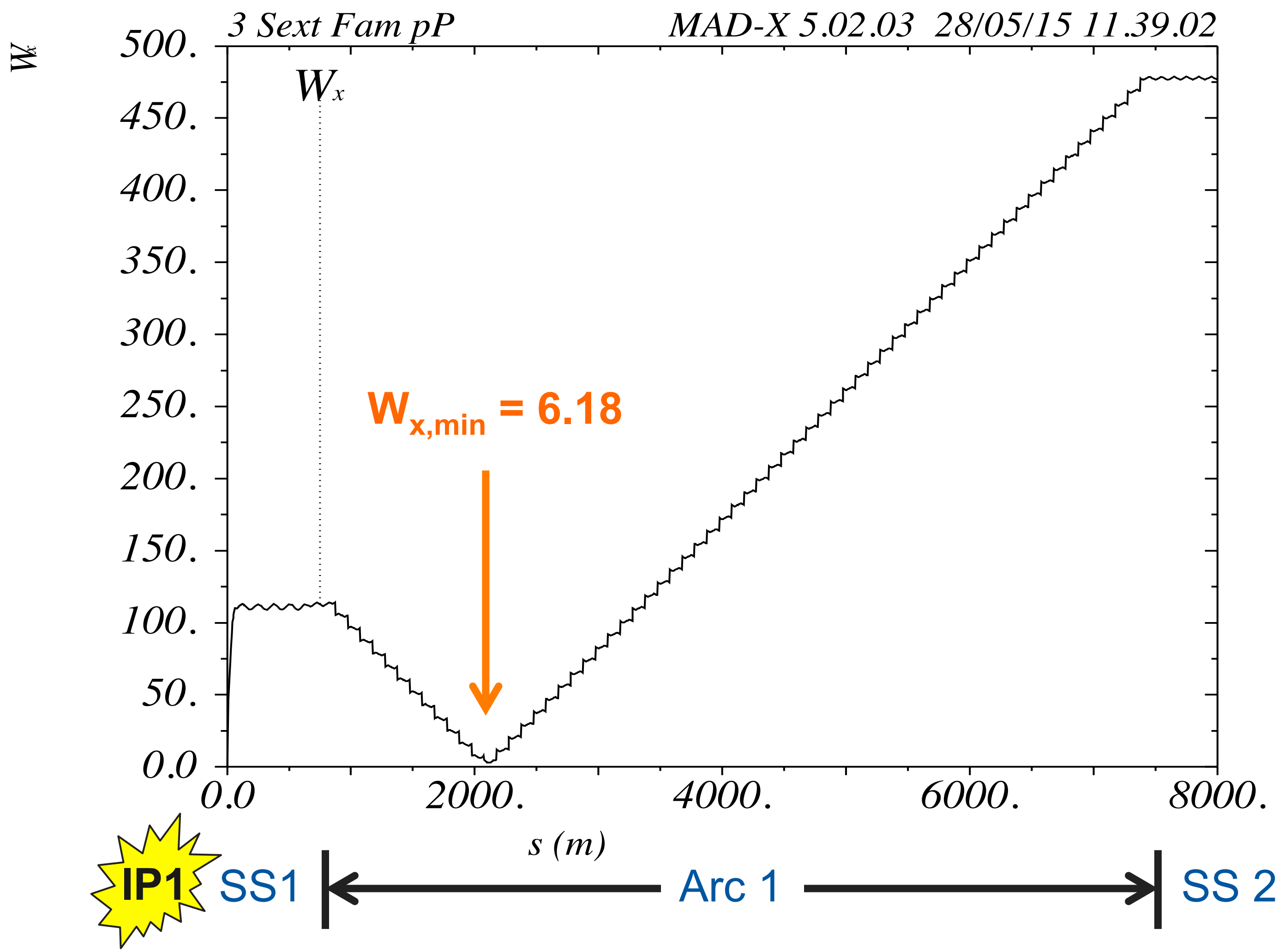
K2SF1 = 5
K2SF2 = 0

Phase mismatch

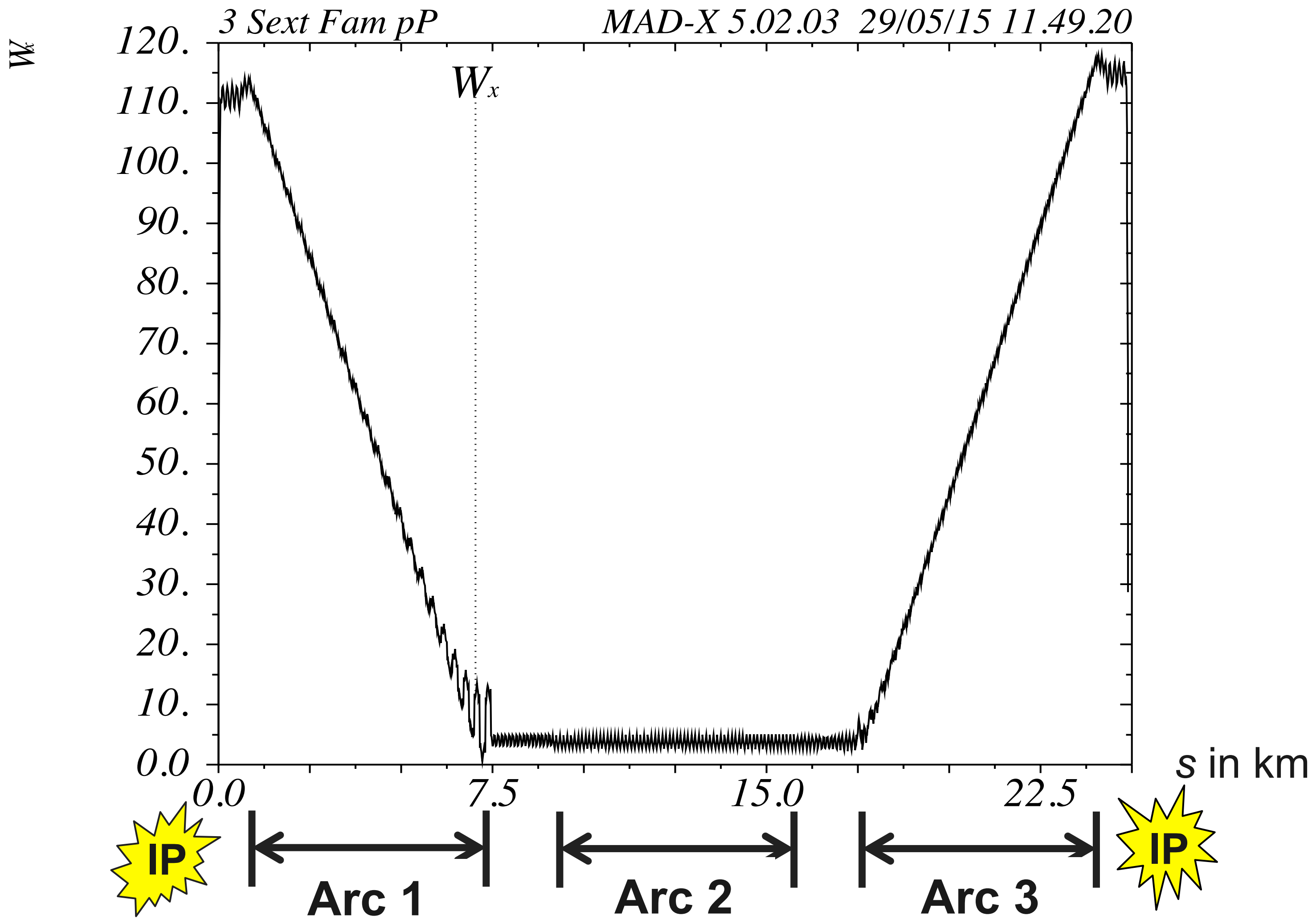




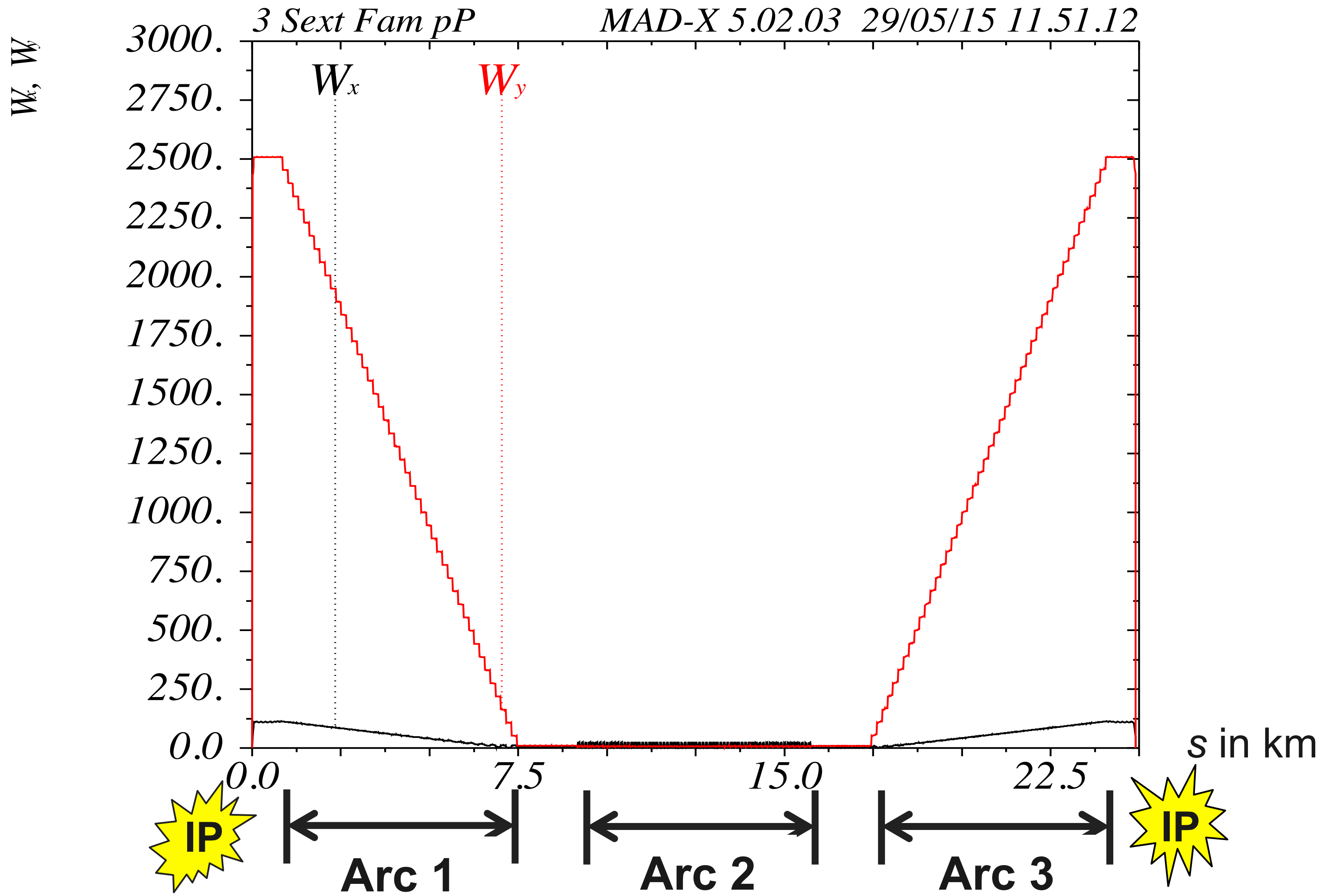
Matched phase



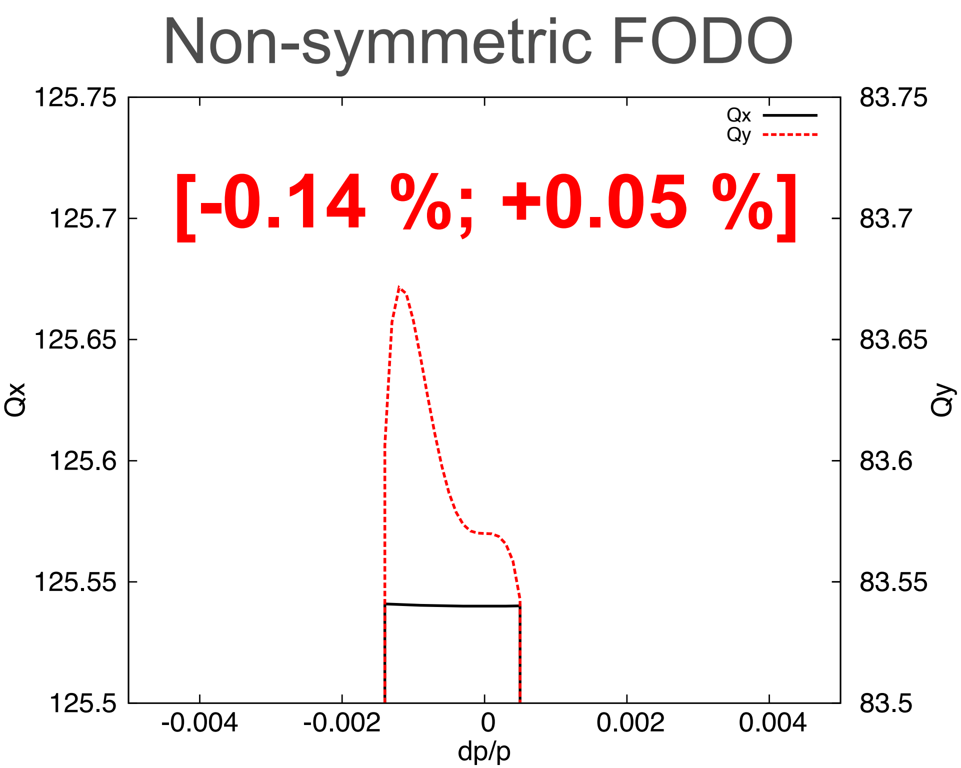
Hor. W function in the 1st quarter

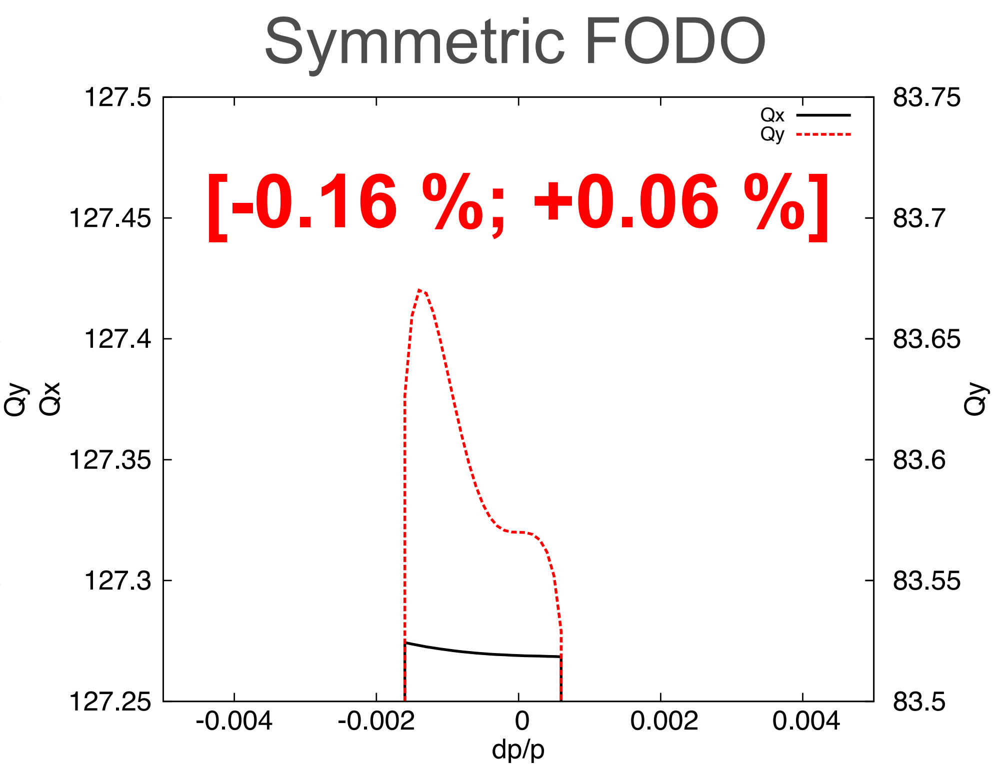


Both W functions in the 1st quarter



Momentum acceptance

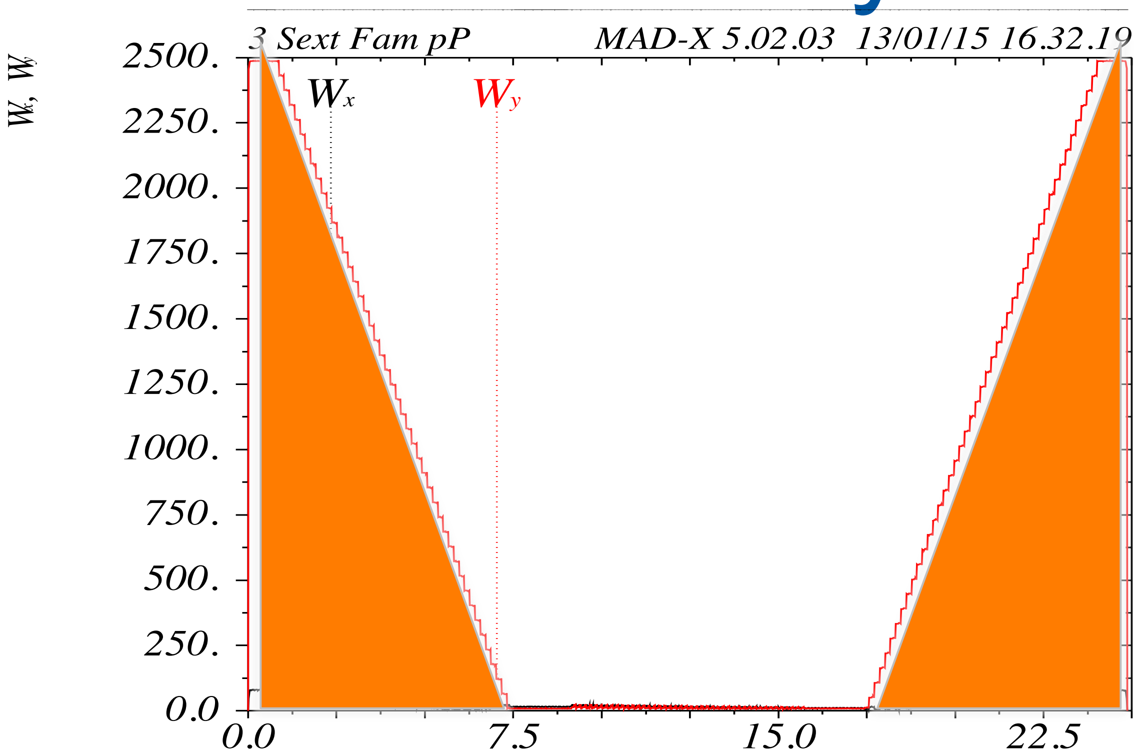




Corrected Chromaticity

	Non-symmetric FODO			Symmetric FODO		
	Nat. Chrom.	Corr. Chrom.	ΔQ (δ=0.05 %)	Nat. Chrom.	Corr. Chrom.	ΔQ (δ=0.05 %)
Q_x	502.16	502.16		506.16	509.08	
Q_x'	-603.80	5.7e-05	2.83e-08	-629.88	-4.20	-2.10e-03
Q_x''	-8.3e+03	3.5e+03	4.41e-04	-1.6e+04	6.6e+03	8.19e-04
Q_x'''	-1.4e+08	-5.5e+05	-1.14e-05	-2.7e+08	-1.5e+07	-3.13e-04
Q_x''''	-2.1e+12	-8.5e+09	-2.20e-05	-4.1e+12	-2.9e+10	-6.73e-05
Q_y	334.28	334.28		334.28	334.28	
Q_y'	-2044.43	2.8e-01	1.39e-04	-2059.23	6.7e-02	3.36e-05
Q_y''	-8.4e+06	-1.2e+04	-1.53e-03	-8.6e+06	-9.8e+03	-1.22e-03
Q_y'''	-2.0e+11	-3.4e+09	-7.00e-02	-2.0e+11	-2.5e+09	-5.11e-02
Q_y''''	-6.5e+15	3.6e+10	9.25e-05	-6.7e+15	-1.5e+12	-3.92e-03

3rd order chromaticity

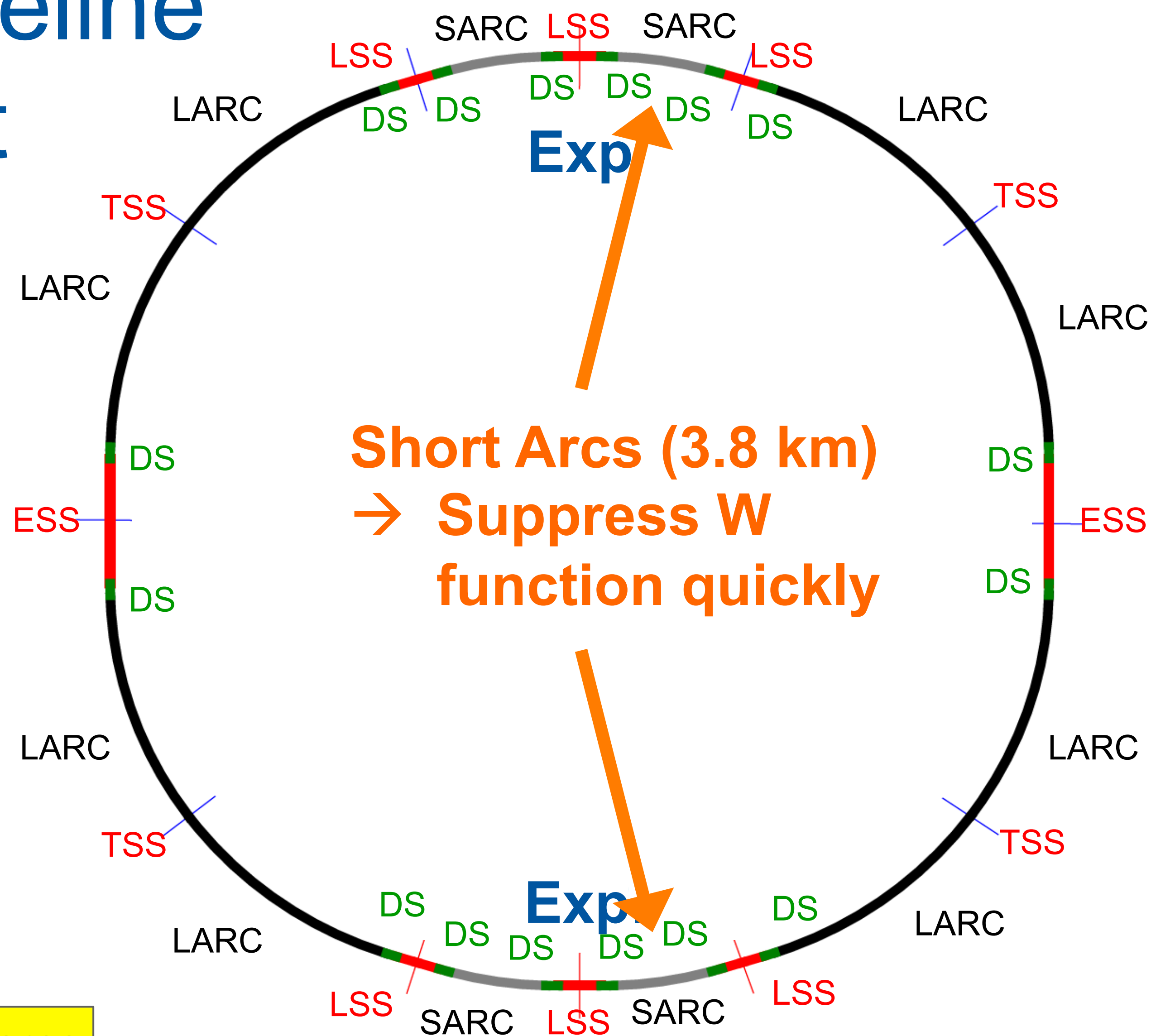


~W²

$$\frac{\partial^3 \varphi_y}{\partial \delta^3} = 6\frac{\partial \varphi_y}{\partial \delta} - \int_0^{\Pi} \beta_y (K_1 - K_2\eta_0)(a_{y,1}^2 + b_{y,1}^2) ds + \\ + 3\int_0^{\Pi} \beta_y (K_2\eta_1 - K_2\eta_2) ds + \frac{3}{2}\int_0^{\Pi} \beta_y b_{y,2} (K_1 - K_2\eta_0) ds$$

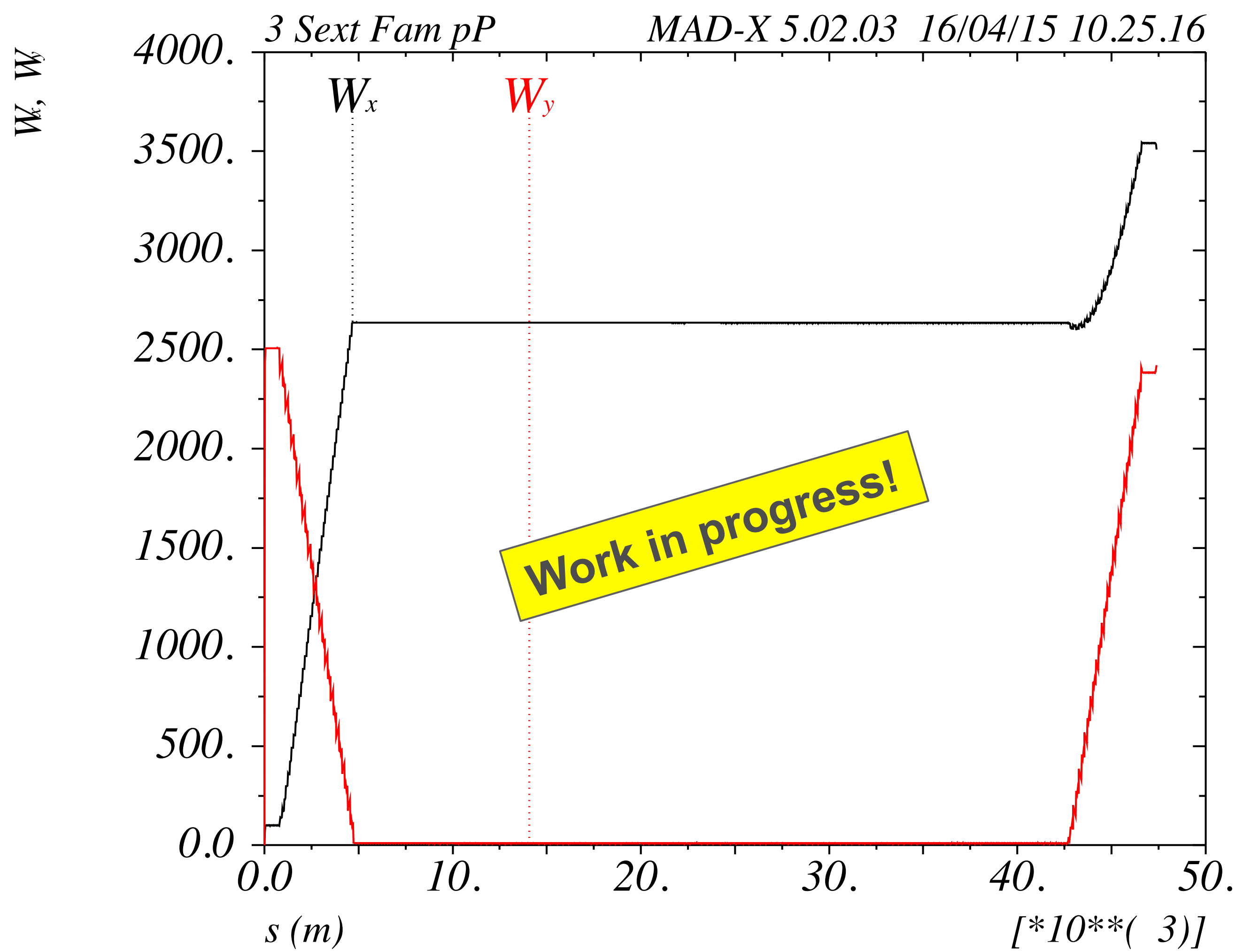
Anton Bogomyagkov

2) Baseline Layout



P. Lebrun & J. Osborne

W functions: baseline layout



Next steps:

- 1 sextupole and splitted quadrupoles
- 60°/60° and 90°/90° phase advance plus non-interleaved sextupole scheme
- Better targeted higher order correction?
- CC with individual sextupole pairs
- Discussion with BINP colleagues about how to combine the local CCS with the arcs

Phase functions in MAD-X

MAD-X Manual:

MUX Phase function $\mu_x = \int ds/\beta_x$, $[2\pi]$

PHIX Chromatic phase function $\Phi_x = \arctan(a_x/b_x)$, $[2\pi]$

$$b_x = \frac{1}{\beta_x}\frac{\partial \beta_x}{\partial p_t}, \qquad a_x = \frac{\partial \alpha_x}{\partial p_t} - \frac{\alpha_x}{\beta_x}\frac{\partial \beta_x}{\partial p_t}$$

Which one has to be matched?

→ I would guess PHIX…

Chromatic phase

Theoretically PHIX should be twice MUX…

Table 1.3: Comparison of the difference of both the MAD-X phase function μ and the MAD-X chromatic phase function φ between final doublet quadrupole of the respective plane and the first SD sextupole.

	Before optimising $\Delta\mu_x$:	After optimising $\Delta\mu_x$:
$\Delta\mu_x$	4.999999996 +0.125	5.118999996 +0.125
$\Delta\mu_y$	3.000000002	3.000000002
$\Delta\varphi_x$	9.021745692 +0.25	9.259497576 +0.25
$\Delta\varphi_y$	6.000035054	6.000078839

→ Where does the difference come from?

Discussion/Open questions

- Which influence has the working point?
- Matching tolerance/matching order
- Which parameters are worth to compare?
- How could additional sextupoles for higher order correction be added in the arc?
- How is an interleaved sextupole scheme be matched?