Limited acceptance and baryon number conservation vs. cumulants of net proton/baryon distribution

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Based on:
AB, V. Koch, V. Skokov, PRC 87 (2013) 014901
AB, V. Koch, PRC 86 (2012) 044904
AB, V. Koch, PRC 91 (2015) 027901

Outline

- Baryon number conservation
    - calculation
    - new observable
- Limited acceptance
    - required vs. actual acceptance
    - results, problems
- Local efficiency corrections
- Conclusions
- Backup

To make a long story short we hope to see a nontrivial dependence of net baryon/proton or charge cumulant ratios as a function of energy

$$c_1 = \langle N_B - N_{\bar{B}} \rangle$$

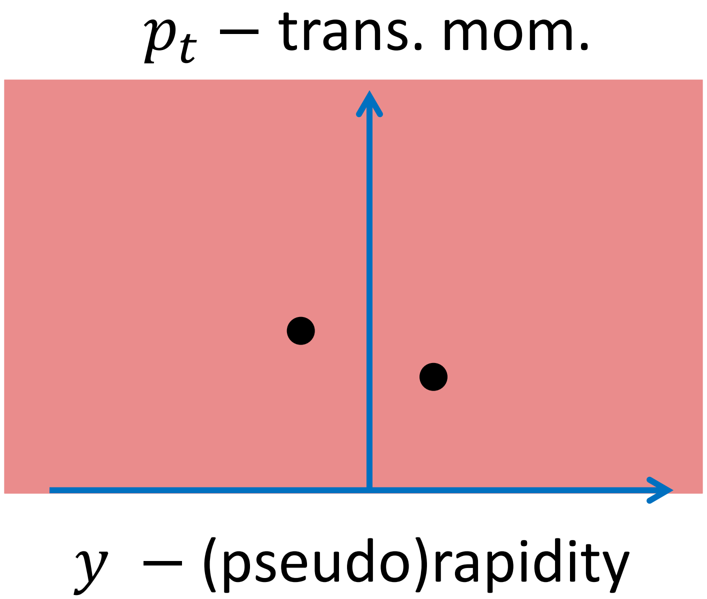
$$c_2 = \langle (N_B - N_{\bar{B}})^2 \rangle - \langle N_B - N_{\bar{B}} \rangle^2$$

$c_3, c_4, c_5, c_6, \ldots$

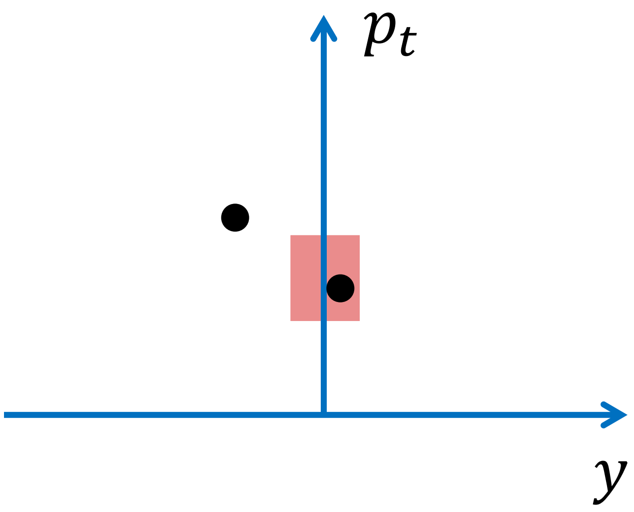
or $B \to Q$

electric charge

Obvious problem:
what is the optimal phase-space region to make measurement

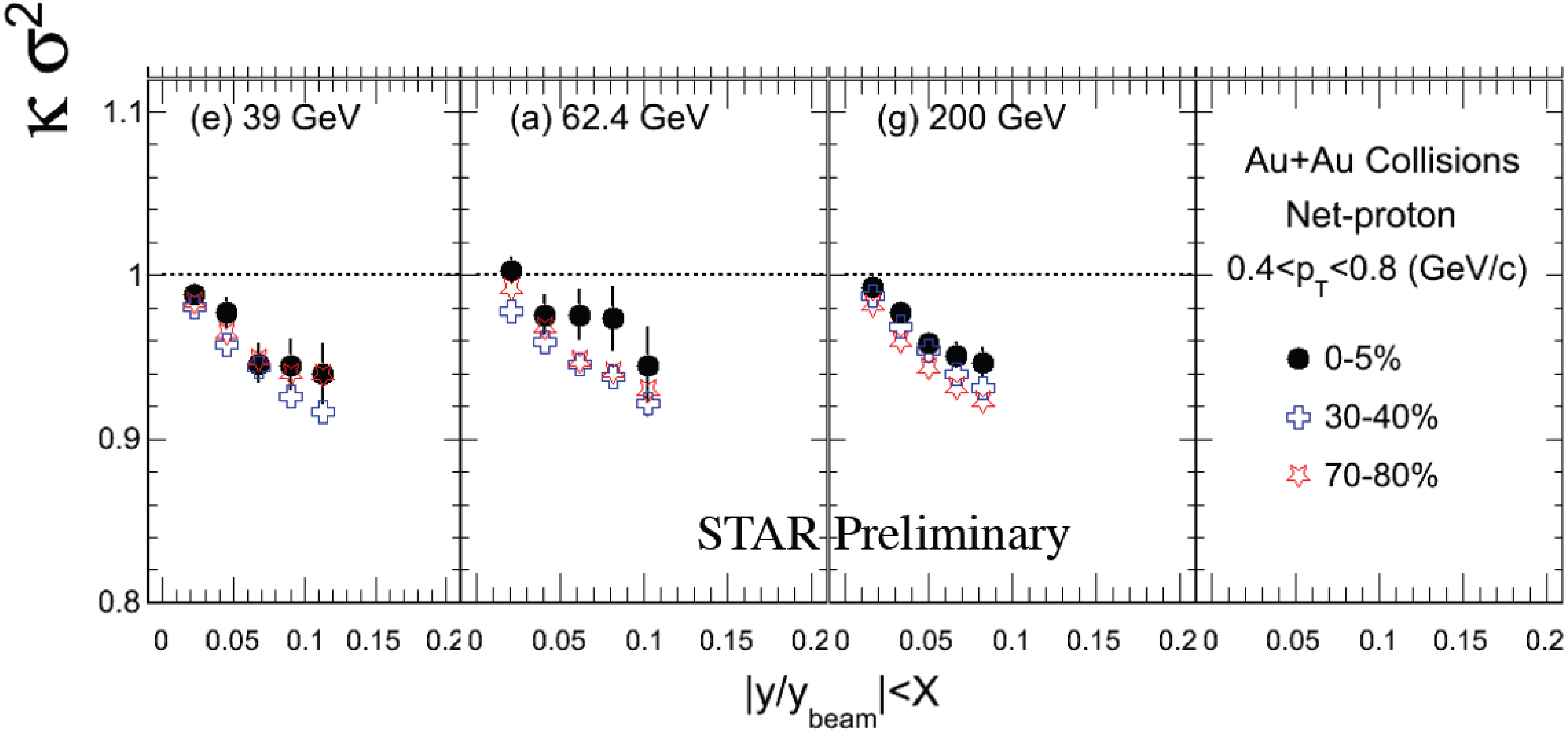


if we measure all baryons there are no fluctuations of $N_B - N_{\bar{B}}$ and $c_2 = 0$ etc.



if we take very narrow bin to suppress baryon conservation we lose interesting physics

See, e.g., c_4/c_2 as a function of rapidity cut



It looks like the baryon number conservation

Baryon number conservation

AB, V. Koch, V. Skokov, PRC 87 (2013) 014901

Skellam

$$\boxed{\begin{array}{c} P(n_B, n_{\bar{B}}) = P(n_B)P(n_{\bar{B}}) \\ \downarrow \\ P_\Delta(n_B - n_{\bar{B}}) \end{array}}$$

$P(x)$ – Poisson dist.,

n_B – measured # of baryons

$P_\Delta(n_B - n_{\bar{B}})$ - Skellam distribution

Baryon conservation

$$\boxed{\begin{aligned} P_B(n_B, n_{\bar{B}}) \sim \sum P(N_B) P(N_{\bar{B}}) \delta_{N_B - N_{\bar{B}} - B} \times \\ \times B(N_B, n_B; p_B) B(N_{\bar{B}}, n_{\bar{B}}, p_{\bar{B}}) \end{aligned}}$$

$$B(N_B, n_B; p_B) = \frac{N_B!}{n_B!\,(N_B - n_B)!} (p_B)^{n_B} (1 - p_B)^{N_B - n_B}$$

$$P_B(n_B, n_{\bar{B}}) \rightarrow P_{\Delta,B}(n_B - n_{\bar{B}})$$

$B(\dots)$ – binomial dist.

N_B – total # of baryons,

We assume that $P(N_B)$ is give by Poisson. We start with a system without any correlations whatsoever and enforce global baryon conservation.

All calculation can be done analytically

$$\boxed{\begin{aligned} P_B(n) = {} & \left(\frac{p_B}{p_{\bar{B}}}\right)^{n/2} \left(\frac{1-p_B}{1-p_{\bar{B}}}\right)^{(B-n)/2} \\ & \times \frac{I_n(2z\sqrt{p_B p_{\bar{B}}})I_{B-n}\boldsymbol{(}2z\sqrt{(1-p_B)(1-p_{\bar{B}})}\boldsymbol{)}}{I_B(2z)} \end{aligned}}$$

n – net baryon $\qquad z = \sqrt{\langle N_B \rangle \langle N_{\bar{B}} \rangle} \qquad B = \langle N_B \rangle - \langle N_{\bar{B}} \rangle$

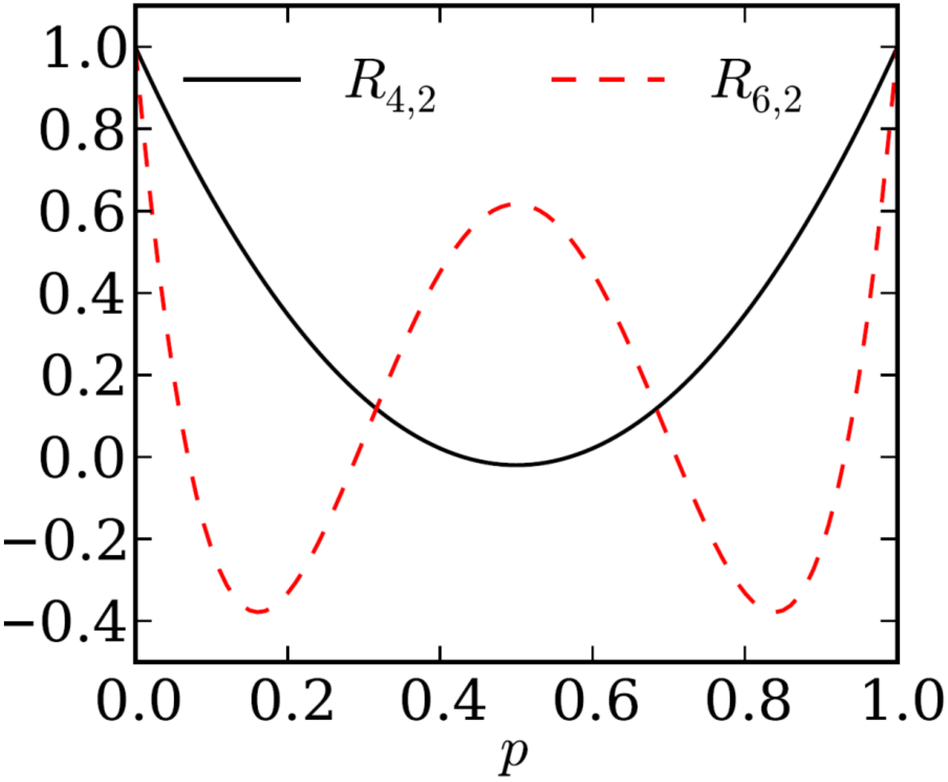
binomial parameter

$$p = \frac{\#\ of\ measured\ protons/baryons}{total\ \#\ of\ baryons}$$

$p_{max} \approx \frac{1}{2}$ if only protons are measured

$p_{max} = 1$ if baryons are measured

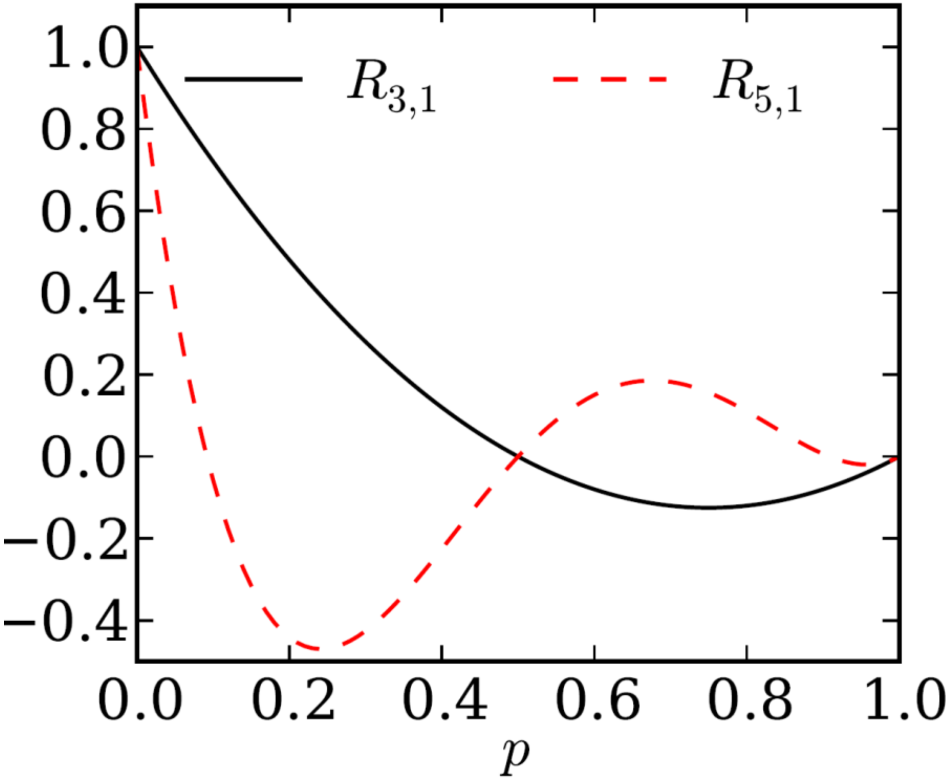
Results for $\langle N_B \rangle = 400,\ \langle N_{\bar{B}} \rangle = 100 \quad \rightarrow \quad B = 300$



$$R_{n,m} = \frac{c_n}{c_m}$$

$$p = \frac{\#\ of\ measured\ protons/baryons}{total\ \#\ of\ baryons}$$

Results for $\langle N_B \rangle = 400$, $\langle N_{\bar{B}} \rangle = 100 \quad \rightarrow \quad B = 300$

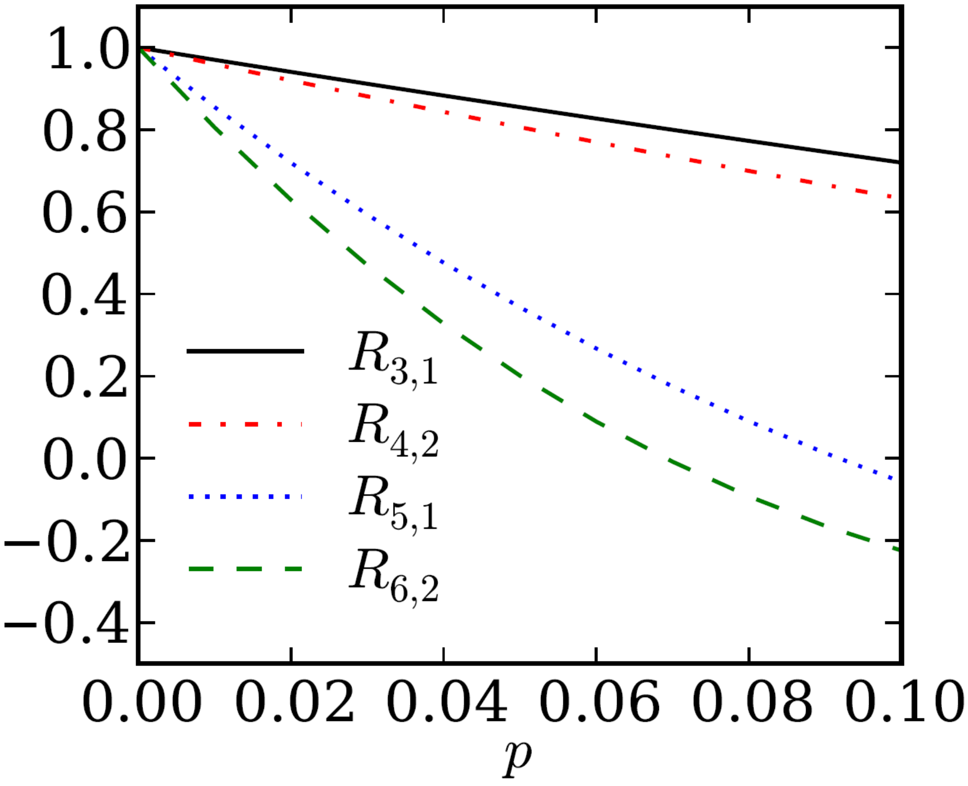


$$R_{n,m} = \frac{c_n}{c_m}$$

$$p = \frac{\#\ of\ measured\ protons/baryons}{total\ \#\ of\ baryons}$$

small p

STAR 200 GeV,
$p \approx 0.013$



We obtain ($B = 350$):

200 GeV: $R_{4,2} \approx 0.95$, $R_{6,2} \approx 0.77$

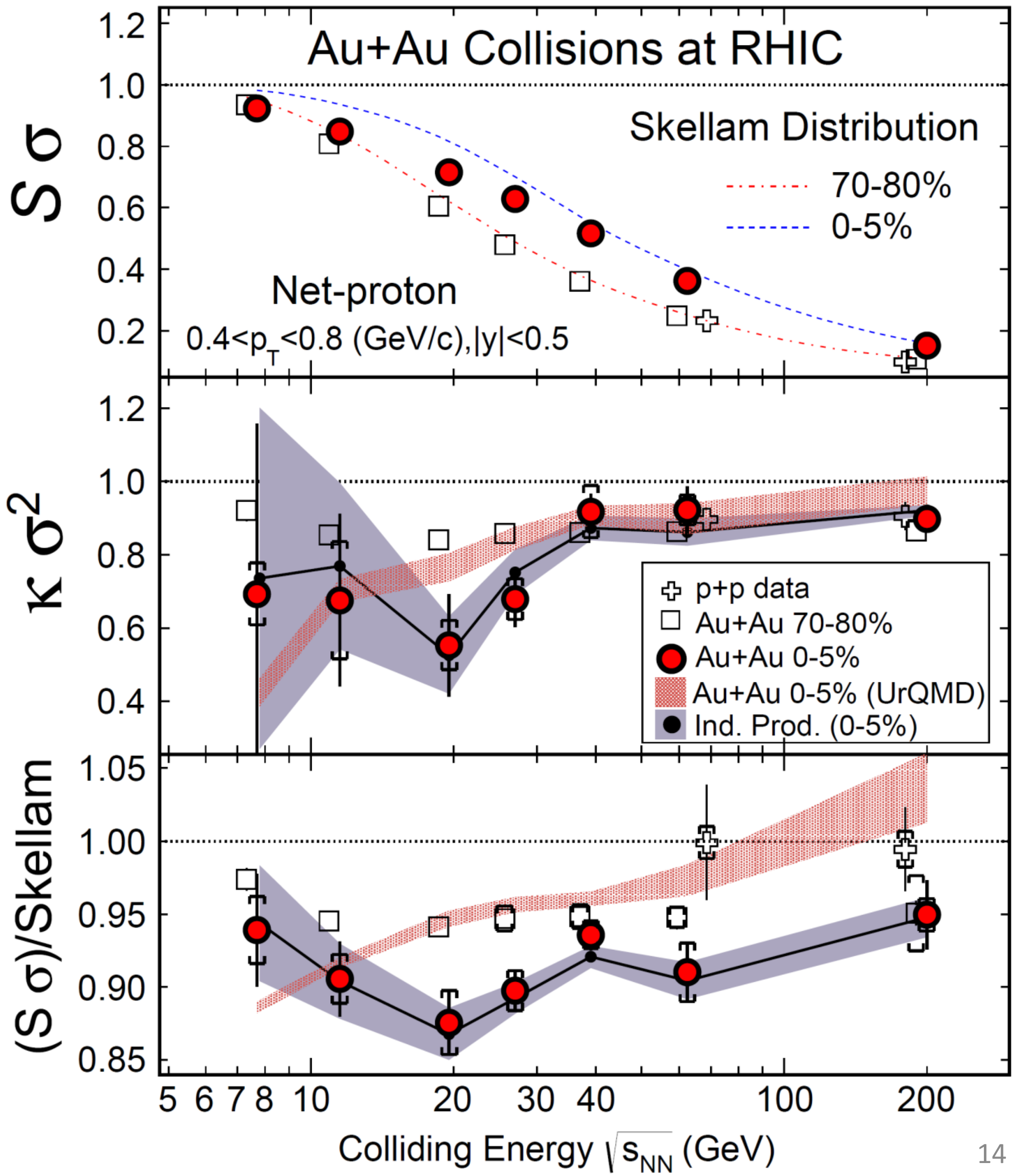
5 GeV: $R_{4,2} \approx 0.85$, $R_{6,2} \approx 0.32$

PRL 112 (2014) 032302

STAR data

Baryon conservation is there and should be considered carefully

Perhaps what we see in UrQMD is just baryon conservation



$$c_1 = pB,$$
$$c_3 = c_1(1-p)(1-2p),$$
$$c_5 = c_3[1-12p(1-p)].$$

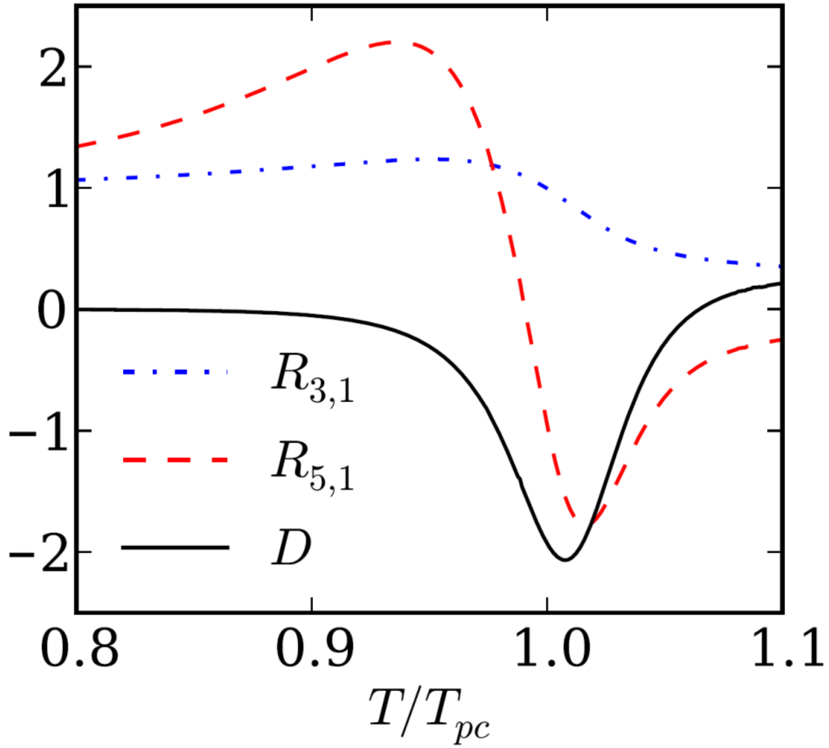
New observable

$$D = R_{5,1} - R_{3,1}\left[1 - \frac{3}{4}(1+\gamma)(3-\gamma)\right]$$

$$\gamma = \sqrt{1 + 8R_{3,1}}$$

$D = 0$ for a system with only baryon conservation

PQM calculation →



$\mu_B/T = 0.5$

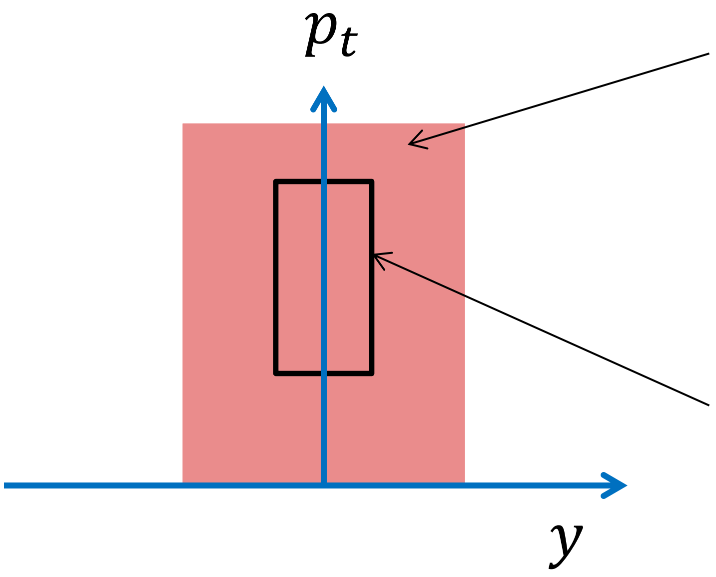
T_{pc} – crossover temperature

PQM = Polyakov loop-extended Quark-Meson model

Limited acceptance

AB, V. Koch, PRC 86 (2012) 044904

Definitions



Required (desired) acceptance.
If we measure all relevant particles in this acceptance we will capture the desired physics.

Actual acceptance.
In addition we usually cannot measure all relevant particles (neutrons, detector efficiency).

K_n – cumulants in the required acceptance

c_n – cumulants in the actual acceptance

Calculation

$$p(n_1, n_2) = \sum P(N_1, N_2)\, B(N_1, n_1; p_1) B(N_2, n_2; p_2)$$

↓ c_n — what we measure

↓ K_n, $F_{i,k}$ — what we would like to measure

$$\boxed{p_1 = p_2 = 1\text{:}\ c_n = K_n}$$

factorial moments $F_{i,k} = \left\langle \frac{N_1! N_2!}{(N_1 - i)!(N_2 - k)!} \right\rangle$

$B(\ldots) -$ binomial dist.

It turns out one cannot relate cumulants K_n solely through cumulants c_m

This can be done for factorial moments

$$\left\langle \frac{N_1!\,N_2!}{(N_1-i)!\,(N_2-k)!} \right\rangle = \frac{1}{p_1^i p_2^k} \left\langle \frac{n_1!\,n_2!}{(n_1-i)!\,(n_2-k)!} \right\rangle$$

$$F_{i,k} = \frac{1}{p_1^i p_2^k}\, f_{i,k}$$

So we express required cumulants through factorial moments $F_{i,k}$, which are know from the above equality ($f_{i,k}$ is measured).

Binomial parameter

$$p = \frac{\#\ of\ measured\ particles}{total\ \#\ of\ particles\ that\ should\ be\ measured}$$

this *p* is different than *p* in baryon conservation

If we want to study net-baryon cumulants but we measure net-proton cumulants then $p_{max} \approx 1/2$.
Detector efficiency, cuts in rapidity or transverse momentum lead to $p < 1/2$.

STAR: $p < 1/2$ and probably $p \approx 1/5$ (nobody really knows)

Relations between K_n and c_n (required vs. actual acceptance).
Here $p_1 = p_2 = p$.

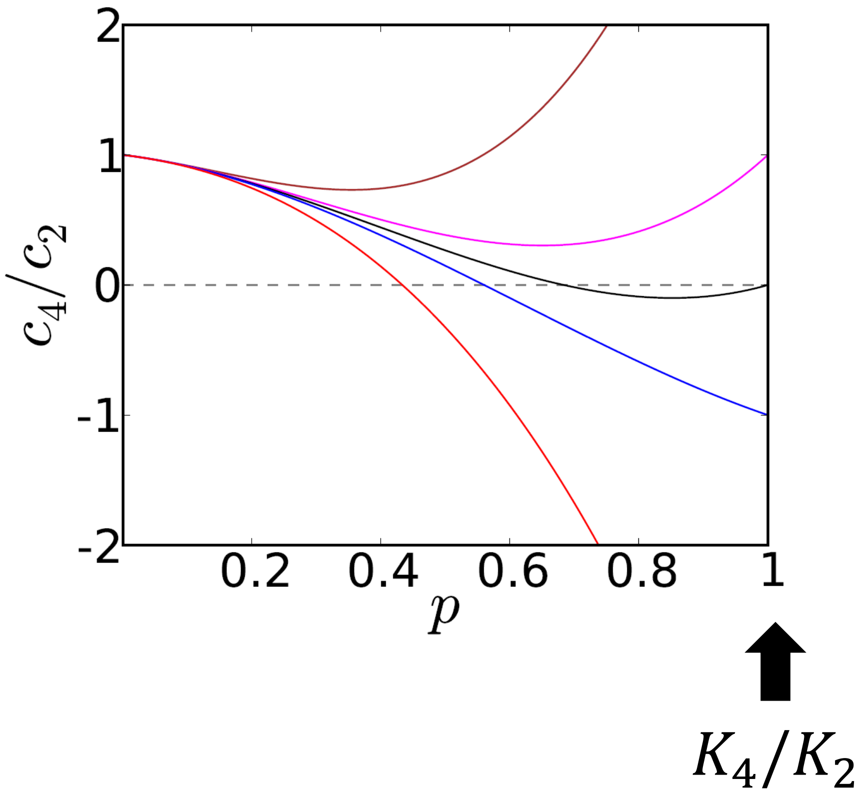
$$
\begin{aligned}
pK_1 &= c_1, \\
p^2K_2 &= c_2 - n(1-p), \\
p^3K_3 &= c_3 - c_1(1-p^2) - 3(1-p)(f_{20} - f_{02} - nc_1) \\
p^4K_4 &= c_4 - np^2(1-p) - 3n^2(1-p)^2 - 6p(1-p) \\
&\quad \times (f_{20} + f_{02}) + 12c_1(1-p)(f_{20} - f_{02}) - (1-p^2) \\
&\quad \times \left(c_2 - 3c_1^2\right) - 6n(1-p)\left(c_1^2 - c_2\right) - 6(1-p) \\
&\quad \times (f_{03} - f_{12} + f_{02} + f_{20} - f_{21} + f_{30}).
\end{aligned}
$$

$f_{i,k}$ – measured factorial moments $\qquad n \equiv \langle n_1 \rangle + \langle n_2 \rangle$

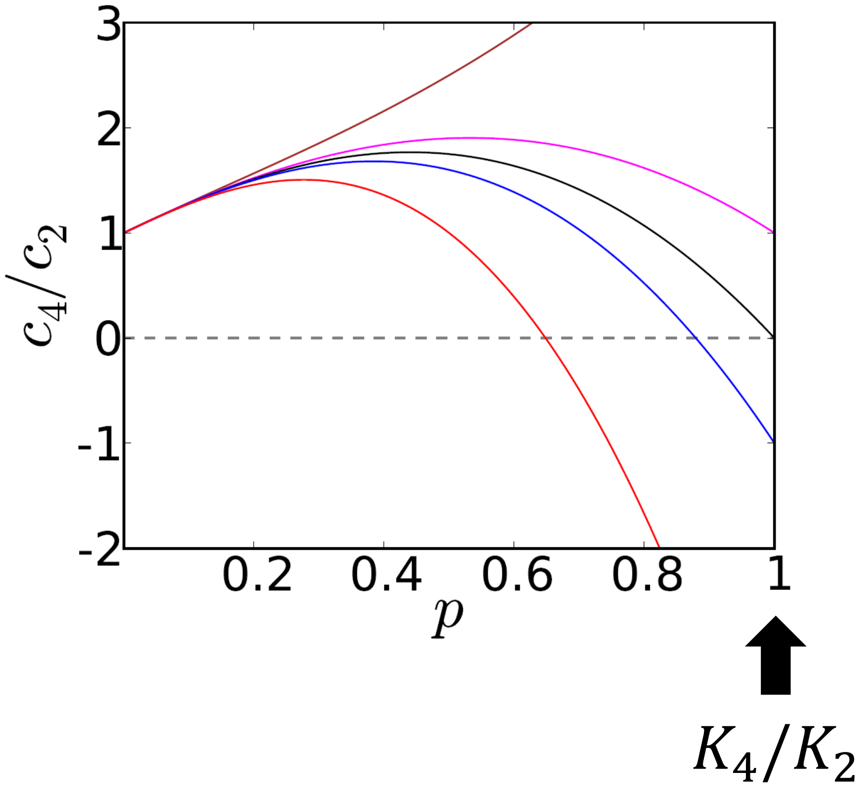
General case $p_1 \neq p_2$, see PRC 86 (2012) 044904

Illustration

multiplicity distr. narrower
than Poisson



multiplicity distr. broader
than Poisson

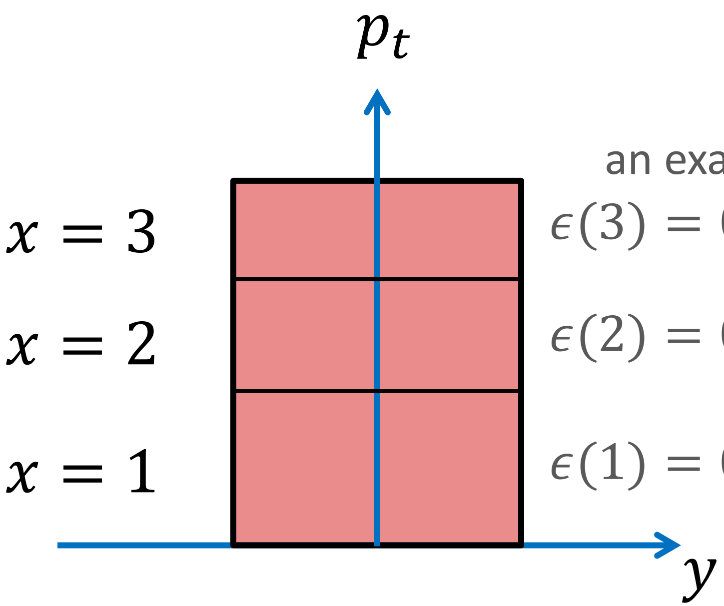


$$\frac{K_4}{K_2} = 5, 1, 0, -1, -5$$

Local efficiency corrections

AB, V. Koch, PRC 91 (2015) 027901

Suppose our machine detects particles with probabilities that depend on p_t (in general p_t, y, ϕ).



an example:

$\epsilon(3) = 0.5, \bar{\epsilon}(3) = 0.4$

$\epsilon(2) = 0.8, \bar{\epsilon}(2) = 0.8$

$\epsilon(1) = 0.9, \bar{\epsilon}(1) = 0.8$

ϵ ($\bar{\epsilon}$) -- probabilities to detect baryons (antibaryons) or positive (negative) charges

We measure $\underbrace{f_{i,k} \text{ and } c_n}_{\text{observed}}$ but we want to know true $\underbrace{F_{i,k} \text{ and } K_n}_{\text{produced}}$

Calculation

$$\langle N \rangle = \sum_{x=1,2,3} \langle N(x) \rangle \qquad \langle N(x) \rangle = \frac{1}{\epsilon(x)} \langle n(x) \rangle$$

$$F_{1,1} = \langle N\bar{N} \rangle = \sum_{x=1,2,3} \sum_{\bar{x}=1,2,3} \langle N(x)\bar{N}(\bar{x}) \rangle$$

$$\langle N(x)\bar{N}(\bar{x}) \rangle = \frac{1}{\epsilon(x)\bar{\epsilon}(\bar{x})} \langle n(x)\bar{n}(\bar{x}) \rangle$$

Once we know $F_{i,k}$ we can construct cumulnats K_n

and

$$F_{2,0} = \langle N(N-1) \rangle = \sum_{x_1=1,2,3} \sum_{x_2=1,2,3} \left\langle N(x_1)[N(x_2) - \delta_{x_1,x_2}] \right\rangle$$

$$\left\langle N(x_1)[N(x_2) - \delta_{x_1,x_2}] \right\rangle = \frac{1}{\epsilon(x_1)\epsilon(x_2)} \left\langle n(x_1)[n(x_2) - \delta_{x_1,x_2}] \right\rangle$$

$\delta_{x_1,x_2} = 1$ if $x_1 = x_2$ and zero otherwise

See backup or PRC 91 (2015) 027901 for general equations

Conclusions

- Baryon number conservation results in a comparable signal as the experimental data for net proton cumulants

- Limited acceptance/efficiency is the most serious problem that makes any interpretation of net proton cumulants challenging

- Data should be corrected (at least checked) for local efficiency

Backup

Cumulants

$$g(t) = \ln\left(\sum_n P_B(n) e^{nt}\right)$$

cumulant generating function

$$g(t) = \sum_{k=1}^{\infty} c_k \frac{t^k}{k!}$$

n-th derivative with respect to t (at $t = 0$) gives c_n

$P_B(n)$ – net baryon/proton/charge distribution

Cumulants vs. factorial moments

$$
\begin{aligned}
K_1 =\ & \langle N_1 \rangle - \langle N_2 \rangle\,, \\
K_2 =\ & N - K_1^2 + F_{02} - 2F_{11} + F_{20}, \\
K_3 =\ & K_1 + 2K_1^3 - F_{03} - 3F_{02} + 3F_{12} + 3F_{20} - 3F_{21} + F_{30} - 3K_1(N + F_{02} - 2F_{11} + F_{20}), \\
K_4 =\ & N - 6K_1^4 + F_{04} + 6F_{03} + 7F_{02} - 2F_{11} - 6F_{12} - 4F_{13} + 7F_{20} - 6F_{21} + 6F_{22} + 6F_{30} - 4F_{31} + F_{40} \\
& + 12K_1^2(N + F_{02} - 2F_{11} + F_{20}) - 3(N + F_{02} - 2F_{11} + F_{20})^2 - 4K_1(K_1 - F_{03} - 3F_{02} + 3F_{12} + 3F_{20} - 3F_{21} + F_{30})
\end{aligned}
$$

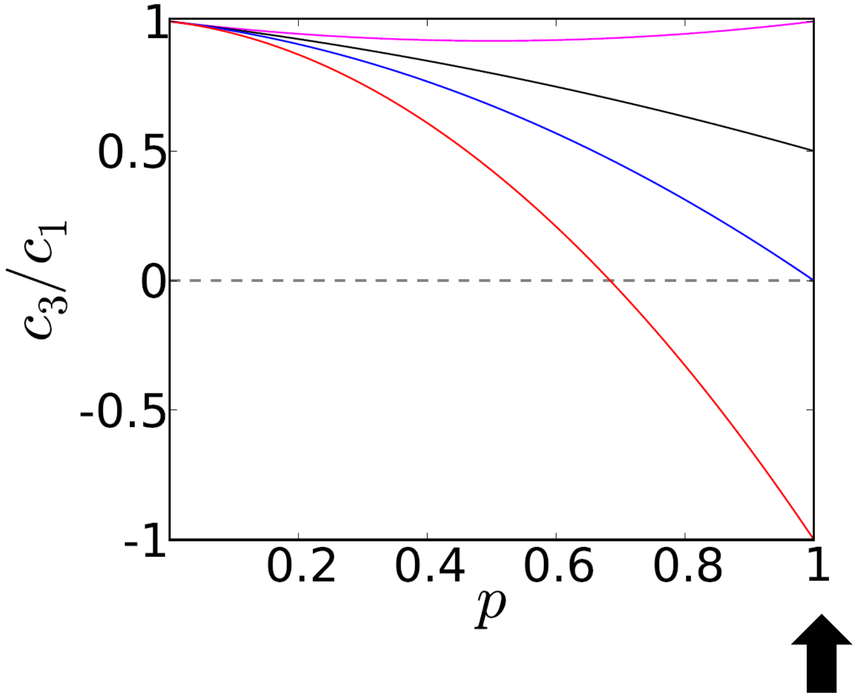
$$
\begin{aligned}
K_5 =\ & K_1 + 24K_1^5 - F_{05} - 10F_{04} - 25F_{03} - 15F_{02} + 15F_{12} + 20F_{13} + 5F_{14} + 15F_{20} - 15F_{21} - 10F_{23} + 25F_{30} \\
& - 20F_{31} + 10F_{32} + 10F_{40} - 5F_{41} + F_{50} - 60K_1^3(N + F_{02} - 2F_{11} + F_{20}) + 30K_1(N + F_{02} - 2F_{11} + F_{20})^2 \\
& + 20K_1^2(K_1 - F_{03} - 3F_{02} + 3F_{12} + 3F_{20} - 3F_{21} + F_{30}) - 10(N + F_{02} - 2F_{11} + F_{20})(K_1 - F_{03} \\
& - 3F_{02} + 3F_{12} + 3F_{20} - 3F_{21} + F_{30}) - 5K_1(N + F_{04} + 6F_{03} + 7F_{02} - 2F_{11} - 6F_{12} - 4F_{13} + 7F_{20} - 6F_{21} \\
& + 6F_{22} + 6F_{30} - 4F_{31} + F_{40})
\end{aligned}
$$

$$
\begin{aligned}
K_6 =\ & N - 120K_1^6 + F_{06} + 15F_{05} + 65F_{04} + 90F_{03} + 31F_{02} - 2F_{11} - 30F_{12} - 80F_{13} - 45F_{14} - 6F_{15} + 31F_{20} - 30F_{21} \\
& + 30F_{22} + 30F_{23} + 15F_{24} + 90F_{30} - 80F_{31} + 30F_{32} - 20F_{33} + 65F_{40} - 45F_{41} + 15F_{42} + 15F_{50} - 6F_{51} + F_{60} \\
& + 360K_1^4(N + F_{02} - 2F_{11} + F_{20}) - 270K_1^2(N + F_{02} - 2F_{11} + F_{20})^2 + 30(N + F_{02} - 2F_{11} + F_{20})^3 - 120K_1^3(K_1 \\
& - F_{03} - 3F_{02} + 3F_{12} + 3F_{20} - 3F_{21} + F_{30}) + 120K_1(N + F_{02} - 2F_{11} + F_{20})(K_1 - F_{03} - 3F_{02} + 3F_{12} \\
& + 3F_{20} - 3F_{21} + F_{30}) - 10(K_1 - F_{03} - 3F_{02} + 3F_{12} + 3F_{20} - 3F_{21} + F_{30})^2 + 30K_1^2(N + F_{04} + 6F_{03} + 7F_{02} - 2F_{11} \\
& - 6F_{12} - 4F_{13} + 7F_{20} - 6F_{21} + 6F_{22} + 6F_{30} - 4F_{31} + F_{40}) - 15(N + F_{02} - 2F_{11} + F_{20})(N + F_{04} + 6F_{03} \\
& + 7F_{02} - 2F_{11} - 6F_{12} - 4F_{13} + 7F_{20} - 6F_{21} + 6F_{22} + 6F_{30} - 4F_{31} + F_{40}) - 6K_1(K_1 - F_{05} - 10F_{04} - 25F_{03} - 15F_{02} \\
& + 15F_{12} + 20F_{13} + 5F_{14} + 15F_{20} - 15F_{21} - 10F_{23} + 25F_{30} - 20F_{31} + 10F_{32} + 10F_{40} - 5F_{41} + F_{50}).
\end{aligned}
$$

$$N = \langle N_1 \rangle + \langle N_2 \rangle \qquad K_1 = \langle N_1 \rangle - \langle N_2 \rangle \qquad F_{ik} = \frac{1}{p_1^i p_2^k} f_{ik}$$

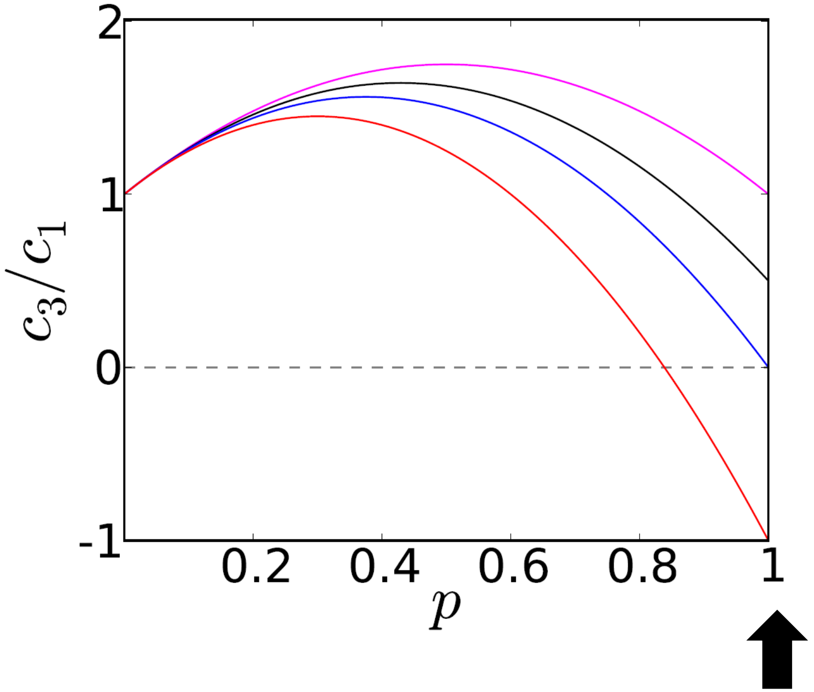
c_3/c_1 as a function of binomial parameter *p*

multiplicity distr. narrower than Poisson



K_4/K_2

multiplicity distr. broader than Poisson



K_4/K_2

$$\frac{K_4}{K_2} = -1, 0, 1/2, 1$$

Local efficiency – general expressions

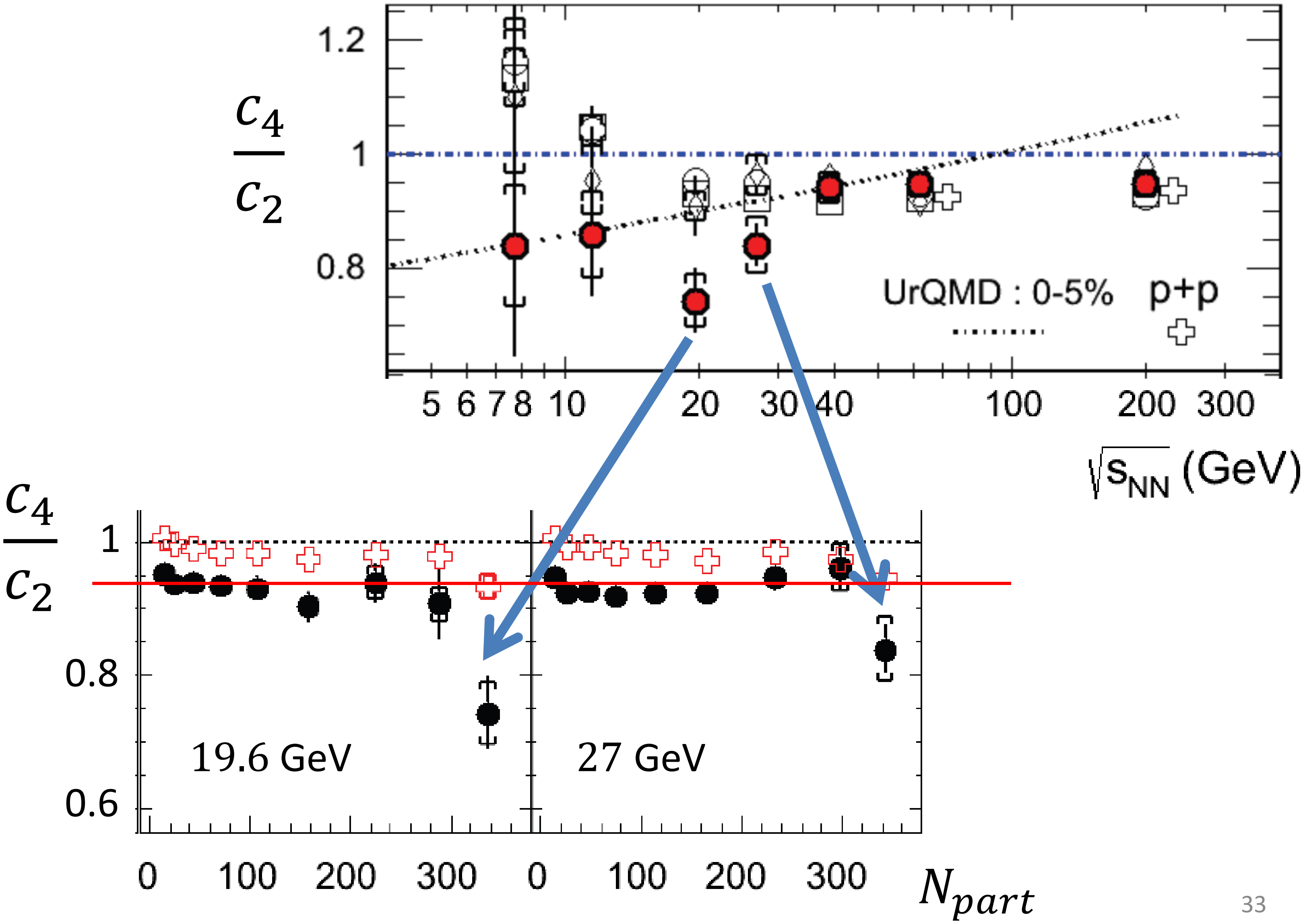
$$A_{i,k}\,(x_1,\ldots,x_i;\bar{x}_1,\ldots,\bar{x}_k) = \langle N(x_1)[N(x_2)-\delta_{x_1,x_2}]\cdots[N(x_i)-\delta_{x_1,x_i}-\cdots-\delta_{x_{i-1},x_i}]$$
$$\bar{N}(\bar{x}_1)[\bar{N}(\bar{x}_2)-\delta_{\bar{x}_1,\bar{x}_2}]\cdots[\bar{N}(\bar{x}_k)-\delta_{\bar{x}_1,\bar{x}_k}-\cdots-\delta_{\bar{x}_{k-1},\bar{x}_k}]\rangle$$

$$a_{i,k}\,(x_1,\ldots,x_i;\bar{x}_1,\ldots,\bar{x}_k) = \langle n(x_1)[n(x_2)-\delta_{x_1,x_2}]\cdots[n(x_i)-\delta_{x_1,x_i}-\cdots-\delta_{x_{i-1},x_i}]$$
$$\bar{n}(\bar{x}_1)[\bar{n}(\bar{x}_2)-\delta_{\bar{x}_1,\bar{x}_2}]\cdots[\bar{n}(\bar{x}_k)-\delta_{\bar{x}_1,\bar{x}_k}-\cdots-\delta_{\bar{x}_{k-1},\bar{x}_k}]\rangle$$

$$a_{i,k} = \epsilon(x_1)\cdots\epsilon(x_i)\bar{\epsilon}(\bar{x}_1)\cdots\bar{\epsilon}(\bar{x}_k)A_{i,k}$$

$$F_{i,k} = \sum_{x_1,\ldots,x_i}\sum_{\bar{x}_1,\ldots,\bar{x}_k}\frac{a_{i,k}\,(x_1,\ldots,x_i;\bar{x}_1,\ldots,\bar{x}_k)}{\epsilon(x_1)\ldots\epsilon(x_i)\bar{\epsilon}(\bar{x}_1)\ldots\bar{\epsilon}(\bar{x}_k)}$$

STAR results for c_4/c_2. The minimum is present only in the most central collisions.



Peculiar centrality dependence only for 19.6 and 27 GeV