Higher moments of particle distributions in infinite hadronic matter

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HGS-HIRe for FAIR Helmholtz Graduate School for Hadron and Ion Research
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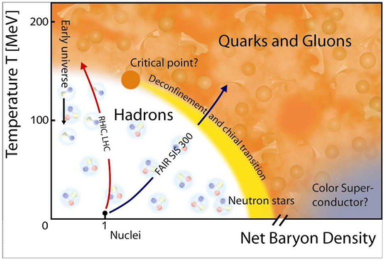
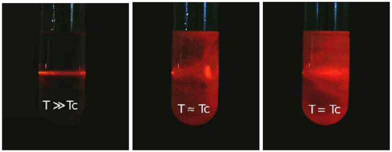
Fluctuations and phase transitions

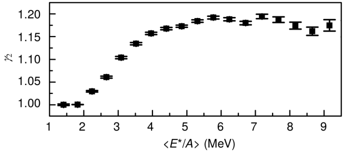
- In physical chemistry
- Nuclear liquid-gas transition
  - $\gamma_2 = \frac{m_2 m_0}{m_1^2}$
    m_i - i-th moment of charge multiplicity, not including largest fragment
  - $NVE = \frac{\sigma^2_{E_{kin}/A}}{\langle E_{kin}/A \rangle}$

Ma et al., Phys.Rev. C69 (2004) 031604

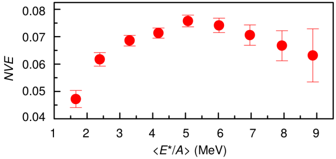
- Heavy ion collisions







γ_2 of the QP system formed in Ar+Ni as a function of excitation energy. From TAMU data.



NVE of the QP system formed in Ar+Ni as a function of excitation energy in our TAMU data.

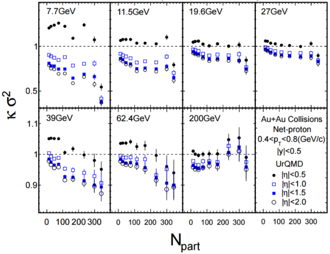
Fluctuations in heavy ion collisions

- What fluctuates?
  - Multiplicities
  - Energy
  - Conserved charges
- Challenges
  - Volume fluctuations
  - Conservation laws effects
  - Kinematic cuts
  - Detector efficiencies
  - Finite statistics
  - "Stopping fluctuations"

Role of dynamical simulations in fluctuations studies

- Transport models provide means to get handle on:
  - Charge diffusion
  - Effects of resonances
  - Kinematic cuts
  - Conservation laws effect
- With transport models one can test new correction methods
- Transport models are already used for these purposes

J. Phys. G: Nucl. Part. Phys. 40 105104

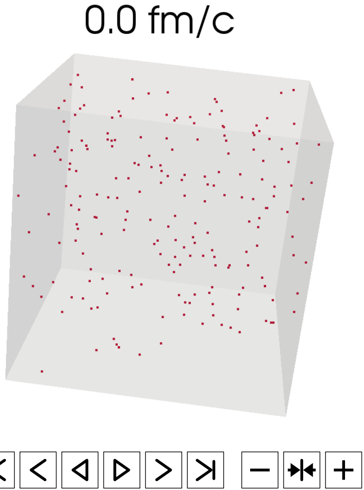


What biases can be brought be the transport model itself?

SMASH transport code

- Non-equilibrium relativistic hadron transport
- Resonances up to 2 GeV
- Geometrical collision criterion $d_{ij} \leq (\sigma_{ij}/\pi)^{1/2}$
- Parametrized resonance cross-sections, mass-dependent width
- In this presentation a simple setup is used:
  - Stable particles of one sort
  - Only elastic collisions
  - Infinite matter calculation - box with periodic boundaries

- Example:



$\sigma_{el} = 30$ mb, $V = (10 \text{ fm})^3$, $T = 0.2$ GeV

Analytical expectation for the moments of a gas in a box

- Rectangular box, volume V, M uniformly distributed particles
- Probability to find n particles in a subvolume v ($p \equiv v/V$, $q \equiv 1 - p$)

$$w(n) = C_M^n p^n q^{M-n}$$

- Central moments of binomial distribution $\mu_r \equiv \langle (n - \langle n \rangle)^r \rangle$

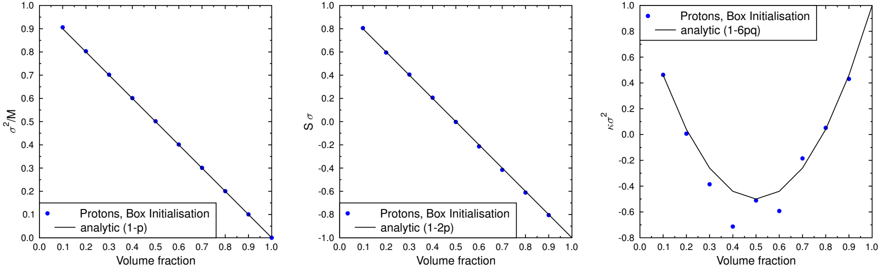
$$\mu_2 = Mpq$$
$$\mu_3 = Mpq(1 - 2p)$$
$$\mu_4 = 3M^2 p^2 q^2$$

- Variance, skewness, kurtosis

$$\sigma^2/M \equiv \mu_2/M = q$$
$$S\sigma \equiv \mu_3/\sigma^2 = 1 - 2p$$
$$\kappa\sigma^2 \equiv (\mu_4/\mu_2^2 - 3)\mu_2 = 1 - 6pq$$

Moments of gas in the box: statistical effects

Initialize box with uniformly distributed 200 protons, $N_{event} = 5 \cdot 10^6$



- Why does one need so much statistics?
- Is code result consistent with expectation? Error bars needed.
  - Simulate N_{event} events N times, compute variance of σ, S, κ?
  - What do we do for μ_1? Variance $\mu_2(\mu_1) = \mu_2$.
  - In a similar way $\mu_2(\mu_{2,3,4})$ can be expressed for an arbitrary distribution
    P. R. Rider, *Moments of moments*, Proc N.A.S., Vol. 15, 1929

On relative statistical errors of mean, σ^2, S, κ

Assume $N \equiv N_{events} \gg 1$, expressions simplify considerably.

i	$\sqrt{\mu_2(\mu_i)}/\mu_i$	$\sqrt{\mu_2(\mu_i)}/\mu_i$ for binomial
1	$\sqrt{\mu_2}/\mu_1$	$N^{-1/2}\sqrt{(1-p)/p}$
2	$N^{-1/2}\frac{\sqrt{\mu_4-\mu_2^2}}{\mu_2}$	$\sqrt{2}N^{-1/2}$
3	$N^{-1/2}\frac{\sqrt{A}}{\mu_3}$	$\sqrt{6p(1-p)}/(1-2p)$
4	$N^{-1}\frac{\sqrt{B}}{\mu_4}$	$4\sqrt{2/3}N^{-1/2}$

$A = \mu_6 - 6\mu_4\mu_2 - \mu_3^2 + 9\mu_2^3$
$B = N\mu_8 + 40\mu_2\mu_6 - 8N\mu_5\mu_3 - N\mu_4^2 - 120\mu_4\mu_2^2 + 16N\mu_3^2\mu_2 + 72\mu_2^4$

Why does skewness still converge in the simulation?
Why do we need so much statistics for kurtosis?

Moments of gas in the box: systematic effects

Procedure:

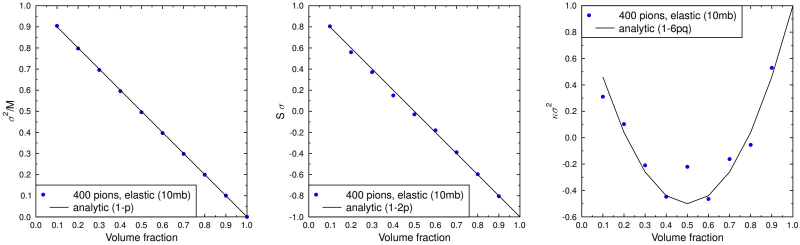
- Initialize box with pions
  - uniform in space
  - thermal momentum distribution
  - $5 \cdot 10^6$ events
  - can only interact elastically with cross-section σ
  - periodic boundaries, interaction through periodic boundaries included
- Run until momentum distribution stabilizes - 10 fm/c is enough
- Look at higher moments

How do moments change with

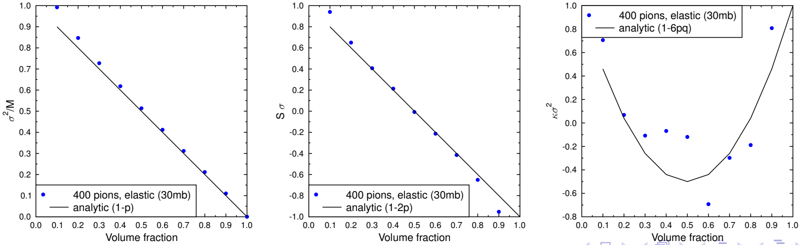
- cross-section σ
- density
- mean free path

Effect of the cross-section

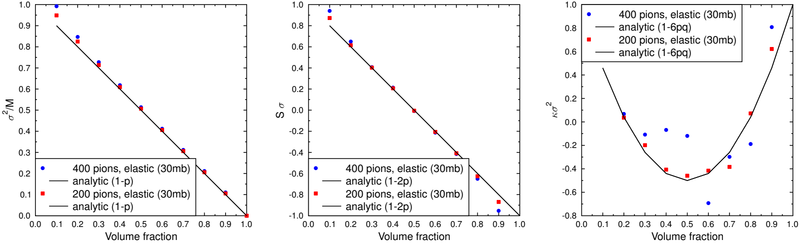
$\sigma = 10mb$



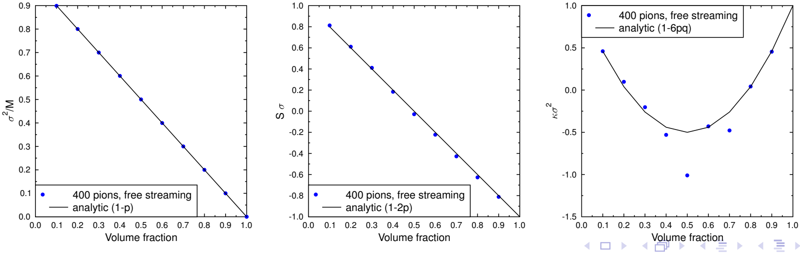
$\sigma = 30mb$: systematic deviations



Effect of density



Are these effects physical/numerical/statistical? Compare to free streaming, $\sigma = 0$.

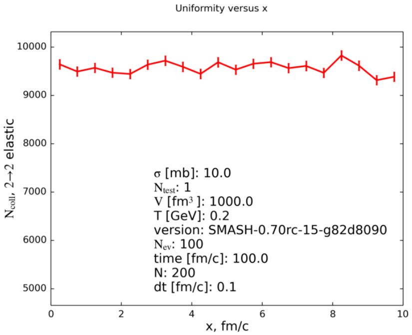


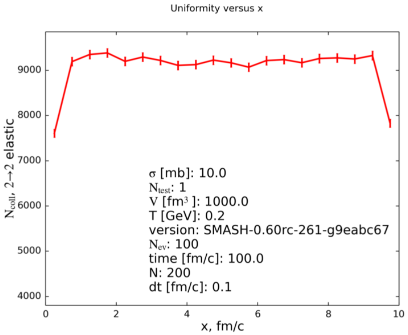
Summary

- Box with elastically interacting particles was simulated with transport code
  - Higher moments extremely sensitive to statistics
  - Systematical effects of interactions on variance and skewness are seen, increasing with σ and particle density
- Work on errors of higher moments in progress
- Future plans
  - Systematic study of mean free path effect
  - Isospin diffusion in a box with protons, neutrons and resonances
  - π-ρ system with charge conservation
  - Effects of kinematic cuts

Backup: SMASH box

Scattering through boundaries implemented:





With scattering through periodic boundaries

No scattering through periodic boundaries