FIAS Frankfurt Institute for Advanced Studies

HIC for FAIR Helmholtz International Center

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Fluctuations and correlations in dynamical models

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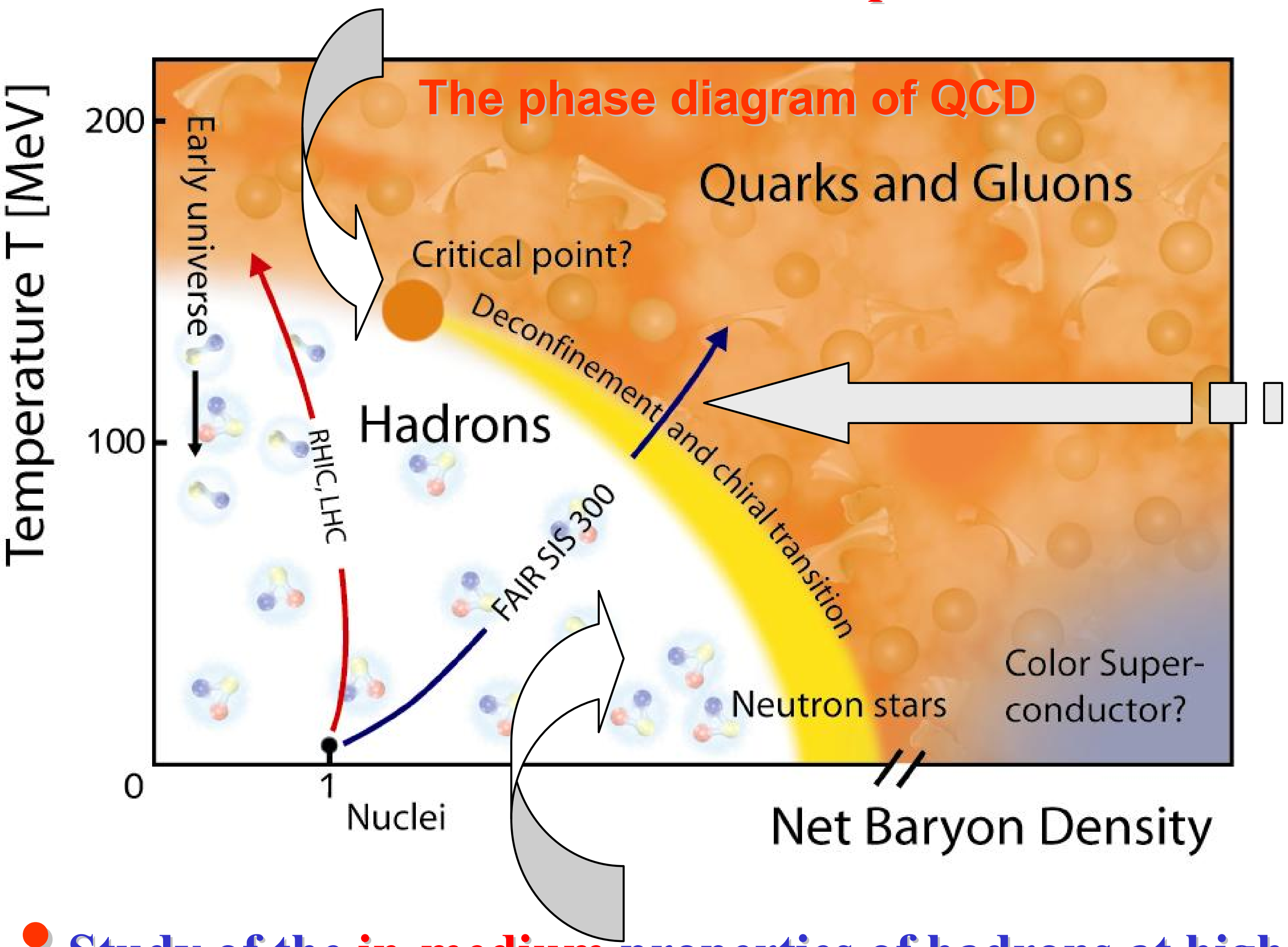
HIC for FAIR Workshop on Fluctuation and Correlation Measures in Nuclear Collisions 2015

The holy grail of HIC

- Search for the critical point



- Study of the phase transition from hadronic to partonic matter – Quark-Gluon-Plasma

- Study of the in-medium properties of hadrons at high baryon density and temperature
- Study of the partonic medium beyond the phase boundary

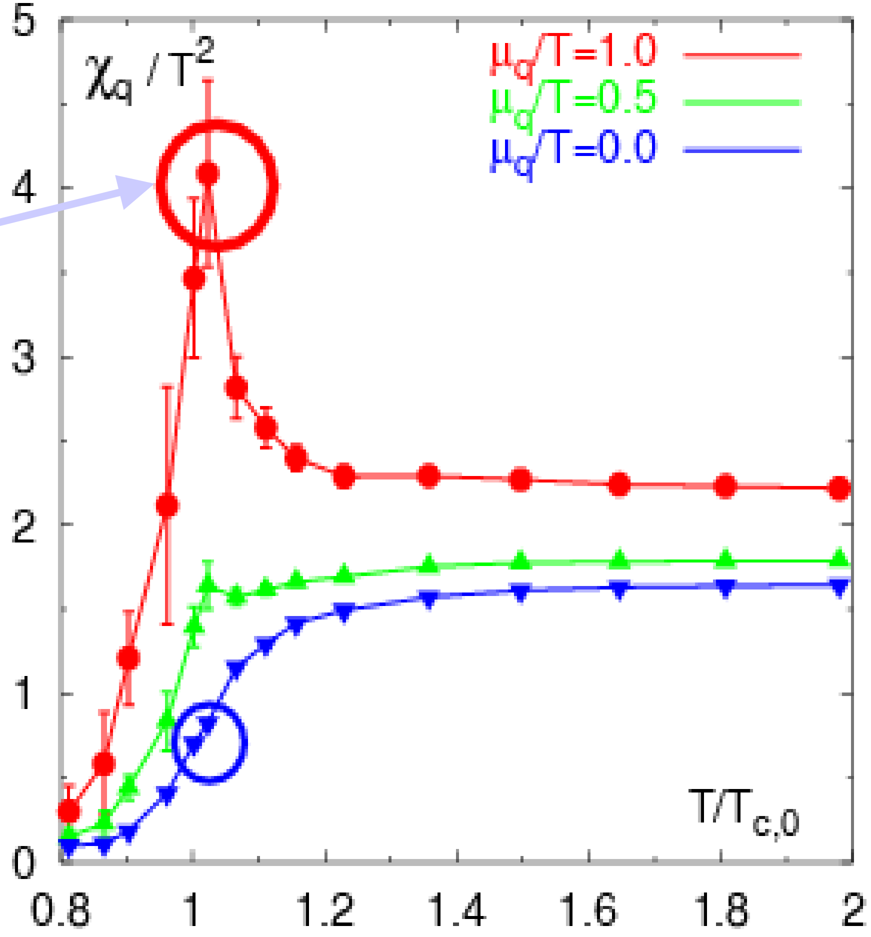
Lattice QCD: Critical Point

Fluctuations of the quark number density (susceptibility) at $\mu_q>0$

[F. Karsch et al.]

$$\frac{\chi_q}{T^2} = \left[\frac{\partial^2}{\partial(\mu_q/T)^2} \frac{P}{T^4} \right]_{T_{fixed}}$$

Lattice QCD predictions:
χ_q (quark number density fluctuations) will diverge at the critical chiral point =>



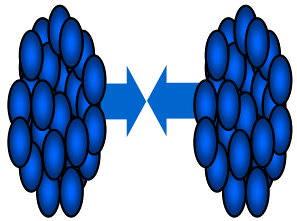
Experimental observation – look for non-monotonic behavior of the observables near the critical point :

- baryon number fluctuations
- charge number fluctuations
- multiplicity fluctuations
- particle ratio fluctuations (K/π, K/p, ...)
- mean p_T fluctuations
- 2 particle correlations
- ...

"Background" Fluctuations

Many factors lead to "background" fluctuations that can mask the signal of the critical point and therefore have to be carefully studied and accounted for:

- limited size of the colliding system
- fluctuations of initial conditions in heavy-ion collisions
- event-by-event fluctuations of the collision geometry
- experimental acceptance
- statistical fluctuations
- …



In order to understand the "background" fluctuations we apply models, where no phase transition is implemented :

- wounded nucleon model
- statistical model of hadron-resonance gas
- transport models HSD and UrQMD
- …

cf. review: Konchakovski et al., J. Phys. G37 (2010) 073101

Study of fluctuations within transport models

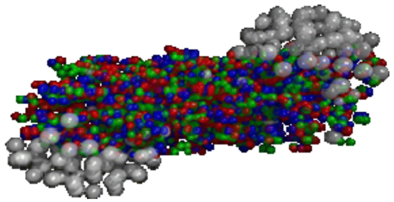
HSD – **H**adron-**S**tring-**D**ynamics transport approach

UrQMD – **U**ltra-**r**elativistic-**Q**uantum-**M**olecular-**D**ynamics

Transport models allow to study:

- statistical and dynamical fluctuations
- event-by-event analysis - similar to the experiment
- the centrality dependence
- the energy dependence of fluctuations
- the influence of the experimental acceptance

on the final results on fluctuations

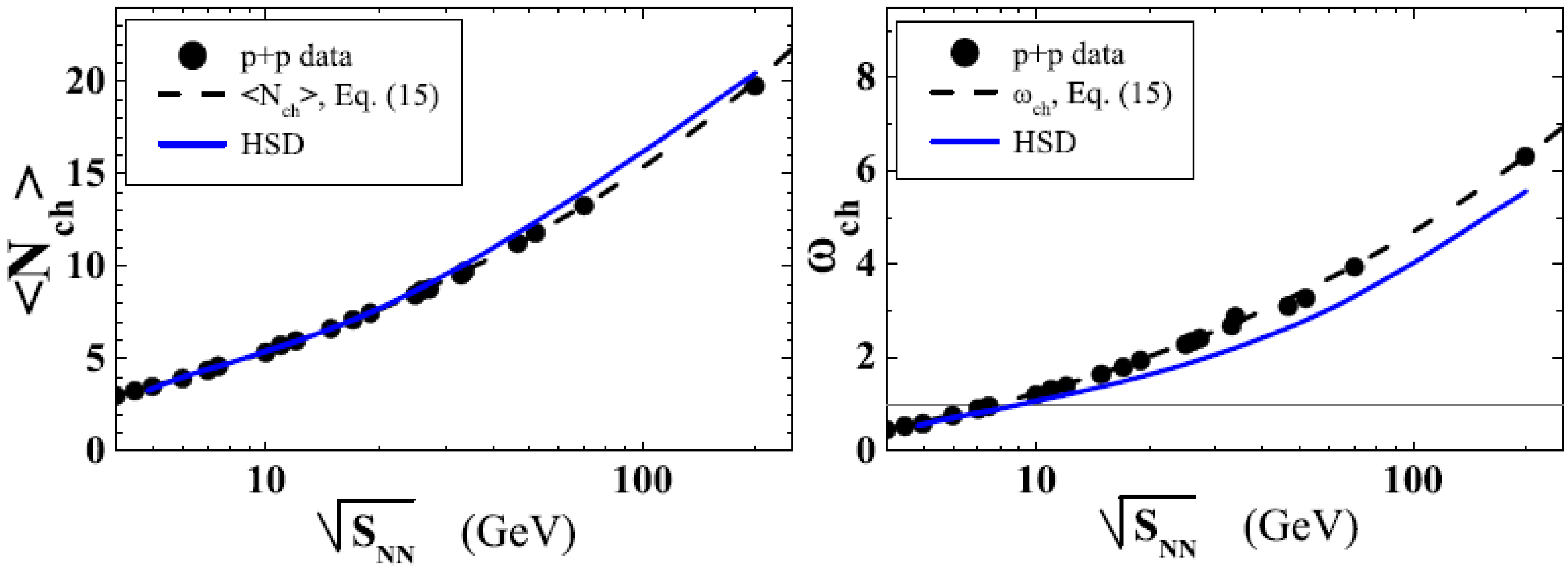


cf. review: Konchakovski et al., J. Phys. G37 (2010) 073101

Multiplicity fluctuations in p+p

- Scaled variance - multiplicity fluctuations in some acceptance (charge, strangeness, etc.):

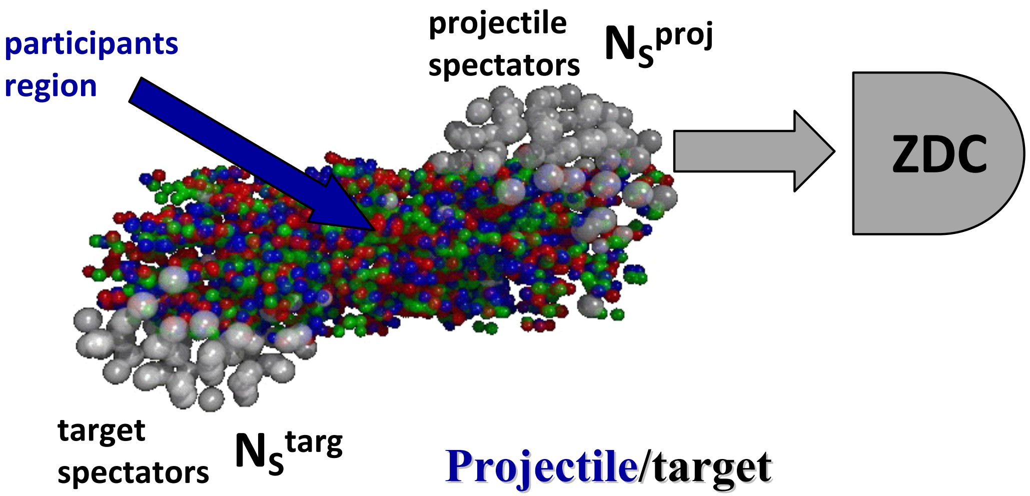
$$\omega = \frac{Var(N)}{\langle N \rangle} = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$



- The excitation functions of N_{ch} and charge multiplicity fluctuations ω_{ch} from HSD are approximately in line with experimental data

Fluctuations in the number of participants

Fixed-target experiment:



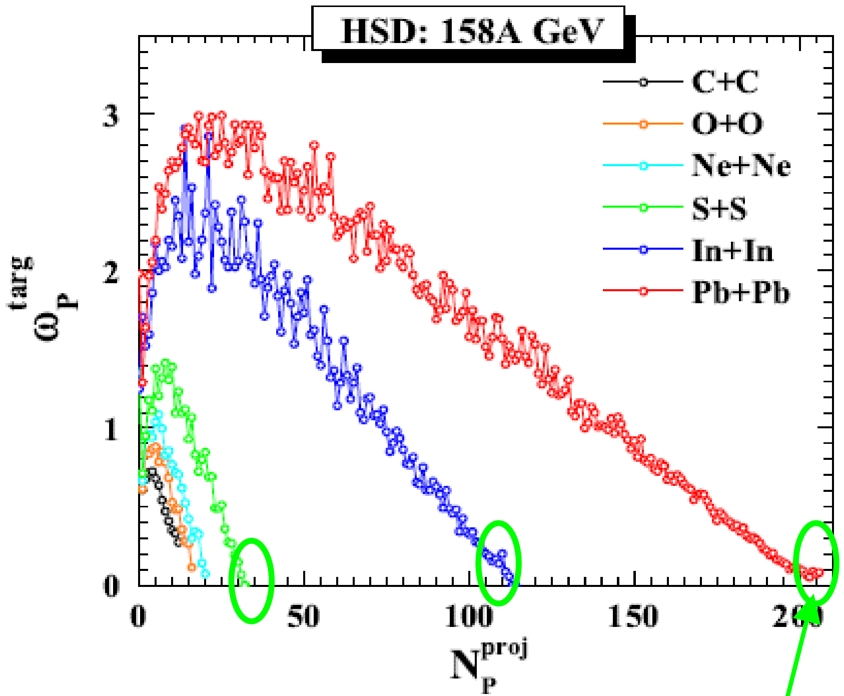
Projectile/target participants:

$N_p^{proj} = A - N_s^{proj}$

$N_p^{targ} = A - N_s^{targ}$

Even for a fixed number of projectile participants N_p^{proj} the full number of participants N_p can fluctuate due to participant fluctuations in the target N_p^{targ}

Participant number fluctuations are reflected in the observable fluctuations (e. g. multiplicity fluctuations)



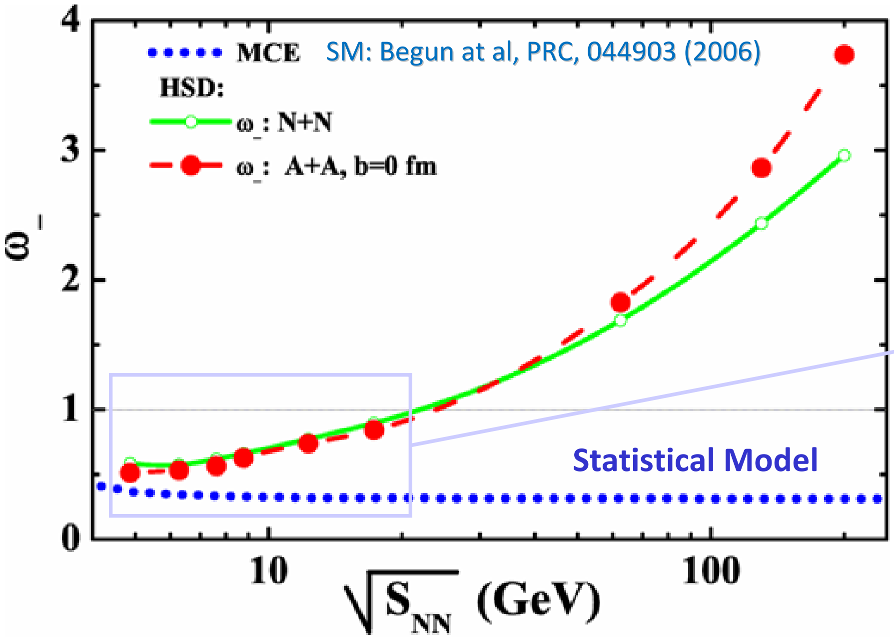
To get rid of the fluctuations in the participant number one needs to consider only the most central collisions!

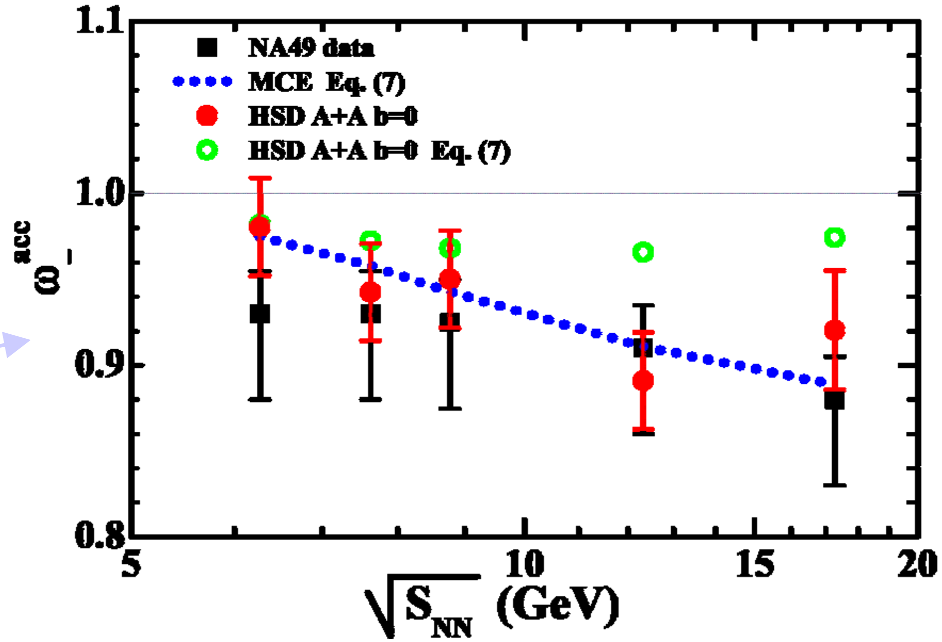
Konchakovski et al., Phys. Rev. C73 (2006) 034902; C78 (2008) 024906

Multiplicity fluctuations in N+N and central A+A

Konchakovski, Gorenstein, Bratkovskaya, Phys. Lett. B 651, 114 (2007)

- ❑ Fluctuations in p+p and central A+A are very close within HSD due to the small participant number fluctuation in central A+A
- ❑ Statistical model shows very small and energy independent fluctuations and contradicts to the transport calculations where ω reaches significant values for large energies

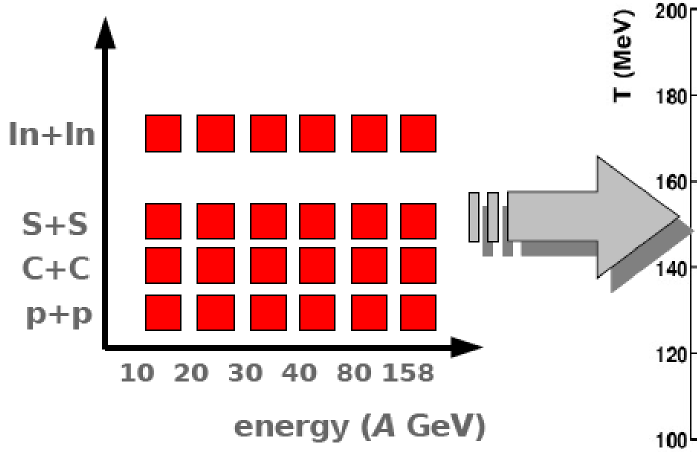


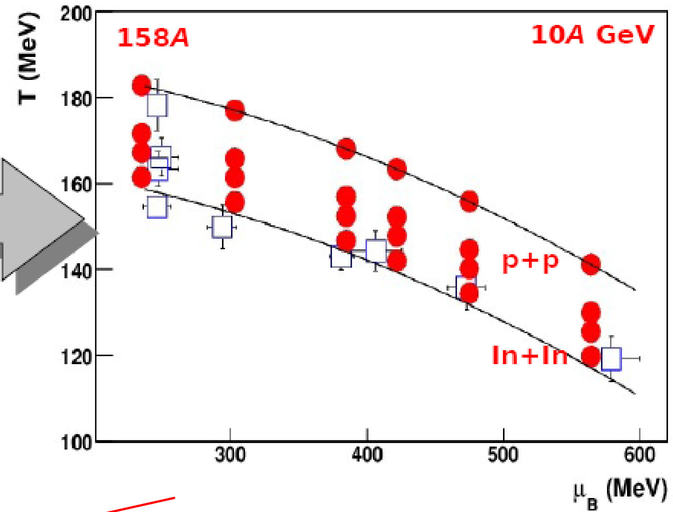


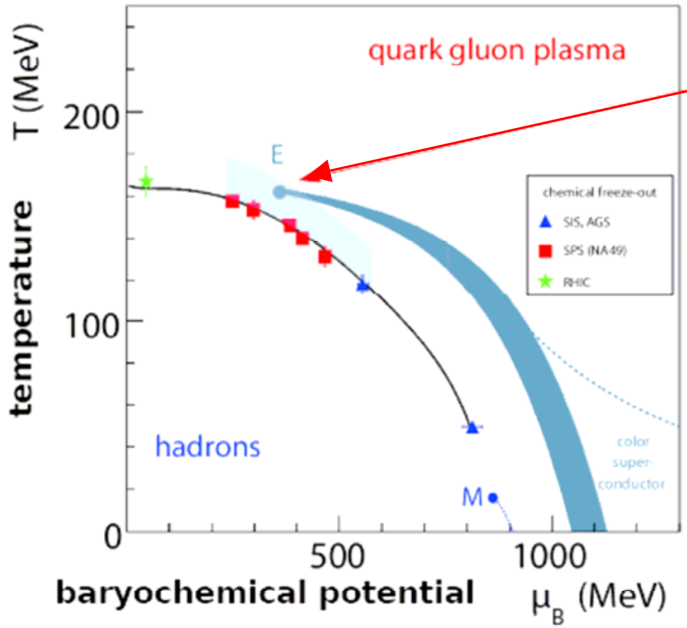
- ❑ NA49 ➔ one cannot clearly distinguish between statistical and transport models because of small acceptance and small differences between the model predictions in this range of energy

Fluctuation program of NA61/SHINE Collaboration

NA61/SHINE Collaboration provides a comprehensive energy and system size scan of the phase diagram at the CERN SPS







- **The critical point should lead to an increase of multiplicity fluctuations in the two dimensional plane:**

(E, A) energy - system size

or equivalently

(T,μ_B) temperature - baryon-chemical potential

Gazdzicki, PoS CPOD2006:016

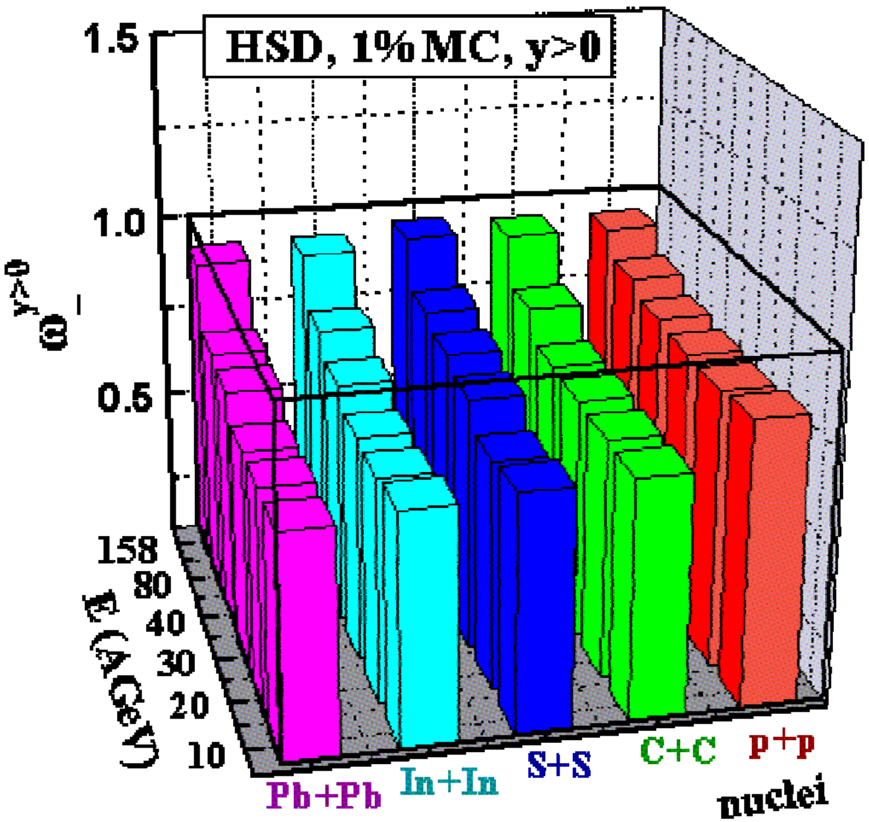
Fluctuations and CP: Stephanov, Rajagopal, Shuryak, Phys. Rev. D 60, 114028

Freeze-out points: Becattini et al., Phys. Rev. C 73, 044905

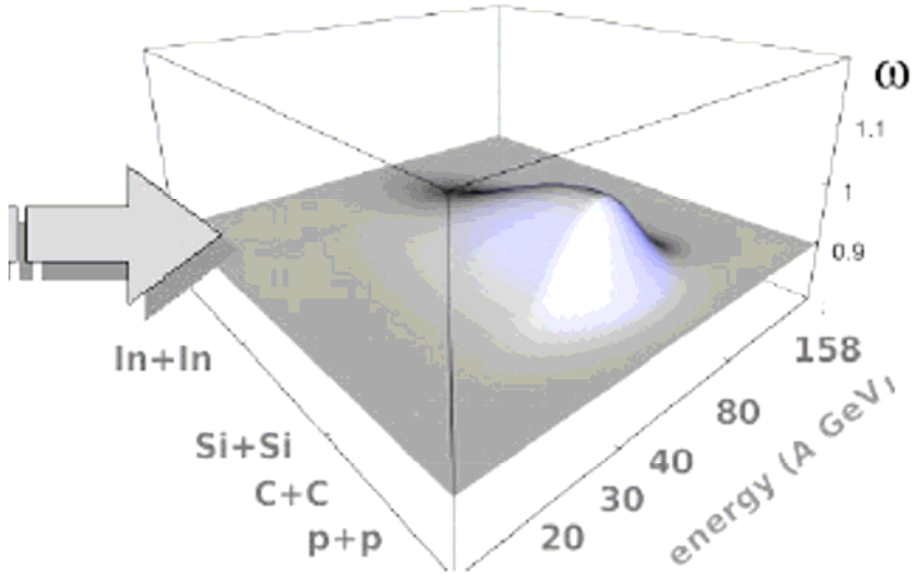
Multiplicity fluctuations for 1%MC

Konchakovski, Lungwitz, Gorenstein, Bratkovskaya, Phys. Rev. C78 (2008) 024906

rapidity y>0



❑ Multiplicity fluctuations for 1%MC practically do not **depend on atomic mass for y>0** and only slightly grow with increasing collision energy.



Gazdzicki, PoS CPOD2006:016

❑ **HSD and UrQMD show a plateau** on top of which the SHINE Collaboration expects to find increasing multiplicity fluctuations as a "signal" for the critical point !

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Konchakovski, Lungwitz, Gorenstein, Bratkovskaya, Phys. Rev. C78 (2008) 024906

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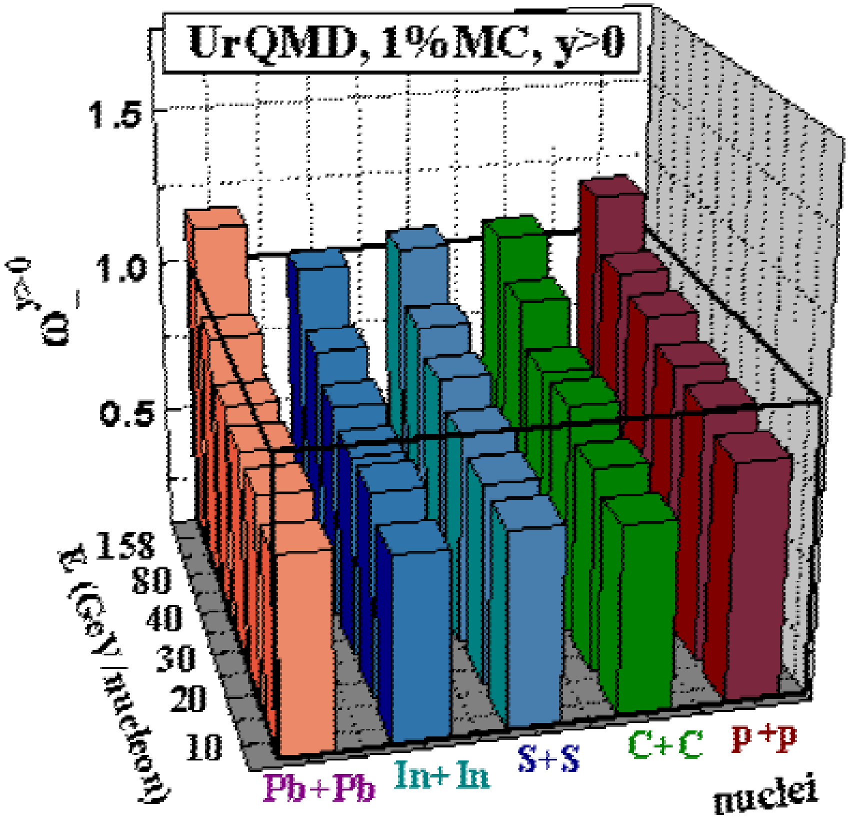

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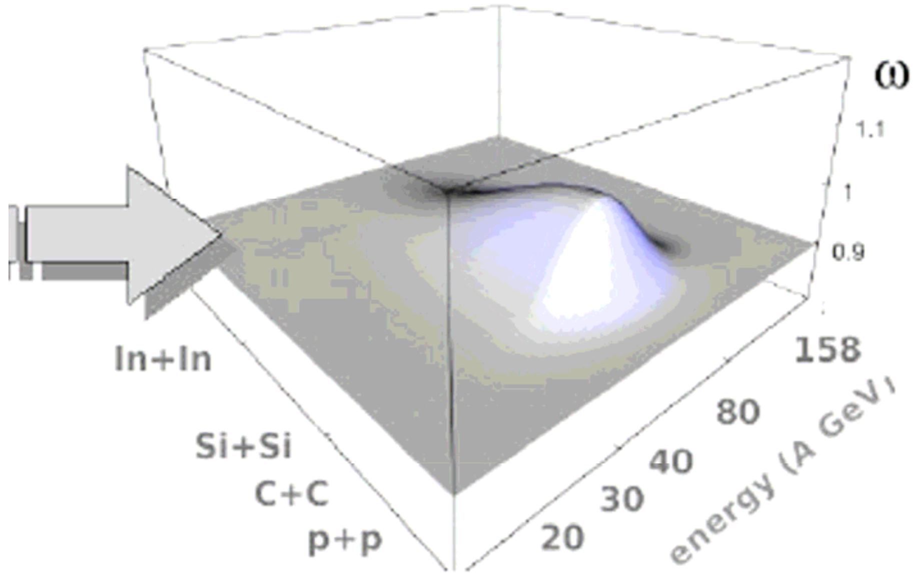

Gazdzicki, PoS CPOD2006:016

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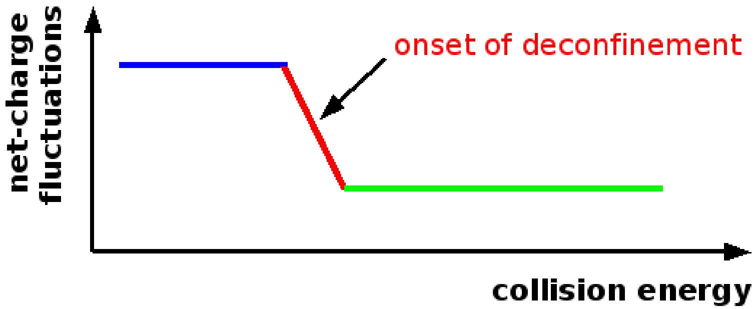
Charge fluctuations

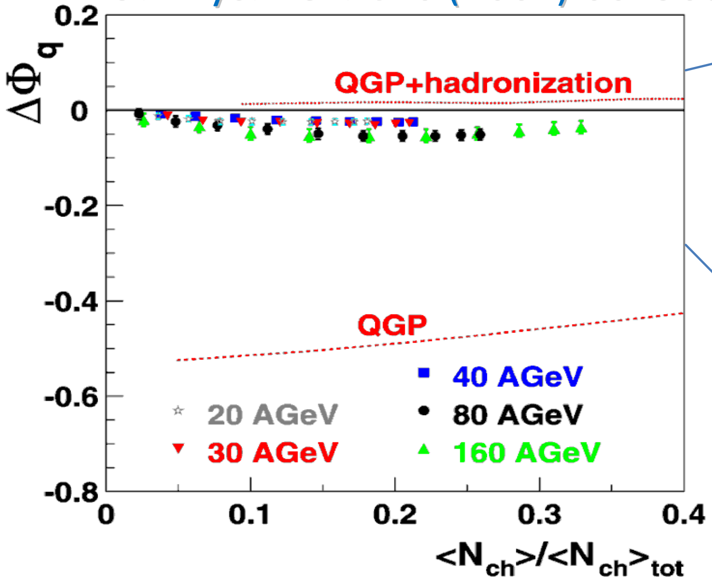
❑ sensitive to the **EoS** at the early stage of the collision and to its changes in the deconfinement phase transition region



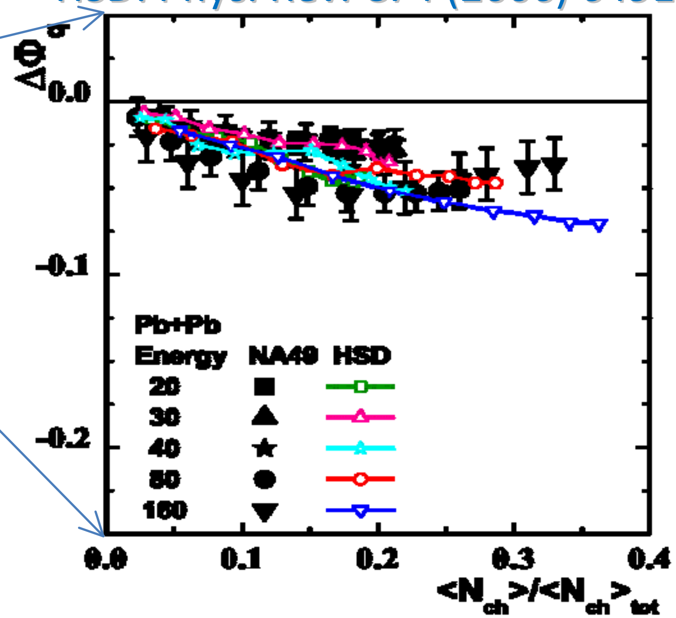
Jeon, Koch, PRL85 (2000) 2076
Asakawa, Heinz, Muller PRL85 (2000) 2072

❑net-charge fluctuations are smaller in QGP than in a hadron gas

NA49: Phys. Rev. C70 (2004) 064903



HSD: Phys. Rev. C74 (2006) 64911



❑The **decay of resonances** strongly modifies the initial QGP fluctuations!

Event-by-event particle ratio fluctuations

- **Ratio fluctuations:** $$\sigma^2 \equiv \frac{\langle \Delta (N_A/N_B)^2 \rangle}{\langle N_A/N_B \rangle^2}$$

In assumption $|\Delta N_A| \ll \langle N_A \rangle, \quad |\Delta N_B| \ll \langle N_B \rangle$

it can be rewritten as: $$\sigma^2 \cong \frac{\omega_A}{\langle N_A \rangle} + \frac{\omega_B}{\langle N_B \rangle} - 2\rho_{AB} \left[\frac{\omega_A \omega_B}{\langle N_A \rangle \langle N_B \rangle} \right]^{1/2}$$

with correlation parameter $$\rho_{AB} \equiv \frac{\langle \Delta N_A \Delta N_B \rangle}{\left[\langle (\Delta N_A)^2 \rangle \langle (\Delta N_B)^2 \rangle \right]^{1/2}}$$

- **In GCE for ideal Boltzman gas:**

$$\omega_A = \omega_B = 1 \text{ and } \rho_{AB} = 0 \Rightarrow \sigma^2 = \frac{1}{\langle N_A \rangle} + \frac{1}{\langle N_B \rangle}$$

- **Dynamical fluctuations:**

after subtraction of σ for mixed events one gets:

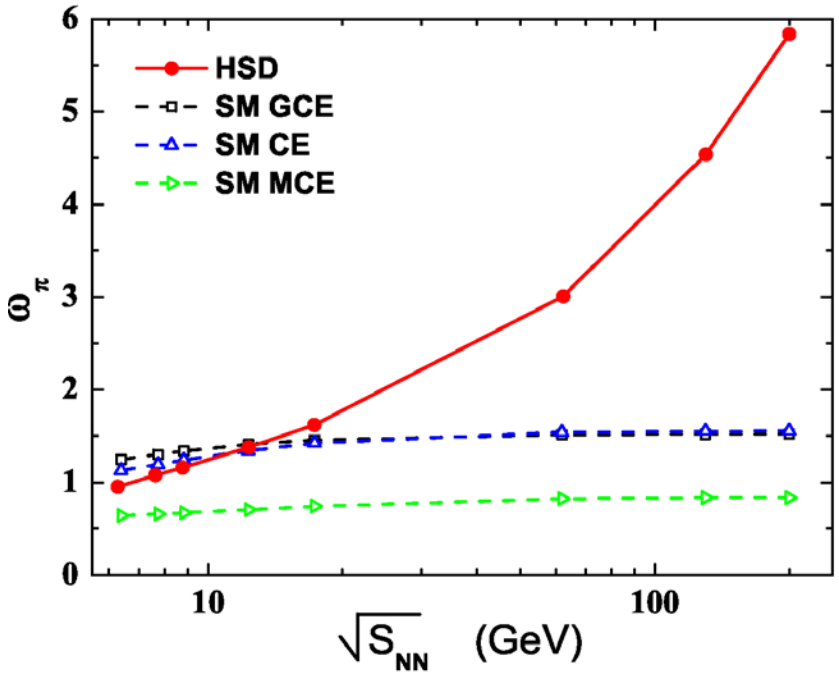
$$\sigma_{dyn} \equiv \mathrm{sign}\left(\sigma^2 - \sigma^2_{mix}\right) \left|\sigma^2 - \sigma^2_{mix}\right|^{1/2}$$

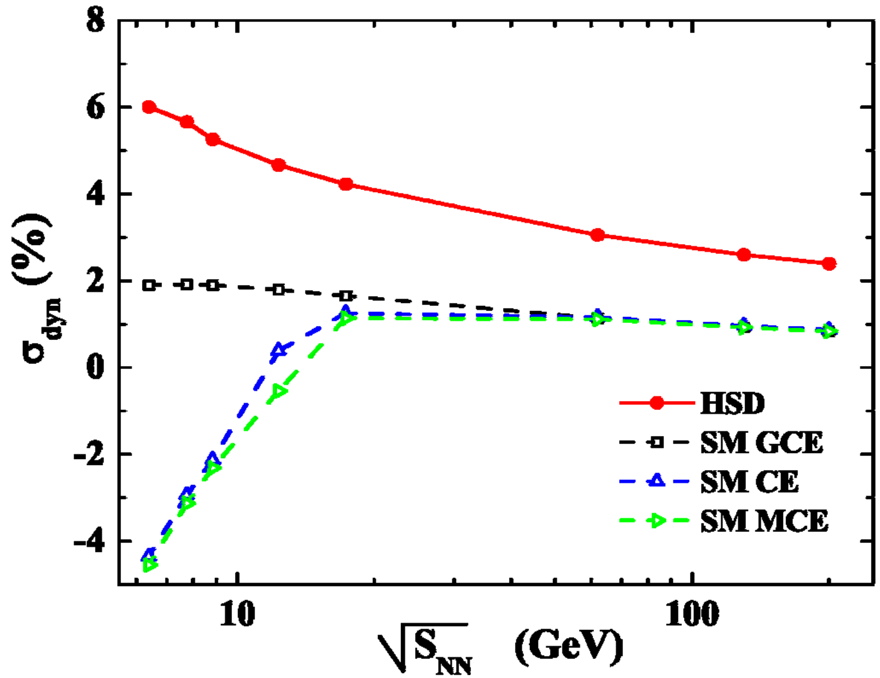
- **ν_{dyn} fluctuations:**

independent of fluctuations of the number of participants

$$\nu_{\mathrm{dyn},K\pi} = \frac{\langle N_K (N_K - 1) \rangle}{\langle N_K \rangle^2} + \frac{\langle N_\pi (N_\pi - 1) \rangle}{\langle N_\pi \rangle^2} - 2 \frac{\langle N_K N_\pi \rangle}{\langle N_K \rangle \langle N_\pi \rangle}$$

Statistical and HSD Model Results for Ratio Fluctuations



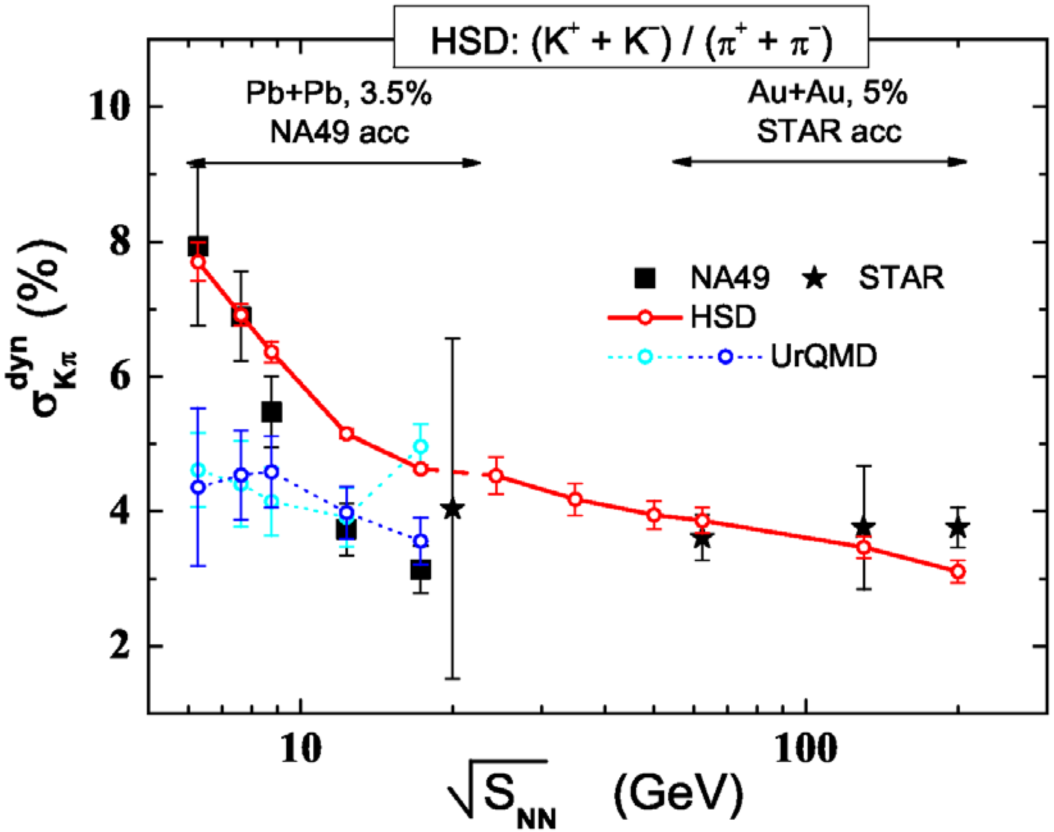


- Large difference in SM and the transport model predictions for ω_π with increasing energy!

- For σ_{dyn} SM and HSD differ at low energies in contrast to ω!

Gorenstein, Hauer, Konchakovski, Bratkovskaya, Phys. Rev. C 79(2009) 024907

K/π-ratio fluctuations: Transport models vs Data

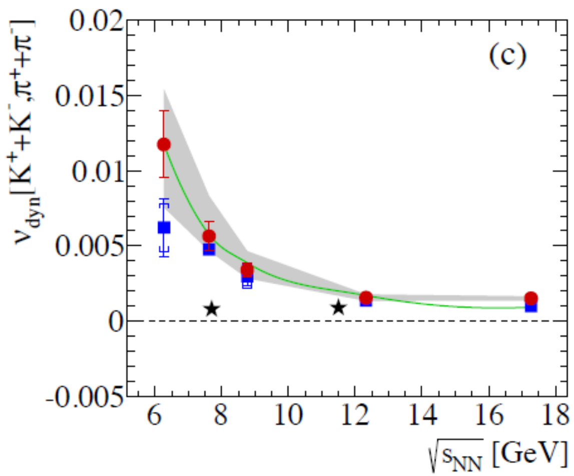


- **Exp. data show a plateau from top SPS up to RHIC energies and an increase towards lower SPS energies**
  ➔ evidence for a critical point at low SPS energies ?

- **but the HSD (without QGP!) results shows the same behavior ➔**
  K/p-ratio fluctuation is driven by hadronic sources

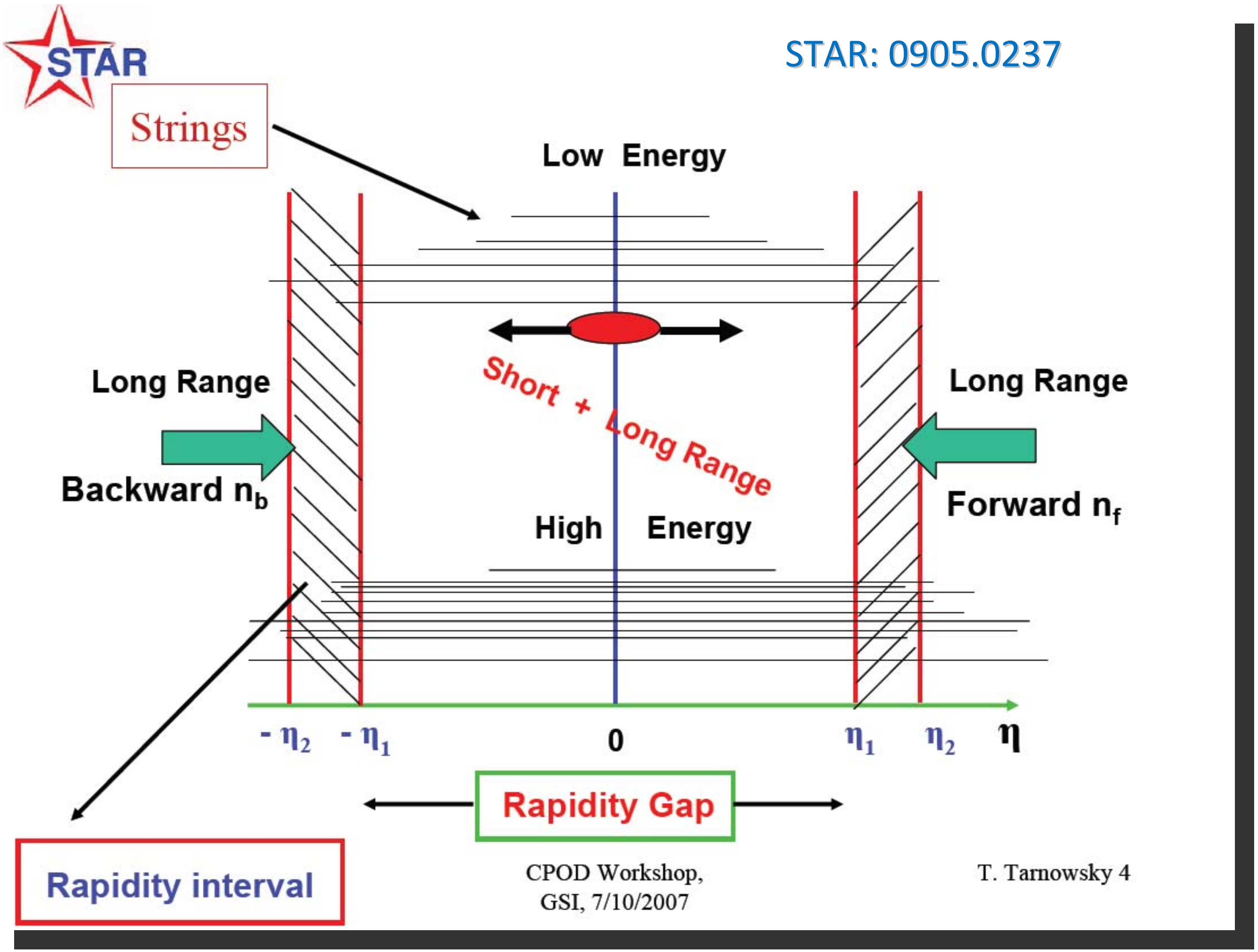
! K/p ratio fluctuation is sensitive to the acceptance!

Cf. talk by Anar Rustamov

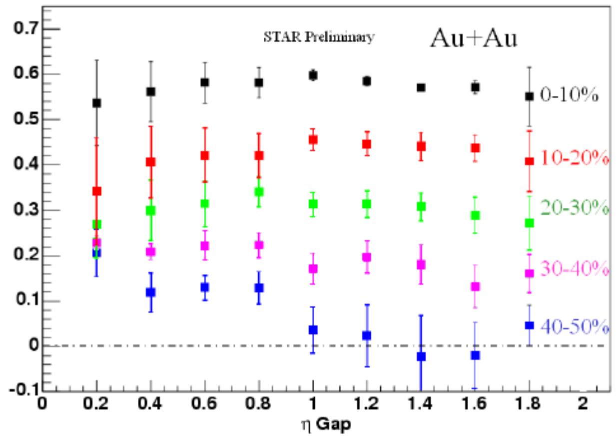


HSD: Phys. Rev. C 79 (2009) 024907
UrQMD: J. Phys. G 30 (2004) S1381, PoS CFRNC2006,017
NA49: 0808.1237
STAR: 0901.1795

Forward-Backward Correlations in Nucleus-Nucleus Collisions: Baseline Contributions from Geometrical Fluctuations?



Au+Au √s = 200 GeV



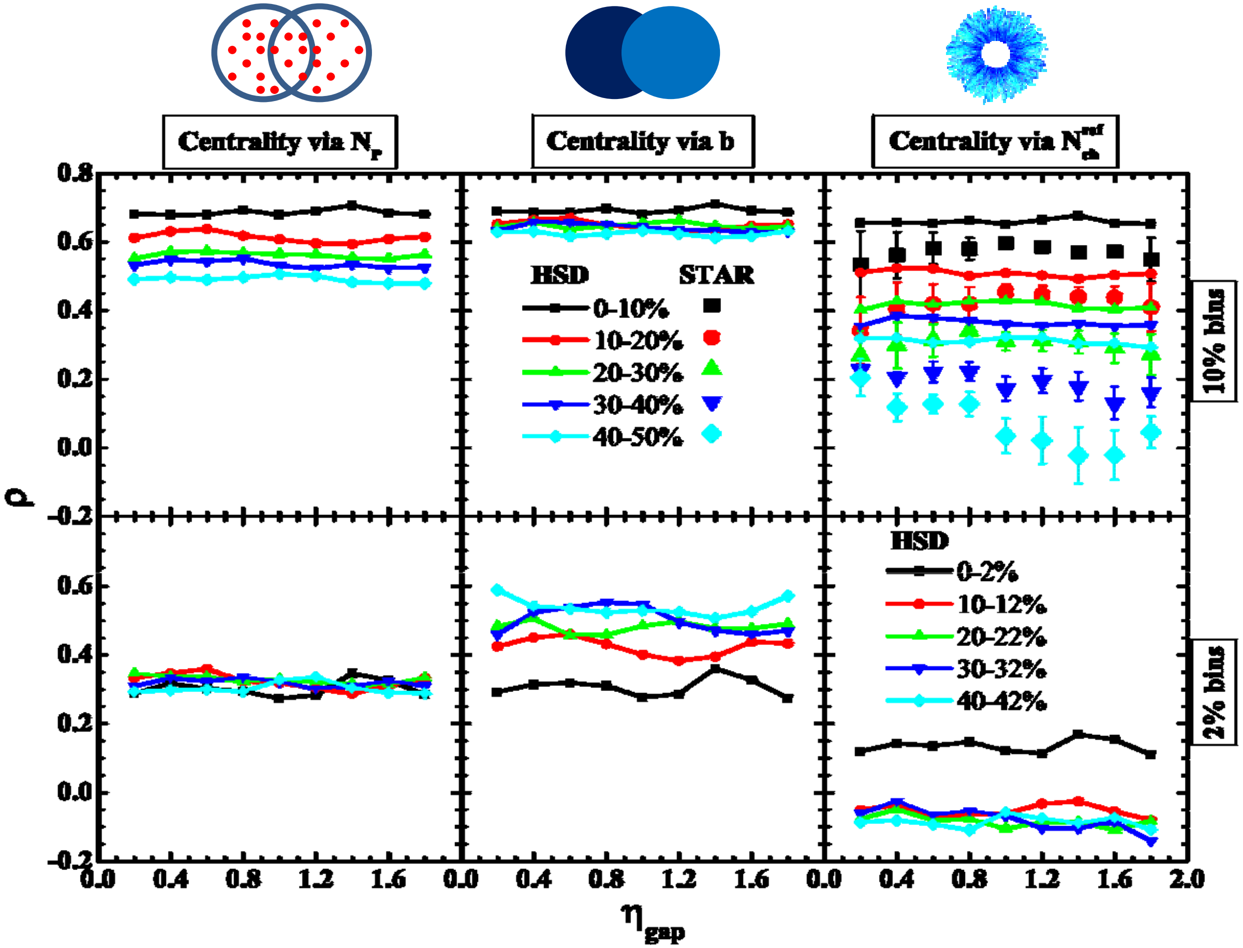
STAR:
More central collision -> higher correlations

Interpretation:

- **Dual Partonic Model?**
- **CGC?**
- **Geometrical fluctuations?**

$$\rho_{fb} \equiv \frac{\langle \Delta N_f \cdot \Delta N_b \rangle^{\eta gap}}{\sqrt{\langle (\Delta N_f)^2 \rangle^{\eta gap} \ \langle (\Delta N_b)^2 \rangle^{\eta gap}}}$$

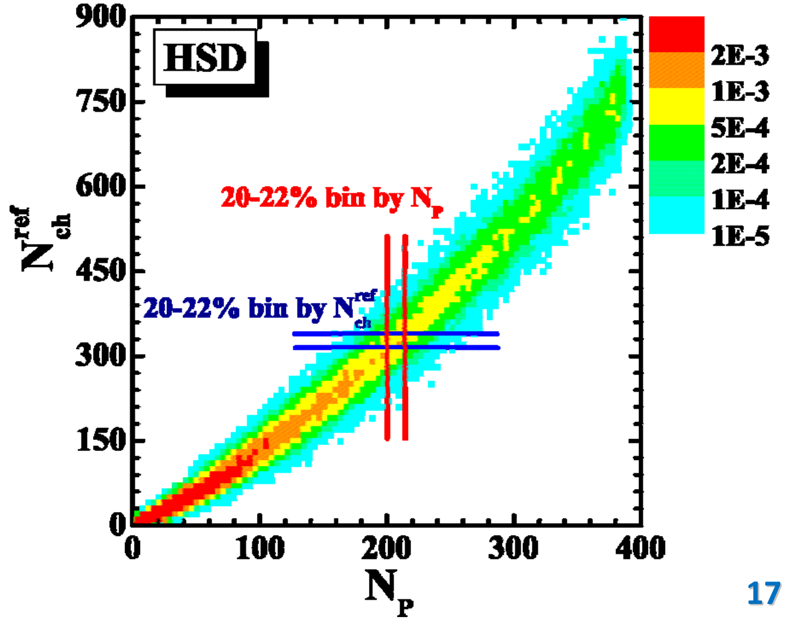
Forward-Backward Correlations: Au+Au 200 GeV



• Correlation coefficient strongly depends on centrality definition

• When decreasing the width of centrality bins the FB correlation becomes weaker

• Different centrality definitions lead to different event samples in the same centrality class. This is crucial for small centrality bins!



Konchakovski, Hauer, Torrieri, Gorenstein, Bratkovskaya, PRC79, 034910 (2009)

Summary I

- The fluctuations in the number of target participants – for fixed projectile participants - strongly influence all observable fluctuations

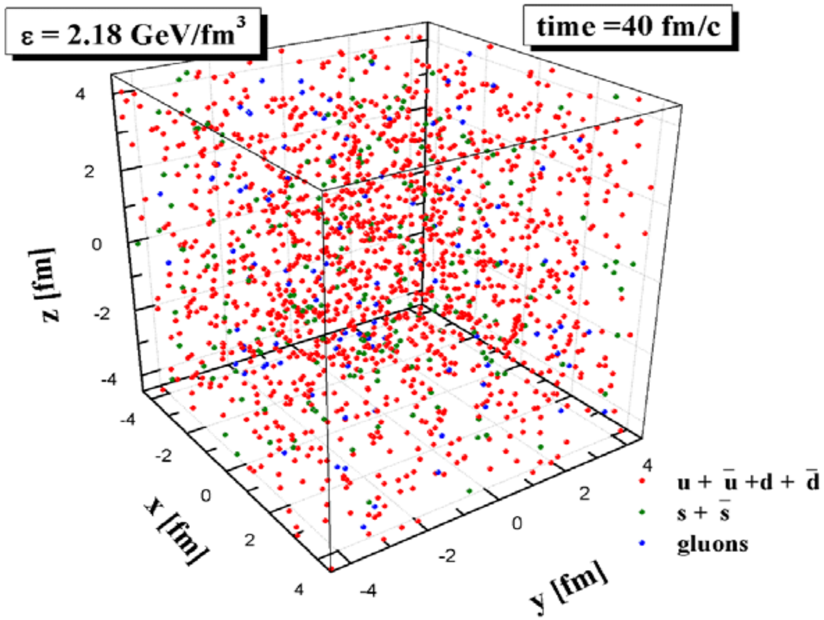
- Statistical and transport models show different results in central A+A collisions for multiplicity fluctuations versus energy. To distinguish between models ➔ exp. bin energy scan

- Transport models show a smooth energy and atomic number dependence for the multiplicity fluctuations.
  Thus, the hadron-string models (without explicit phase transition!) demonstrate that the expected enhanced fluctuations - attributed to the critical point and phase transition - may be observed experimentally on top of a monotonic and smooth 'background'

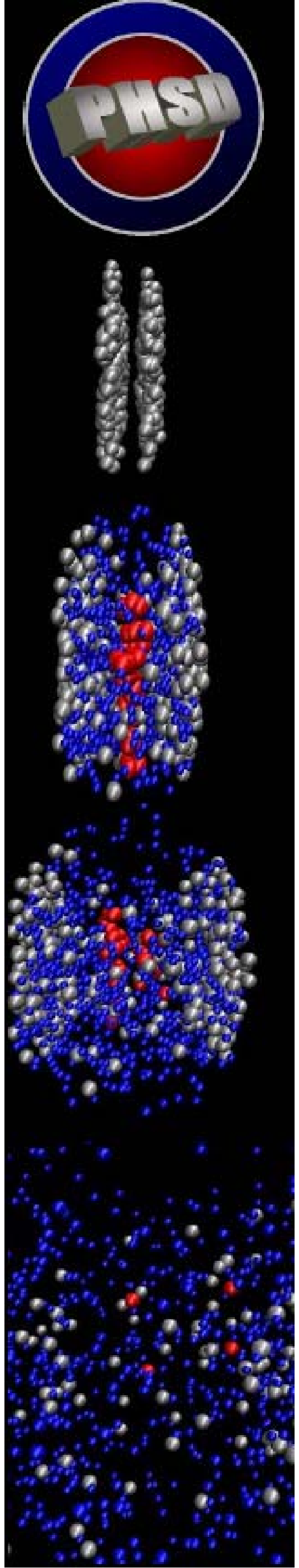
- HSD results for the K/π ratio fluctuations show that it grows at low SPS energies similar to the NA49 data;
  strong sensitivity to exp.acceptance!

- Forward-backward correlations show a large sensitivity on the initial collisional geometry and centrality bin definition!

Fluctuations in-equilibrium QGP using PHSD



Parton-Hadron-String-Dynamics (PHSD)



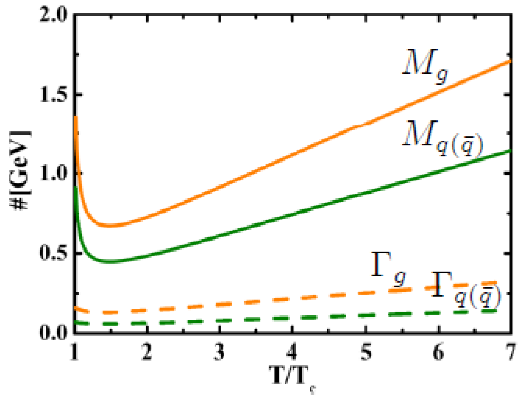
PHSD is a **non-equilibrium transport model** with

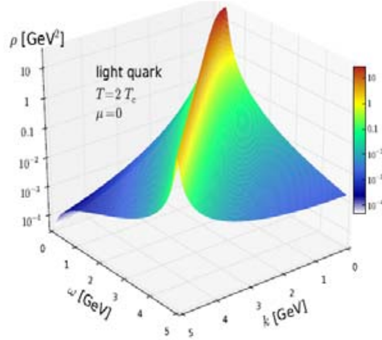
- explicit **phase transition** from hadronic to partonic degrees of freedom
- **lQCD EoS** for the partonic phase (‚crossover' at μ_q=0)
- explicit **parton-parton interactions** - between quarks and gluons
- dynamical **hadronization**

❑ **QGP phase is described by the Dynamical QuasiParticle Model (DQPM) matched to reproduce lattice QCD**

A. Peshier, W. Cassing, PRL 94 (2005) 172301;
W. Cassing, NPA 791 (2007) 365: NPA 793 (2007)

- **strongly interacting quasi-particles:** massive quarks and gluons (g,q,q_{bar}) with sizeable collisional widths in self-generated **mean-field potential**





- **Spectral functions:**

$$\rho_i(\omega,T) = \frac{4\omega\Gamma_i(T)}{\left(\omega^2 - \vec{p}^2 - M_i^2(T)\right)^2 + 4\omega^2\Gamma_i^2(T)} \quad (i = q,\bar{q},g)$$

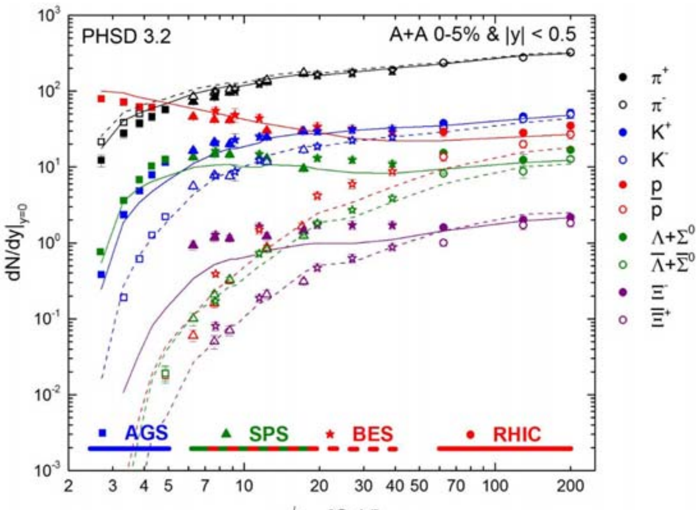
❑ **Transport theory**: **generalized off-shell transport equations** based on the 1st order gradient expansion of Kadanoff-Baym equations (**applicable for strongly interacting system!**)

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; W. Cassing, EPJ ST 168 (2009) 3

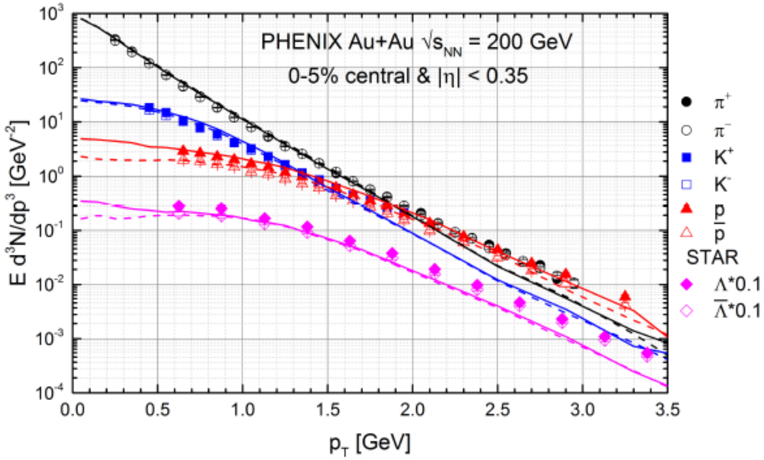
Description of A+A with PHSD

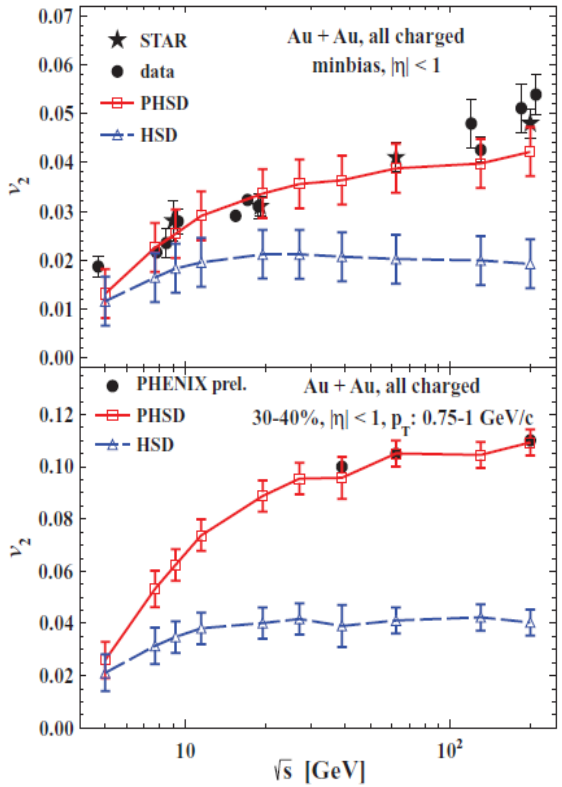
Important: to be conclusive on charm observables, the **light quark dynamics** must be well under control!

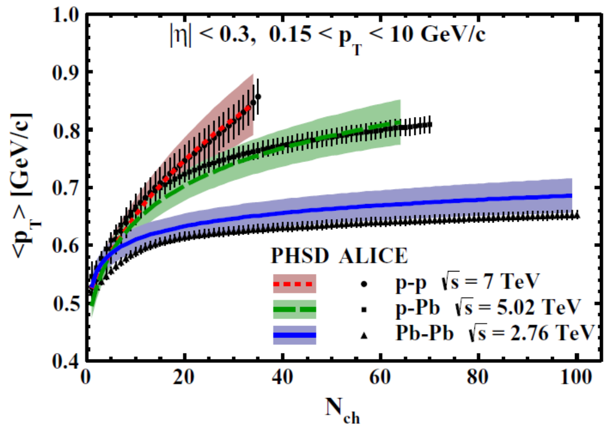
Cf. talk by Pierre Moreau, Tu, 16:20; Alessia Palmese, Fr, 16:00

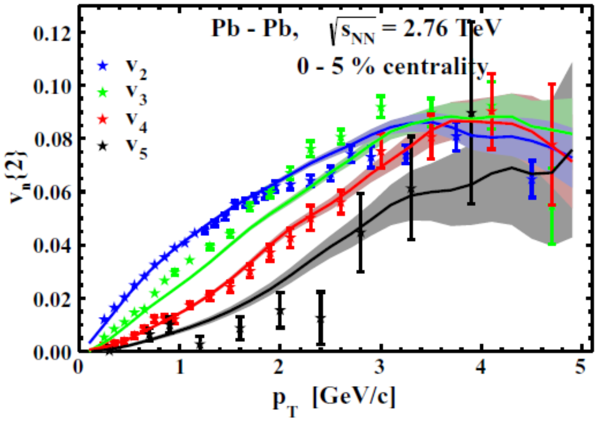


PHSD: P. Moreau









V. Konchakovski et al.,
PRC 85 (2012) 011902; JPG42 (2015) 055106

PHSD provides a **good description of ‚bulk‘ observables** (y-, p_T-distributions, flow coeficients v_n, …) from SPS to LHC

Properties of parton-hadron matter in-equilibrium

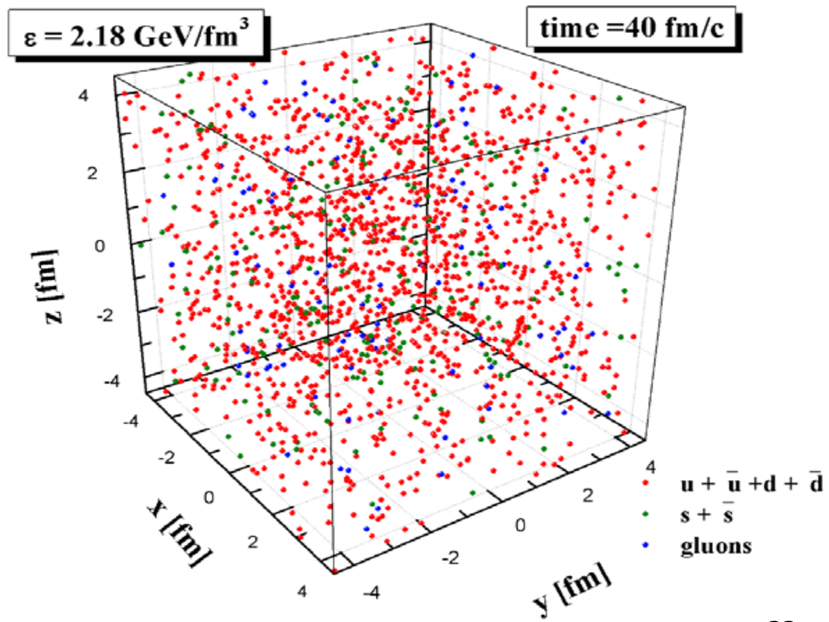
V. Ozvenchuk et al., PRC 87 (2013) 024901, arXiv:1203.4734
V. Ozvenchuk et al., PRC 87 (2013) 064903, arXiv:1212.5393

The goal:

- study of the dynamical equilibration of QGP within the non-equilibrium off-shell PHSD transport approach
- transport coefficients (shear and bulk viscosities) of strongly interacting partonic matter
- particle number fluctuations (scaled variance, skewness, kurtosis)

Realization:

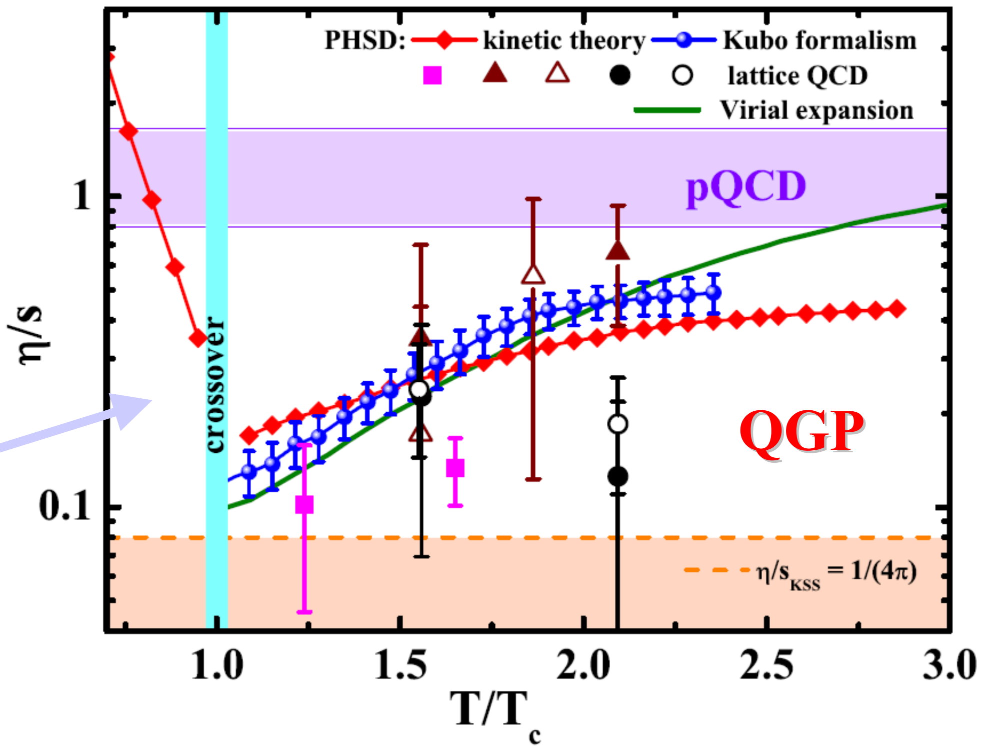
- Initialize the system in a finite box with periodic boundary conditions with some energy density ε and chemical potential μ_q
- Evolve the system in time until equilibrium is achieved



Properties of parton-hadron matter – shear viscosity

η/s using **Kubo formalism** and the **relaxation time approximation** (‚kinetic theory')

- ❑ $T=T_C$: η/s shows **a minimum (~0.1)** close to the critical temperature
- ❑ $T>T_C$: QGP - **pQCD limit** at higher temperatures
- ❑ $T<T_C$: fast increase of the ratio η/s for **hadronic matter**→
  - lower interaction rate of hadronic system
  - smaller number of degrees of freedom (or entropy density) for hadronic matter compared to the QGP



Virial expansion: S. Mattiello, W. Cassing, Eur. Phys. J. C 70, 243 (2010).

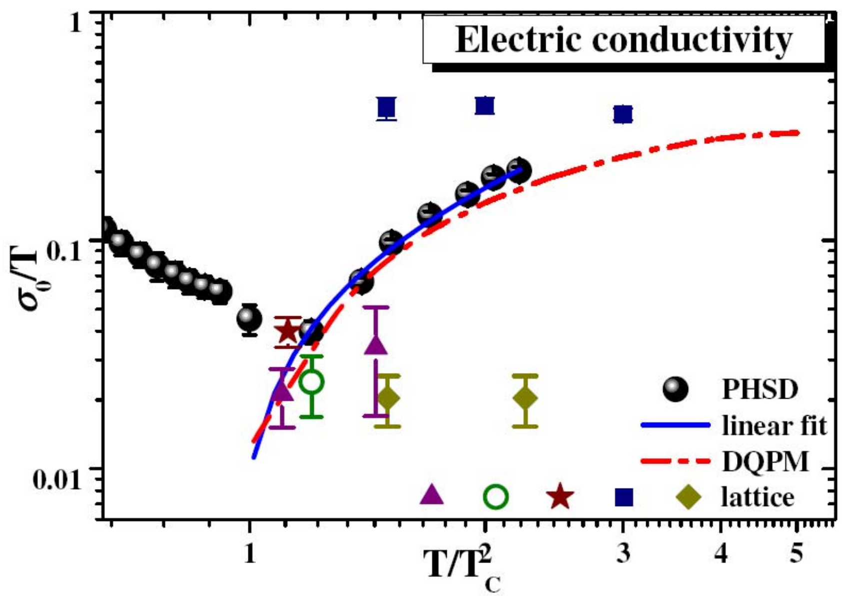
QGP in PHSD = strongly-interacting liquid

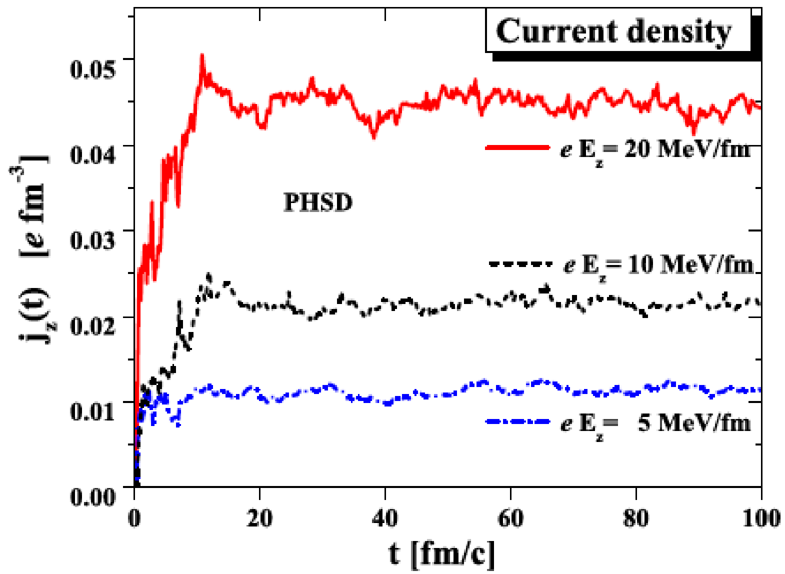
V. Ozvenchuk et al., PRC 87 (2013) 064903

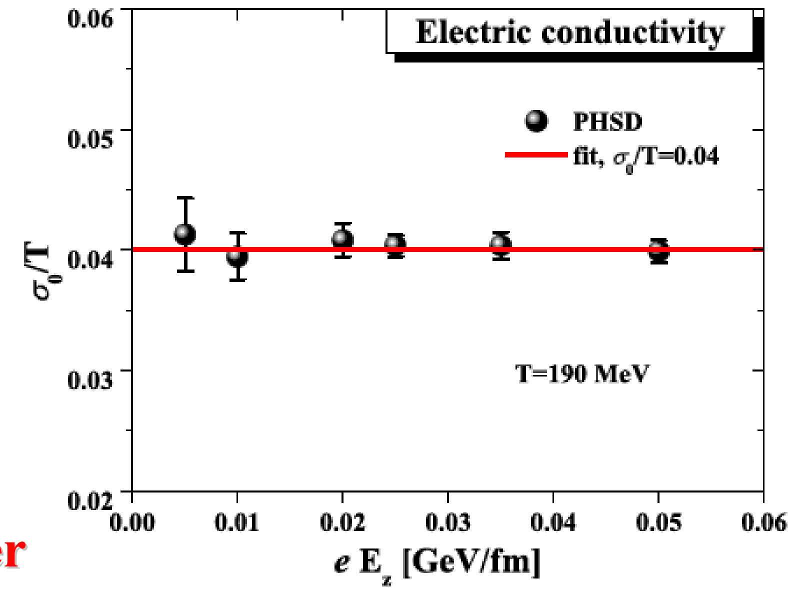
Properties of parton-hadron matter – electric conductivity

- The response of the strongly-interacting system in equilibrium to an **external electric field eE_z** defines the **electric conductivity σ_0**:

$$\frac{\sigma_0}{T} = \frac{j_{eq}}{E_z T}, \qquad j_z(t) = \frac{1}{V}\sum_j eq_j \frac{p_z^j(t)}{M_j(t)},$$







- the **QCD matter** even at $T \sim T_c$ is a **much better electric conductor than Cu or Ag** (at room temperature) by a factor of 500 !

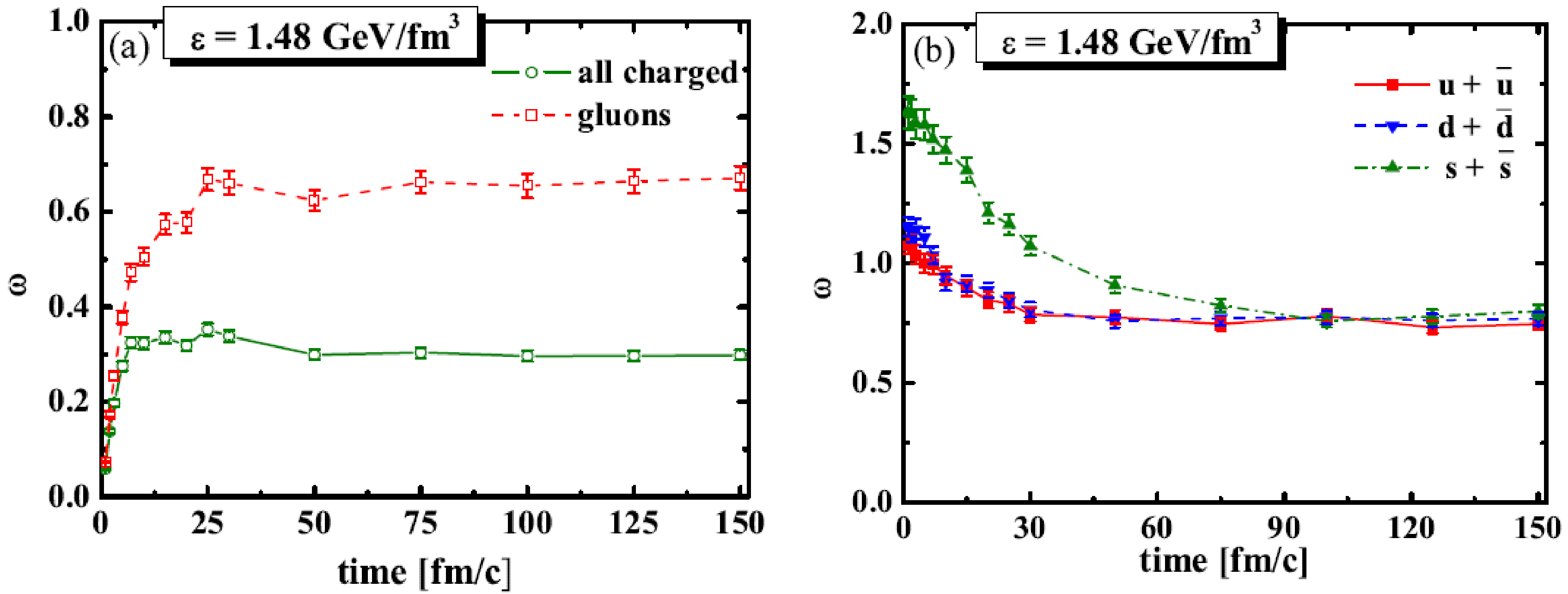
W. Cassing et al., PRL 110(2013)182301

Scaled variance

- **scaled variance:** $\omega = \dfrac{\sigma^2}{\mu}$

where μ is the mean value of the observable x averaged over N events:

σ^2 is the sample variance: $\sigma^2 = \dfrac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu)^2$ $\qquad \mu = \dfrac{1}{N}\sum_{i=1}^{N} x_i$



➔ **scaled variances reach a plateau in time for all observables**

➔ **equilibrium values are less than 1 (as in GCE) for all ω ➔ MCE**

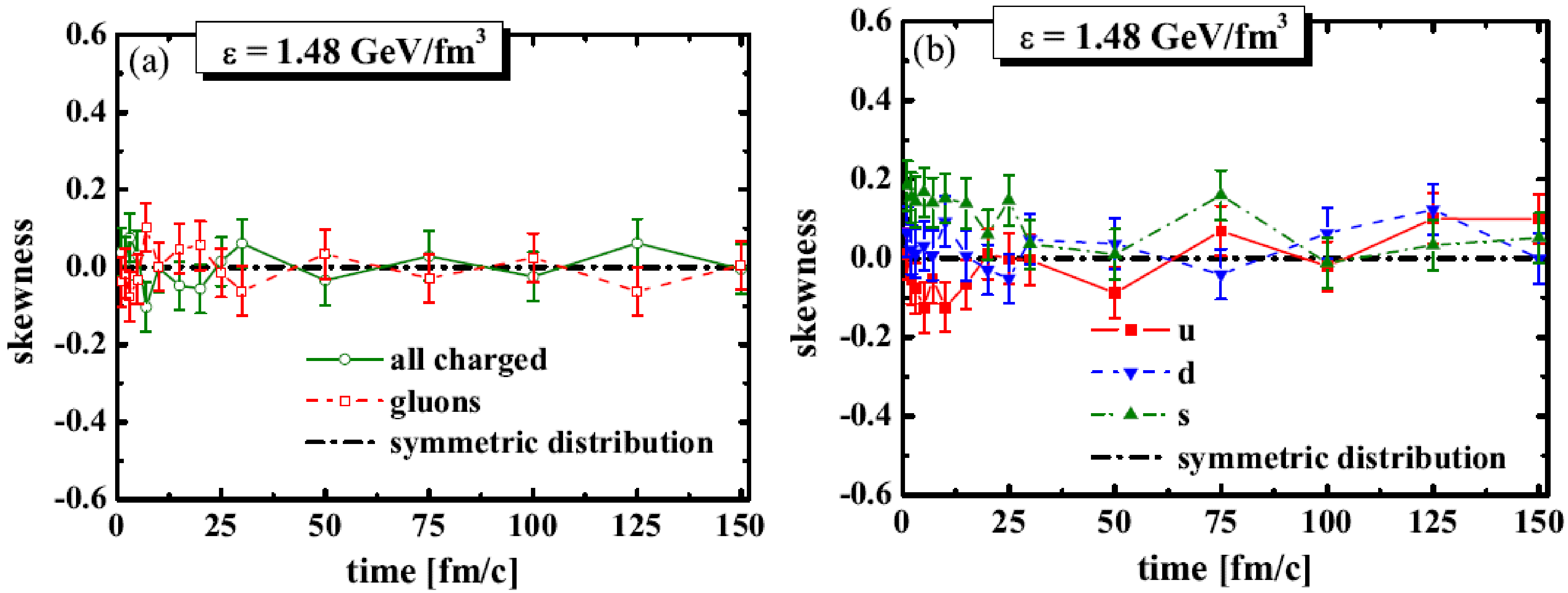
➔ **particle number fluctuations are flavor blind**

V. Ozvenchuk et al., PRC 87 (2013) 024901, arXiv:1203.4734

Skewness

- skewness $g_1 = \frac{m_3}{m_2^{3/2}} = \frac{m_3}{\sigma^3}$, $m_3 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^3$

- skewness characterizes the asymmetry of the distribution function with respect to its average value

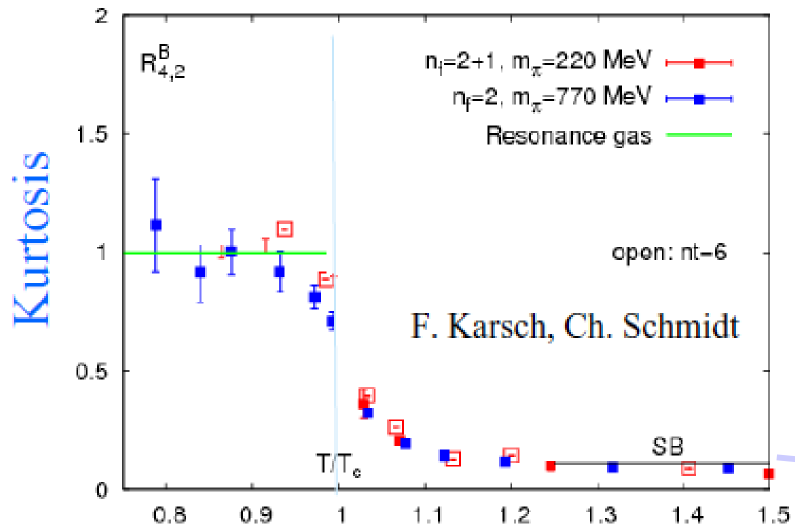


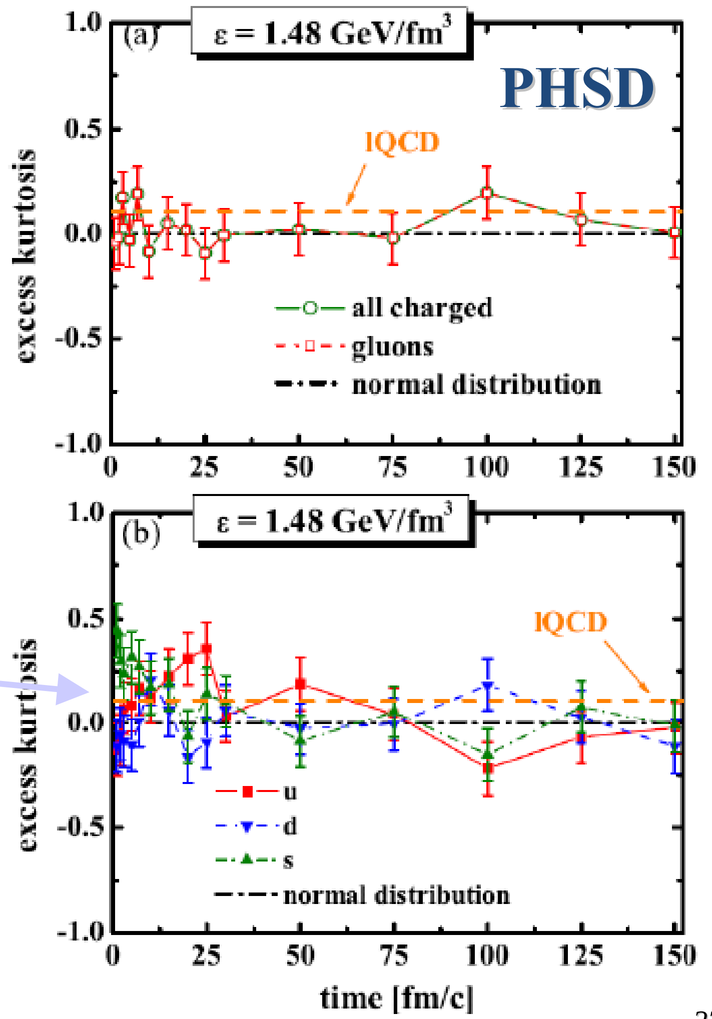
V. Ozvenchuk et al., PRC 87 (2013) 024901, arXiv:1203.4734

Kurtosis

- **kurtosis:** $$\beta_2 = \frac{m_4}{m_2^2} = \frac{m_4}{\sigma^4}, \qquad m_4 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^4$$
- **β_2 is equal to 3 for normal distribution → excess kurtosis** $g_2 = \beta_2 - 3$
- **Kurtosis as a probe of deconfinement:**

lQCD: Ejiri, Karsch, Redlich, Phys. Lett. B 633, 275 (2006)





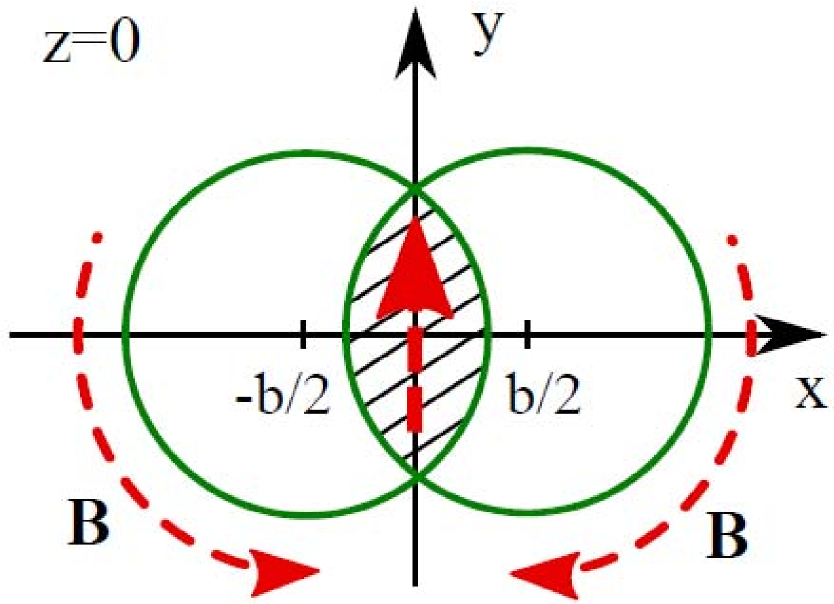
- **Kurtosis in PHSD is compatible with lQCD for QGP**

V. Ozvenchuk et al., PRC 87 (2013) 024901, arXiv:1203.4734

Summary II (parton-hadron matter in-equilibrium)

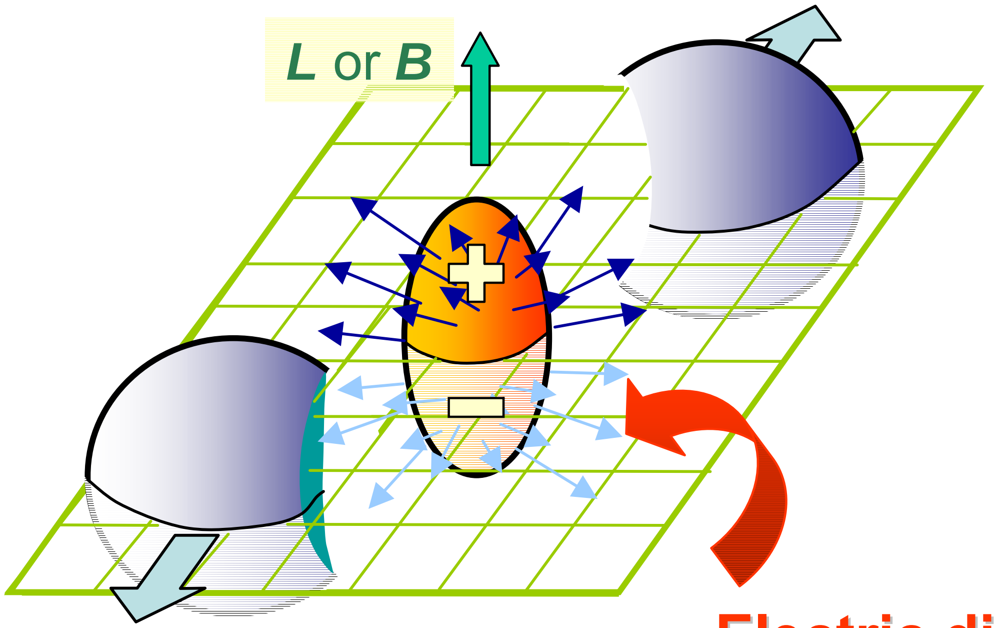
- η/s ➔ QGP in PHSD behaves as a strongly-interacting liquid
- significant rise of the bulk viscosity to entropy density ratio ζ/s in the vicinity of the critical temperature when including the scalar mean-field from PHSD
- scaled variances ω for the different particle number fluctuations in the box reach equilibrium values in time and behave as in a micro-canonical ensemble
- skewness for all observables are compatible with zero
- excess kurtosis is compatible with lQCD results for gluons and charged particles

Chiral magnetic effect and evolution of the electromagnetic field in relativistic heavy-ion collisions



Charge separation in HIC: CP violation signal

Magnetic field through the axial anomaly induces a parallel electric field which will separate different charges



Non-zero angular momentum (or equivalently strong magnetic field) in heavy-ion collisions make it possible for P- and CP-odd domains to induce charge separation
➔ ‚chiral magnetic effect' (CME)
D.Kharzeev, PLB 633 (2006) 260

Electric dipole moment of QCD matter !

Measuring the charge separation with respect to the reaction plane - S.Voloshin, PRC 70 (2004) 057901

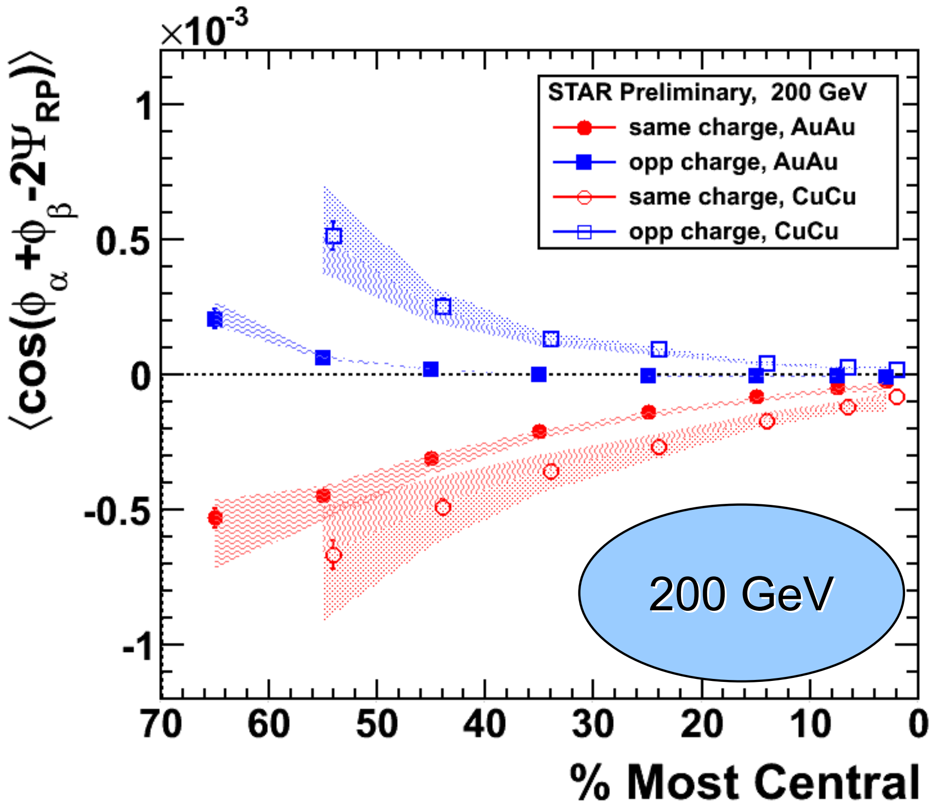
$$\langle\cos(\psi_\alpha + \psi_\beta - 2\Psi_{RP})\rangle =$$
$$= \langle\cos(\psi_\alpha + \psi_\beta - 2\psi_c)\rangle / v_{2,c} = v_{1,\alpha}v_{1,\beta} - a_\alpha a_\beta$$

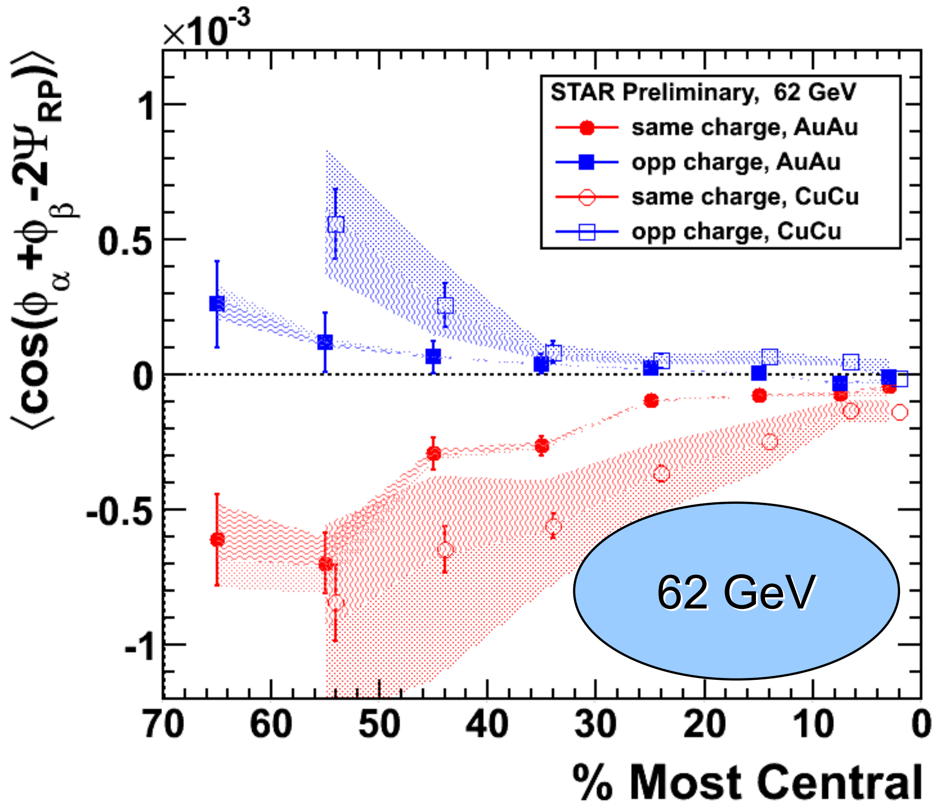
➔ Combination of intense B-field and deconfinement is needed for a spontanuous parity violation signal !

Charge separation in RHIC experiments

STAR Collaboration, PRL 103 (2009) 251601

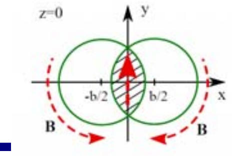
$\langle\cos(\varphi_a+\varphi_b-2\psi_{RP})\rangle$





Combination of intense B and deconfinement is needed for a spontaneous parity violation signal

PHSD with electromagnetic fields



❑ Generalized transport equations in the presence of electromagnetic fields*:

$$\dot{\vec{r}} \rightarrow \frac{\vec{p}}{p_0} + \vec{\nabla}_p U \ , \qquad U \sim Re(\Sigma^{ret})/2p_0$$

$$\dot{\vec{p}} \rightarrow -\vec{\nabla}_r U + e\vec{E} + e\vec{v} \times \vec{B}$$

$\Leftarrow$

$$\begin{cases} \vec{B} = \vec{\nabla} \times \vec{A} \\ \vec{E} = -\vec{\nabla}\Phi - \dfrac{\partial \vec{A}}{\partial t} \end{cases}$$

❑ A general solution of the wave equations

$$\begin{cases} \nabla \times \mathbf{E} = -\dfrac{1}{c}\dfrac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \end{cases} \Rightarrow \begin{cases} \vec{A}(\vec{r},t) = \dfrac{1}{4\pi}\displaystyle\int \dfrac{\vec{j}(\vec{r'},t')\ \delta(t-t'-|\vec{r}-\vec{r'}|/c)}{|\vec{r}-\vec{r'}|}\ d^3r'dt' \\ \Phi(\vec{r},t) = \dfrac{1}{4\pi}\displaystyle\int \dfrac{\rho(\vec{r'},t')\ \delta(t-t'-|\vec{r}-\vec{r'}|/c)}{|\vec{r}-\vec{r'}|}\ d^3r'dt' \end{cases}$$

➔ retarded Lienard-Wiechert electric and magnetic potentials:

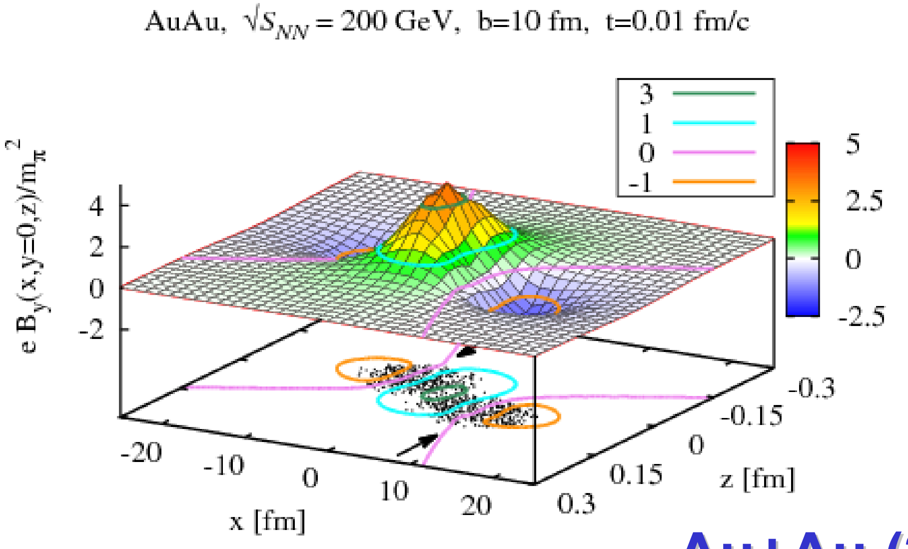
$$e\,\mathbf{E}(\mathbf{r},t) = \alpha \sum_n Z_n \frac{[\mathbf{R}_n - R_n \mathbf{v}_n]}{(R_n - \mathbf{R}_n \cdot \mathbf{v})^3}(1-v^2), \qquad e\,\mathbf{B}(\mathbf{r},t) = \alpha \sum_n Z_n \frac{\mathbf{v} \times \mathbf{R}_n}{(R_n - \mathbf{R}_n \cdot \mathbf{v})^3}(1-v^2),$$

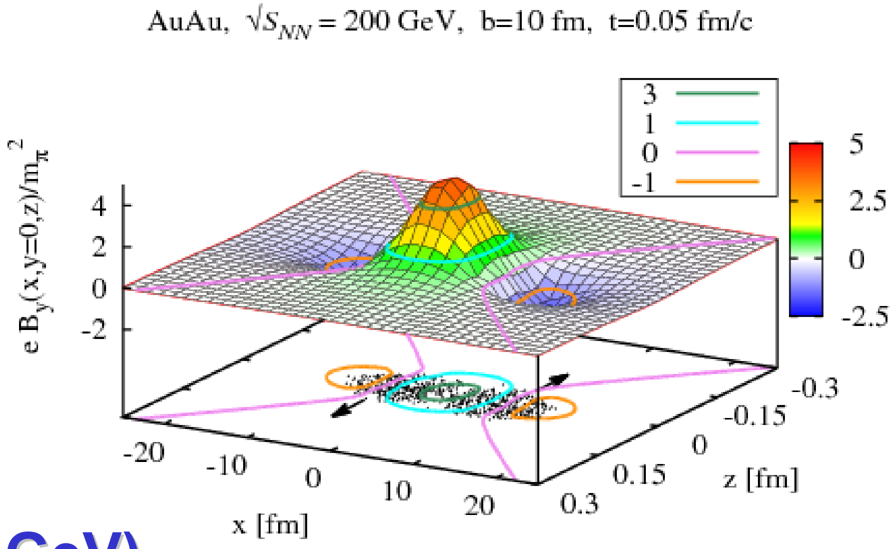
$$\mathbf{R}_n = \mathbf{r} - \mathbf{r}_n$$

* Realized in the PHSD for
hadrons and quarks

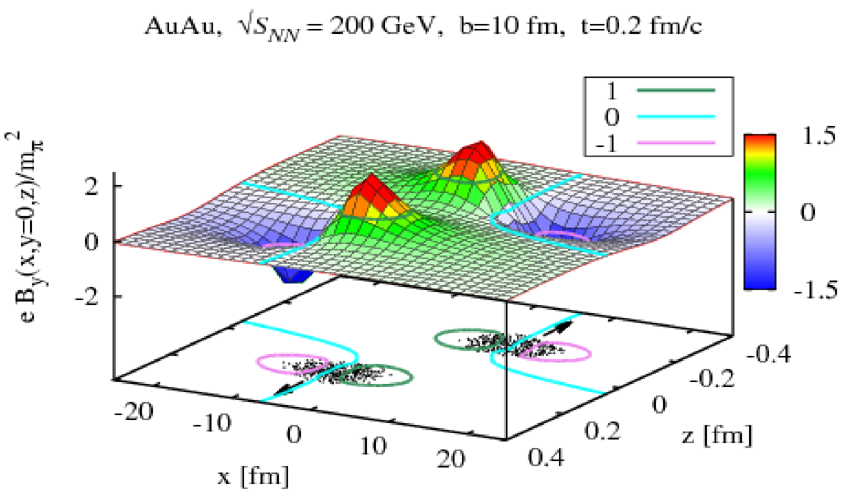
V. Voronyuk, et al., Phys.Rev. C83 (2011) 054911

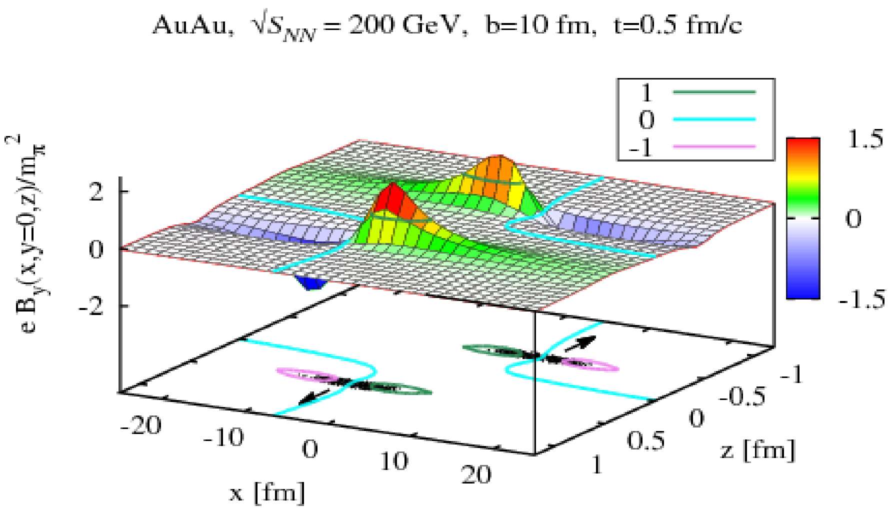
Magnetic field evolution





Au+Au (200 GeV)
b=10 fm

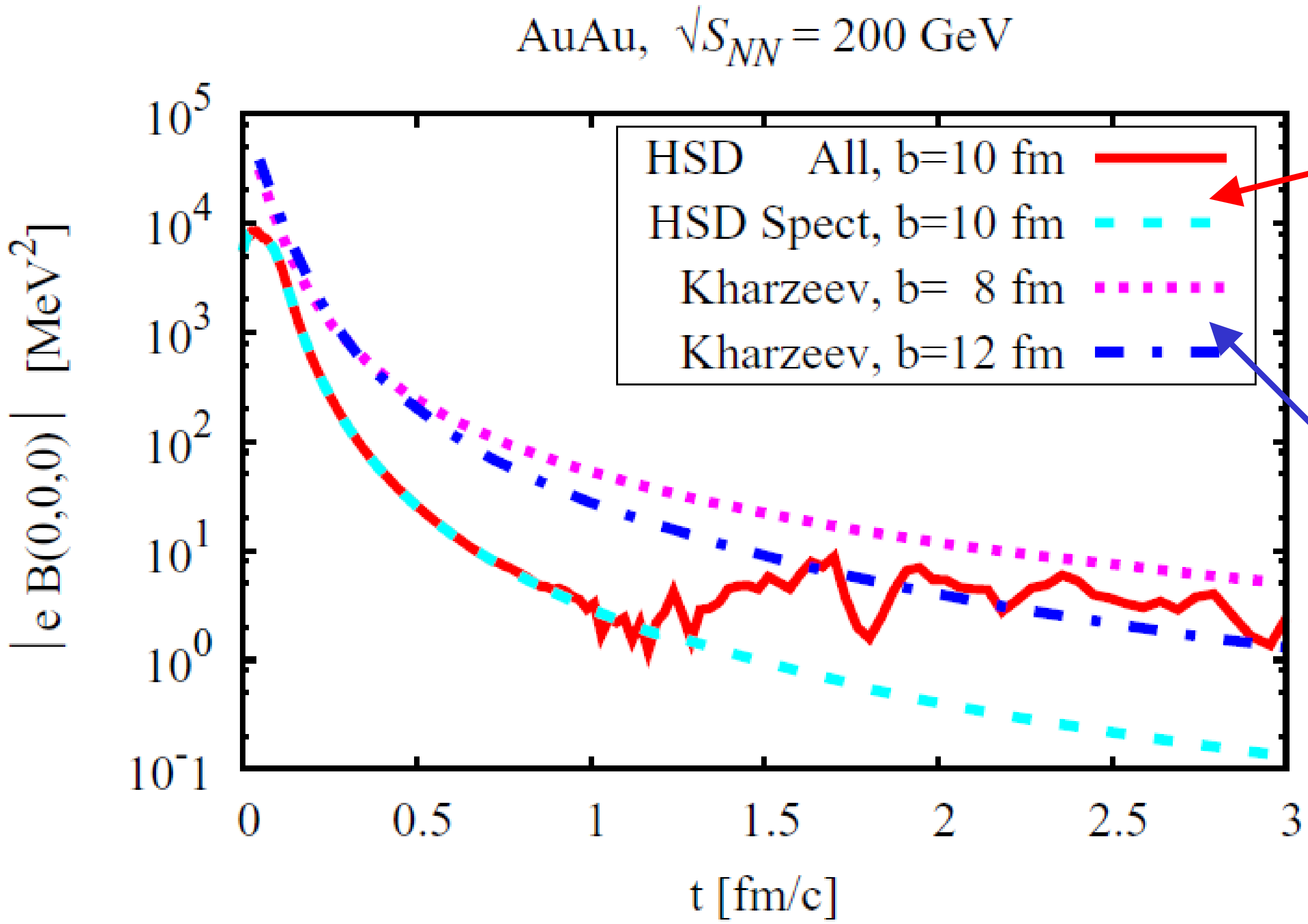




$m_\pi^2 \approx 10^{18}$Gauss

V.Voronyuk, et al., Phys.Rev. C83 (2011) 054911

Time dependence of eB_y



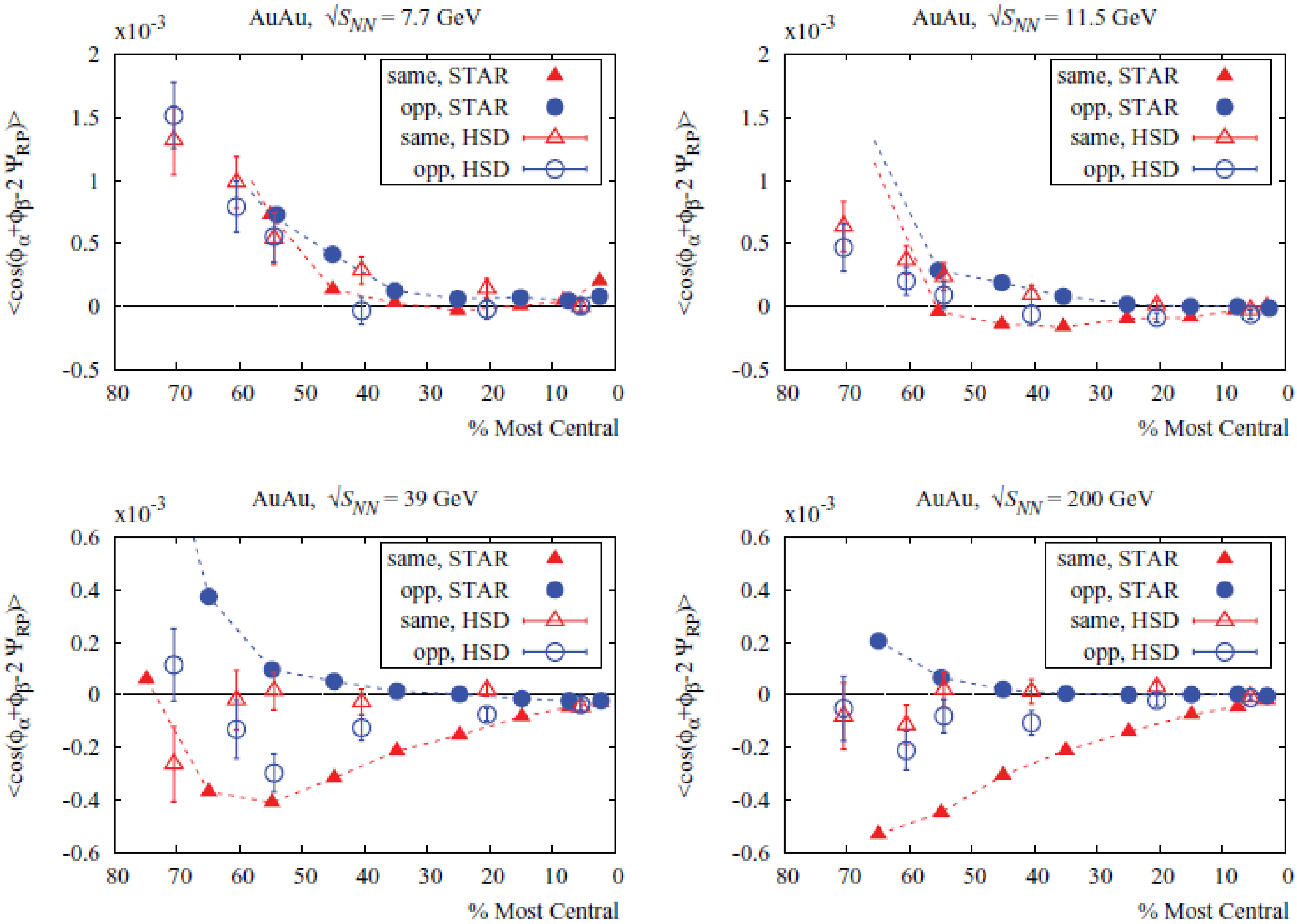
HSD: V.Voronyuk, et al., PRC83 (2011) 054911

Collision of two infinitely thin layers (pancake-like)
D.E. Kharzeev et al., NPA803, 227 (2008)

- **Until t~1 fm/c the induced magnetic field is defined by spectators only**
- **Maximal magnetic field is reached during nuclear overlapping time Δt~0.2 fm/c, then the field goes down exponentially**

V.Voronyuk, et al., Phys.Rev. C83 (2011) 054911

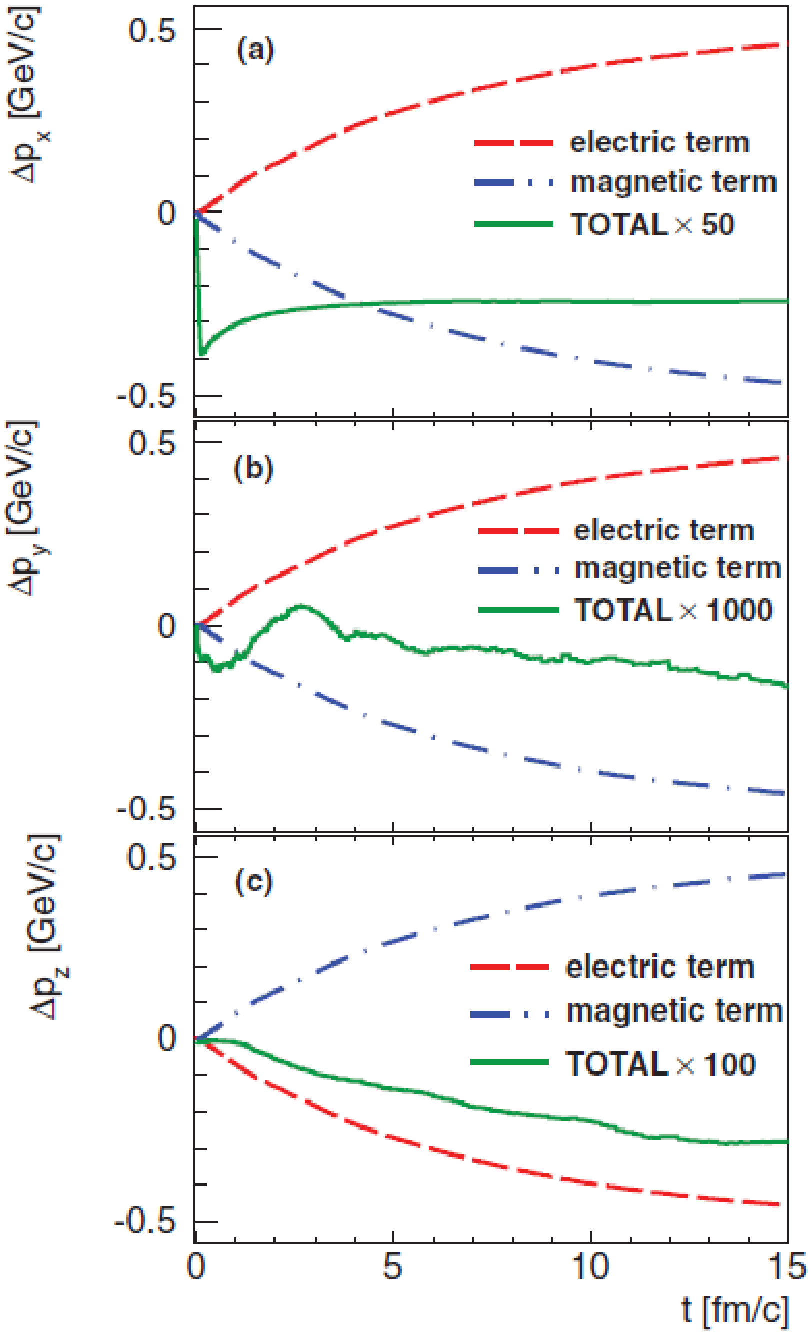
Angular correlation wrt. reaction plane



➔ **Angular correlation is of hadronic origin up to $s^{1/2}$ = 11 GeV !**

V. D. Toneev et al., PRC 85 (2012) 034910, PRC 86 (2012) 064907

Compensation of magnetic and electric forces



Au+Au, $s^{1/2}$ = 11 GeV, b=10 fm

$$\dot{\vec{p}} \rightarrow e\vec{E} + e\vec{v} \times \vec{B}$$

Momentum increment: $\Delta \mathbf{p}(t) = \sum_{t_i}^{t} \langle d\mathbf{p}(t_i) \rangle$

(for p_Z>0)

➔ strong magnetic and electric forces compensate each other!

V. D. Toneev et al., PRC 85 (2012) 034910, PRC 86 (2012) 064907

Summary III (CME – angular correlations)

The PHSD transport model with retarded electromagnetic fields shows :

- creation of strong electric and magnetic fields at heavy-ion collisions
- strong magnetic and electric forces compensate each other ➔small effect on observables
- low-energy experiments within the RHIC BES program at $\sqrt{s_{NN}}$ = 7.7 and 11.5 GeV can be explained within *hadronic scenario* without reference to the spontaneous local CP violation.
- PHSD doesn't reproduce the exp. data on angular correlations $\langle\cos(\varphi_a+\varphi_b-2\psi_{RP})\rangle$ at $\sqrt{s_{NN}}$= 39 - 200 GeV ➔ indication for CME?

Thank you!