Event-by-Event Fluctuations in Relativistic Nucleus-Nucleus Collisions

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I. Global Conservation Laws. Statistical Ensembles

II. Strongly Intensive Measures of Fluctuations

III. Fluctuations with Incomplete Particle Identifications

IV. Van der Waals Equation of State in the GCE and Fluctuations in a Vicinity of the Critical Point

I. Global Conservation Laws. Statistical Ensembles

$E \longleftrightarrow T$ $V \longleftrightarrow p$ $Q \longleftrightarrow \mu_Q$

$2^3 = 8$

E, V, Q MCE

T, V, Q CE

T, V, μ_Q GCE

E, V, μ_Q MGCE

E, p, Q T, p, Q

T, p, μ_Q E, p, μ_Q

} Pressure Ensembles

M.I.G. J. Phys. G (2008)

GCE and CE with Q=0

Rafelski, Danos, Phys. Lett. B (1980)
Redlich, Turko, Z. Phys. C (1980)

$$Z_{gce} = \sum_{N_+,N_-=0}^{\infty} \frac{z^{N_-}}{N_-!} \frac{z^{N_+}}{N_+!} = \exp(2z)$$

$$\mu = 0 \rightarrow \langle Q \rangle = 0 \qquad z = \frac{V}{2\pi^2} T m^2 \, K_2(m/T)$$

$$Z_{ce} = \sum_{N_+,N_-=0}^{\infty} \frac{z^{N_-}}{N_-!} \frac{z^{N_+}}{N_+!} \delta(N_+ - N_-) = I_0(2z)$$

$$\omega^- = \frac{\langle N_-^2 \rangle - \langle N_- \rangle^2}{\langle N_- \rangle}, \qquad \langle N_- \rangle_{gce} = z, \qquad \omega^-_{gce} = 1$$

$$\langle N_- \rangle_{ce} = z \frac{I_1(2z)}{I_0(2z)}, \qquad \omega^-_{ce} = 1 - z \left[\frac{I_1(2z)}{I_0(2z)} - \frac{I_2(2z)}{I_1(2z)} \right]$$

M.I.G., Gazdzicki, Greiner,
Phys. Lett. B (2000) **CE** for antibaryons in p+A

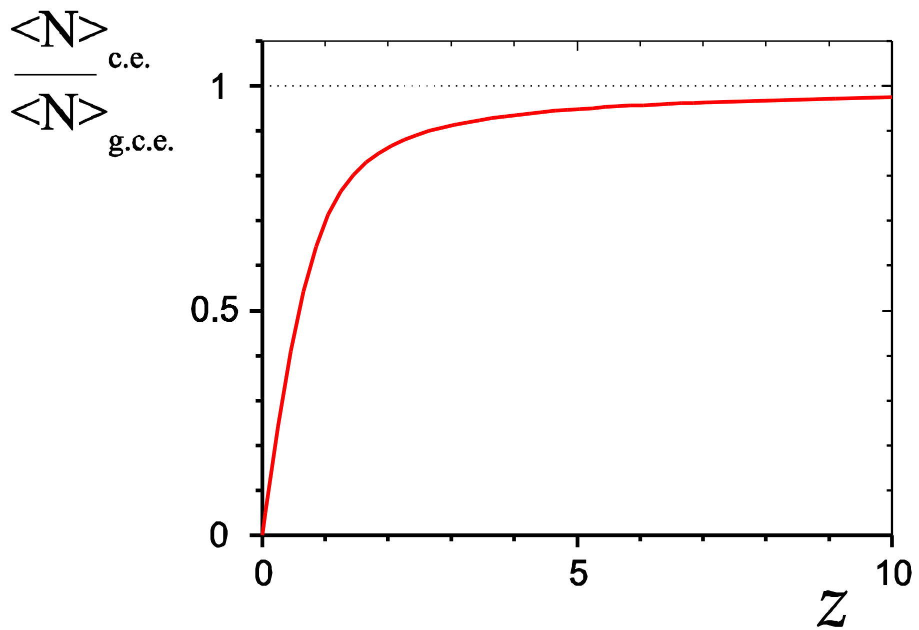
M.I.G., Kostyuk, Stoecker, Greiner,
Phys. Lett. B (2001) **CE** for charmed hadrons

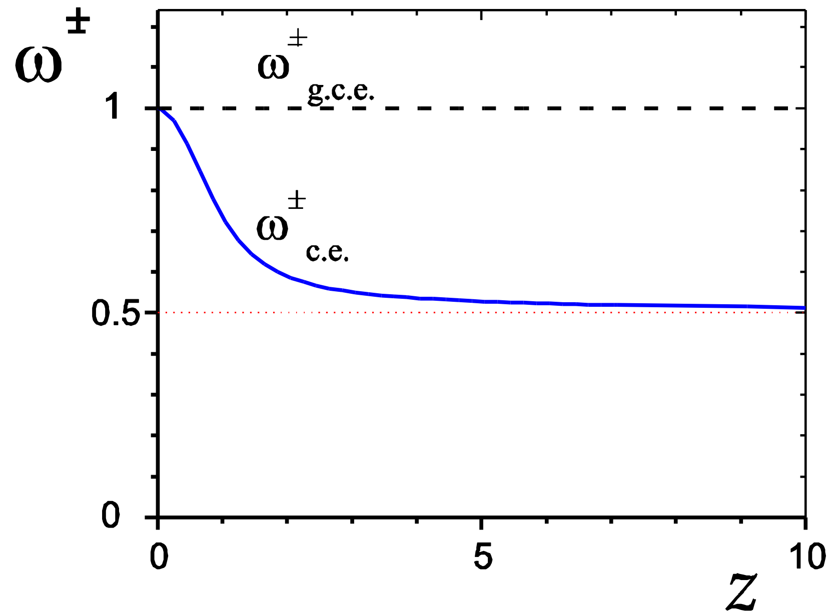
Begun, Gazdzicki, M.I.G., Zozulya,
Phys. Rev. C (2004)

$Q = 0$

GCE: $\mu=0 \to \langle N_+ \rangle = \langle N_- \rangle$,

CE: $\delta(N_+ - N_-) \to N_+ = N_-$





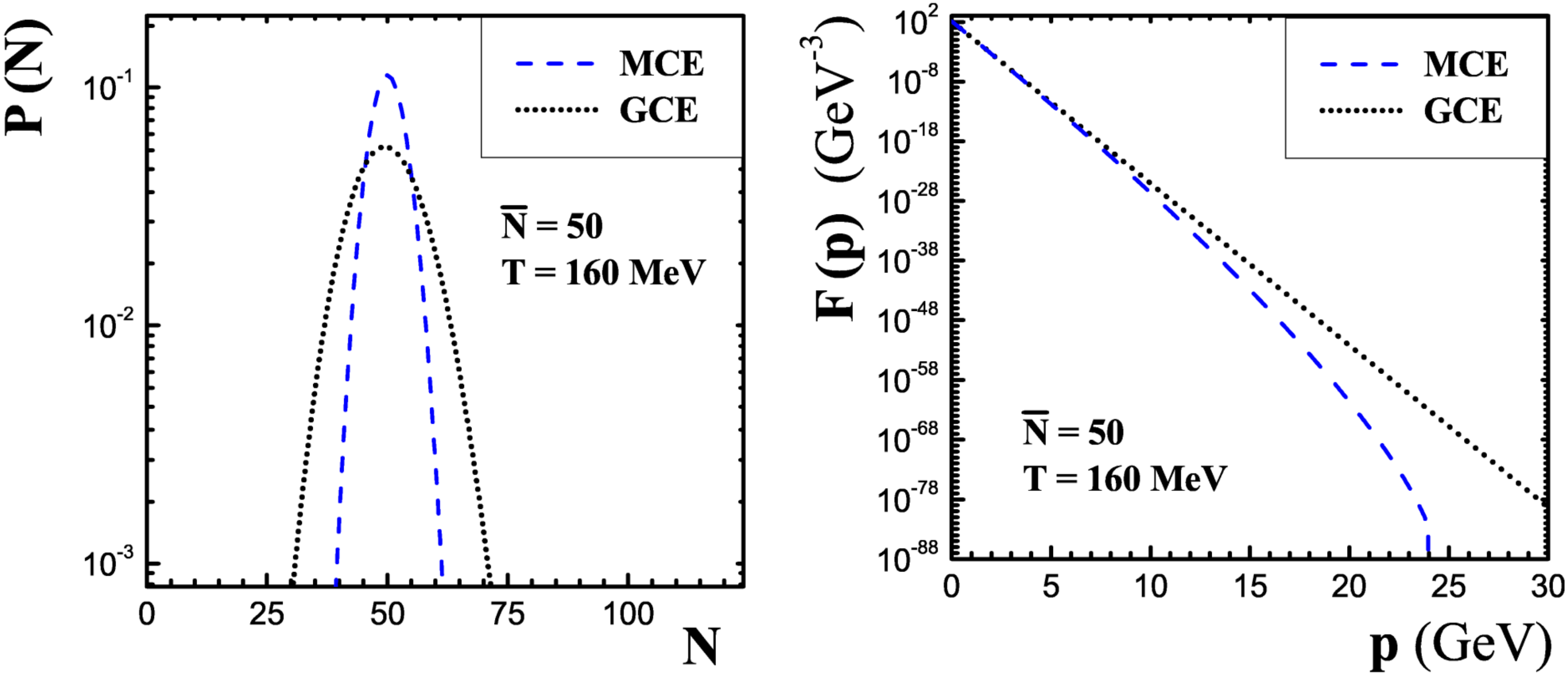
$$z = \frac{V}{2\pi^2} T m^2 K_2(m/T) = \langle N_\pm \rangle_{gce}\ ; \qquad \omega[N] = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}; \qquad \omega_{gce}[N_\pm] = 1$$

$$\frac{\langle N_\pm \rangle_{ce}}{\langle N_\pm \rangle_{gce}} = \frac{I_1(2z)}{I_0(2z)} \to 1 \qquad \omega_{ce}[N_\pm] = 1 - z\left[\frac{I_1(2z)}{I_0(2z)} - \frac{I_2(2z)}{I_1(2z)}\right] \to \frac{1}{2}$$

1980 2004

$$I_n(2z) \cong \frac{\exp(2z)}{\sqrt{4\pi z}}\left[1 - \frac{4n^2 - 1}{16z}\right], \qquad z \to \infty\ ; \qquad I_n(2z) \cong \frac{z^n}{n!}, \qquad z \ll 1$$

Micro Canonical Ensemble of massless neutral particles



$$\omega_{gce} = 1, \qquad \omega_{mce} = \frac{1}{4}$$

Begun, M.I.G., Kostyuk, Zozulya, Phys. Rev. C (2005)

$$\vec{A} = (E, V, Q_1, \ldots, Q_k)$$

Alpha-Enesmble

$$P_\alpha(X) = \int d\vec{A} \; P_\alpha(\vec{A}) \; P_{mce}(X; \vec{A})$$

M.I.G. and Hauer, Phys. Rev. C (2008)

Theory (standard statistical ensembles):

Begun, M.I.G., Kostyuk, Zozulya, Phys. Rev. C (2005), J.Phys. G
Begun, M.I.G., Zozulya, Phys. Rev. C (2005)
Becattini, Ferroni, Eur. Phys. J. C(2005) (2007)
Mekjian, Nucl. Phys. A (2005)
Keranen, Becattini, Begun, M.I.G., Zozulya, J. Phys. G (2005)
Cleymans, Redlich, Turko, Phys. Rev. C (2005), J. Phys. G (2005)
Becattini, Keranen, Ferroni, Gabbriellini, Phys. Rev. C (2005)
Torrieri, Jeon, Rafelski, Phys. Rev. C (2006), Nucleonics (2006)
Hauer, Phys. Rev. C (2008); Hauer, Wheaton, Phys. Rev. C (2009)
Turko, Int. J. Mod. Phys. E (2007), Phys. Part. Nucl. (2008)
Becattini, Ferroni, Eur. Phys. J. C (2007)
Torrieri, J. Phys. G (2006, 2008), Eur. Phys. J. C (2007) , (2012),
Yang, Wang, Phys. Rev. C (2011, 2012)
Torrieri, Bellweid, Market, Westfall (2012)

Theory (generalized statistical ensembles):

M.I.G., Hauer, Phys. Rev. C (2008);
Begun, Gazdzicki, M.I.G., Phys. Rev. C (2008, 2009)
Wilk, Wlodarczyk, Physica (2011)
Biro, Barnafoldi, Van, Eur. Phys. J. A (2013), Physica (2014)

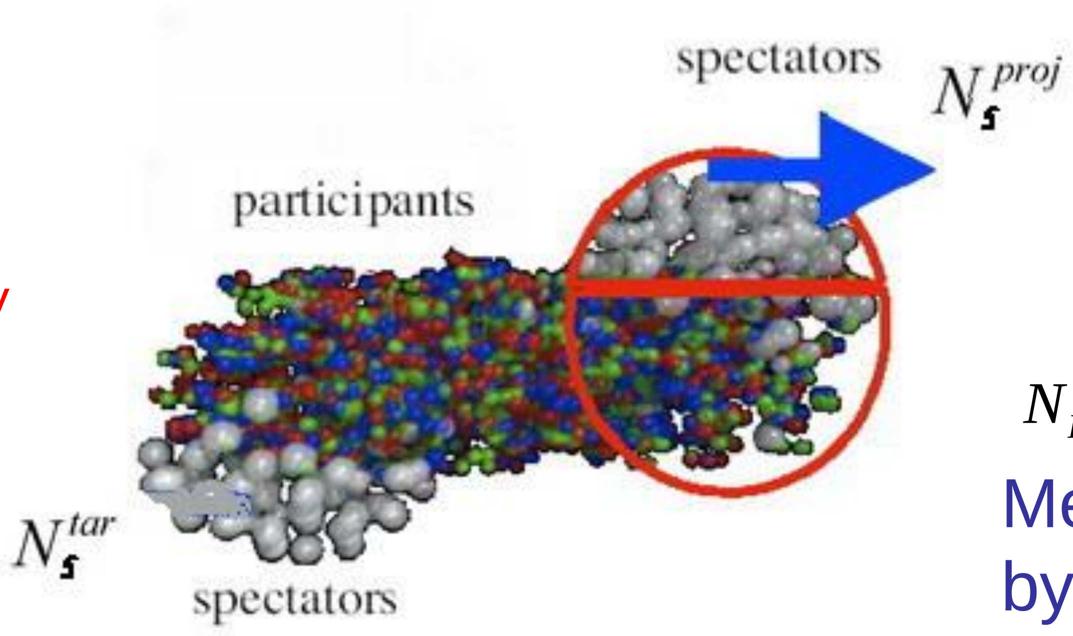
Experiment:

Rybczynski et. al, NA49 Collaboration, J. Phys. Conf. Ser. (2005)
Lungwitz (NA49 Collaboration) (2006, 2007)
Alt, et.al. NA49 Collaboration, Phys. Rev C (2007) (2008)
Center, et. al, NA61 Collaboration, Phys. Atom. Nucl. (2012)

II. Strongly Intensive Measures of Fluctuations

Nucleons: participants and spectators

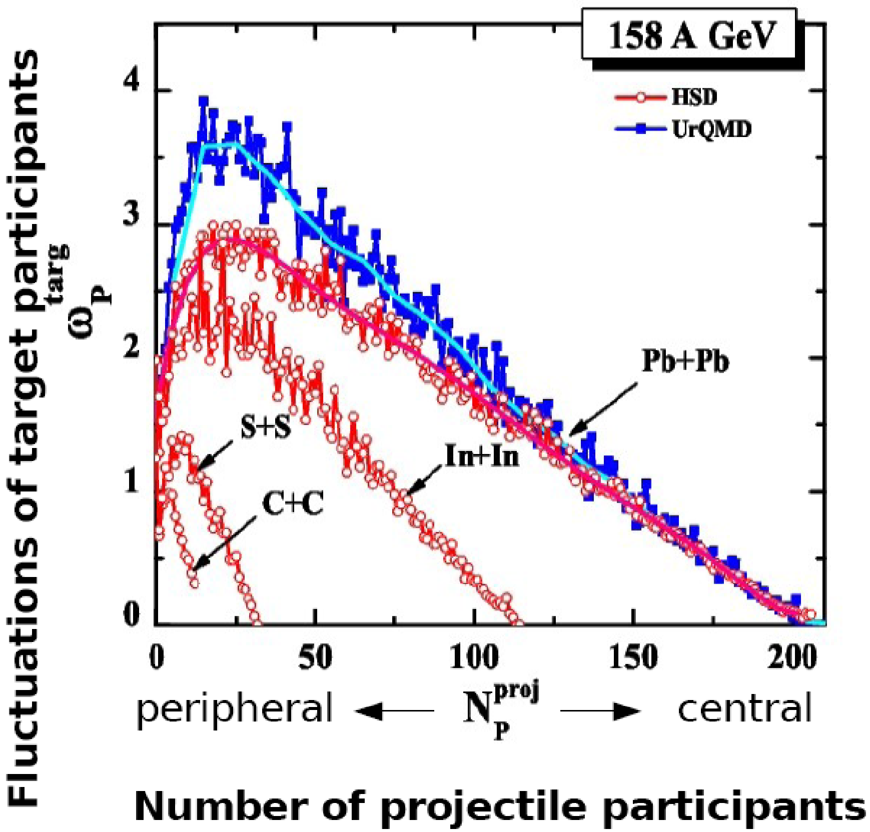
Pb-Pb
158 AGeV



$$N_P^{proj} = A^{proj} - N_S^{proj}$$

Measured
by ZDC

Central collisions of light and medium size nuclei are required for the proposed fluctuation studies



Event-by-event fluctuations in the number of interacting (participant) nucleons are the main source of the background in the fluctuation studies

The fluctuations of the number of projectile participants are suppressed by selecting collisions with fixed number of projectile spectators (in NA49-future measured by PSD)

The fluctuations of the number of target participants can be suppressed only by selection of very central collisions

Konchakovski, M.I.G., et al, Phys. Rev C (2006)

Volume Fluctuations

$F(V)$ event-by-event volume distribution

$A \sim V, \quad B \sim V$ Extensive Quantities

$\Delta[A, B], \Sigma[A, B]$ are independent of the average volume and of volume fluctuations

M.I.G., Gazdzicki, Phys Rev. C (2011)

Examples:

$A = P_T = |p_T^{(1)}| + \ldots + |p_T^{(N)}|$

$B = N$

$A = N_1,$

$B = N_2$

$$\Xi = \exp\left\{\mathbf{V}\sum_{\mathbf{j}}\eta_{\mathbf{j}}\mathbf{d}_{\mathbf{j}}\int\frac{\mathbf{d^3p}}{(\mathbf{2}\pi)^{\mathbf{3}}}\ln\left[1+\lambda_{\mathbf{j}}\eta_{\mathbf{j}}\exp\left(-\sqrt{p^2+m_j^2}/\mathbf{T}\right)\right]\right\}$$

$$\mu_j = b_j\mu_B + s_j\mu_S + q_j\mu_Q, \qquad S=0, \qquad \frac{Q}{B} = 0.4 \div 0.5$$

$$\overline{\mathbf{A}} = \frac{1}{\Xi}\,\lambda_{\mathbf{A}}\,\frac{\partial}{\partial\lambda_{\mathbf{A}}}\,\Xi \;=\; \mathbf{V}\mathbf{n}_{\mathbf{A}}$$

$$\lambda_{\mathbf{j}} = \exp\left(\frac{\mu_{\mathbf{j}}}{\mathbf{T}}\right)$$

$$\eta_{\mathbf{j}} = \pm 1$$

$$\overline{\mathbf{A}^2} = \frac{1}{\Xi}\,\left(\lambda_{\mathbf{A}}\frac{\partial}{\partial\lambda_{\mathbf{A}}}\right)^{\mathbf{2}}\,\Xi \;=\; \mathbf{V}^2\mathbf{n}_{\mathbf{A}}^2$$

$$\mathbf{n}_{\mathbf{A}} = \overline{\mathbf{A}}/\mathbf{V}$$

$$+\mathbf{V}\int\frac{\mathbf{d^3p}}{(\mathbf{2}\pi)^{\mathbf{3}}}\frac{\mathbf{d}_{\mathbf{A}}\lambda_{\mathbf{A}}^{-1}\exp\left(\sqrt{p^2+m_j^2}/\mathbf{T}\right)}{\left[\lambda_{\mathbf{A}}^{-1}\exp\left(\sqrt{p^2+m_j^2}/\mathbf{T}\right)+\eta_{\mathbf{A}}\right]^{\mathbf{2}}}$$

$$\overline{\mathbf{AB}} = \frac{1}{\Xi}\,\lambda_{\mathbf{A}}\frac{\partial}{\partial\lambda_{\mathbf{A}}}\lambda_{\mathbf{B}}\frac{\partial}{\partial\lambda_{\mathbf{B}}}\,\Xi \;=\; \mathbf{V}^2\mathbf{n}_{\mathbf{A}}\mathbf{n}_{\mathbf{B}}$$

$$\omega^*_{\mathbf{A}} = \frac{\overline{\mathbf{A^2}} - \overline{\mathbf{A}}^2}{\overline{\mathbf{A}}} = \mathbf{n}_{\mathbf{A}}^{-1} \int \frac{\mathbf{d^3p}}{(2\pi)^3} \frac{\mathbf{d_A} \lambda_{\mathbf{A}}^{-1} \exp\left(\sqrt{p^2 + m_j^2}/\mathbf{T}\right)}{\left[\lambda_{\mathbf{A}}^{-1} \exp\left(\sqrt{p^2 + m_j^2}/\mathbf{T}\right) + \eta_{\mathbf{A}}\right]^2}$$

$$\int \mathbf{dV} \ \ldots \mathbf{F(V)} = \langle \ldots \rangle$$

$$\langle \mathbf{A} \rangle \ = \ \langle \mathbf{V} \rangle \ \mathbf{n_A} \qquad \langle \mathbf{A^2} \rangle \ = \ \langle \mathbf{V^2} \rangle \ \mathbf{n_A^2} \ + \ \langle \mathbf{V} \rangle \ \mathbf{n_A} \ \omega^*_{\mathbf{A}}$$

$$\langle \mathbf{AB} \rangle \ = \ \langle \mathbf{V^2} \rangle \ \mathbf{n_A n_B}$$

$$\omega_{\mathbf{A}} \ \equiv \ \frac{\langle \mathbf{A^2} \rangle \ - \ \langle \mathbf{A} \rangle^2}{\langle \mathbf{A} \rangle} \ = \ \omega^*_{\mathbf{A}} \ + \ \mathbf{n_A} \ \frac{\langle \mathbf{V^2} \rangle - \langle \mathbf{V} \rangle^2}{\langle \mathbf{V} \rangle}$$

$$< AB > \ - \ < A > \ < B > \ = \ n_A n_B \ \left(< V^2 > \ - \ < V >^2 \right)$$

$$\Delta\left[A, B\right] = \frac{1}{C_\Delta}\left[< B > \ \omega[A] - < A > \ \omega[B]\ \right]$$

$$\Sigma\left[A, B\right] = \frac{1}{C_\Sigma}\ [< B > \ \omega[A] + < A > \ \omega[B]$$

$$- 2\left(< AB > - < A > \ < B >\right)]$$

$$< C_\Delta >, \ < C_\Sigma > \ \sim \ < V >$$

These combinations of second moments $< A^2 >, \ < B^2 >, \ < AB >$
are independent of $< V >$ and $\omega\left[V\right]$

Normalization.

For the Independent Particle Model: $\Delta[A, B] = 1$

IB-GCE ; Mixed Event Model $\Sigma[A, B] = 1$

$$C_\Delta = C_\Sigma = \omega[p_T] < N >$$

$$[A = P_T,\ B = N]$$

$$C_\Delta = < N_1 > - < N_2 >$$

$$C_\Sigma = < N_1 > + < N_2 >$$

$$[A = N_1,\ B = N_2]$$

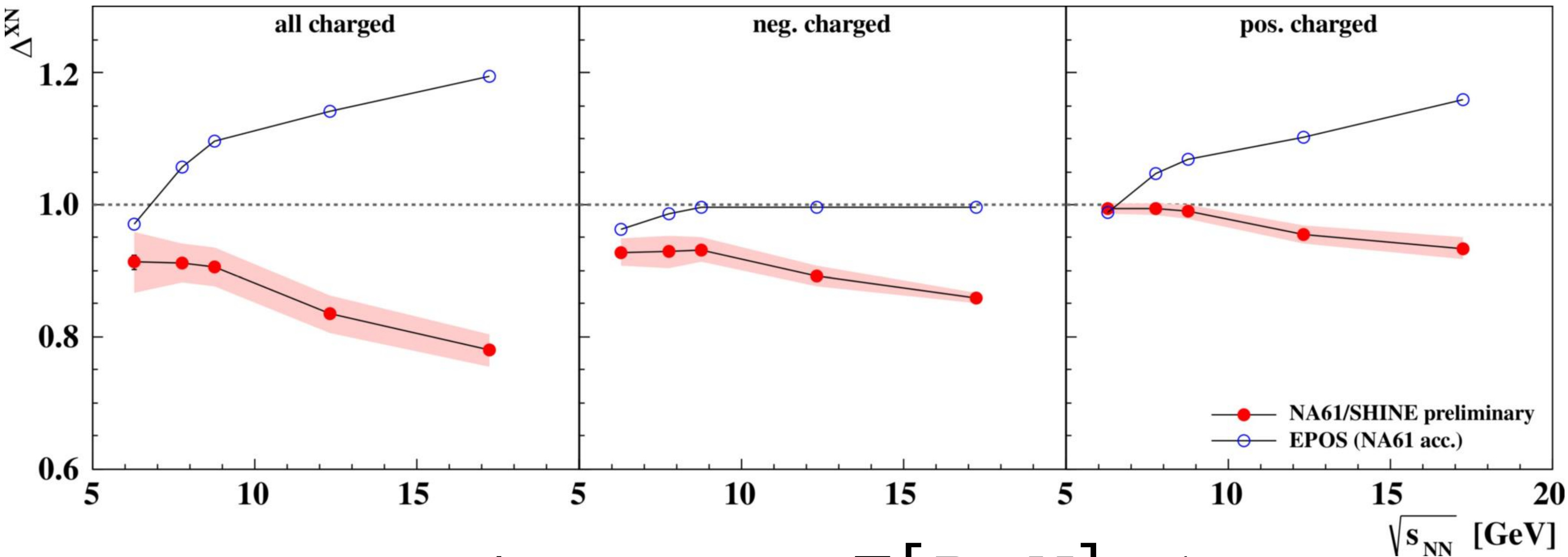
Gazdzicki, M.I.G., Mackowiak-Pawlowska, Phys. Rev. C (2013)

$$T_N = T\left[1+\theta\left(1-\frac{N}{<N>}\right)\right]; \quad T = 160\text{MeV}, \ \ \theta = 0.04$$

M.I.G., Grebieszkow, Phys. Rev. C (2014)

$$\Delta\left[P_T, N\right] \cong 1-4\theta \cong 0.84 , \qquad \Sigma\left[P_T, N\right] \cong 1-2\theta^2 \cong 1.003 .$$

NA61/SHINE data in p+p reactions



p+p at 158 GeV/c

Data: $\Sigma\left[P_T, N\right] \cong 1$

Effects of Quantum Statistics

M.I.G., Rybczynski, Phys. Lett. B (2014),

$$\Delta[P_T, N], \ \Sigma[P_T, N]$$

The strongest effect is in $\Delta[P_T, N]$ of pions

Effects of Resonance Decays

Begun, M.I.G., Grebieszkow, J. Phys. G (2015)

$$R \rightarrow \pi^+ \pi^-$$

$$\frac{< R >}{< \pi^+ > + < \pi^- >} \cong \frac{1 - \Sigma[\pi^+, \pi^-]}{2}$$

Experimental results for the Δ and Σ measures:

Rustamov for the NA61/SHINE and NA49 Collaborations, arXiv:1303.5671

Anticic, et.al Phys. Rev. C (2014)

Mackowiak-Pawlowska, PoS CPOD2013 (2013), J. Phys. Conf. Ser. (2014)

Rybczynski, NA61/SHINE Collaboration, PoS EPS-HEP2013, arXiv:1301.3360

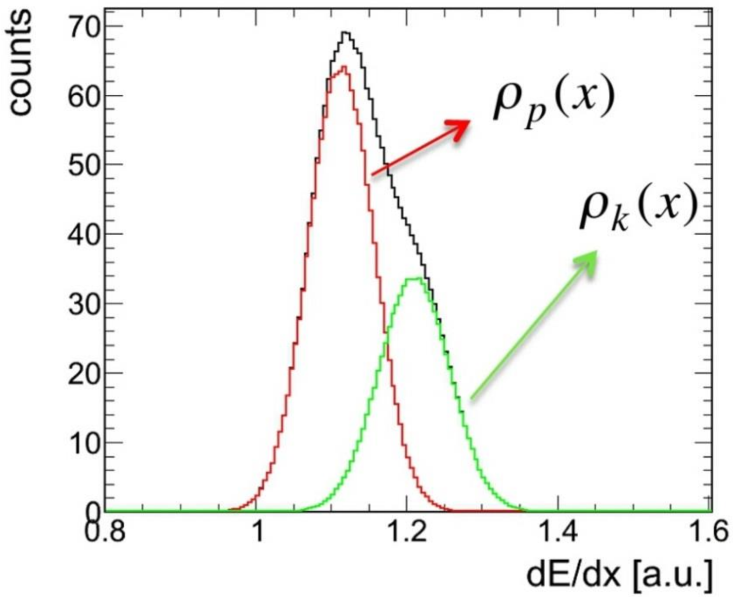
Mackowiak-Pawlowska, Wilczek for the NA61 Collaboration (2013)

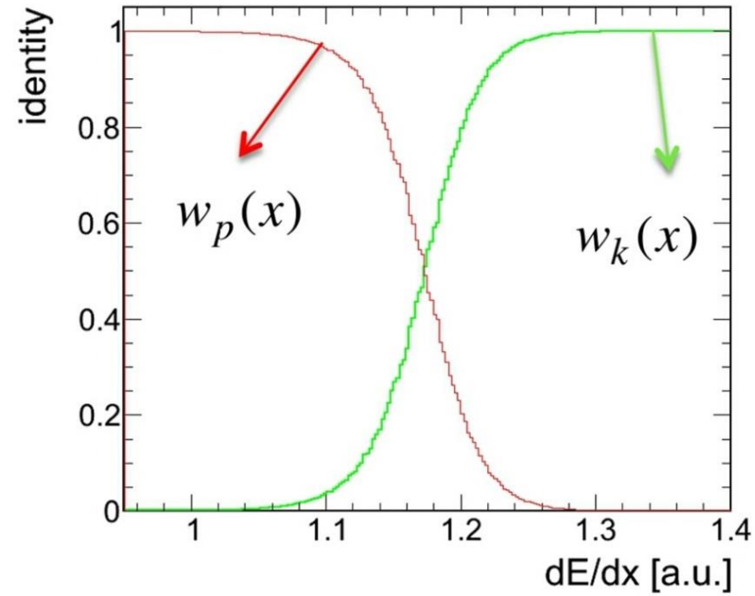
Grebieszkow, Acta Phys. Pol. (2013), PoS CPOD2013 (2013)

Seyboth, arXiv:1402.4619 (2014)

Stefanek, NA61/SHINE and NA49, arXiv:1411.2396

III. Fluctuations with Incomplete Particle Identifications





$$\int dm \rho_j(m) = \langle N_j \rangle,$$

$$\rho(m) = \sum_{i=1}^{k} \rho_i(m)$$

identity variable

$$w_j(m) = \frac{\rho_j(m)}{\rho(m)} \subset [0,1]$$

Complete identification:

$\rho_j(m)$ do not overlap

Gazdzicki, Grebieszkow, Mackowiak, Mrowczynski, Phys. Rev. C (2011)
M.I.G., Phys. Rev. C (2011); Rustamov, M.I.G., Phys. Rev. C (2012)

$$W_j = \sum_{i=1}^{N(n)} w_j(m_i), \qquad W_j^2 = [\sum_{i=1}^{N(n)} w_j(m_i)]^2$$

$$W_p W_q = [\sum_{i=1}^{N(n)} w_p(m_i)][\sum_{i=1}^{N(n)} w_q(m_i)]$$

$N(n) = N_1(n) + ... + N_k(n)$ total multiplicity in n-th event

$$< W_j^2 >= \frac{1}{N_{ev}} \sum_{n=1}^{N_{ev}} W_j^2, \qquad < W_p W_q >= \frac{1}{N_{ev}} \sum_{n=1}^{N_{ev}} W_p W_q$$

N_{ev} is the number of events

In the case of complete identifications: $W_j = N_j$,

$$< W_j^2 > \ = \ < N_j^2 >, \qquad < W_p W_q > \ = \ < N_p N_q >$$

$$\sum_{i=1}^{k} < N_i^2 > u_{ji}^2 + 2 \sum_{1 \le i < l \le k} < N_i N_l > u_{ji} u_{jl} = b_j$$

$$\sum_{i=1}^{k} < N_i^2 > u_{pi} u_{qi} + \sum_{1 \le i < l \le k} < N_i N_l > (u_{pi} u_{ql} + u_{pl} u_{qi}) = b_{pq}$$

$$u_{ji}^s \equiv \frac{1}{< \mathrm{N}_i >} \int dm\, w_j^s, \quad u_{pqi} \equiv \frac{1}{< \mathrm{N}_i >} \int dm\, w_p\ w_q\ \rho_i(m)$$

$$b_j \equiv < W_j^2 > - \sum_{i=1}^{k} < N_i > [u_{ji}^2 - (u_{ij})^2],$$

$$b_{pq} \equiv < W_p W_q > - \sum_{i=1}^{k} < N_i > [u_{pqi} - u_{pi} u_{qi}]$$

$$< N_j^2 >, \quad j = 1, \ldots, k \qquad < N_p N_q >, \quad 1 \le p < q \le k$$

The system of k+k(k-1)/2 linear equations

$$< N_1^2 > = \frac{b_1 u_{22}^2 + b_2 u_{12}^2 - 2 b_{12} u_{12} u_{22}}{(u_{11} u_{22} - u_{12} u_{21})^2}$$

$$< N_2^2 > = \frac{b_2 u_{11}^2 + b_1 u_{21}^2 - 2 b_{12} u_{21} u_{11}}{(u_{11} u_{22} - u_{12} u_{21})^2}$$

$$< N_1 N_2 > = \frac{b_{12}(u_{11} u_{22} + u_{12} u_{21}) - b_1 u_{22} u_{21} - b_2 u_{11} u_{22}}{(u_{11} u_{22} - u_{12} u_{21})^2}$$

Experimental results for fluctuations with the identity method:

Rustamov for the NA61/SHINE and NA49, J. Phys. Ser. and arXiv:1303.5671
Anticic, et.al, Phys. Rev. C (2013, 2014)
Mackowiak-Pawlowska, PoS CPOD2013 (2013), J. Phys. Conf. Ser. (2014)
Grebieszkow, Acta Phys. Pol. (2013), PoS CPOD2013 (2013)
Seyboth, arXiv:1402.4619
Stefanek, NA61/SHINE and NA49, arXiv:1411.2396

IV. Van der Waals Equation of State in the GCE and Fluctuations in a Vicinity of the Critical Point

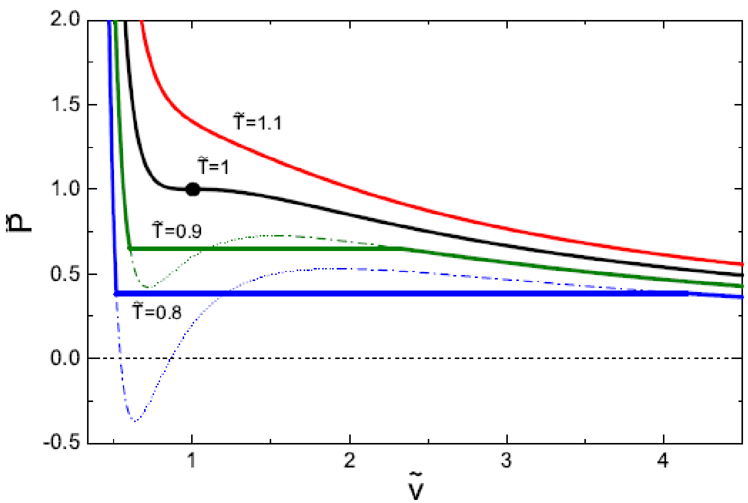
$$p(V,T,N) = \frac{NT}{V-bN} - a\frac{N^2}{V^2} = \frac{nT}{1-bn} - an^2, \quad \text{CE}$$

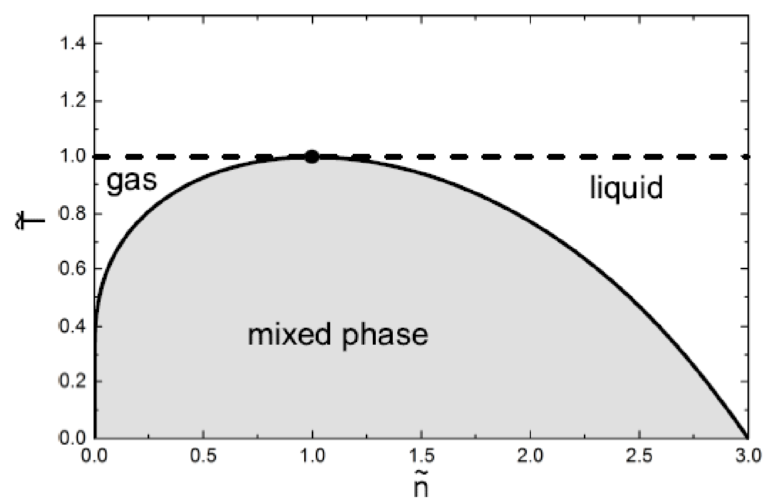
Vovchenko, Anchishkin, M.I.G.

J. Phys. A (2015)
Phys. Rev. C (2015)
arXiv:1506.05763
arXiv:1507.06537

$$T_c = \frac{8a}{27b}, \quad n_c = \frac{1}{3b}, \quad p_c = \frac{a}{27b^2}$$

$$\tilde{p} = \frac{8\tilde{T}\tilde{n}}{3-\tilde{n}} - 3\tilde{n}^2, \qquad \tilde{n} = n/n_c, \quad \tilde{p} = p/p_c, \quad \tilde{T} = T/T_c,$$



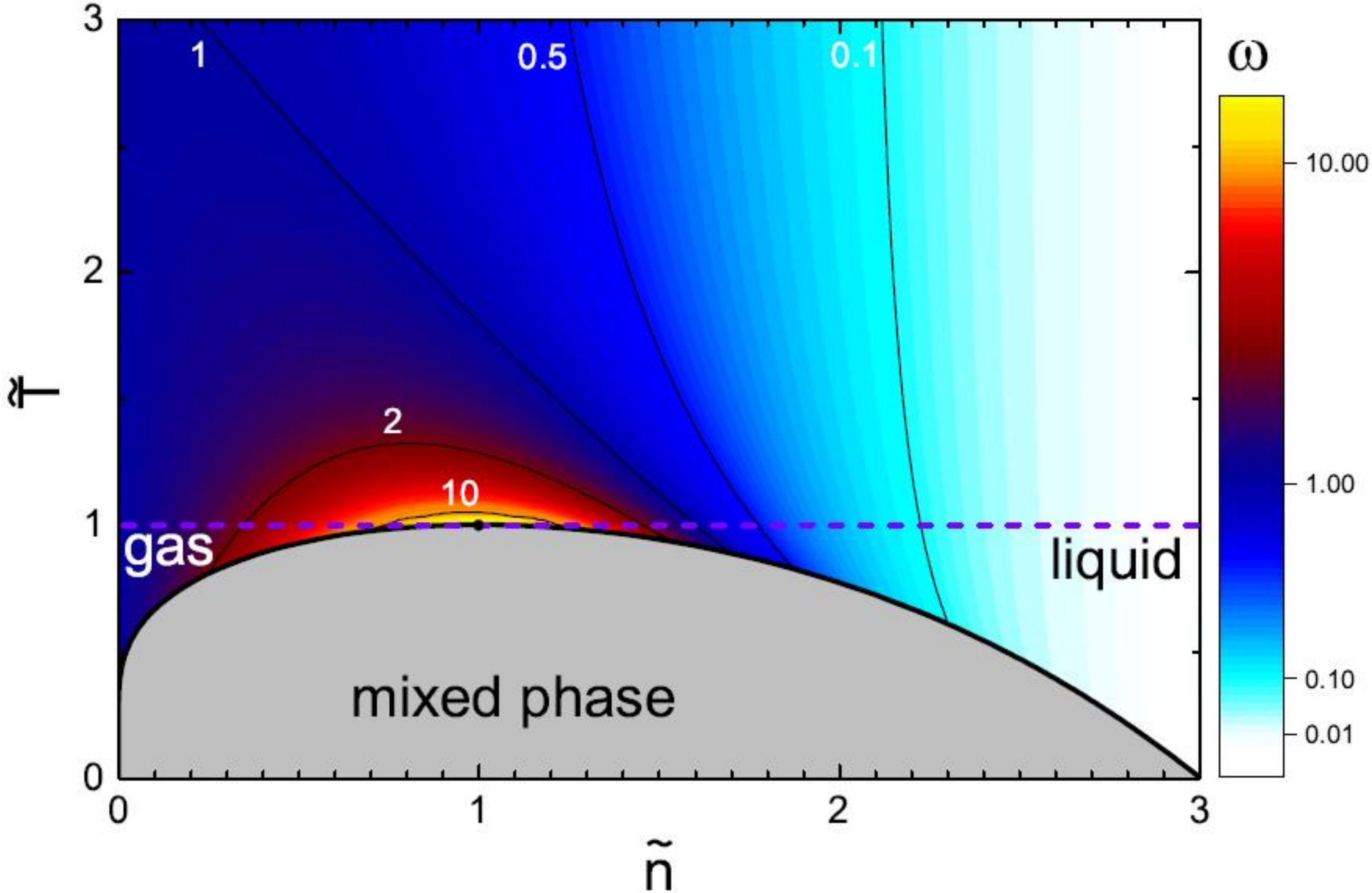


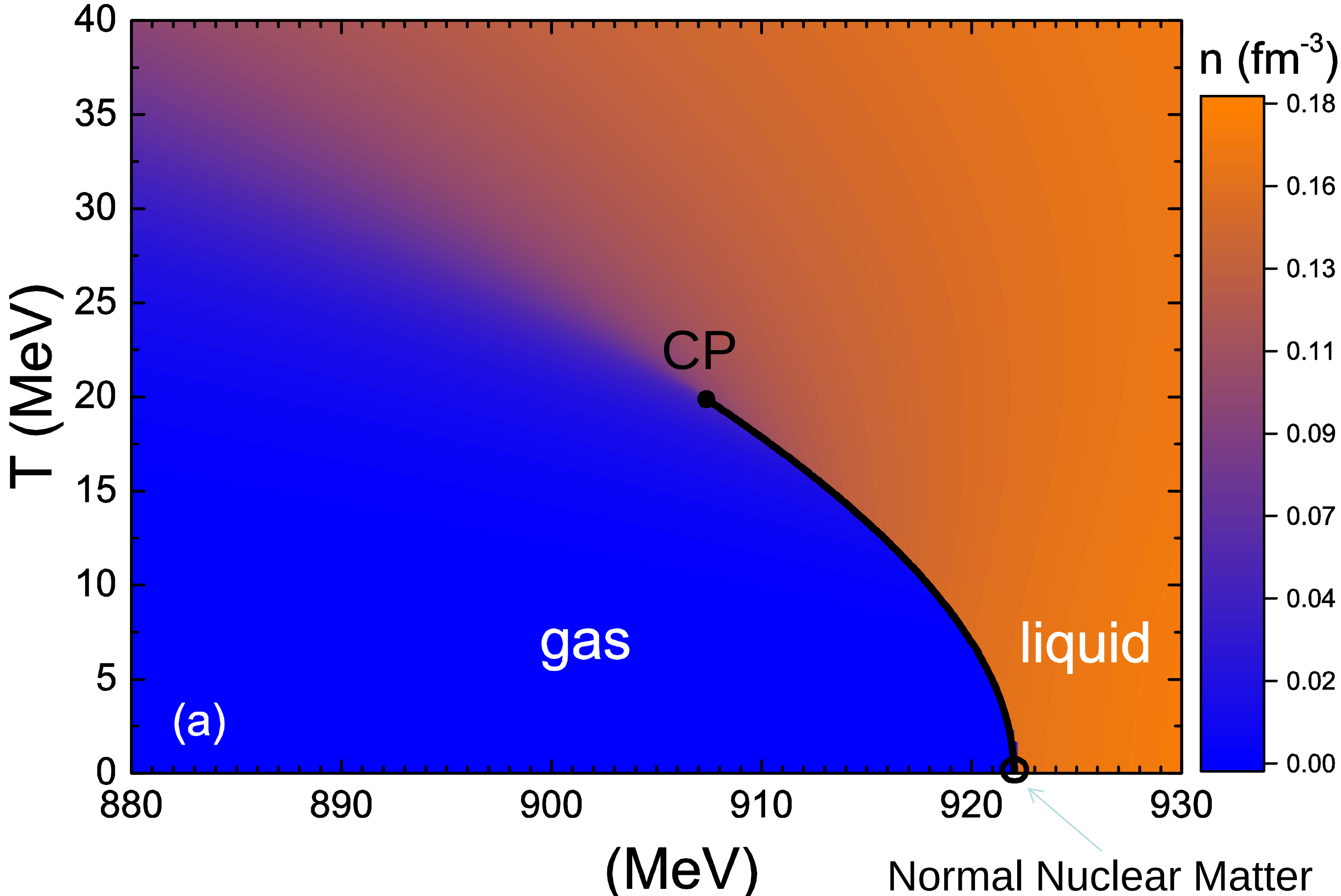
$$p(V,T,N) = \frac{nT}{1-bn} - an^2, \quad \text{CE}$$

$$n(T,\mu) = \frac{n_{\mathrm{id}}(T,\mu^*)}{1+bn_{\mathrm{id}}(T,\mu^*)}, \qquad \mu^* = \mu - b\frac{nT}{1-bn} + 2an, \qquad \text{GCE}$$

$$\omega[N] \equiv \frac{<N^2> - <N>^2}{<N>} = \left[\frac{1}{(1-bn)^2} - \frac{2an}{T}\right]^{-1}$$

$$\omega[N] = \frac{1}{9}\left[\frac{1}{(3-\tilde{n})^2} - \frac{\tilde{n}}{4\tilde{T}}\right]^{-1}$$

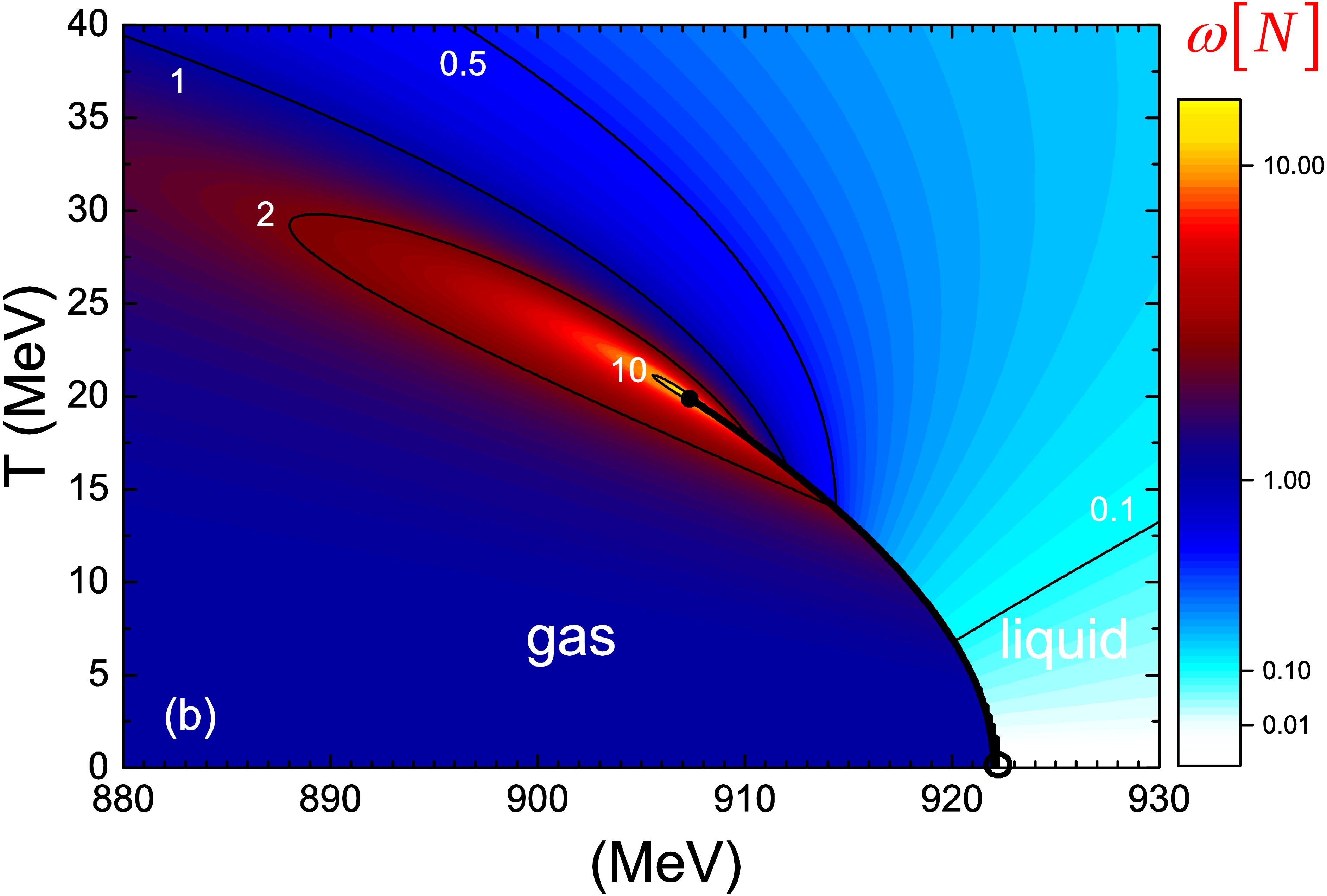




$n_0 \cong 0.16\ \mathrm{fm}^{-3},\ E_B \cong -16\ \mathrm{MeV}$

Nuclear Matter: VDW EoS,

Fermi Statistics, $m = 938$ MeV, $a = 329\ \mathrm{MeVfm}^3,\ r = 0.59\ \mathrm{fm}$

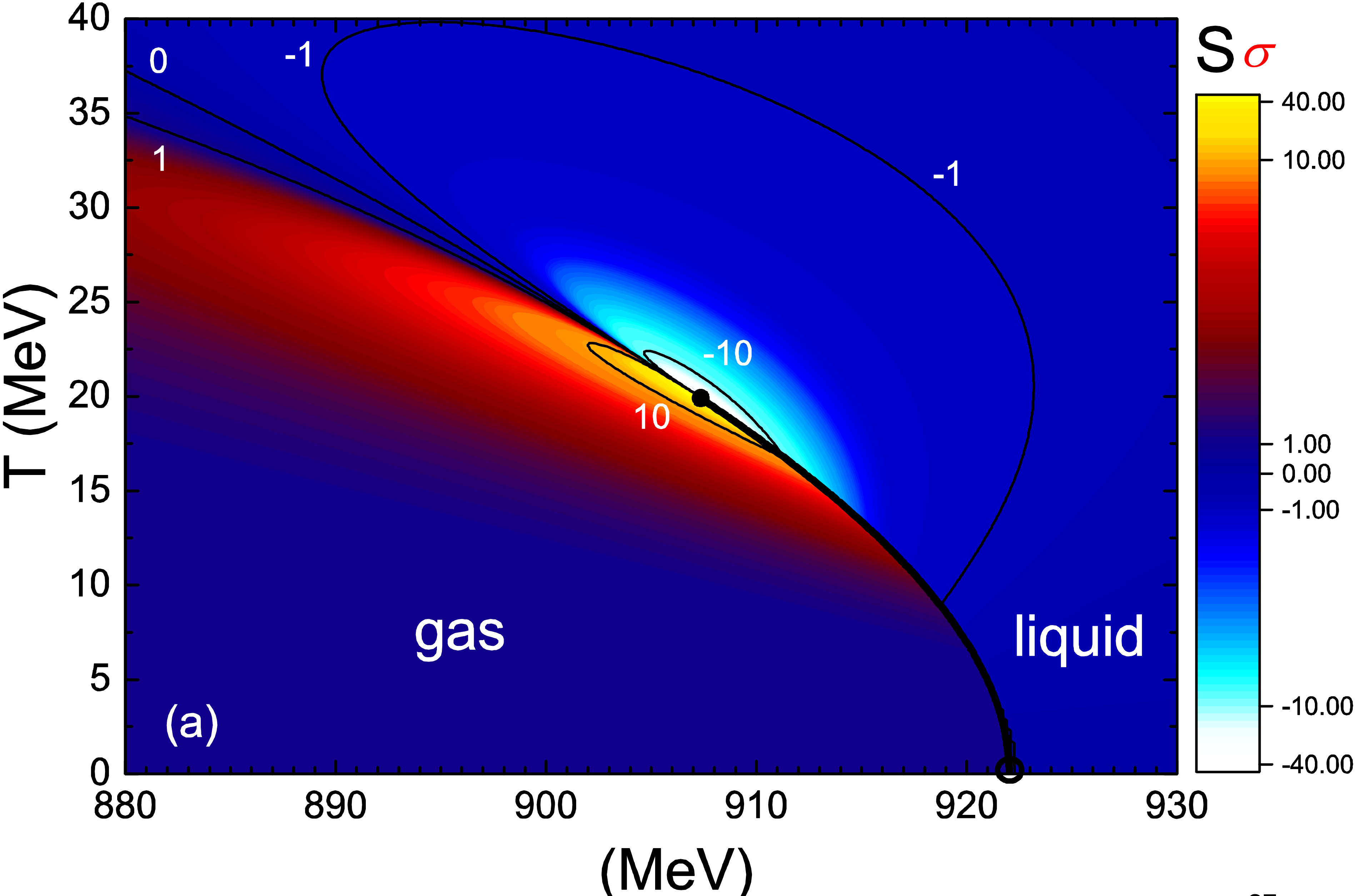


Scaled Variance

$$\omega[N] \equiv \frac{<(\Delta N)^2>}{<N>} \equiv \frac{\sigma^2}{<N>}$$ scaled variance

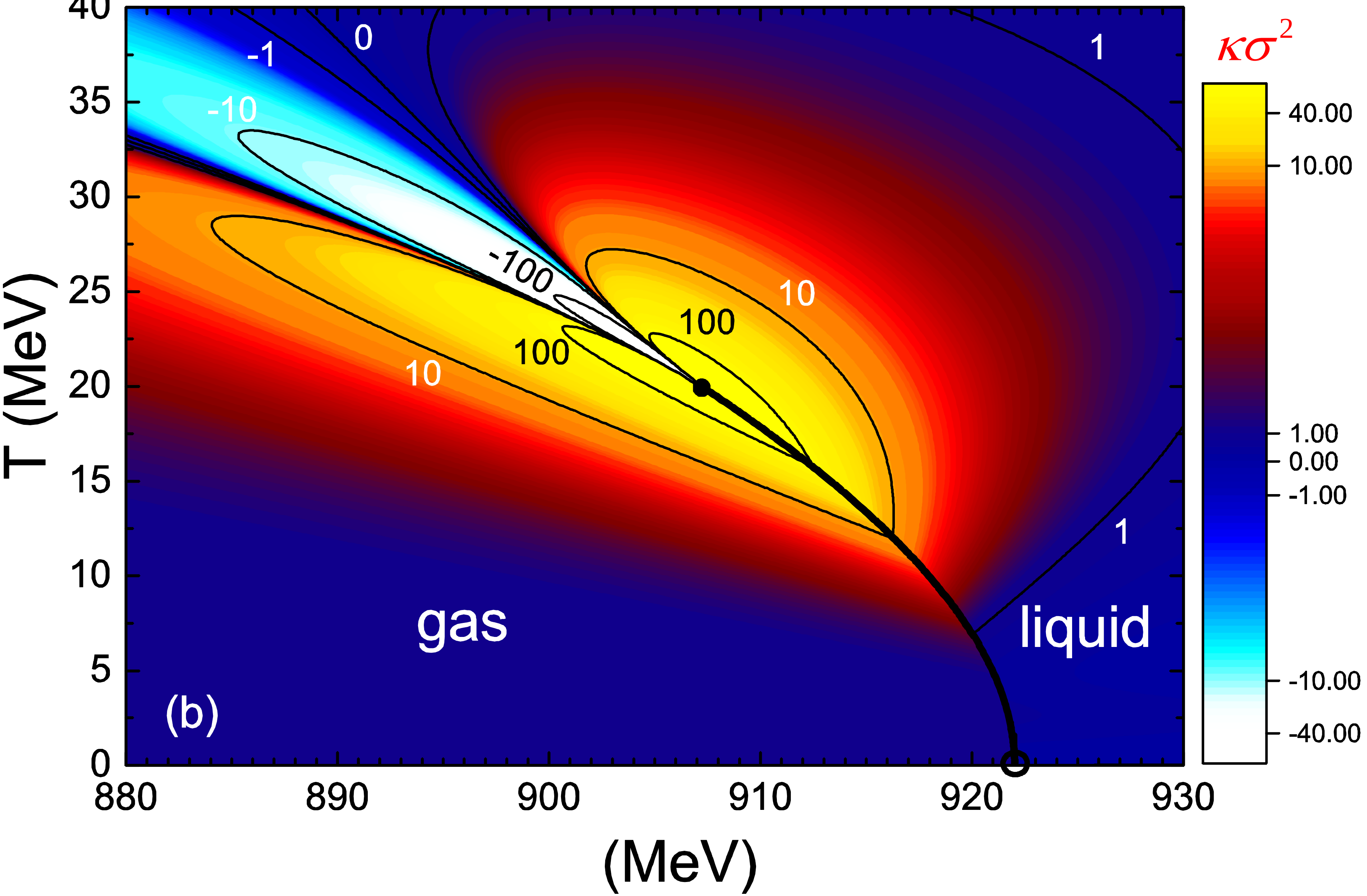
$$S\sigma \equiv \frac{<(\Delta N^3)>}{\sigma^2}$$ skewness

$$\kappa\sigma^2 \equiv \frac{<(\Delta N^4)> - 3<(\Delta N^2)>^2}{\sigma^2}$$ kurtosis



40
35
30
25
20
15
10
5
0
T (MeV)
880
890
900
910
920
930
(MeV)
0
-1
1
-1
-10
10
gas
liquid
(a)
Sσ
40.00
10.00
1.00
0.00
-1.00
-10.00
-40.00

Skewness



κσ²
40.00
10.00
1.00
0.00
-1.00
-10.00
-40.00
T (MeV)
(MeV)
-1
0
-10
1
-100
10
100
gas
liquid
(b)

Kurtosis

Summary

1. Global Conservation Laws

suppress particle number fluctuations and introduce interparticle correlations. This is valid also in the thermodynamic limit $V \to \infty$.

2. Strongly Intensive Measures $\Delta[A,B]$ and $\Sigma[A,B]$

are independent of the average size of the system and of the fluctuations of the size (e.g., GCE, Model of Independent Sources).

3. Identity Method

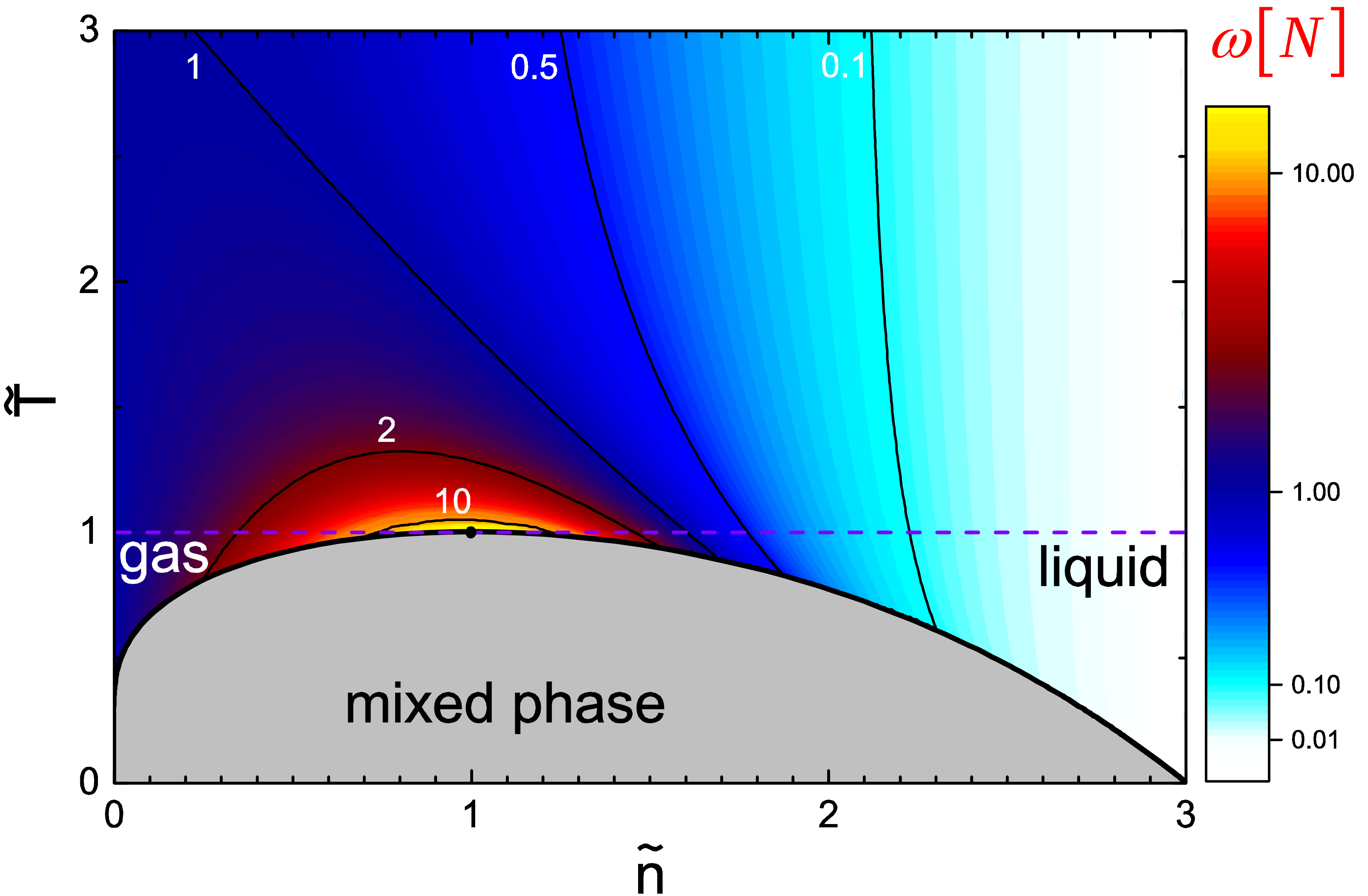
provides the values of all the second and higher moments of identified particle number distributions in a model independent way for the case of incomplete particle identifications.

4. Van der Waals Equation of State and Fluctuations

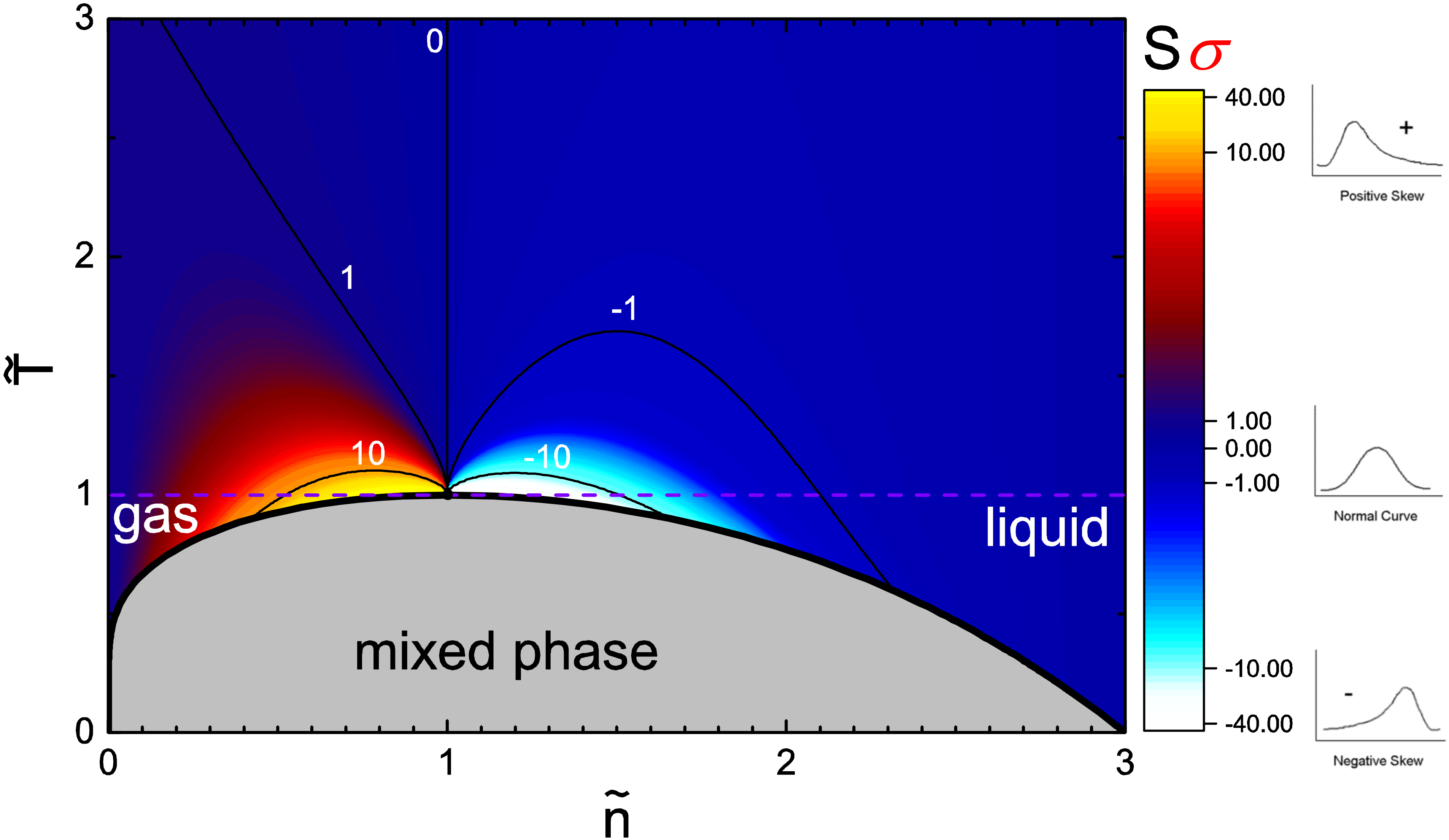
provides an analytical example of the fluctuations in systems with 1st order liquid-gas phase transition and critical point.

Thank you!

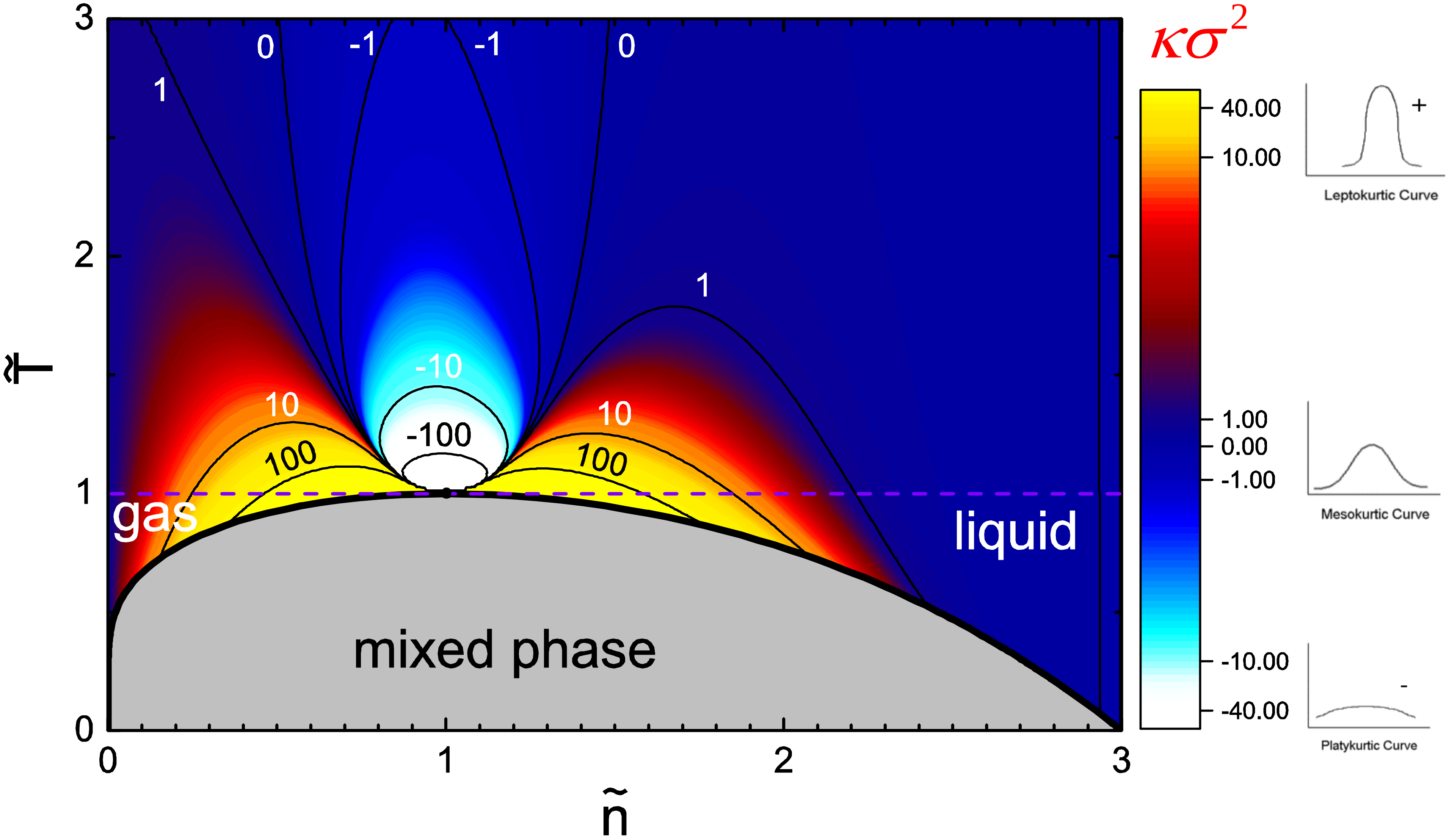
Additional Slides



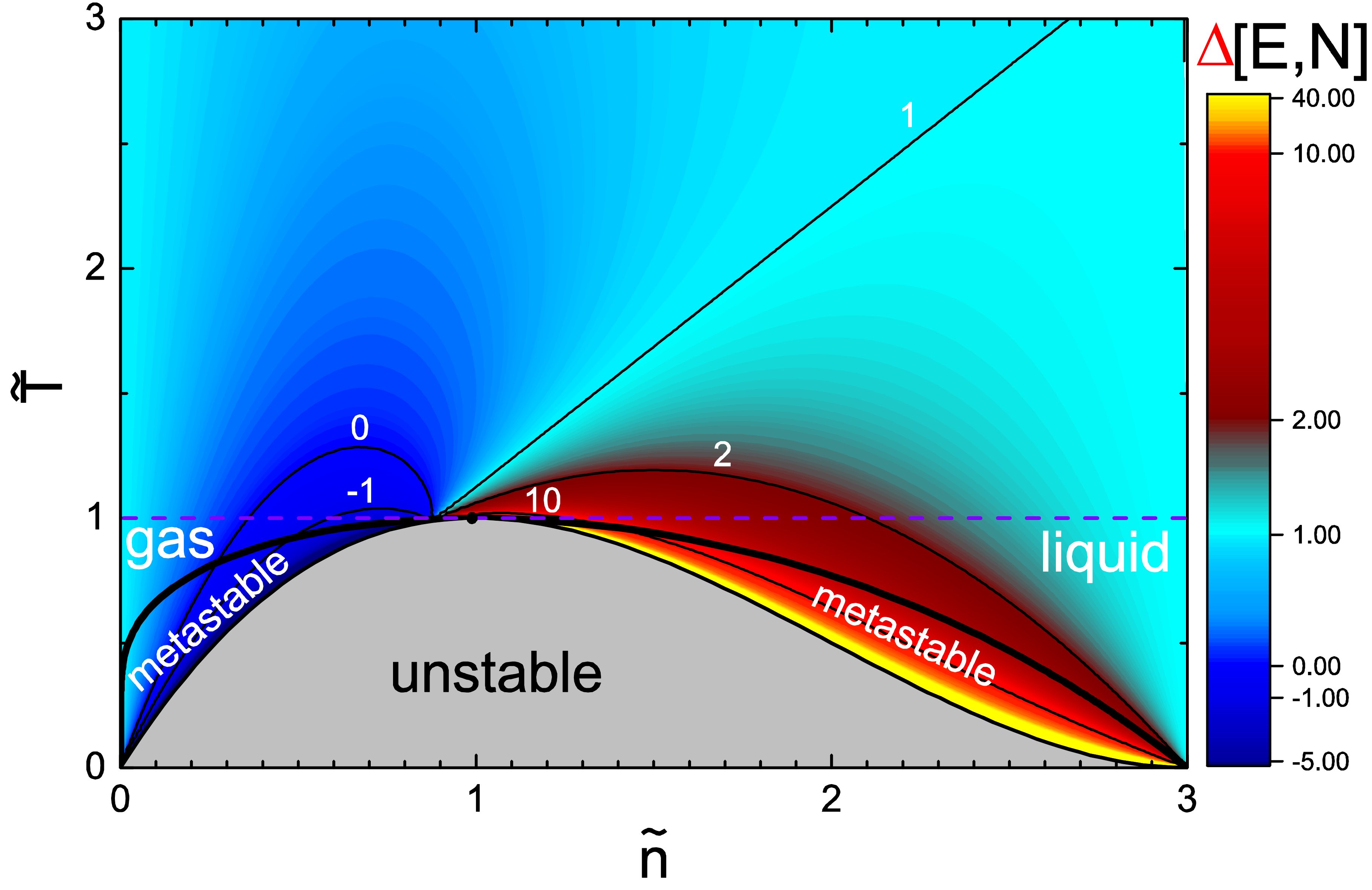
ω[N]
10.00
1.00
0.10
0.01
1
0.5
0.1
2
10
gas
liquid
mixed phase
T̃
ñ
0
1
2
3



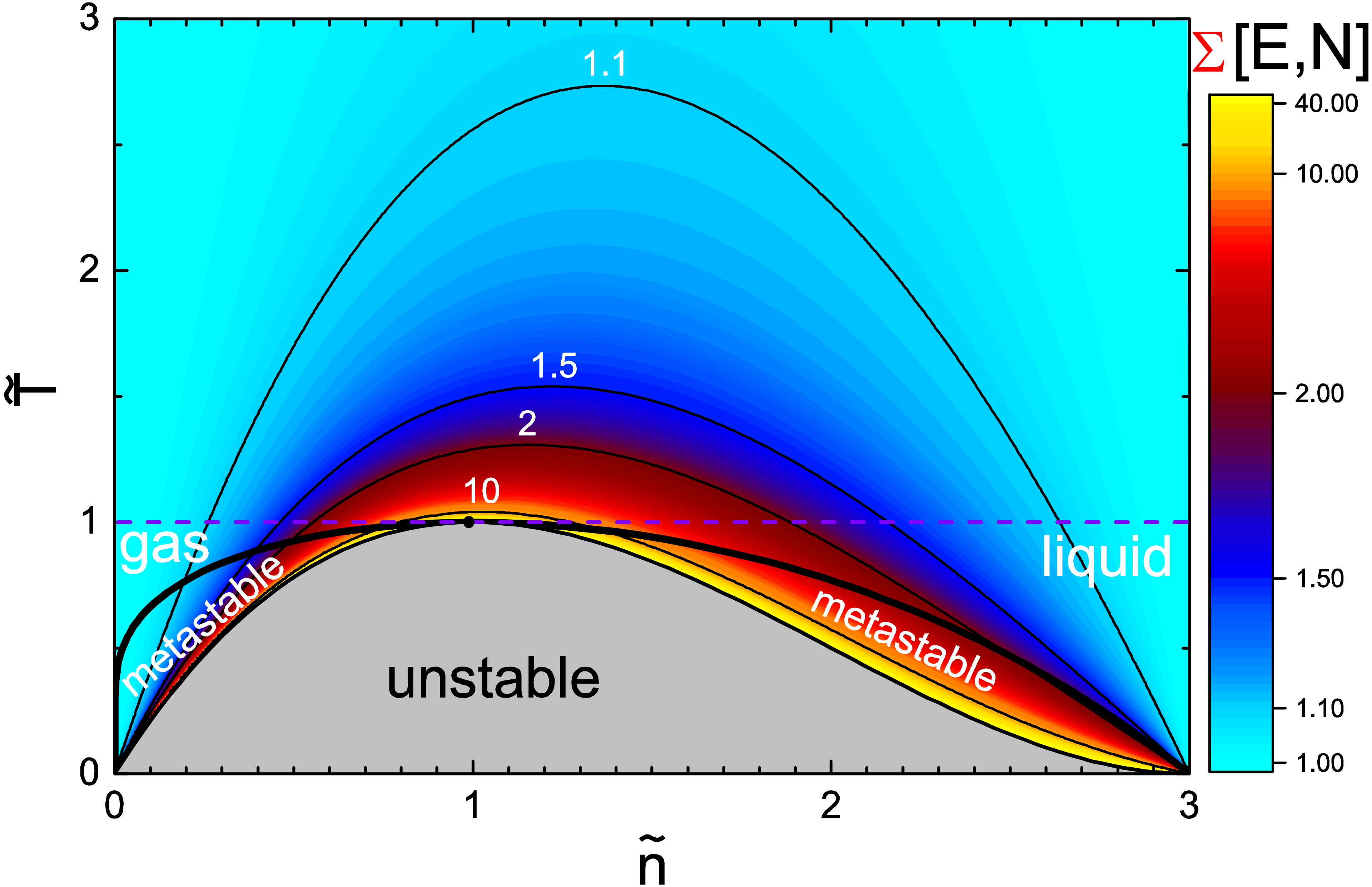
3
2
1
0
$\tilde{T}$
0
1
1
-1
10
-10
gas
liquid
mixed phase
0
1
2
3
$\tilde{n}$
S σ
40.00
10.00
1.00
0.00
-1.00
-10.00
-40.00
+
Positive Skew
Normal Curve
-
Negative Skew



$\kappa\sigma^2$
40.00
10.00
1.00
0.00
-1.00
-10.00
-40.00
Leptokurtic Curve
+
Mesokurtic Curve
Platykurtic Curve
-
$\tilde{T}$
$\tilde{n}$
gas
liquid
mixed phase
1
0
-1
-1
0
1
-10
-100
10
100
10
100
0
1
2
3

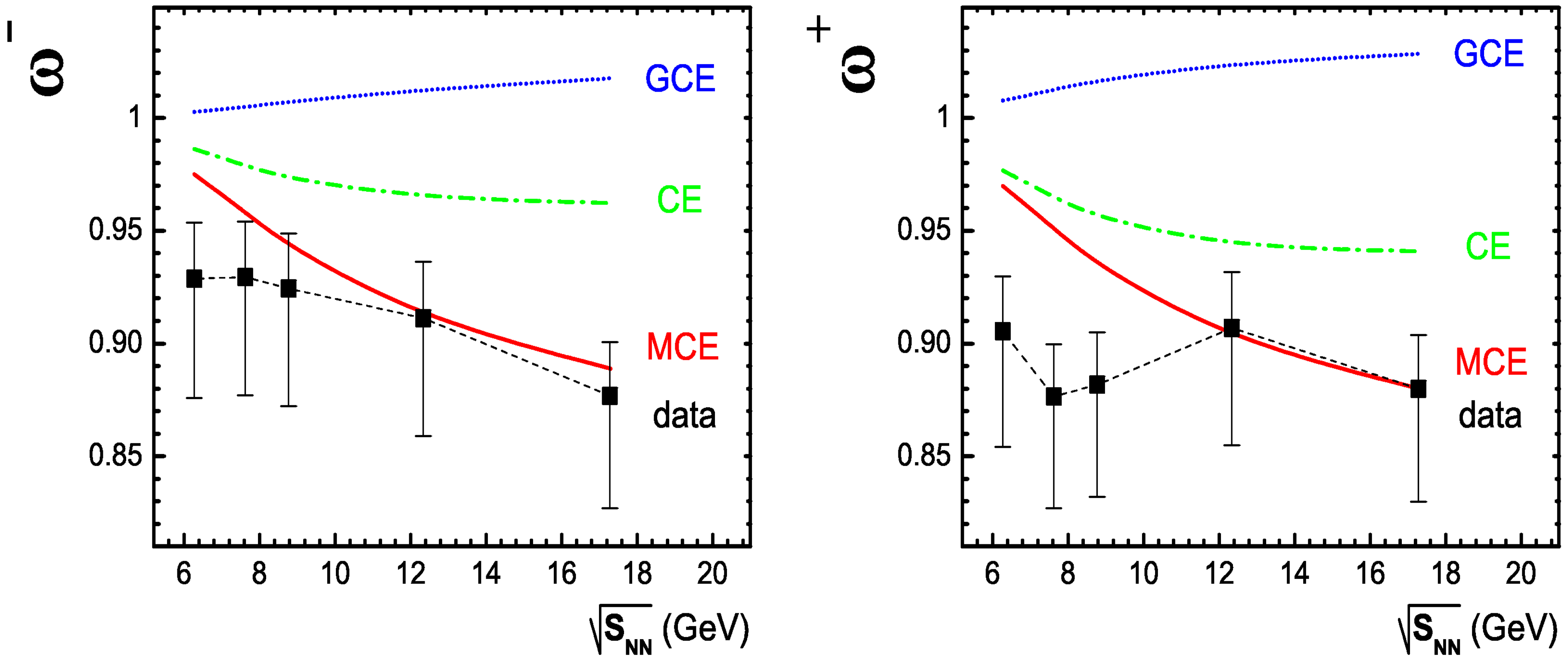


Δ[E,N]
40.00
10.00
2.00
1.00
0.00
-1.00
-5.00
1
0
-1
2
10
gas
liquid
metastable
metastable
unstable
T̃
ñ
0
1
2
3
0
1
2
3


Σ[E,N]
40.00
10.00
2.00
1.50
1.10
1.00
1.1
1.5
2
10
gas
liquid
metastable
metastable
unstable
T̃
ñ
0
1
2
3
0
1
2
3

Comparison with the NA49 data

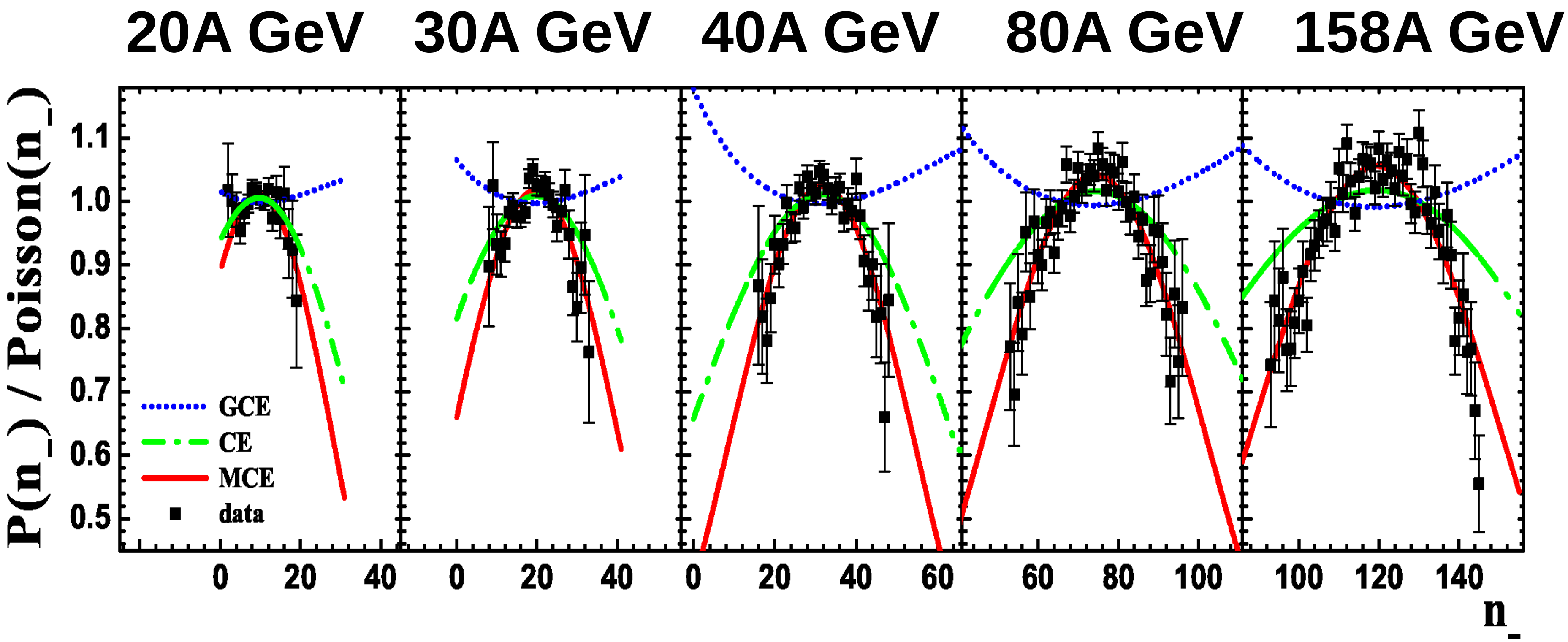
Pb+Pb 1% most central events



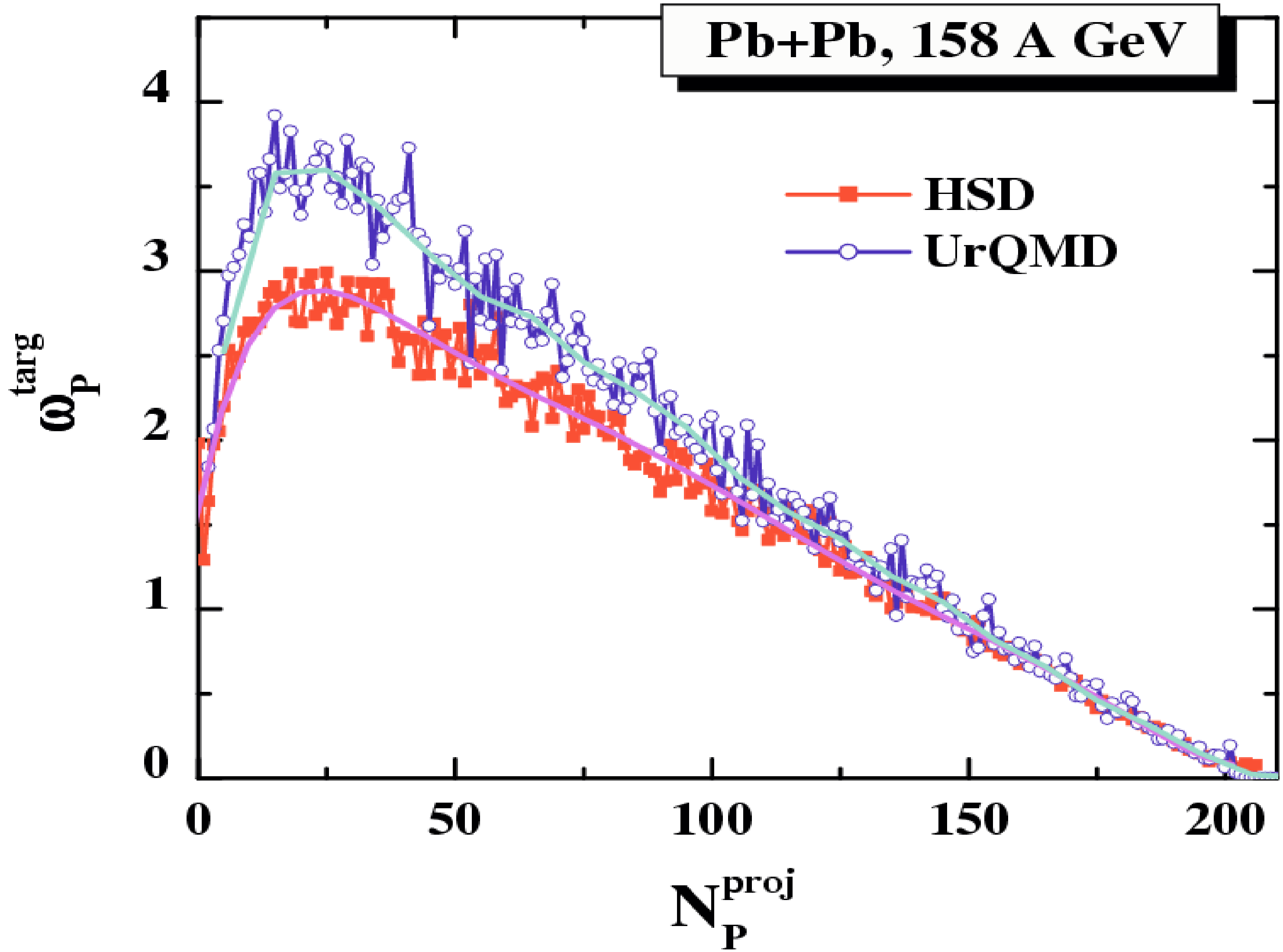
Begun, Gazdzicki, M.I.G., Hauer, Konchakovski, Lungwitz, Phys. Rev. C (2007)

Measured distribution / Poisson distribution

NA49, Pb+Pb, < 1% of most central events



Begun, Gazdzicki, M.I.G., Hauer, Konchakovski, Lungwitz, Phys. Rev. C (2006)



Konchakovski, Hausler, M.I.G., Bratkovskaya, Bleicher, Stoecker,
Phys. Rev. C (2006)

Relation to Other Measures

$$\Phi = \frac{\sqrt{\langle A\rangle\langle B\rangle}}{\langle \mathbf{A}\rangle + \langle \mathbf{B}\rangle}\left[\left(\Sigma^{\mathrm{AB}}\right)^{1/2} - 1\right]$$

Gazdzicki, Mrowczynski (1992)

$$\nu_{\mathrm{dyn}}^{\mathrm{AB}} = \frac{\langle \mathbf{A}(\mathbf{A}-1)\rangle}{\langle \mathbf{A}\rangle^2} + \frac{\langle \mathbf{B}(\mathbf{B}-1)\rangle}{\langle \mathbf{B}\rangle^2} - 2\,\frac{\langle \mathbf{AB}\rangle}{\langle \mathbf{A}\rangle\langle \mathbf{B}\rangle}$$

Pruneau, Gavin, Voloshin (2002)

$$\nu_{dyn}[A,B] = \frac{< A+B >}{< A >< B >}[\Sigma[A,B]-1]$$

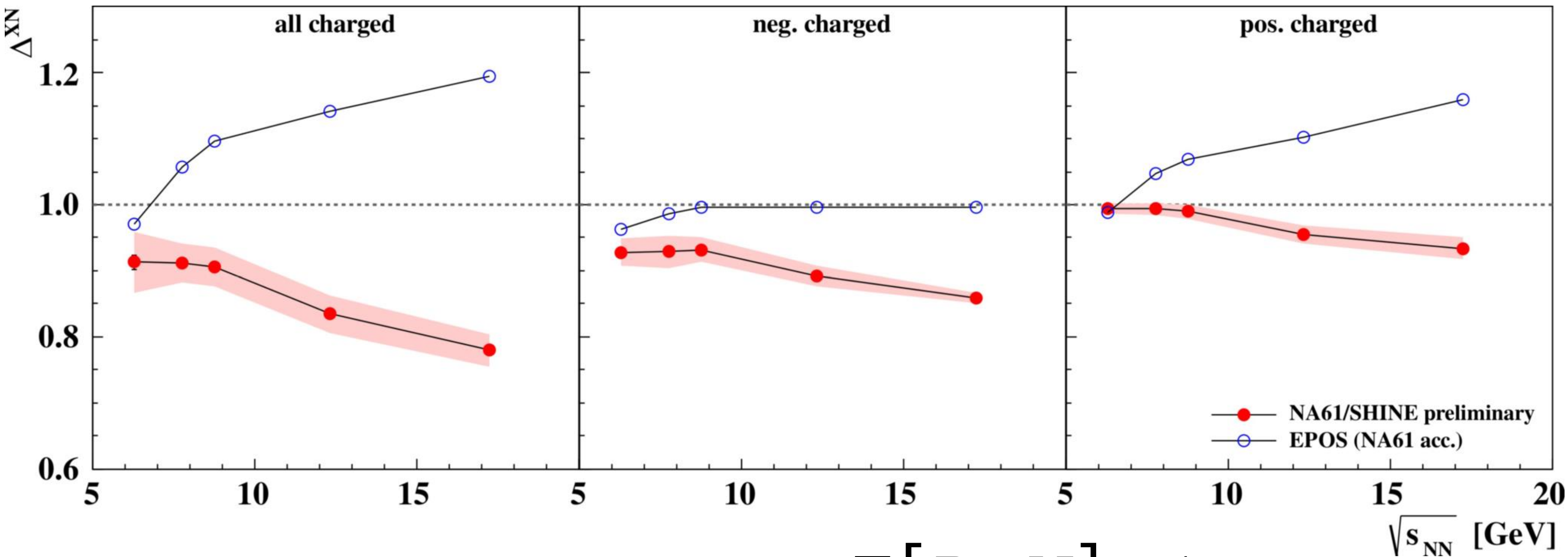
$$\Sigma[P_T, N] \cong 1.01,$$

p+p at 158 GeV/c

$$\Delta[P_T, N] \cong 0.82$$

M.I.G., Grebieszkow,
Phys. Rev.C (2014)

NA61/SHINE data in p+p reactions



Data: $\Sigma[P_T, N] \cong 1$