Rapidity window and centrality dependences of higher order cumulants

Masakiyo Kitazawa

(Osaka U.)

MK, Asakawa, Ono, Phys. Lett. B728, 386-392 (2014)
Sakaida, Asakawa, MK, PRC90, 064911 (2014)
MK, arXiv:1505.04349, Nucl. Phys. A, in press

HIC for FAIR workshop, Frankfurt, 30/Jul./2015

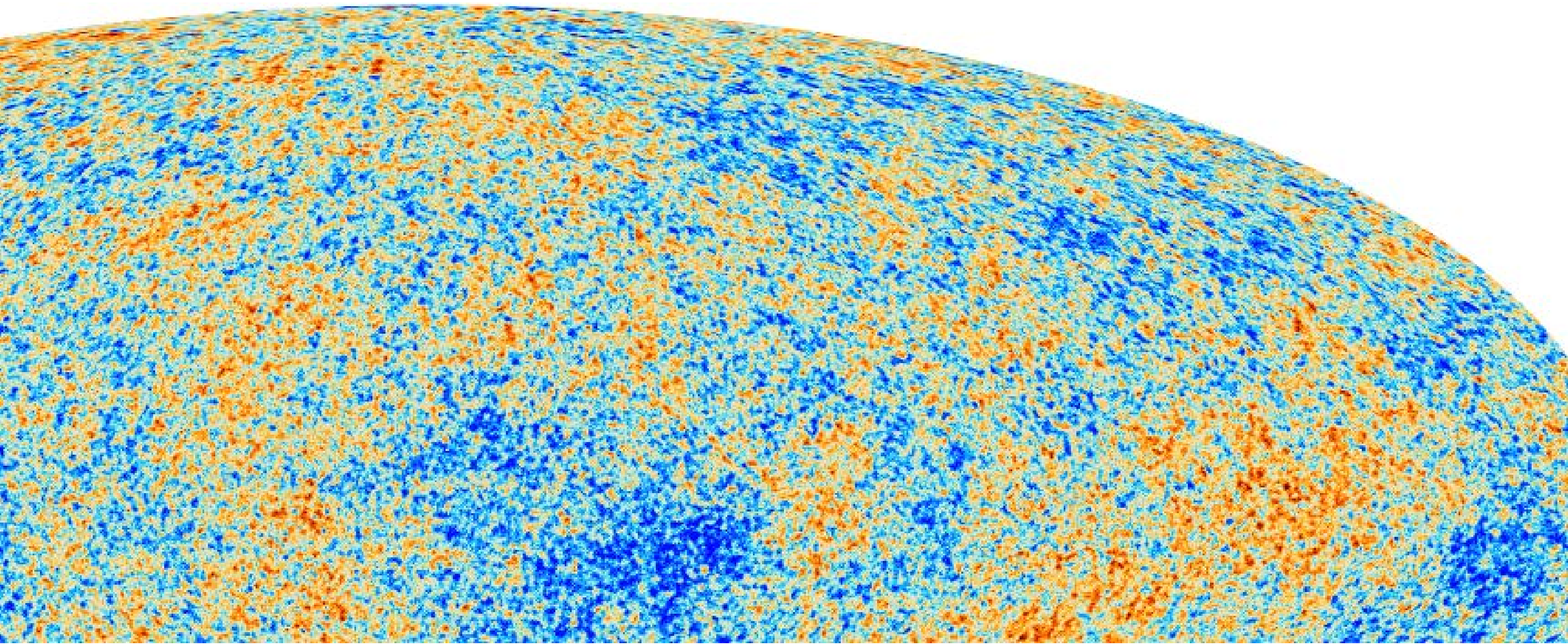
In “haiku”, a Japanese short style poem, a poet wrote…

Even on one blade of grass
the cool wind lives

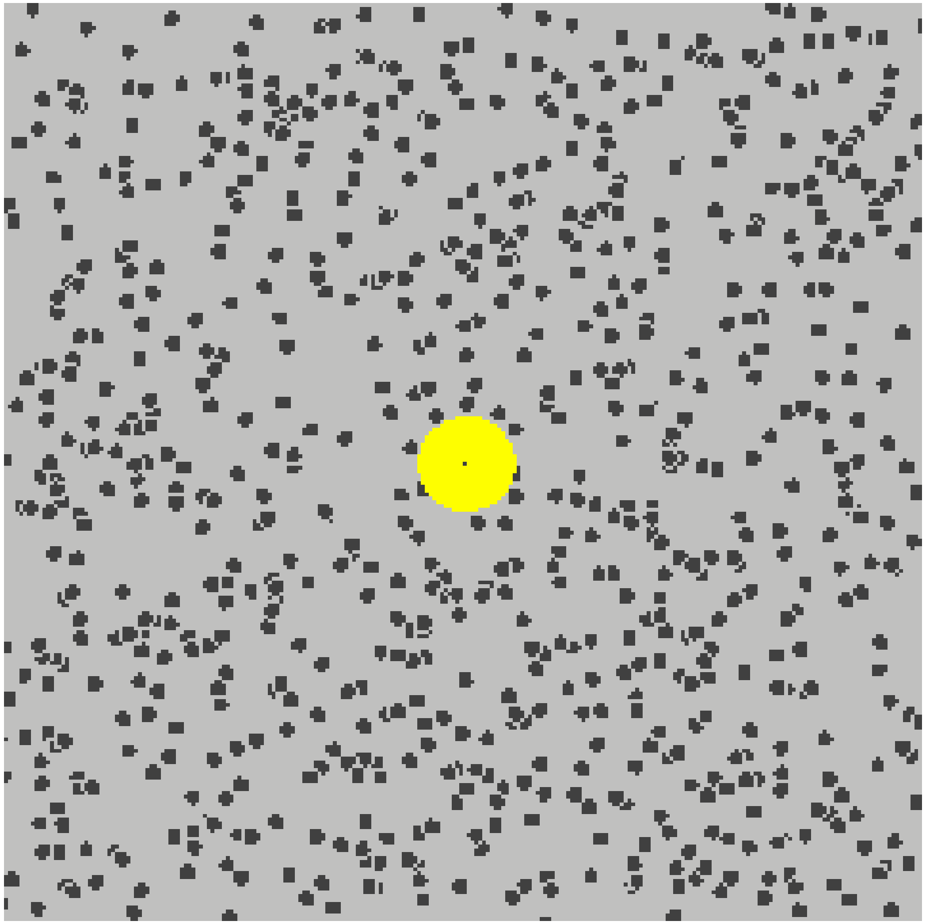
Issa Kobayashi
1814

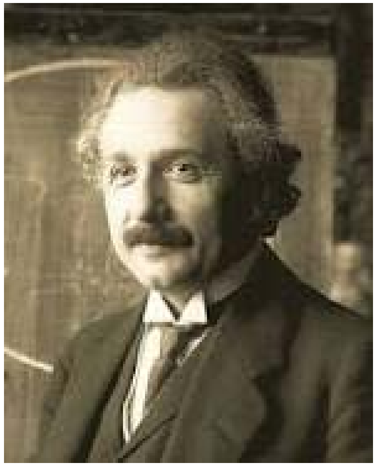
一本の草も涼風宿りけり
小林一茶

Physicists can feel hot early Universe
13 800 000 000 years ago
in tiny fluctuations of
cosmic microwave



Physicists can feel the existence of microscopic atoms behind random fluctuations of Brownian pollens

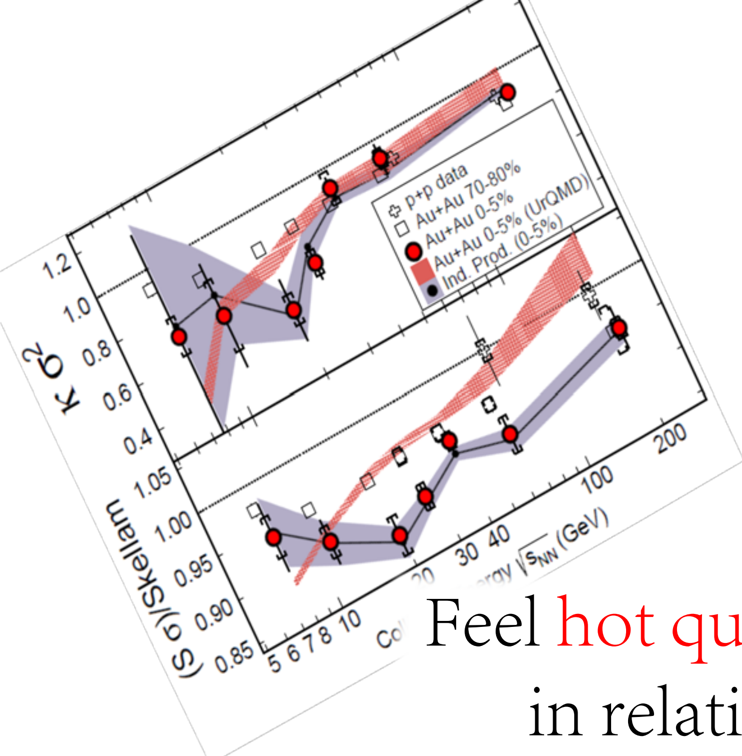


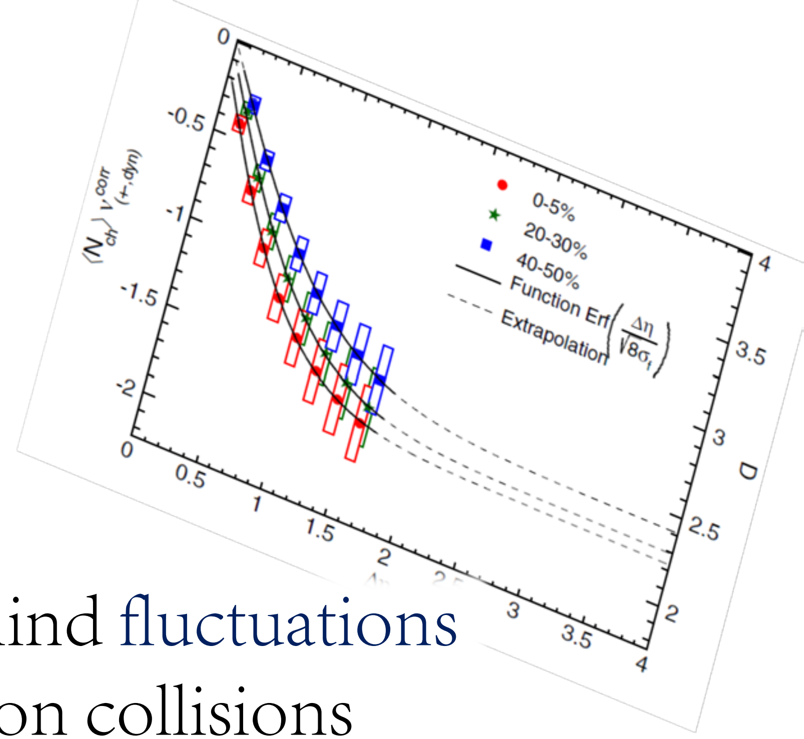


A. Einstein
1905

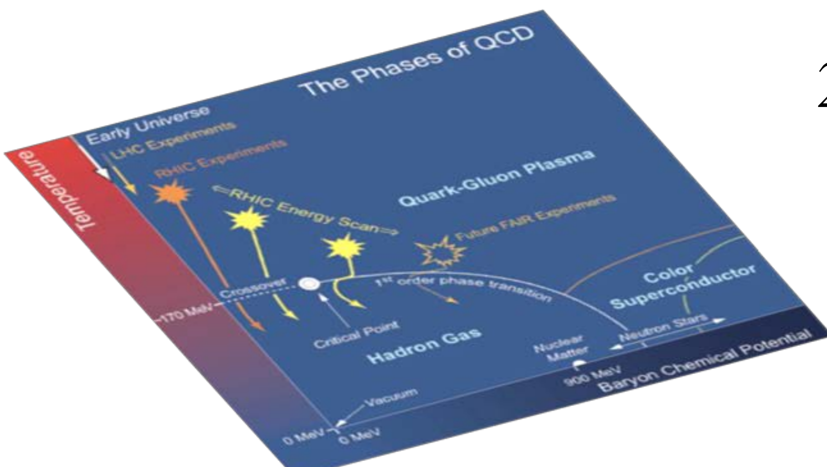
Feel hot quark wind behind fluctuations
in relativistic heavy ion collisions

2010-

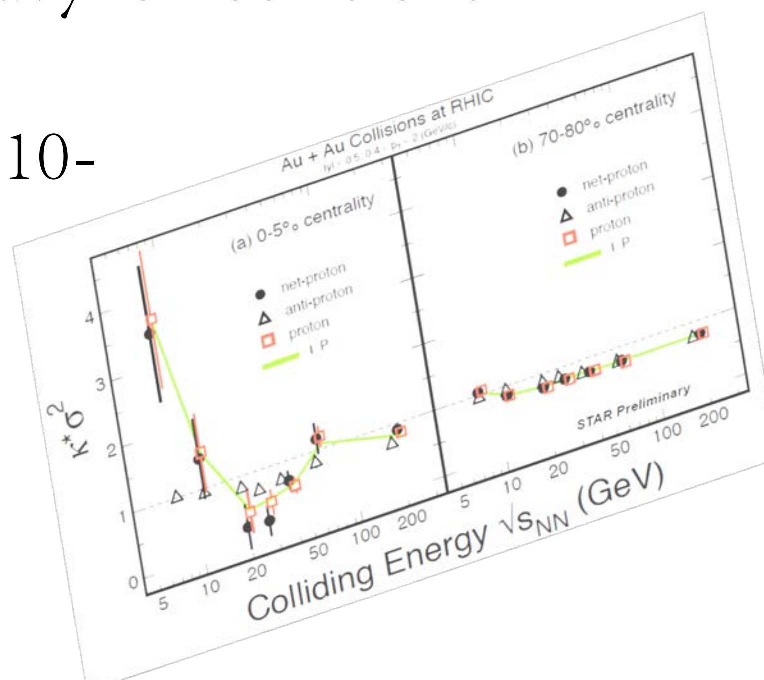




Feel hot quark wind behind fluctuations in relativistic heavy ion collisions



2010-



Outline

1. A poem
2. Electric charge fluctuation @ ALICE
3. Thermal blurring in momentum-space rapidity

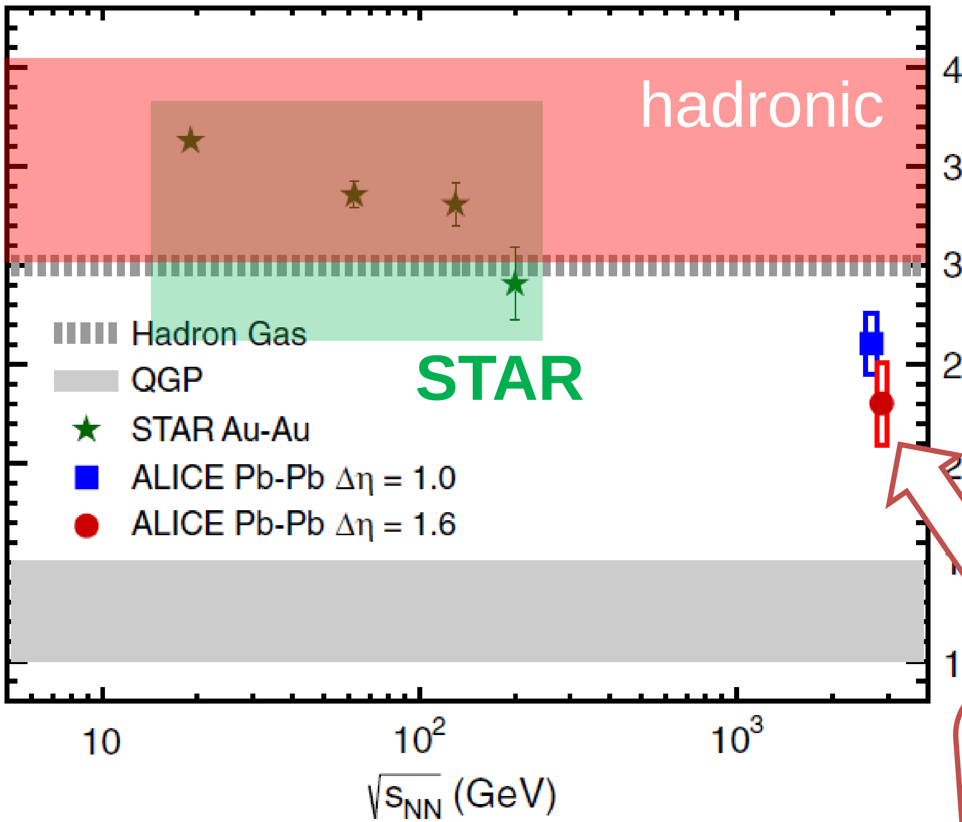
   Ohnishi+, in preparation
4. $\Delta\eta$ dependences of higher order cumulants

   MK, Asakawa, Ono, PLB(2014); MK, NPA(2015)
5. Effect of global charge conservation

   Sakaida, Asakawa, MK, PRC(2014)

Electric charge fluctuations @ ALICE

Charge Fluctuation @ LHC



ALICE, PRL110,152301(2013)

D-measure

$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

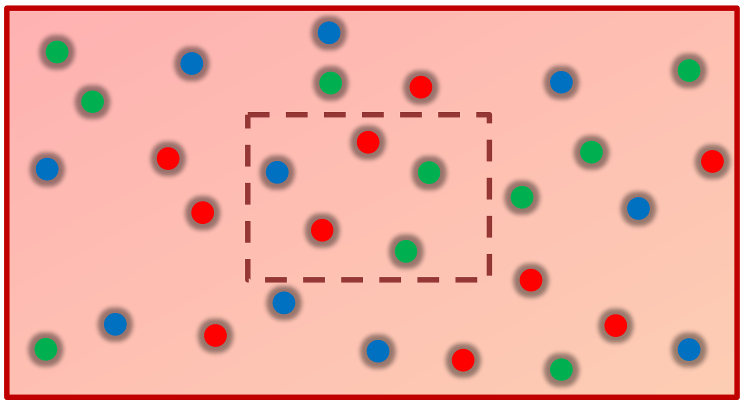
- D ~ 3-4 Hadronic
- D ~ 1-1.5 Quark

Suppression from hadronic value at LHC energy!

$\langle \delta N_Q^2 \rangle$ is not equilibrated at freeze-out at LHC energy!

Fluctuations and Elemental Charge

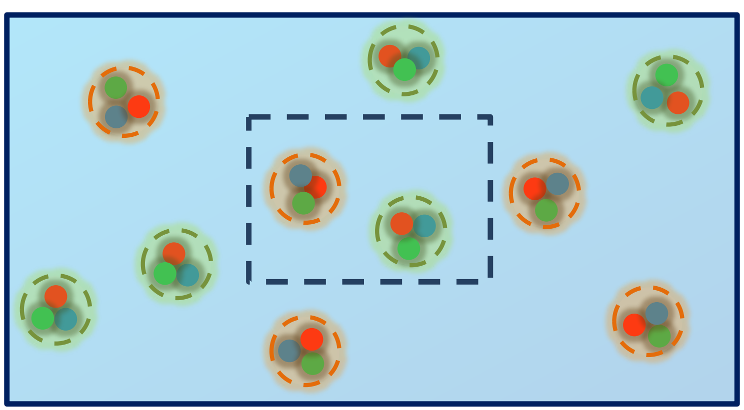
Asakawa, Heinz, Muller, 2000
Jeon, Koch, 2000
Ejiri, Karsch, Redlich, 2005



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$

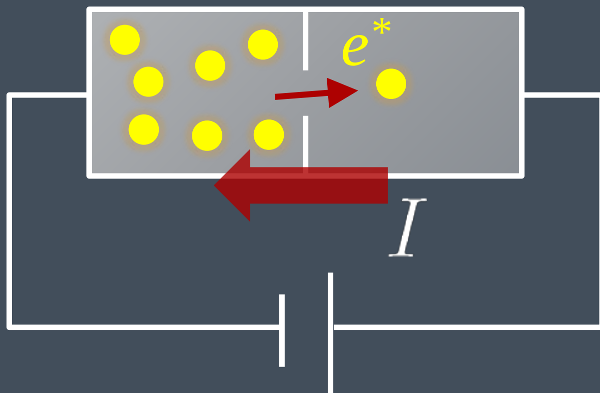


$$\langle \delta N_B^n \rangle_c = \langle N_B \rangle$$

Free Boltzmann → Poisson

$$\langle \delta N^n \rangle_c = \langle N \rangle$$

Shot Noise



$$S_{\text{shot}} \sim \langle \delta I^2 \rangle$$

$$S_{\text{shot}} = 2e^* \langle I \rangle$$

charge of quasi-particles

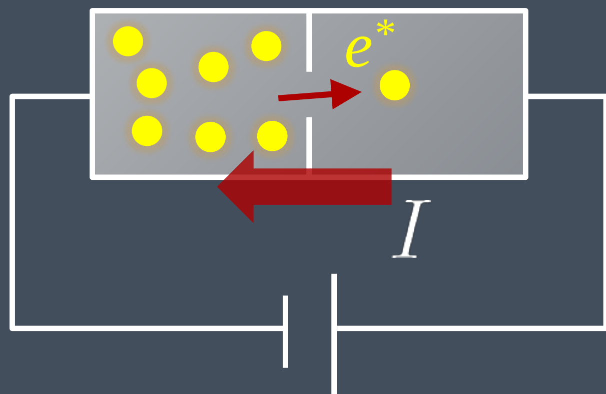
Total charge Q:

$$Q = e\langle N \rangle$$

$$\langle \delta Q^2 \rangle = e^2 \langle \delta N^2 \rangle = e^2 \langle N \rangle = eQ$$

$$\frac{\langle \delta Q^2 \rangle}{Q} = e$$

Shot Noise



$$S_{\text{shot}} \sim \langle \delta I^2 \rangle$$

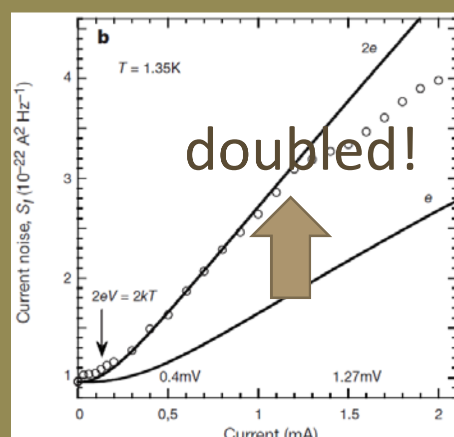
$$S_{\text{shot}} = 2e^* \langle I \rangle$$

charge of quasi-particles

Superconductors with Cooper Pairs

$$e^* = 2e$$

Jehl+, Nature **405**,50 (2000)



Fractional Quantum Hall Systems

$$e^* = \frac{q}{p} e$$

Saminadayar+, PRL**79**,2526 (1997)

Higher order cumulants:

3rd order: ex. Beenakker+, PRL90,176802(2003)

up to 5th order: Gustavsson+, Surf.Sci.Rep.**64**,191(2009)

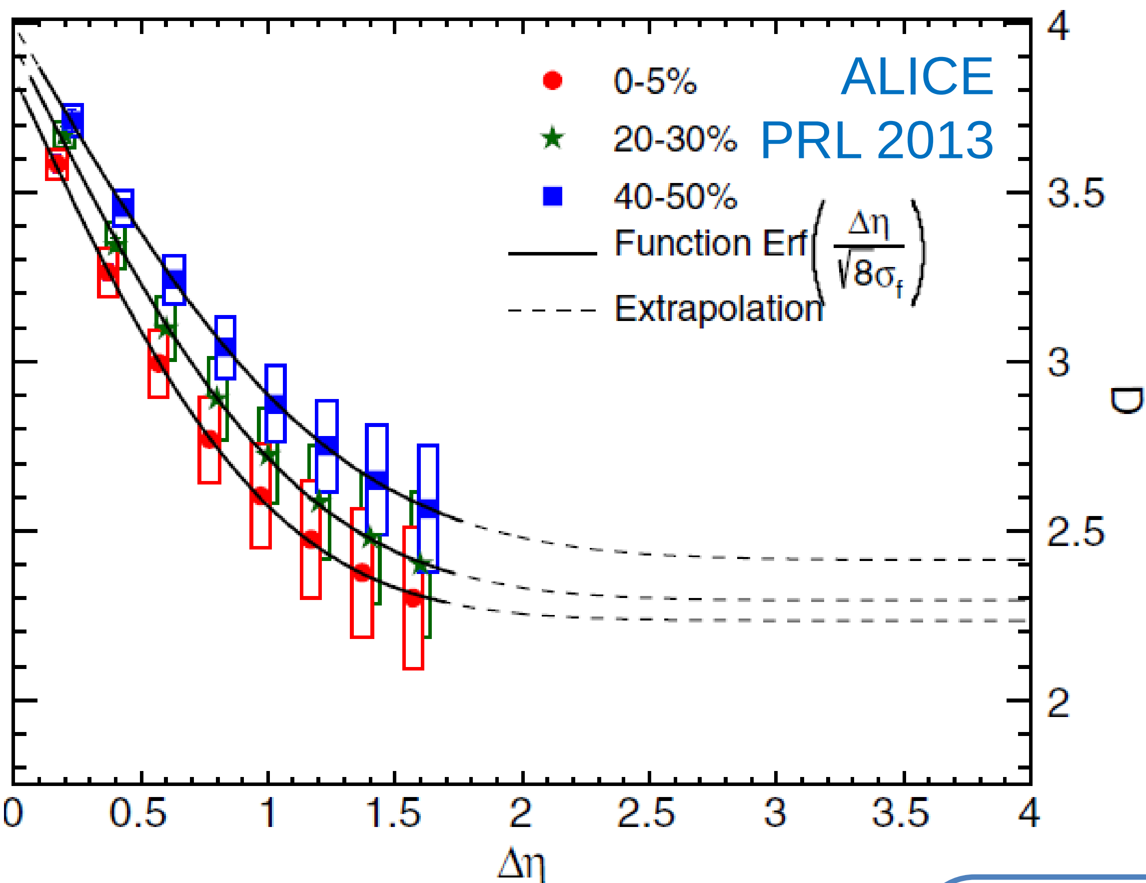
Various Contributions to Fluctuations

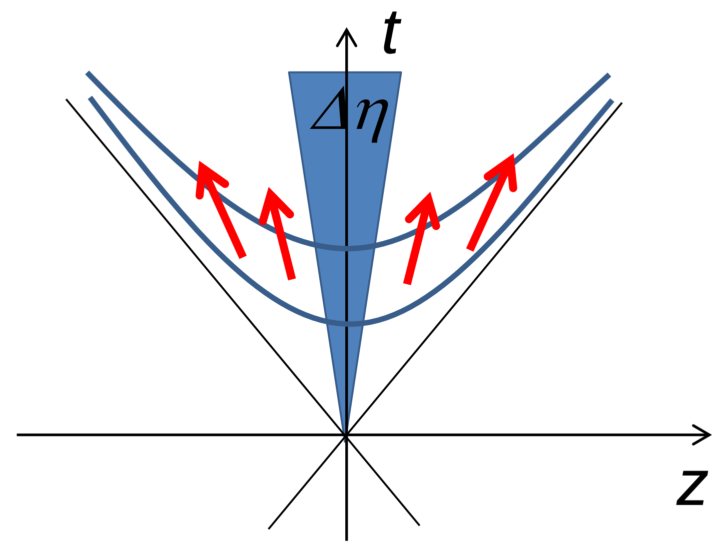
- Initial fluctuations ↑ Enhance
- Effect of jets ↑ Enhance
- Negative binomial (?) ↑ Enhance
- Final state rescattering ↗ **Enhance** to Poisson
- Coordinate vs pseudo rapidities ↗ **Enhance** to Poisson
- Particle missID ↗ **Enhance** to Poisson — Ono, Asakawa, MK PRC(2013)
- Efficiency correction ↗ **Enhance** to Poisson
- Global charge conservation ↓ **Suppress** — Sakaida, Asakawa, MK, PRC(2014)

The suppression is most probably a consequence of the small fluctuation in deconfined medium.

Δη Dependence @ ALICE

ALICE
PRL 2013



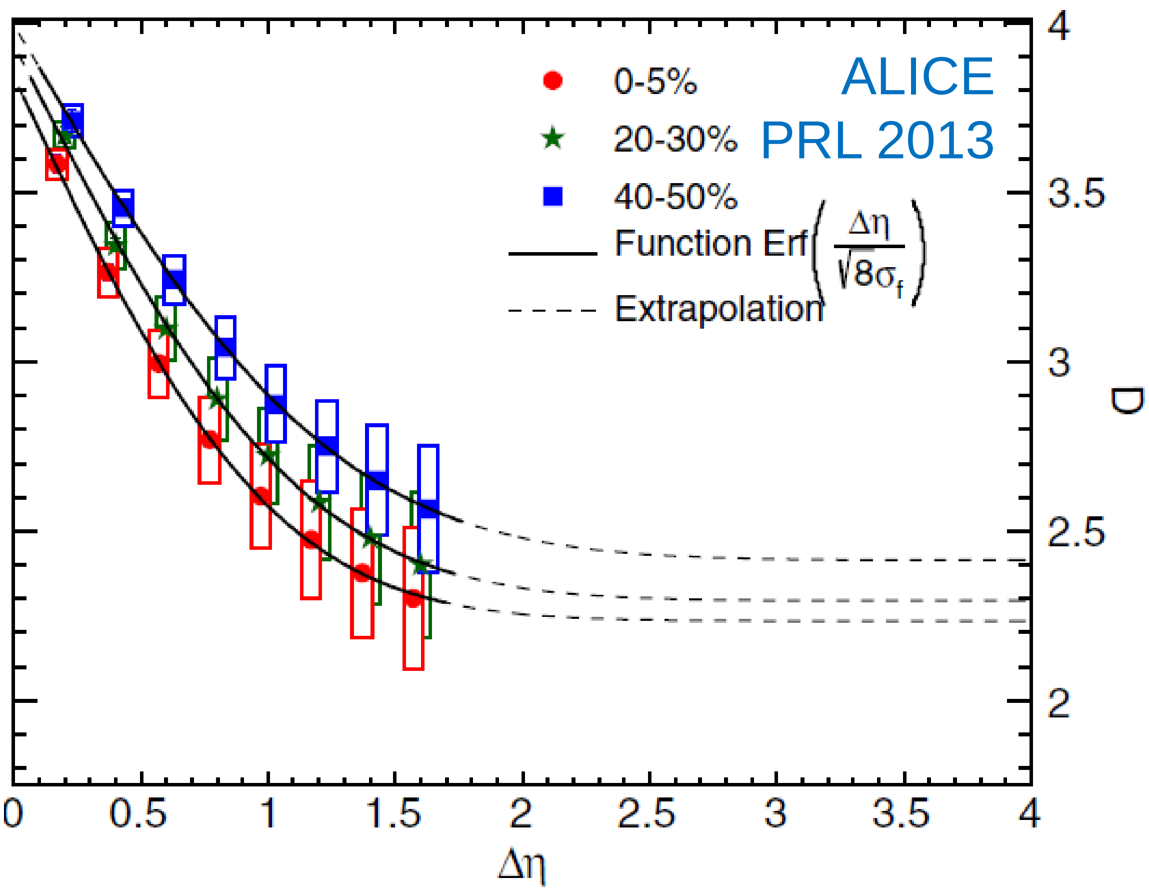


rapidity window

Same information as

- 2 particle corr.: $\langle n(\eta)n(0)\rangle$
- Balance function

Δη Dependence @ ALICE



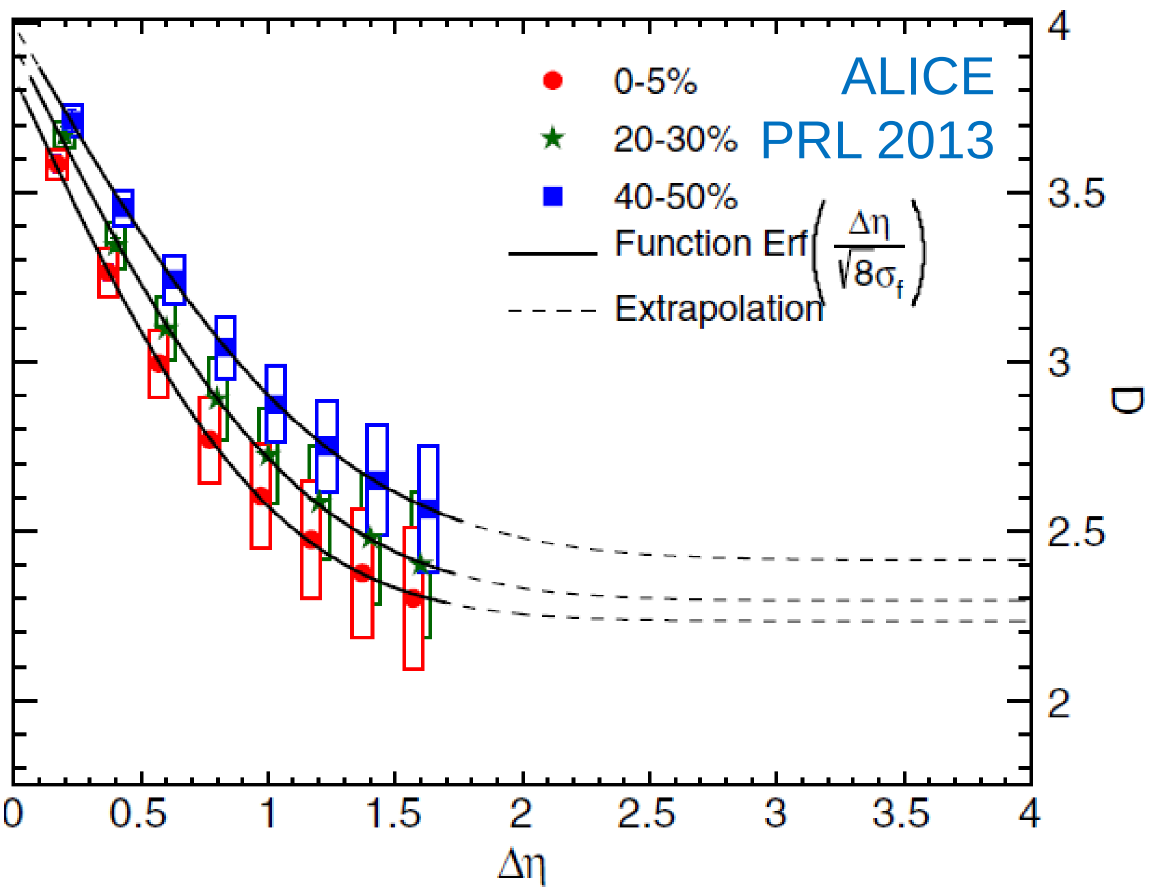
rapidity window

$$D \sim \frac{\langle \delta N_{\mathrm{Q}} \rangle^2}{\Delta\eta}$$

has to be a constant in equil. medium

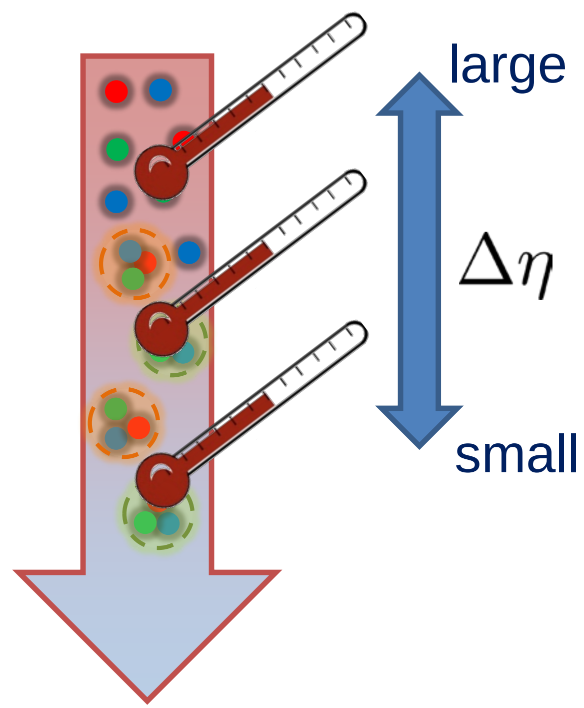
Fluctuation of N_{Q} at ALICE is not the equilibrated one.

Δη Dependence @ ALICE

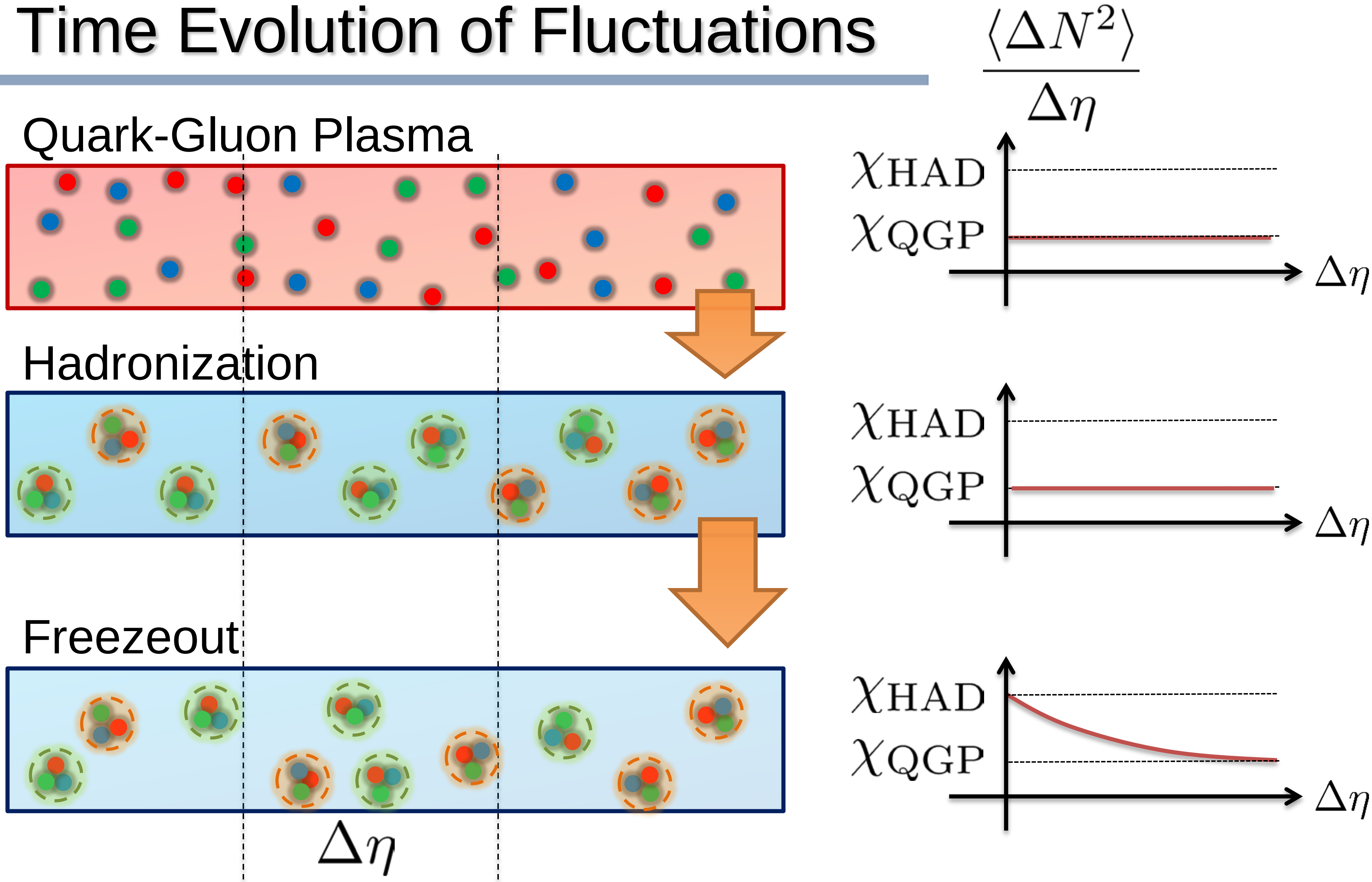


rapidity window

Δη dependent thermometer?

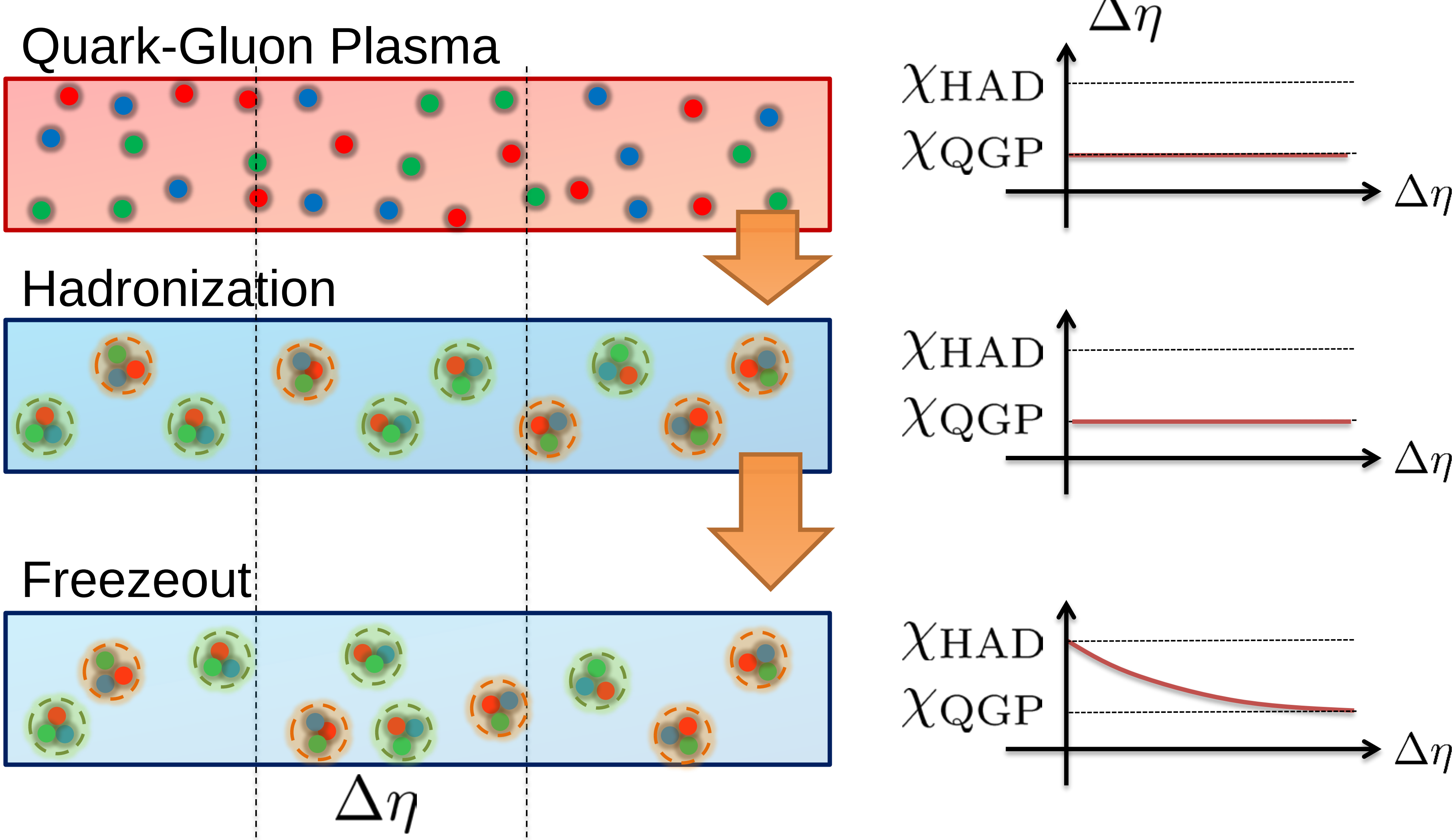


Time Evolution of Fluctuations



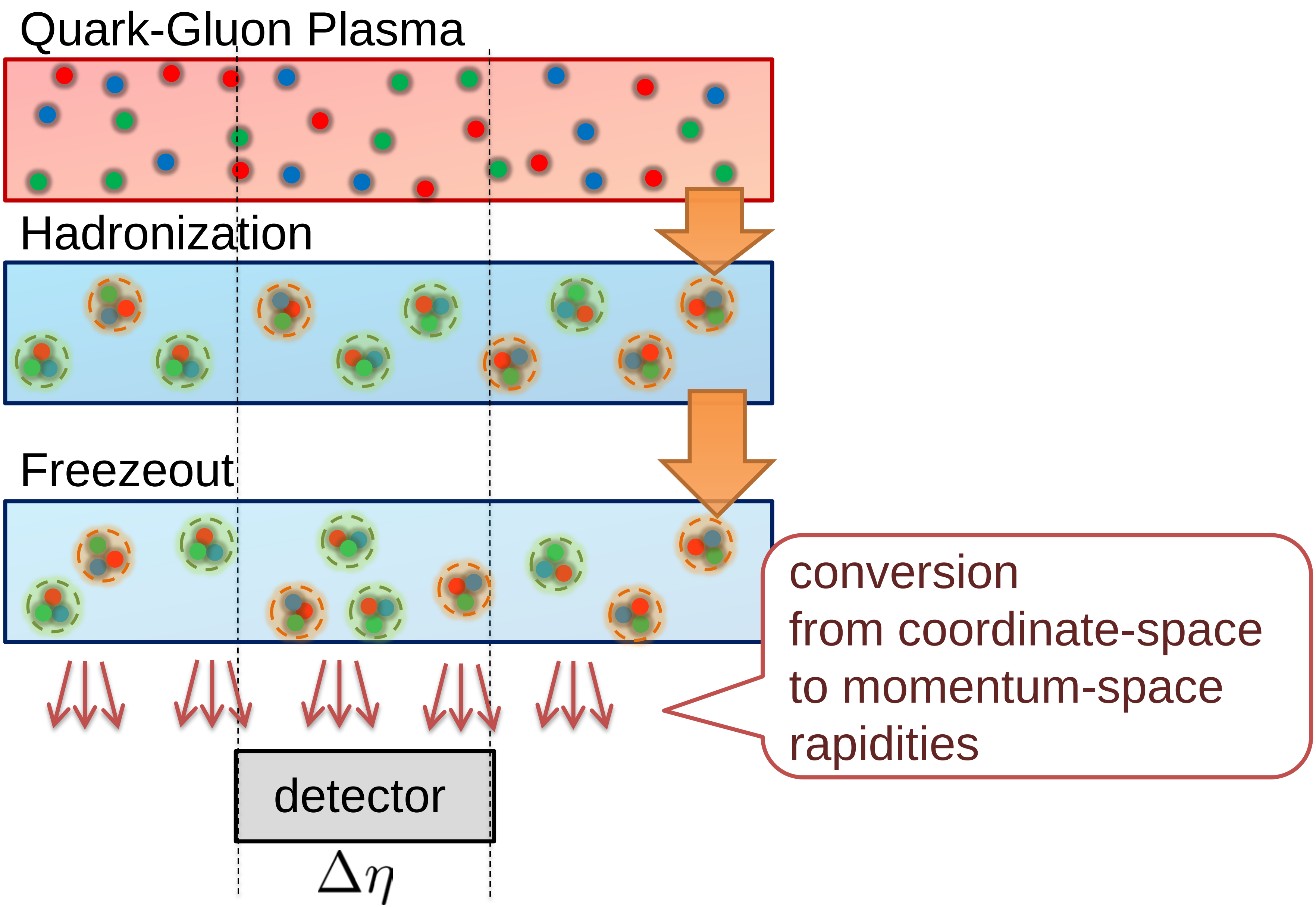
Fluctuations continue to change until kinetic freezeout!!

Time Evolution of Fluctuations

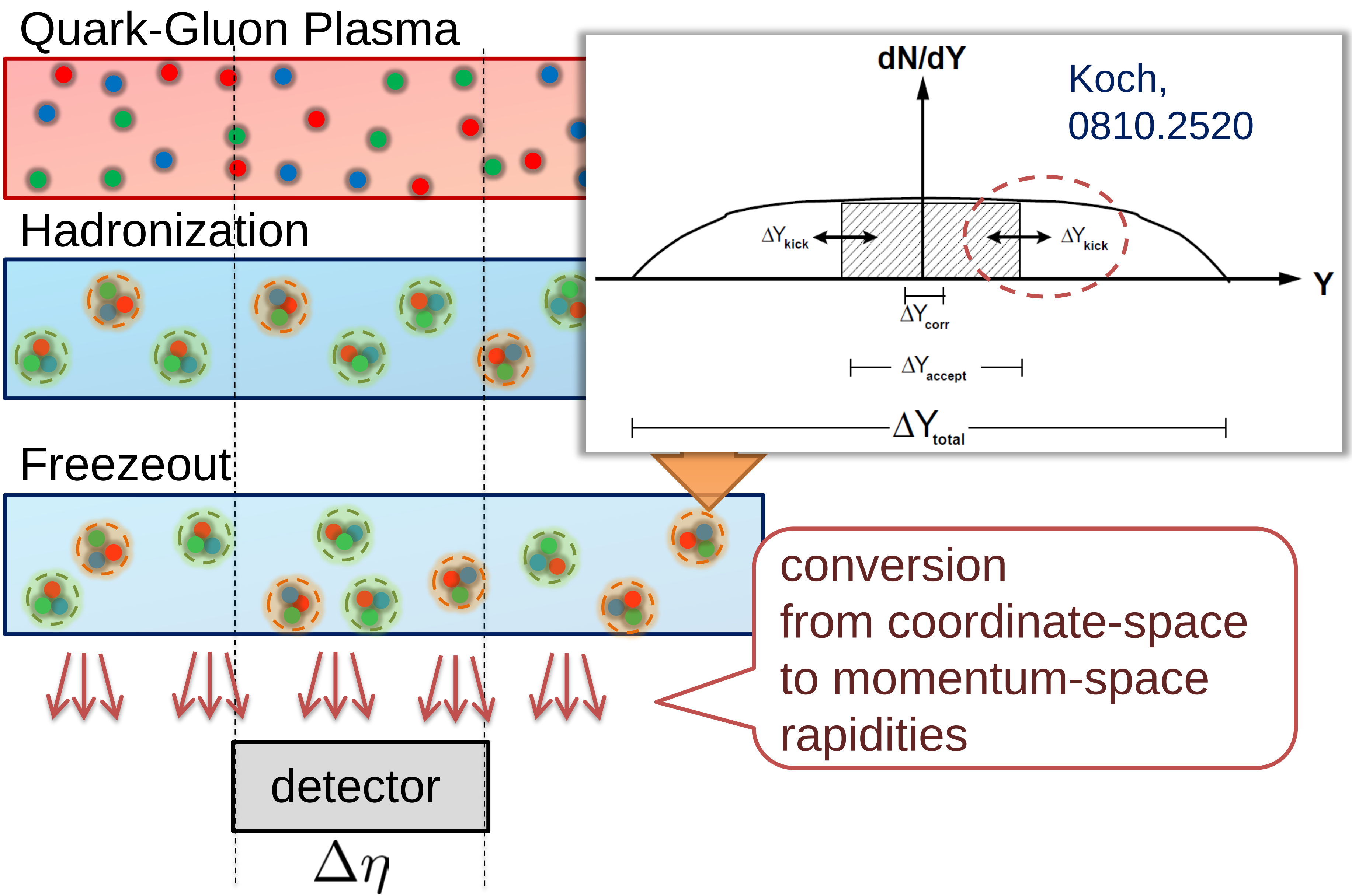


Variation of a conserved charge is achieved only through diffusion. ⇒ The larger $\Delta\eta$, the slower diffusion

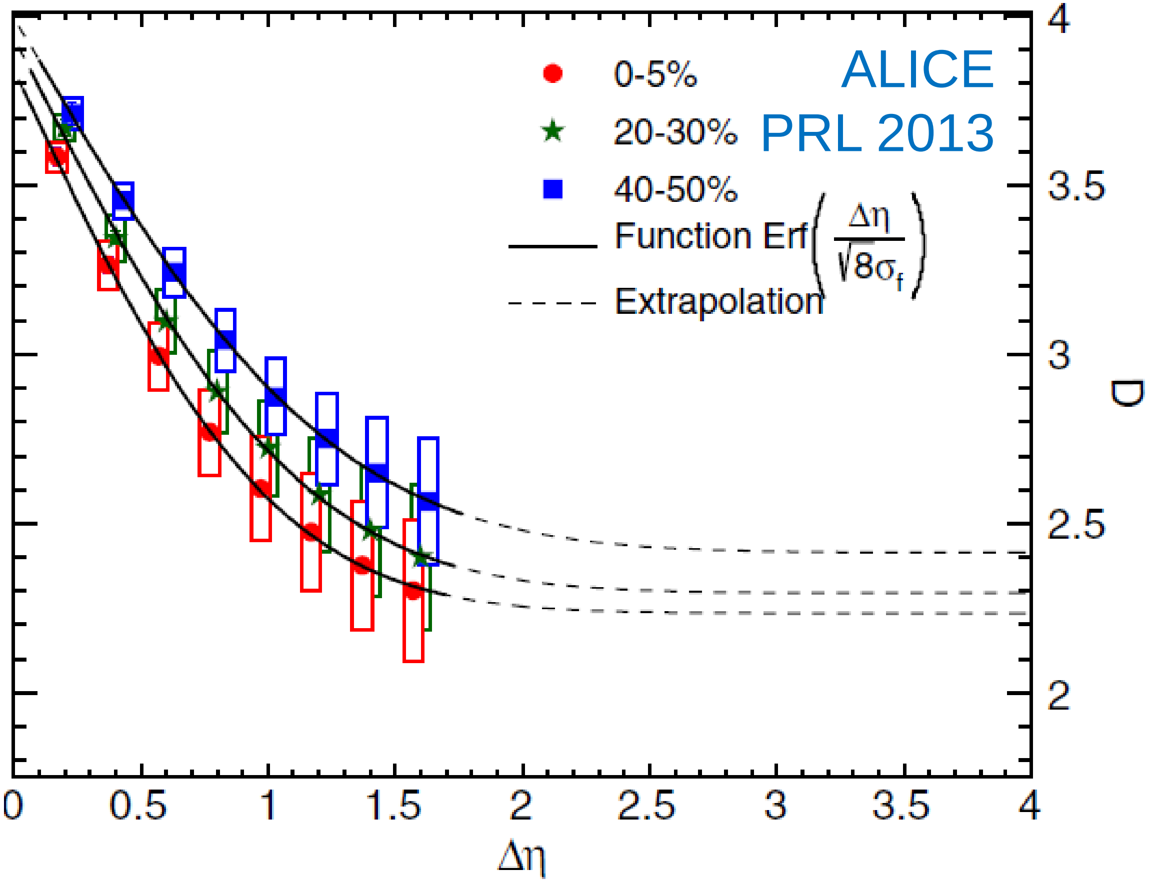
Conversion of Rapidities



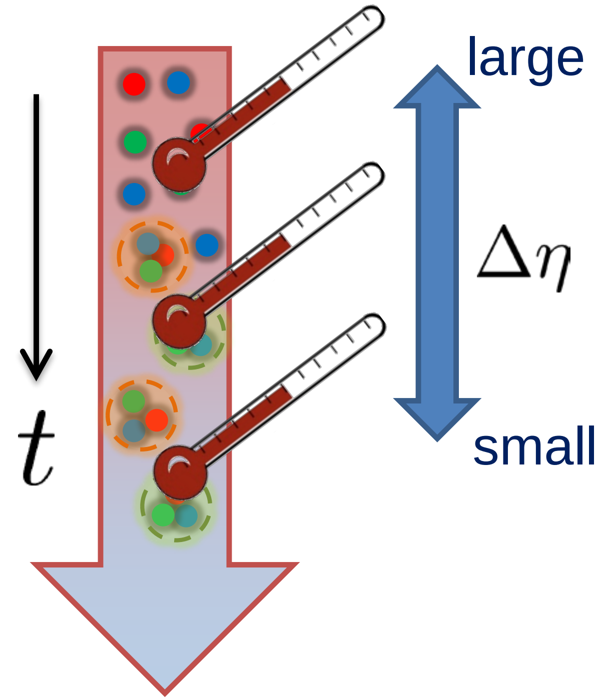
Conversion of Rapidities



Δη Dependence @ ALICE

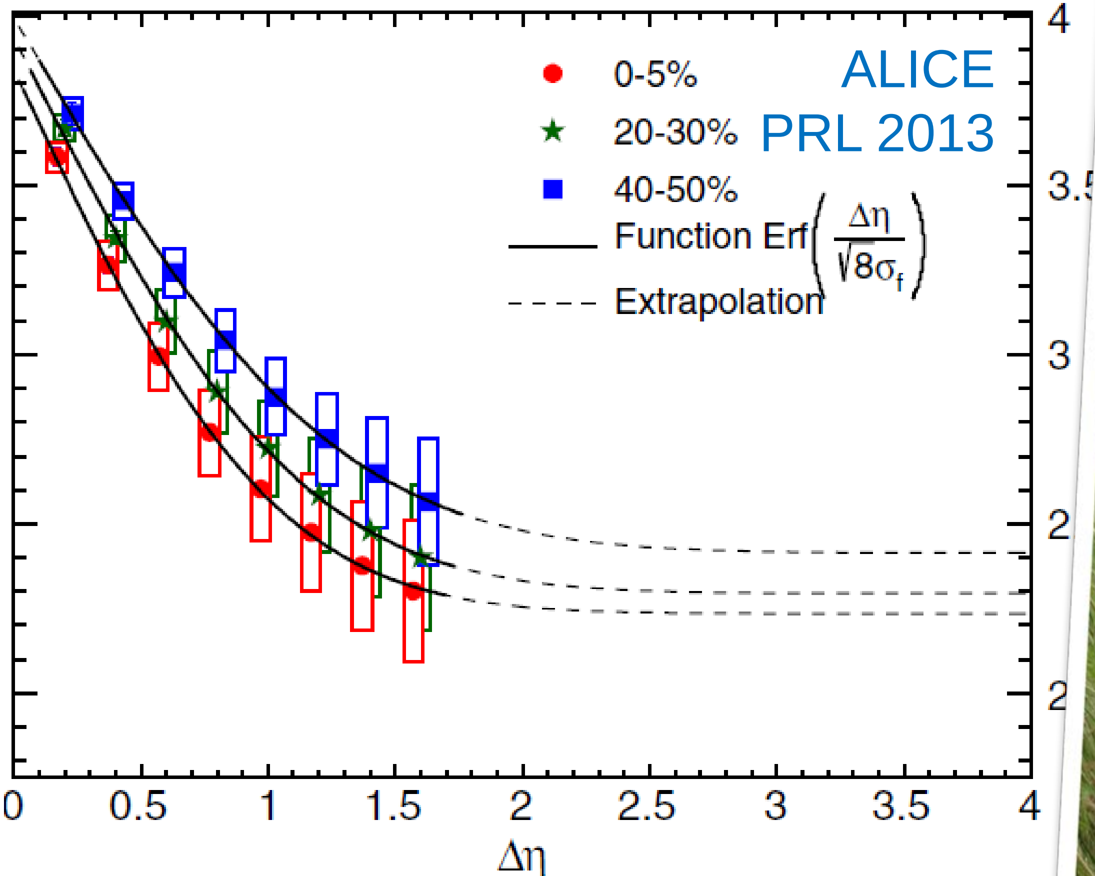


Δη dependent thermometer?



Δη dependences of fluctuation observables encode history of the hot medium!

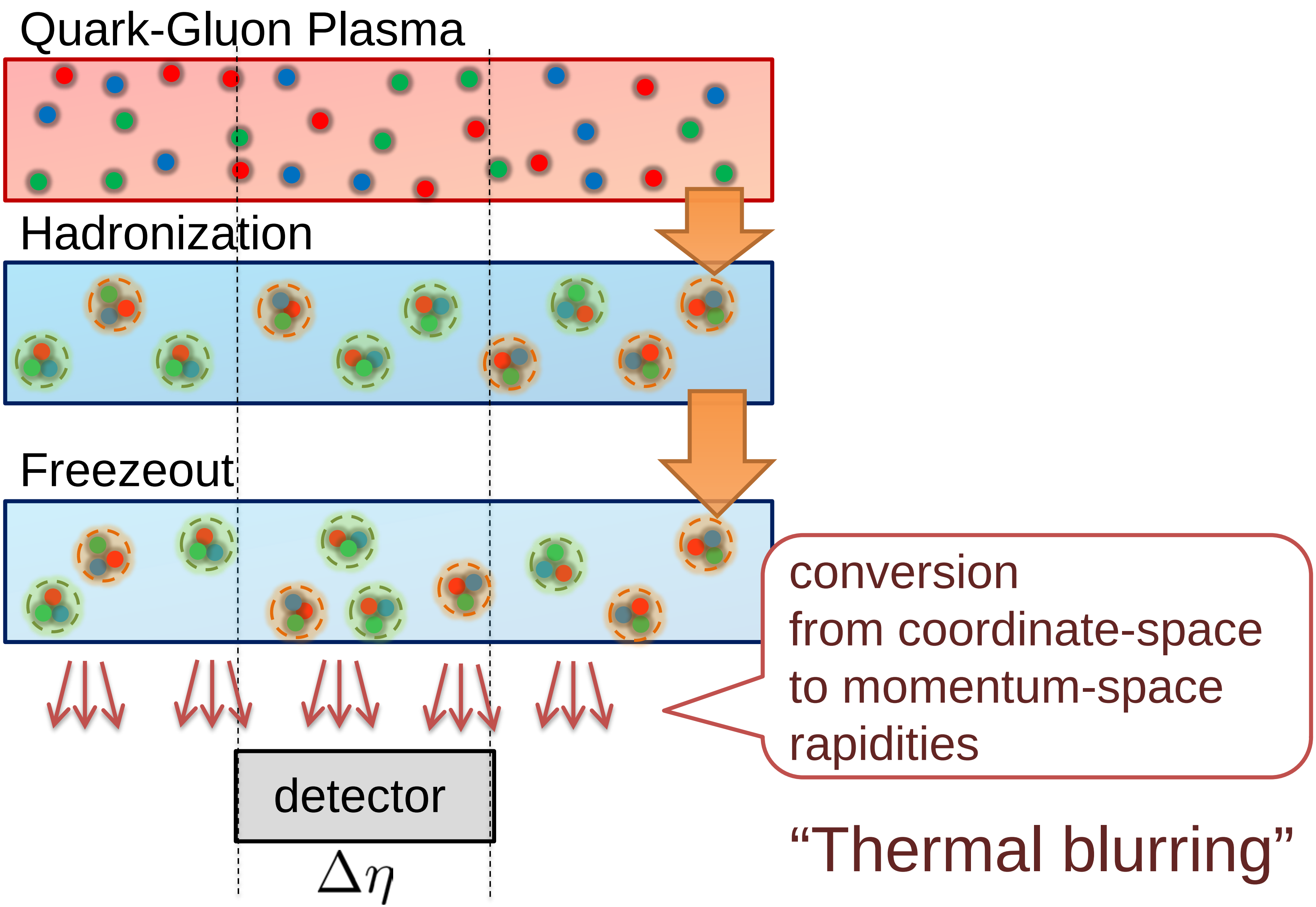
Δη Dependence @ ALICE



Δη dependences of fluctuation observables encode history of the hot medium!

Thermal blurring in momentum-space rapidity

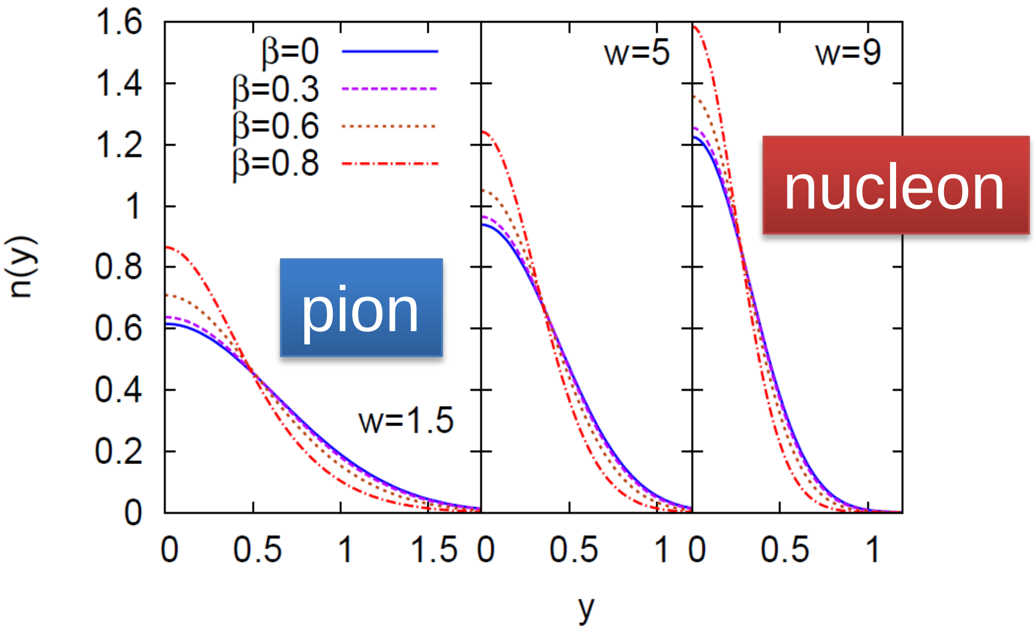
Conversion of Rapidities



"Thermal blurring"

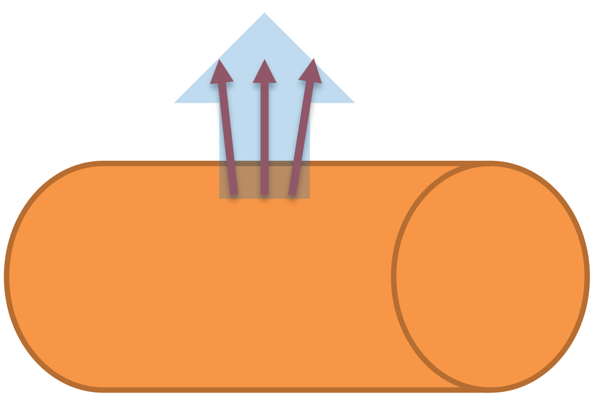
Thermal distribution in η space

Y. Ohnishi, MK,+
in preparation

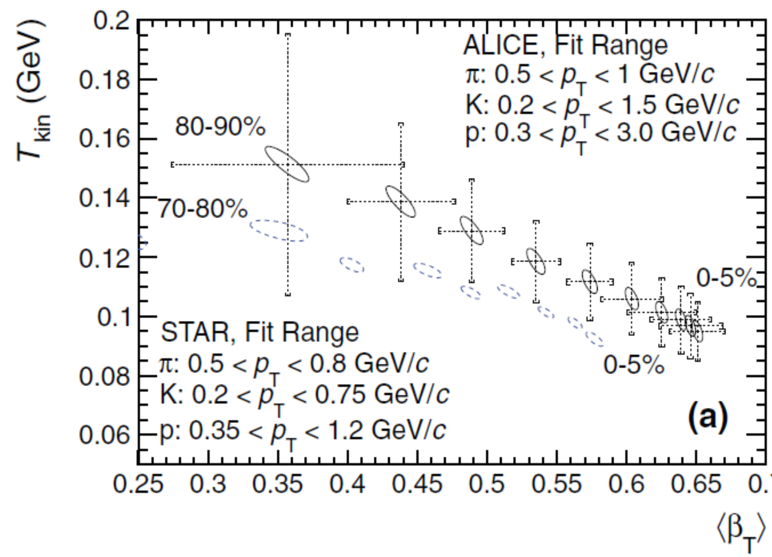


$$w = \frac{m}{T}$$

- pions $w \simeq 1.5$
- nucleons $w \simeq 9$



Blast wave squeezes the distribution in rapidity space



- blast wave
- flat freezeout surface

Thermal distribution in η space

Y. Ohnishi, MK,+
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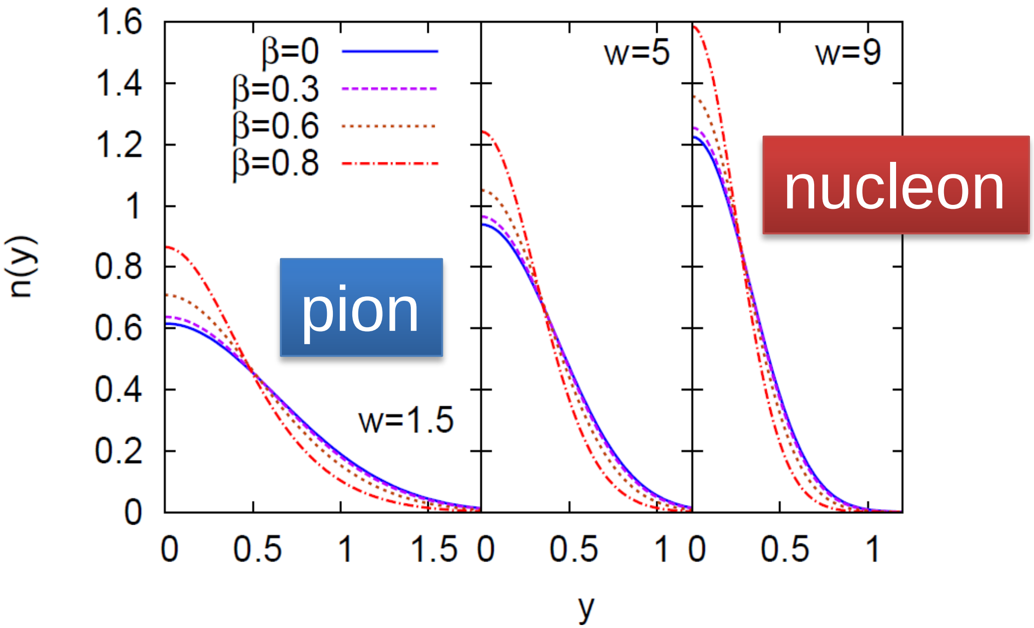

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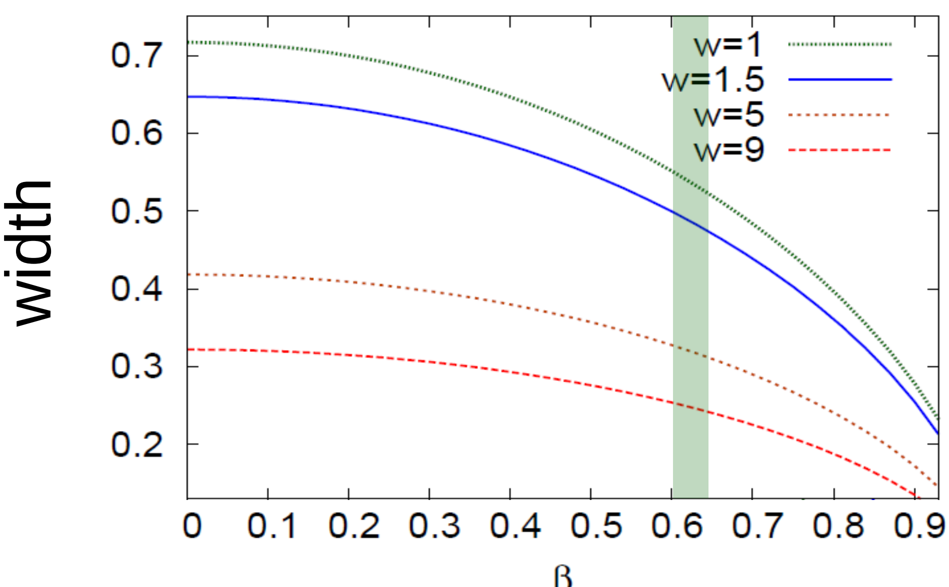
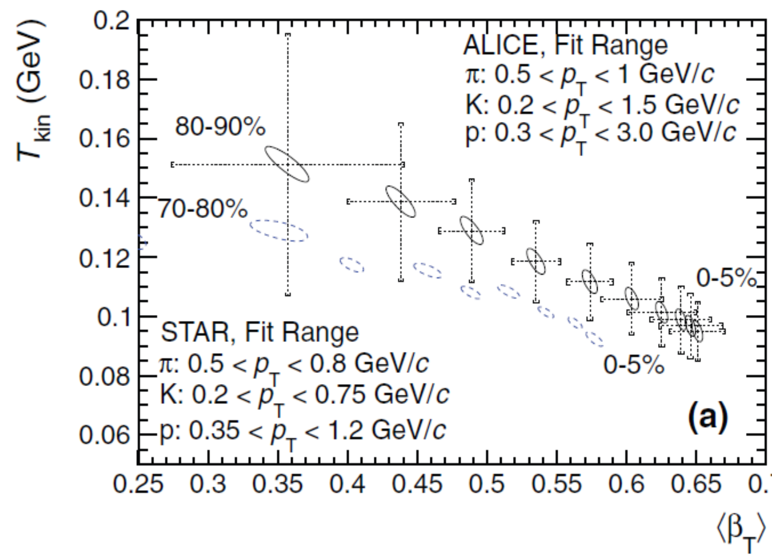
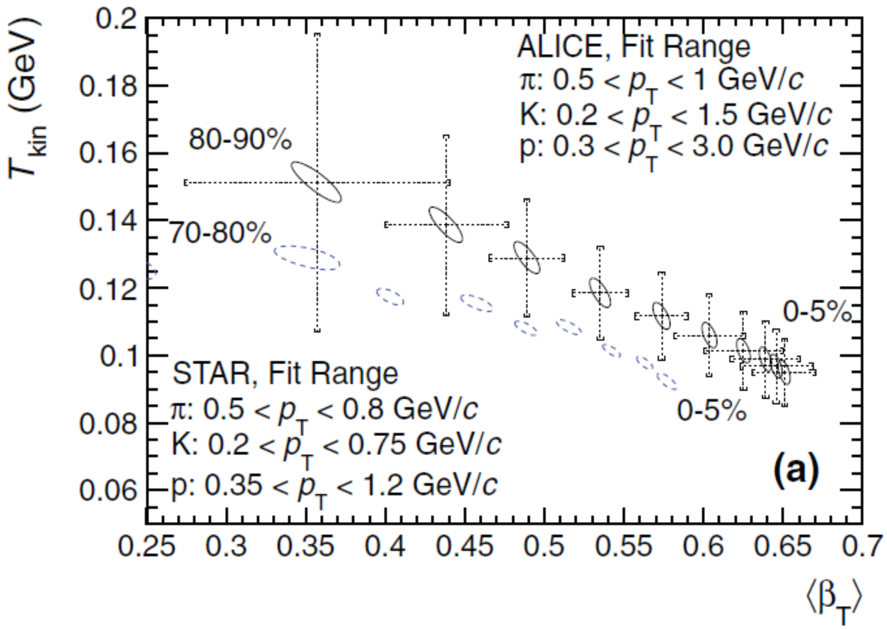
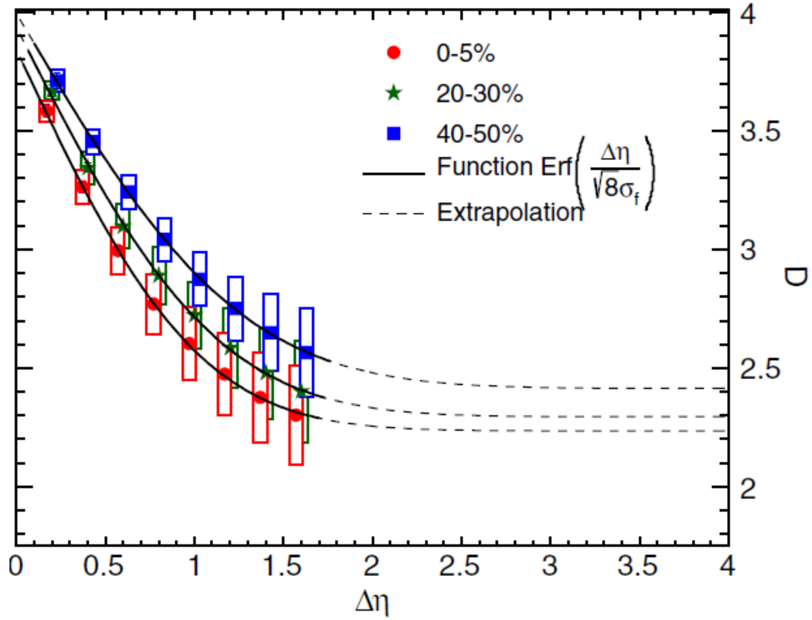




- blast wave
- flat freezeout surface

Centrality Dependence





More central ⇒ { lower T, larger β } ⇒ Smaller blurring

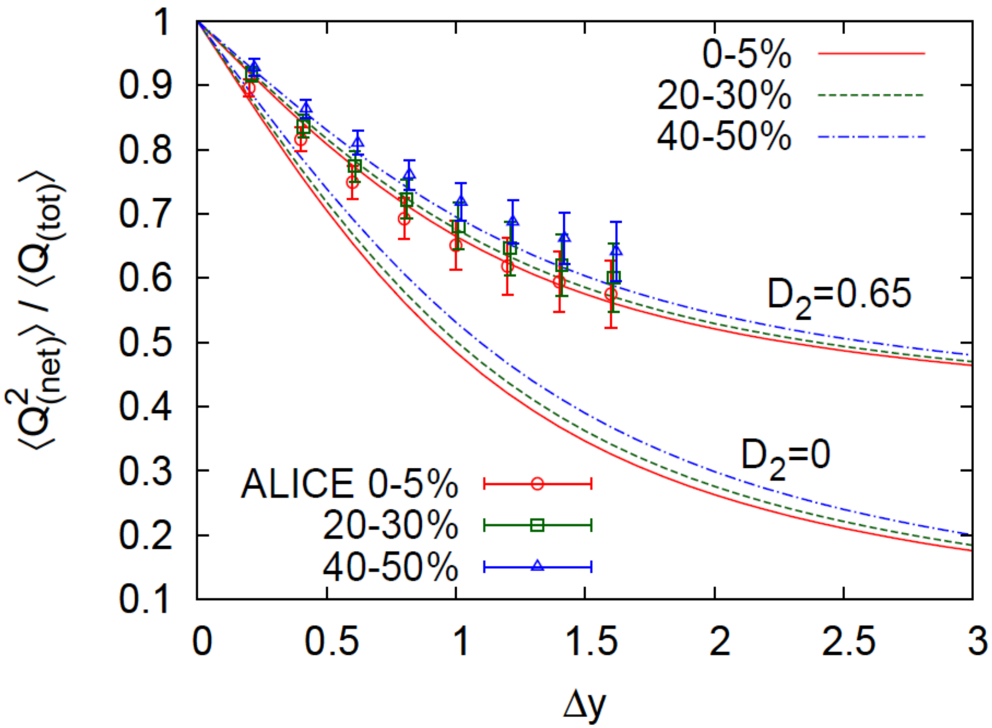
Is the centrality dependence understood solely by the thermal blurring at kinetic f.o.?

Centrality Dependence

$$D_2 = \frac{\langle \delta N_{\rm Q}^2 \rangle}{\langle \delta N_{\rm Q}^2 \rangle_{\rm eq.}}$$

Assumptions:

- Centrality independent cumulant at kinetic f.o.
- Thermal blurring at kinetic f.o.

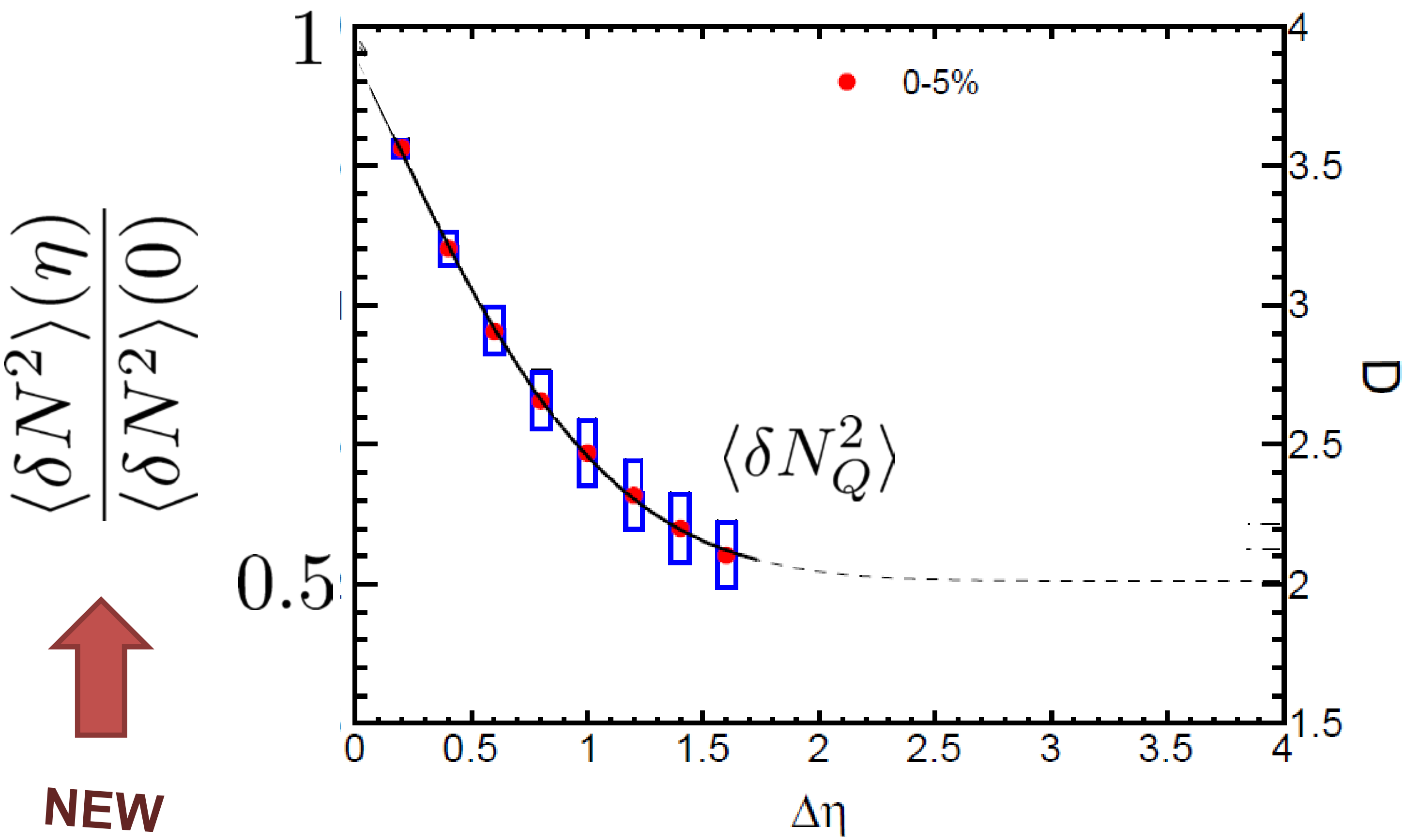


- ☐ Centrality dep. of blast wave parameters may not be large enough to describe the one of $\langle \delta N_{\rm Q}^2 \rangle$
  - ➢ Existence of another physics having centrality dep.
  - ➢ More accurate data is desirable!!

Rapidity Window Dependences of Higher Order Cumulants

$<\delta N_B{}^2>$ and $< \delta N_p{}^2 >$ @ LHC ?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$
should have different Δη dependence.



NEW normalization!

$<\delta N_B^2>$ and $< \delta N_p^2 >$ @ LHC ?

$\langle \delta N_Q^2 \rangle, \langle \delta N_B^2 \rangle, \langle \delta N_p^2 \rangle$
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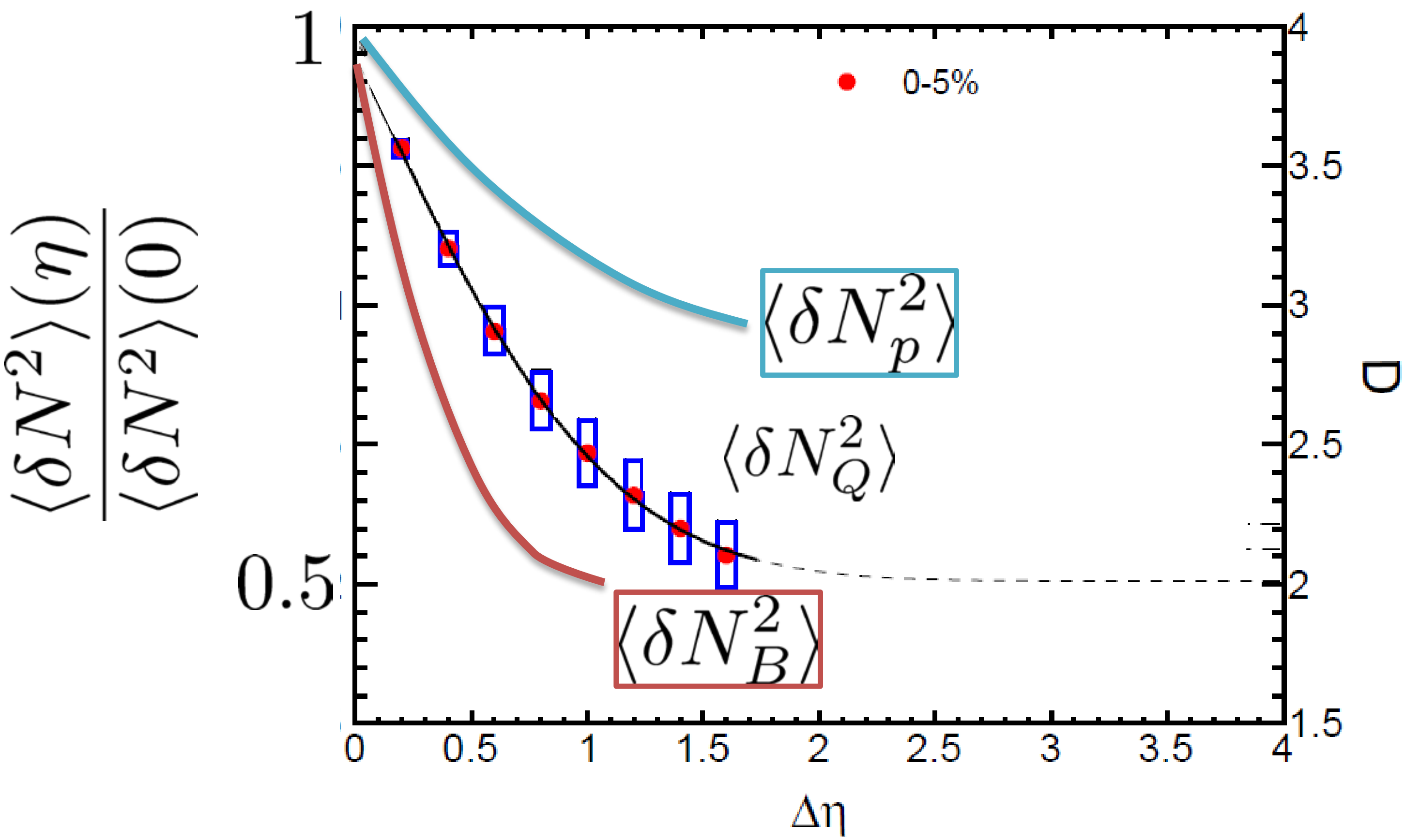
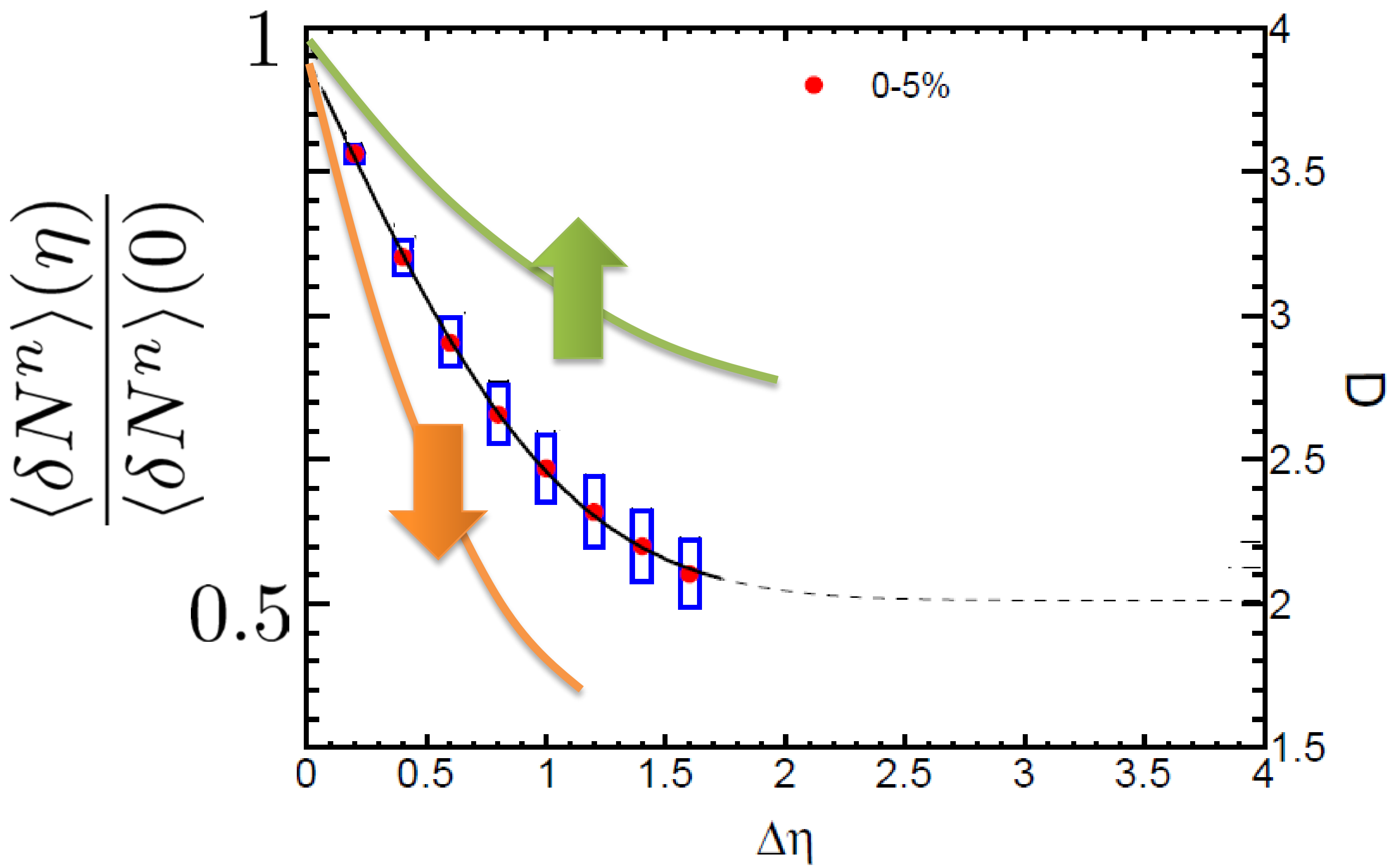


Baryon # cumulants are experimentally observable! MK, Asakawa, 2012

$<\delta N_Q{}^4>$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta\eta$?

suppression or enhancement



Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

Stephanov, Shuryak, 2001

Markov (white noise)
+
continuity

↓

Gaussian noise

cf) Gardiner, "Stochastic Methods"

↓

Fluctuation of *n* is
Gaussian in equilibrium

How to Introduce Non-Gaussianity?

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

- Choices to introduce non-Gaussianity in equil.:
  - *n* dependence of diffusion constant *D*(*n*)
  - colored noise
  - discretization of *n*

How to Introduce Non-Gaussianity?

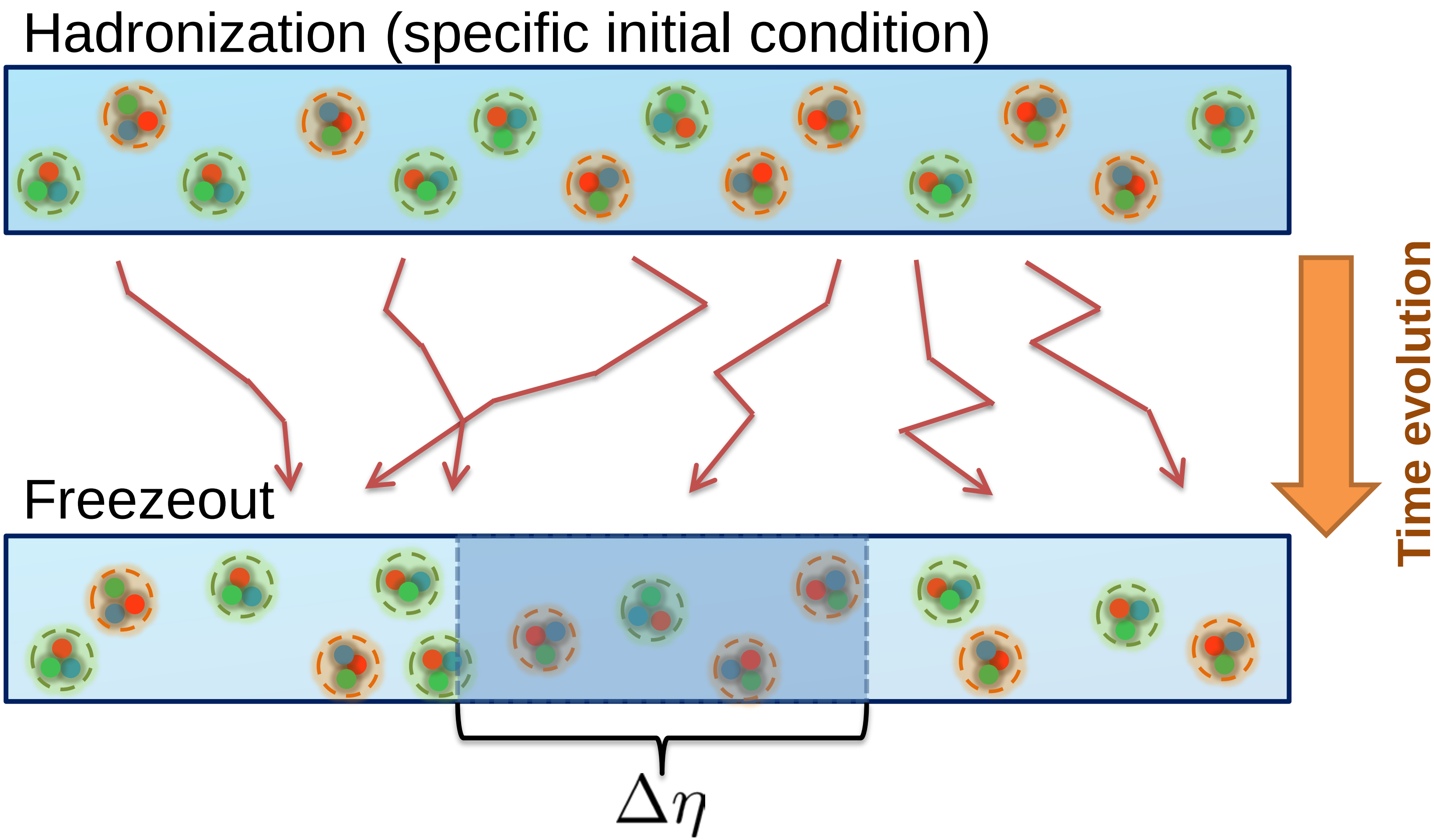
Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

- Choices to introduce non-Gaussianity in equil.:
  - *n* dependence of diffusion constant $D(n)$
  - colored noise
  - discretization of *n* ← **our choice**

REMARK: Fluctuations measured in HIC are almost Poissonian.

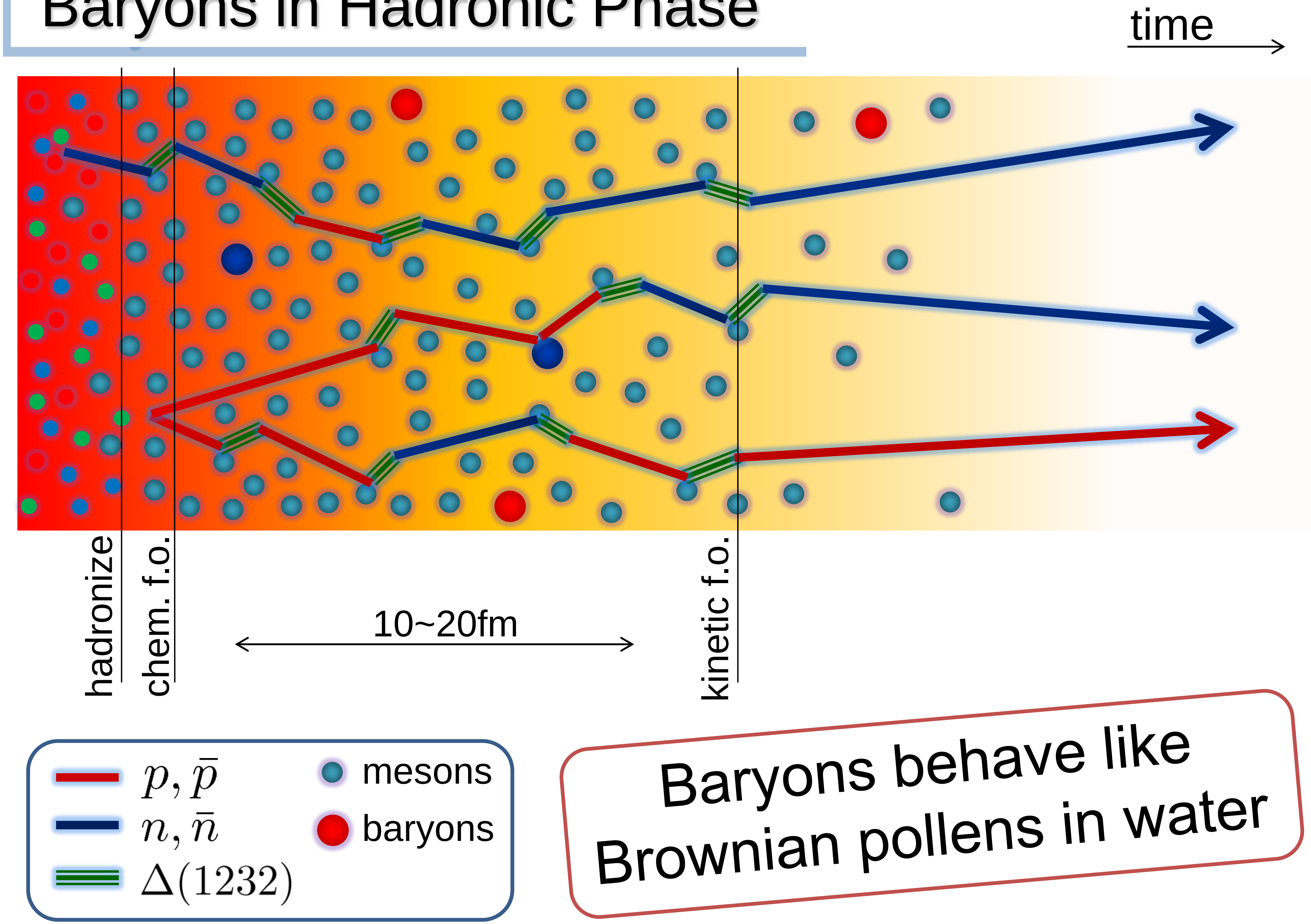
A Brownian Particles' Model



① Describe time evolution of Brownian particles exactly

② Obtain cumulants of particle # in $\Delta\eta$

Baryons in Hadronic Phase

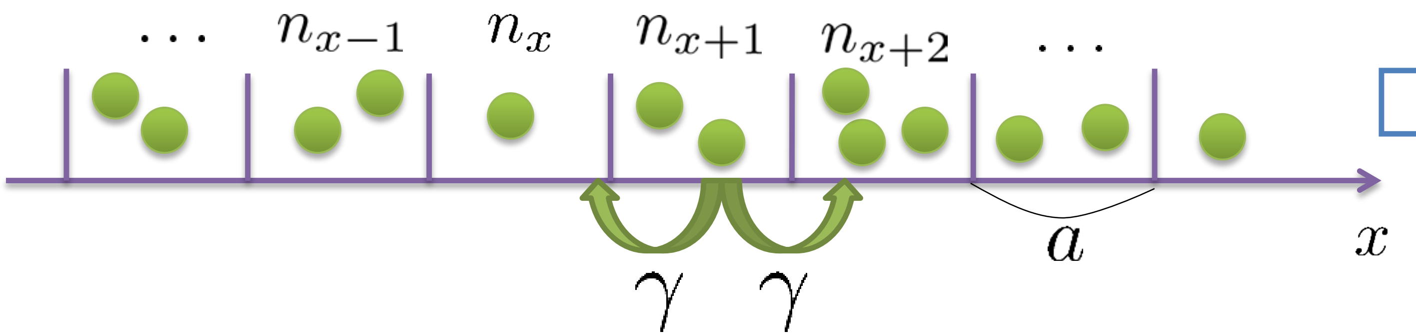


Baryons behave like Brownian pollens in water

Diffusion Master Equation

MK, Asakawa, Ono, 2014
MK, 2015

Divide spatial coordinate into discrete cells



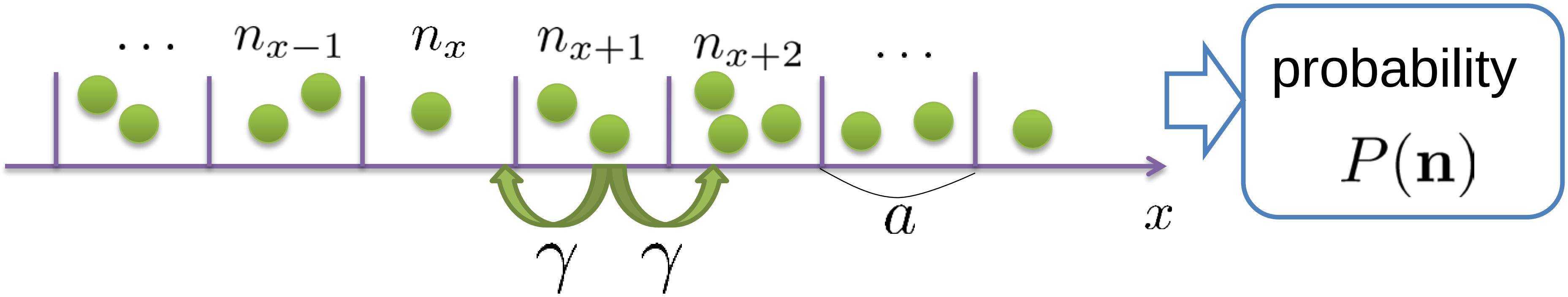
probability

$P(\mathbf{n})$

Diffusion Master Equation

MK, Asakawa, Ono, 2014
MK, 2015

Divide spatial coordinate into discrete cells



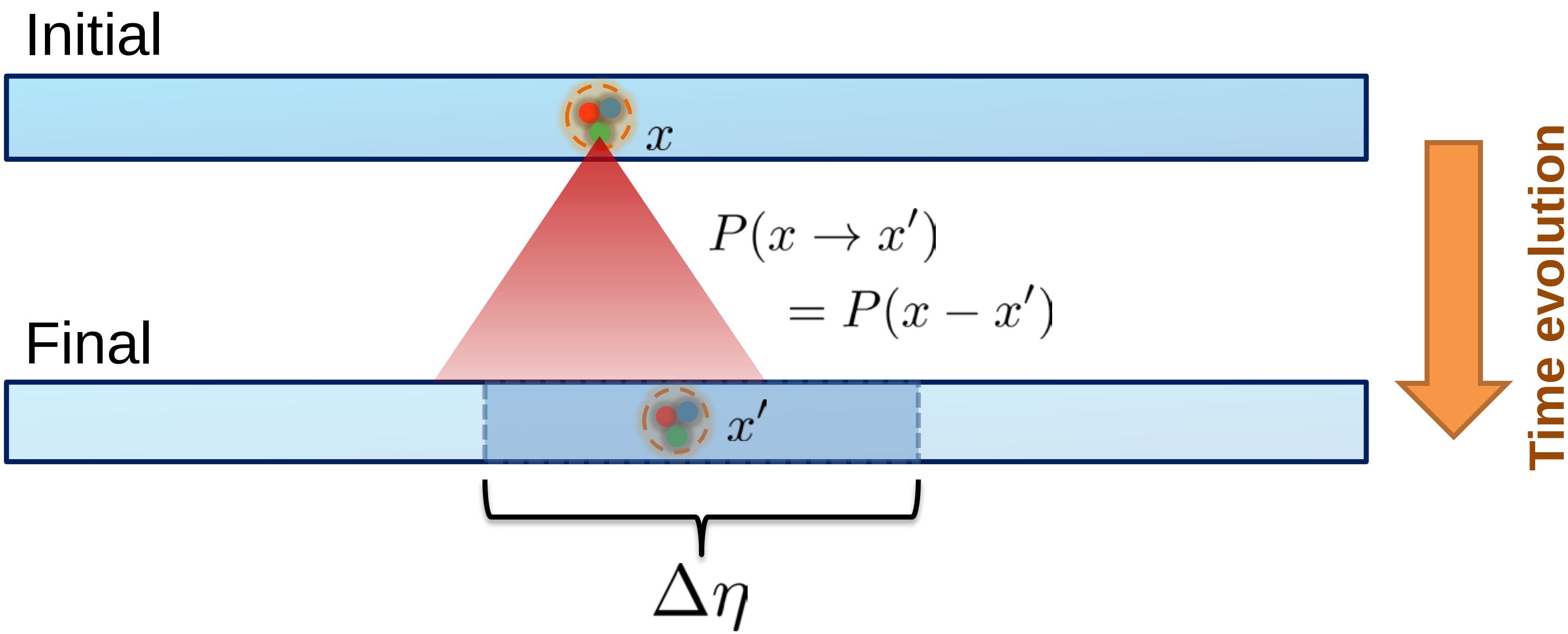
Master Equation for *P*(*n*)

$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma \sum_x [(n_x+1)\left\{P(\mathbf{n}+\mathbf{e}_x-\mathbf{e}_{x+1})+P(\mathbf{n}+\mathbf{e}_x-\mathbf{e}_{x-1})\right\} - 2n_x P(\mathbf{n})]$$

Solve the DME **exactly**, and take *a*→0 limit

No approx., ex. van Kampen's system size expansion

A Brownian Particles' Model



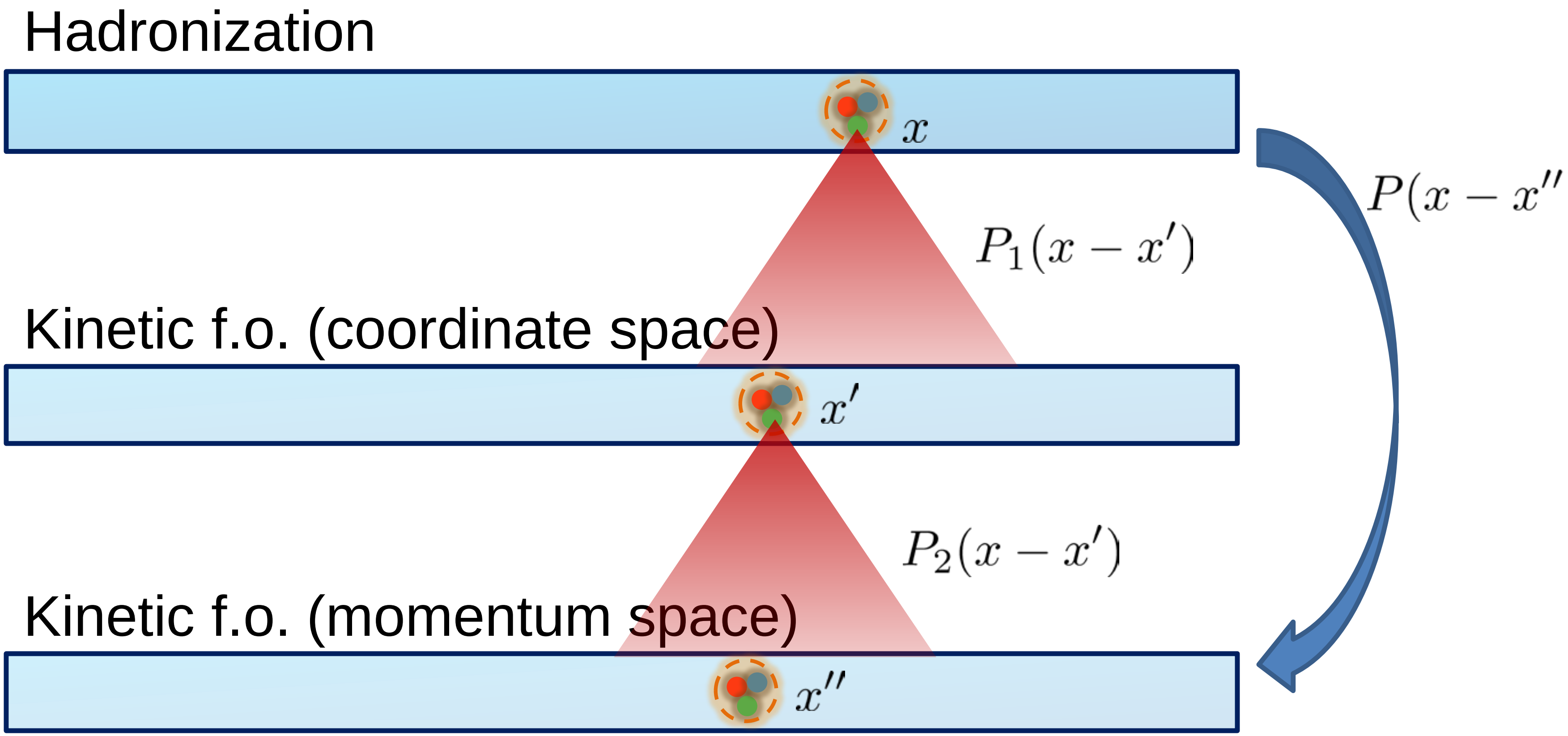
- Each particle are uncorrelated
- A particle moves $x \to x'$ with probability $P(x-x')$

➡ Formula of cumulants

$$\langle \rho(x) \rangle_{\mathrm{final}} = \int dx' P(x - x') \langle \rho(x') \rangle_0$$

$$\cdots$$

Diffusion + Thermal Blurring

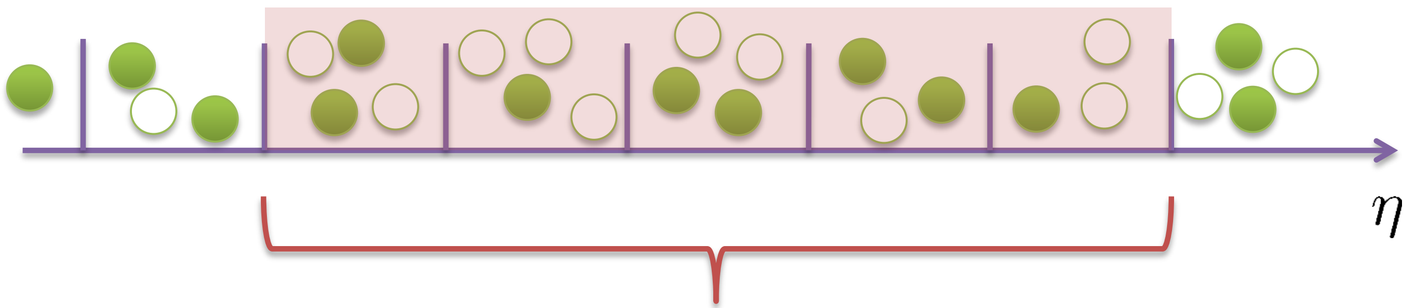


$$P(x - x'') = \int dx' P_1(x - x') P_2(x' - x'')$$

- Diffusion and thermal blurring can be treated simultaneously

Net Charge Number

Prepare 2 species of (non-interacting) particles

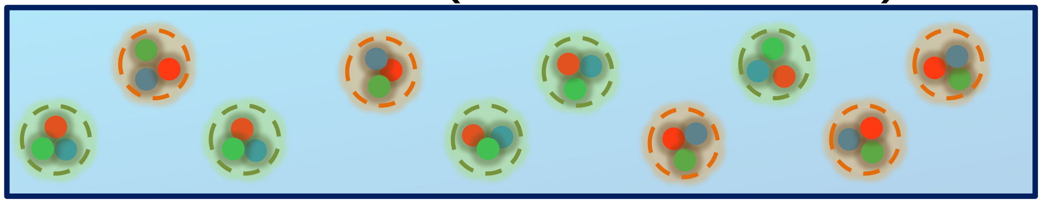


$$\bar{Q}(\tau) = \int_0^{\Delta\eta} d\eta\,(n_1(\eta,\tau) - n_2(\eta,\tau))$$

Time evolution of $\bar{Q}$ up to Gauusianity is consistent with the stochastic diffusion equation

Time Evolution in Hadronic Phase

Hadronization (initial condition)



- Boost invariance / infinitely long system
- Local equilibration / local correlation

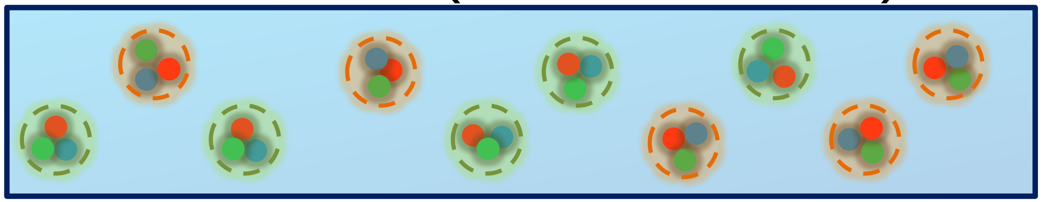
$\langle \bar{Q}^2 \rangle_c, \ \langle \bar{Q}^3 \rangle_c, \ \langle \bar{Q}^4 \rangle_c$ $\quad$ $\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$ $\quad$ $\langle Q^2_{(\mathrm{tot})} \rangle_c, \langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c$

suppression owing to local charge conservation

strongly dependent on hadronization mechanism

Time Evolution in Hadronic Phase

Hadronization (initial condition)



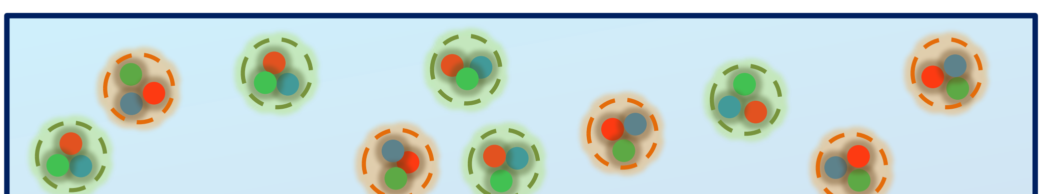
Time evolution via DME

- Boost invariance / infinitely long system
- Local equilibration / local correlation

$\langle \bar{Q}^2 \rangle_c, \ \langle \bar{Q}^3 \rangle_c, \ \langle \bar{Q}^4 \rangle_c$ — $\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$ — suppression owing to local charge conservation

$\langle Q^2_{(\mathrm{tot})} \rangle_c, \langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c$ — strongly dependent on hadronization mechanism

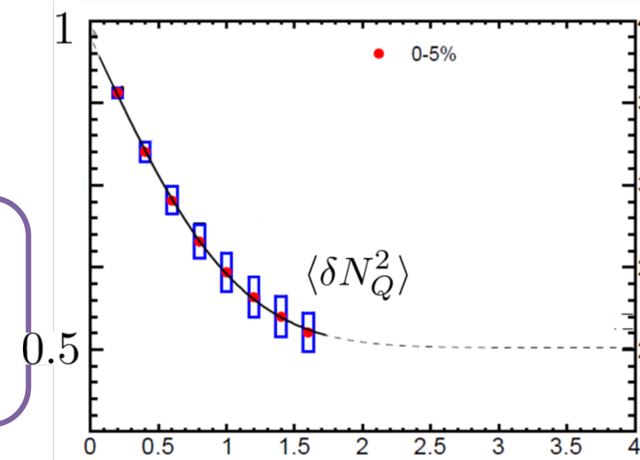
Freezeout

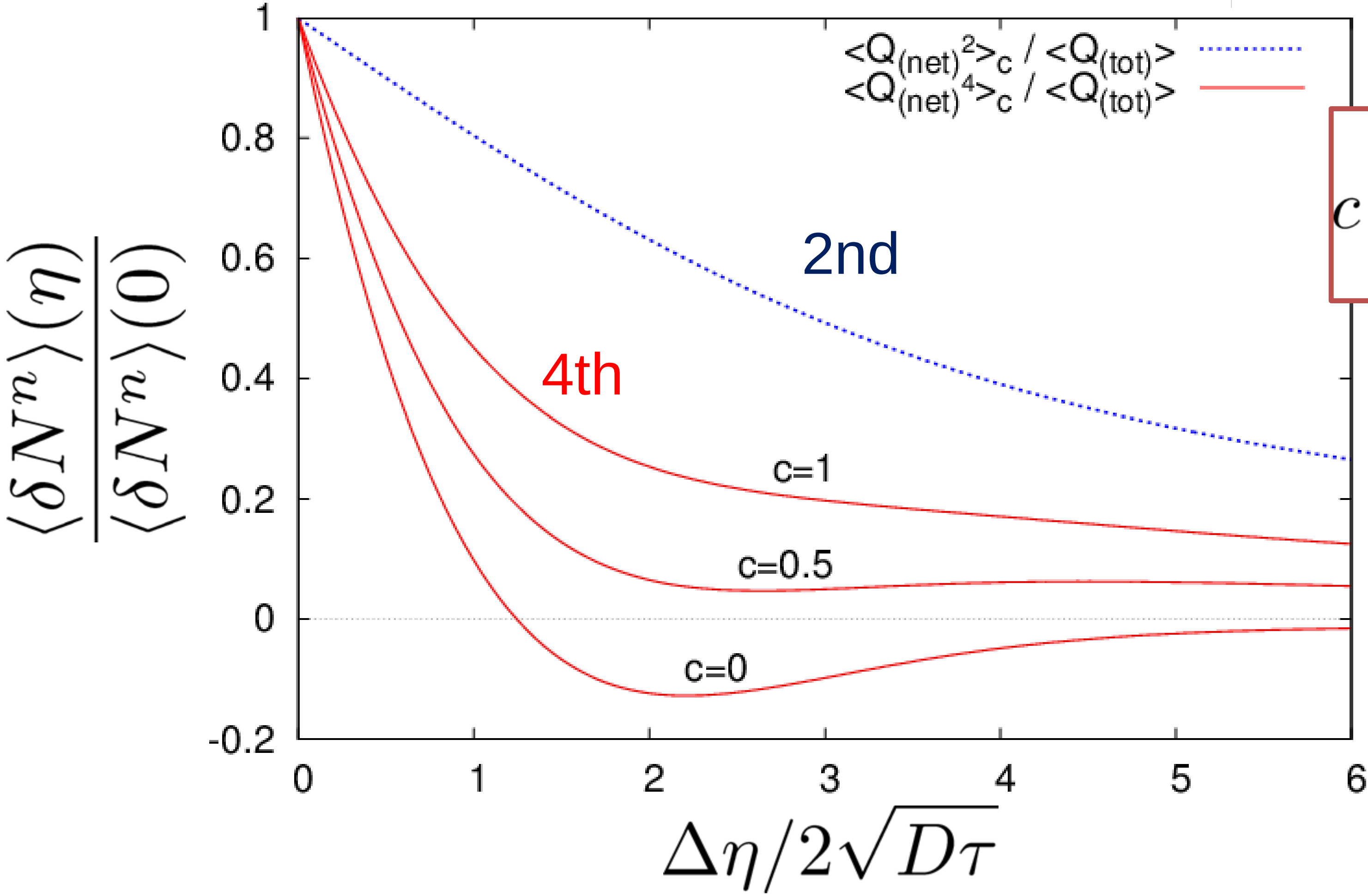


$\Delta\eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0$$



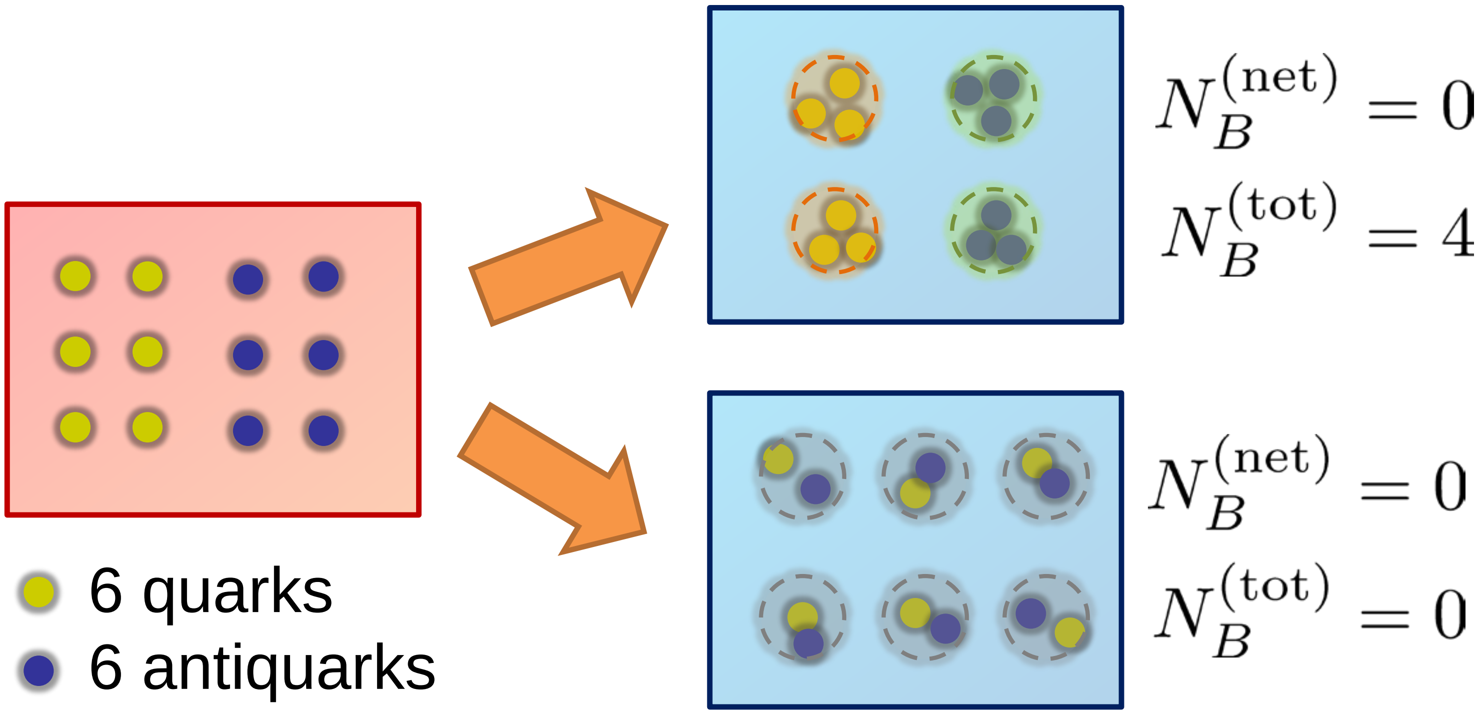


$$c = \frac{\langle Q_{(\mathrm{tot})}^2 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

parameter sensitive to hadronization

Total Charge Number

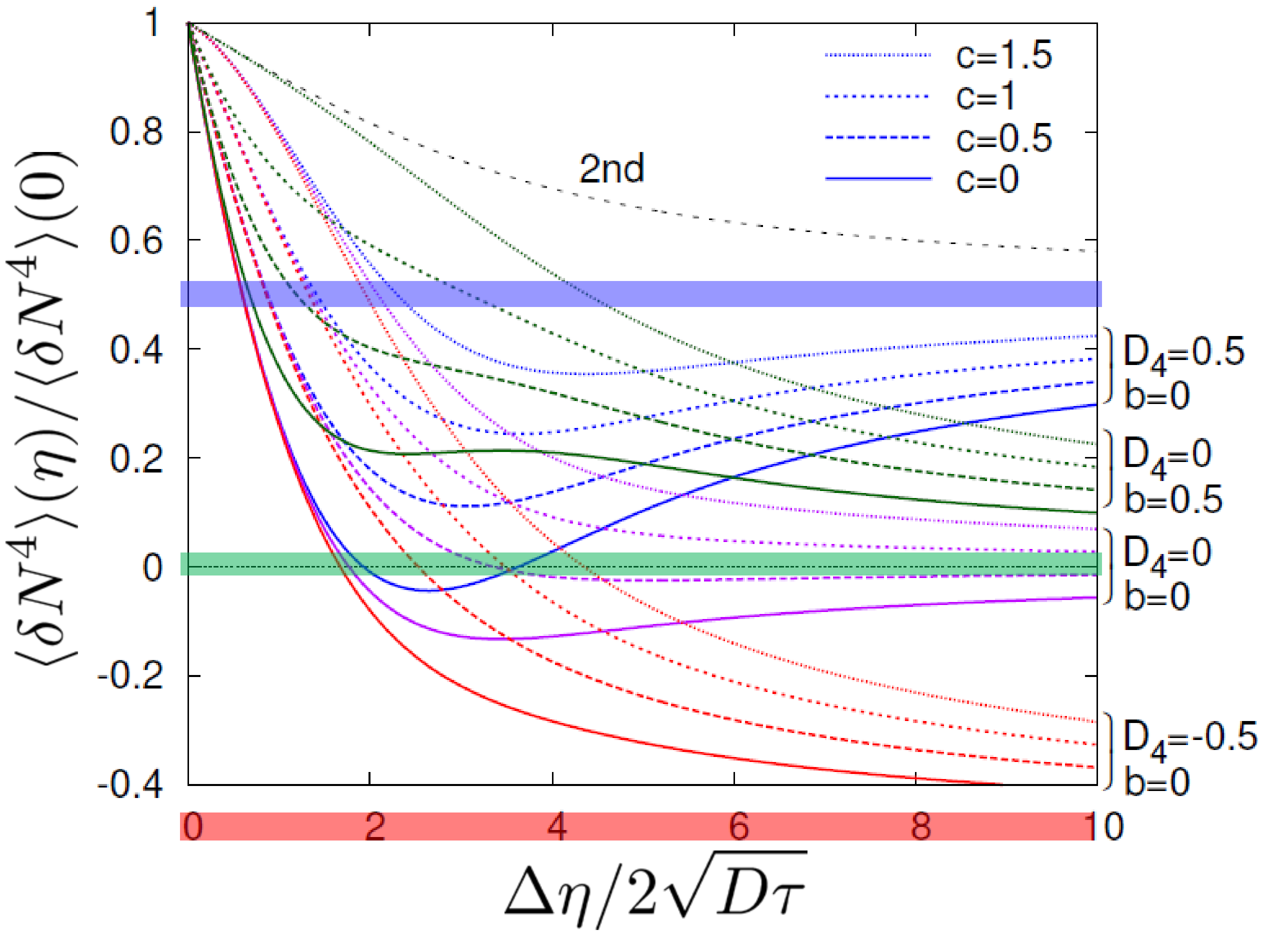
In recombination model,



- $N_B^{(\mathrm{tot})}$ can fluctuate, while $N_B^{(\mathrm{net})}$ does not.

Δη Dependence: 4th order

MK, arXiv:1505.04349



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle}$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{net})}\rangle}$$

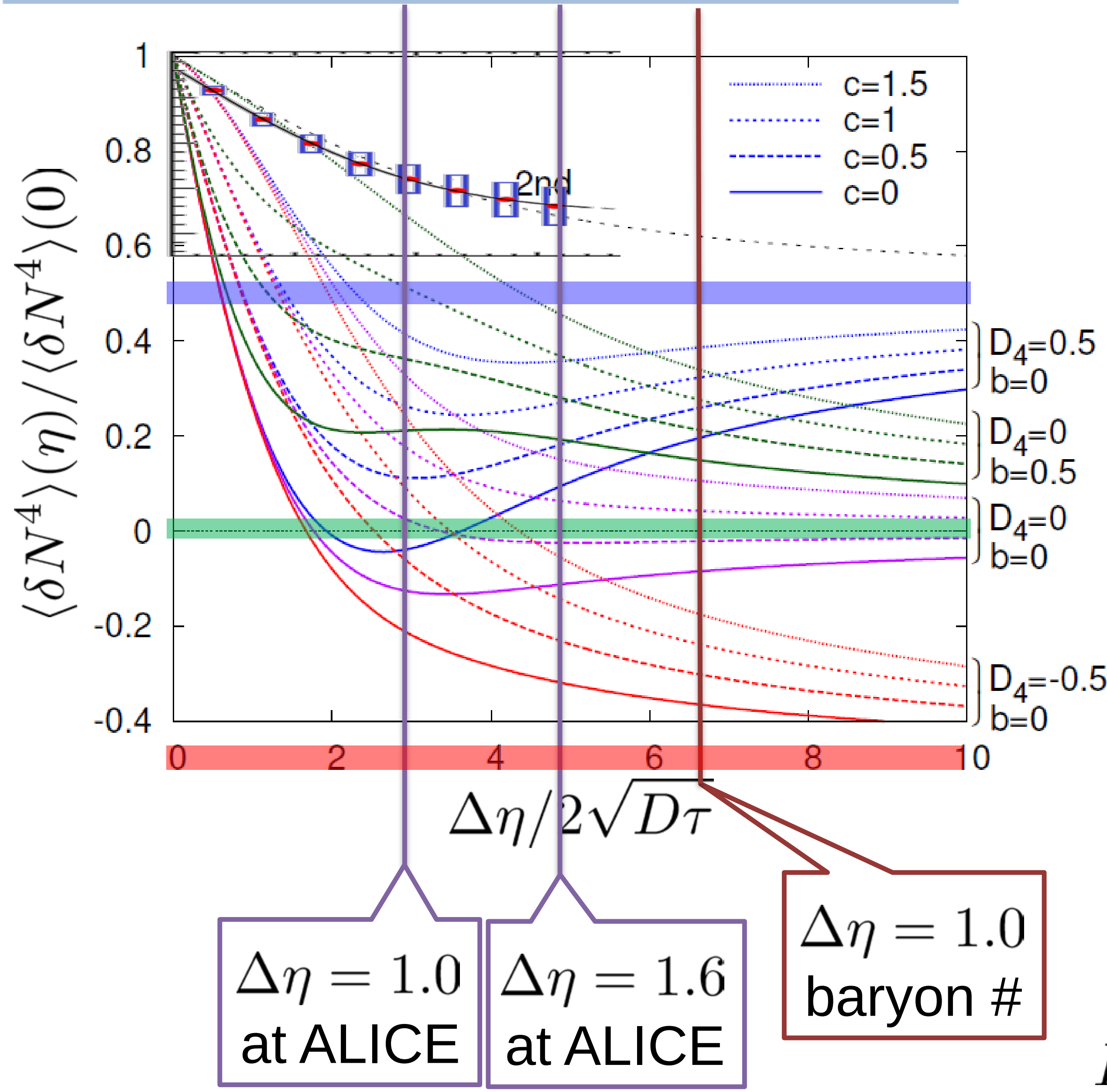
$$c = \frac{\langle Q^2_{(\mathrm{tot})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})}\rangle_c}{\langle Q_{(\mathrm{tot})}\rangle} = 0.5$$

Charcteristic Δη dependences!
Cumulants with a Δη is not the initial value.

Δη Dependence: 4th order

MK, arXiv:1505.04349



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

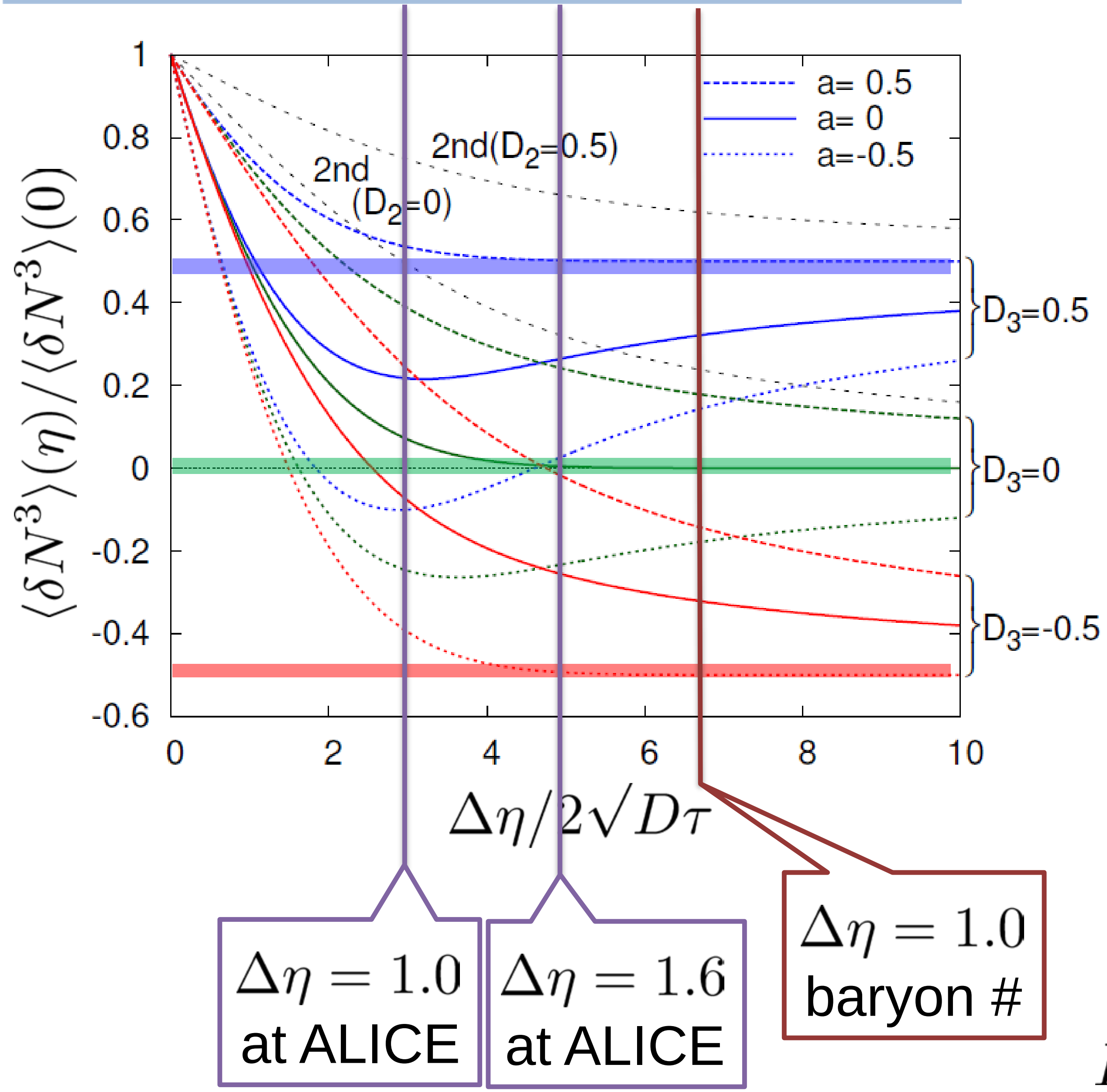
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

$$D \sim M^{-1}$$

Δη Dependence: 3rd order

MK, arXiv:1505.04349



Initial Condition

$$D_3 = \frac{\langle Q^3_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

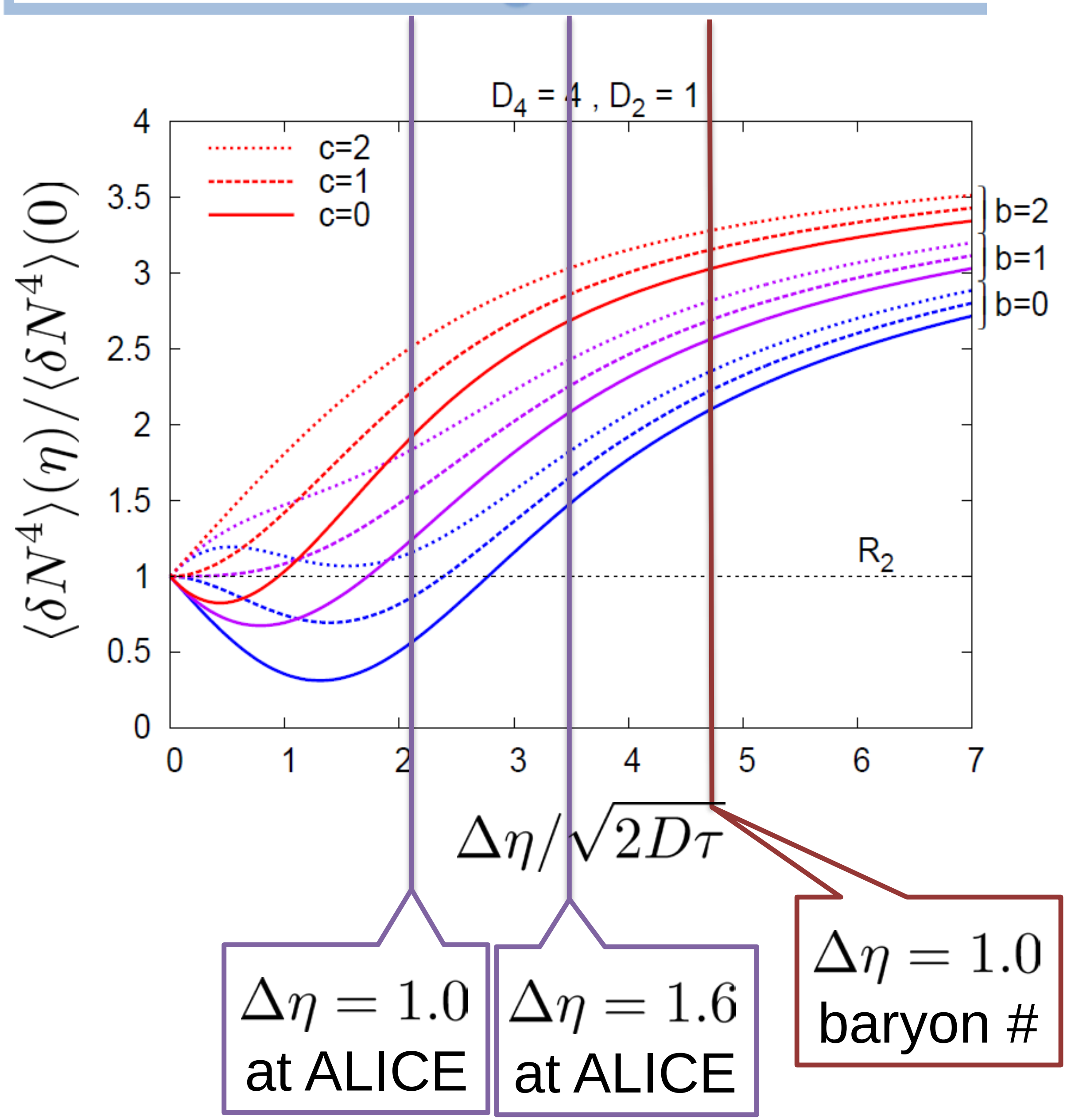
$$a = \frac{\langle Q_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 0.5$$

$$D \sim M^{-1}$$

4^{th} order : Large Initial Fluc.

MK, arXiv:1505.04349



Initial Condition

$$D_4 = \frac{\langle Q^4_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 4$$

$$b = \frac{\langle Q^2_{(\mathrm{net})} Q_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{net})} \rangle}$$

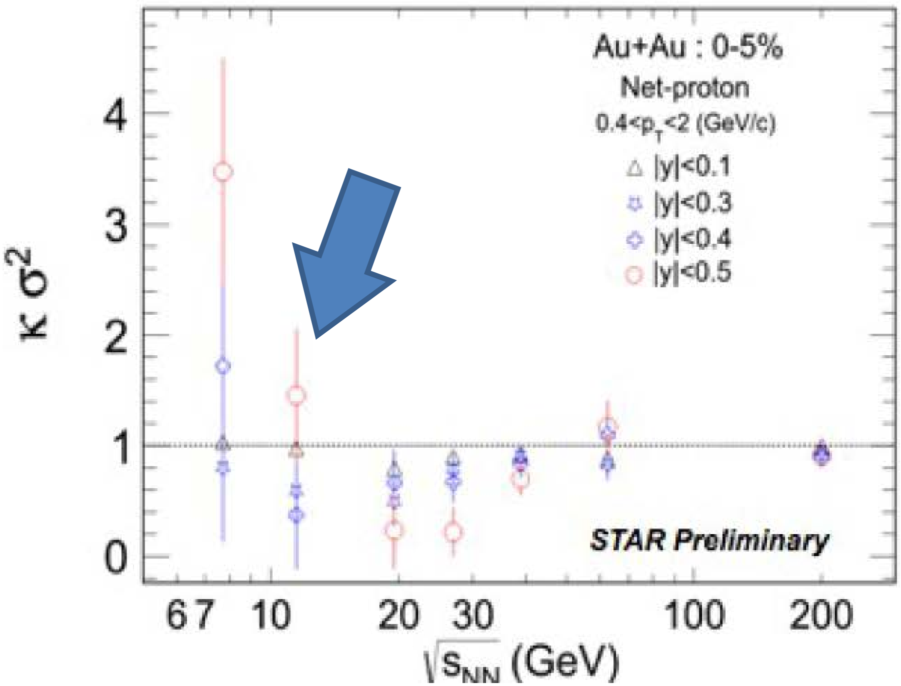
$$c = \frac{\langle Q^2_{(\mathrm{tot})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

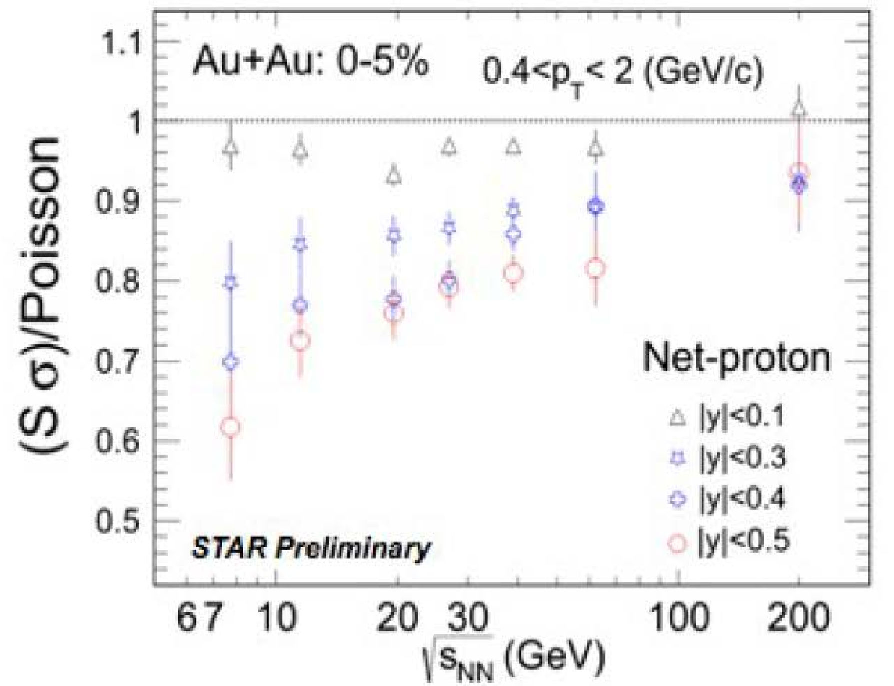
$$D_2 = \frac{\langle Q^2_{(\mathrm{net})} \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle} = 1$$

$$D \sim M^{-1}$$

Δη Dependence @ STAR

Figure from
Jochen Thäder, Yesterday





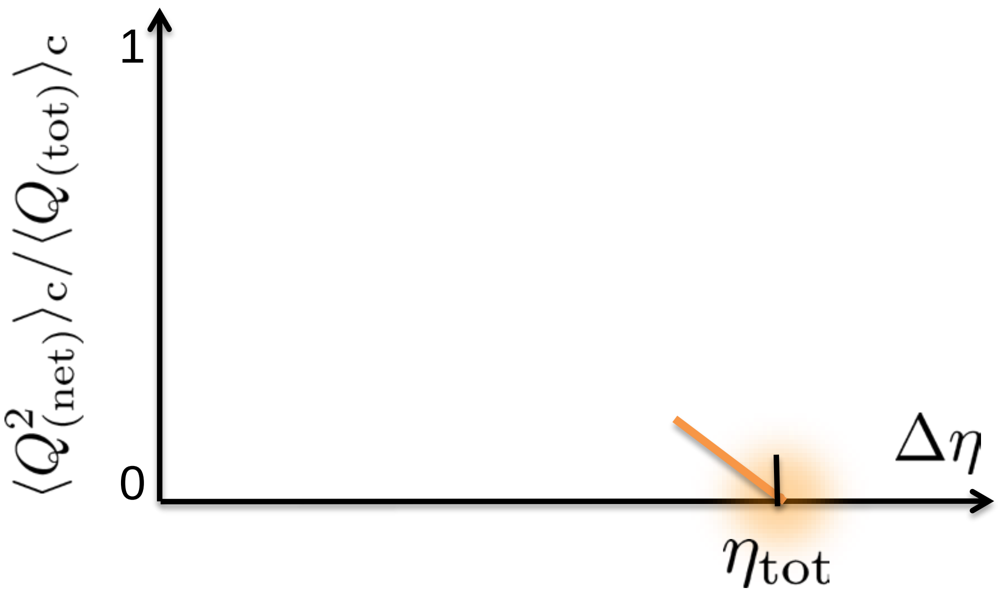
Non monotonic dependence on Δη ?

Effect of Global Charge Conservation (Finite Volume Effect)

Sakaida, Asakawa, MK, PRC, 2014

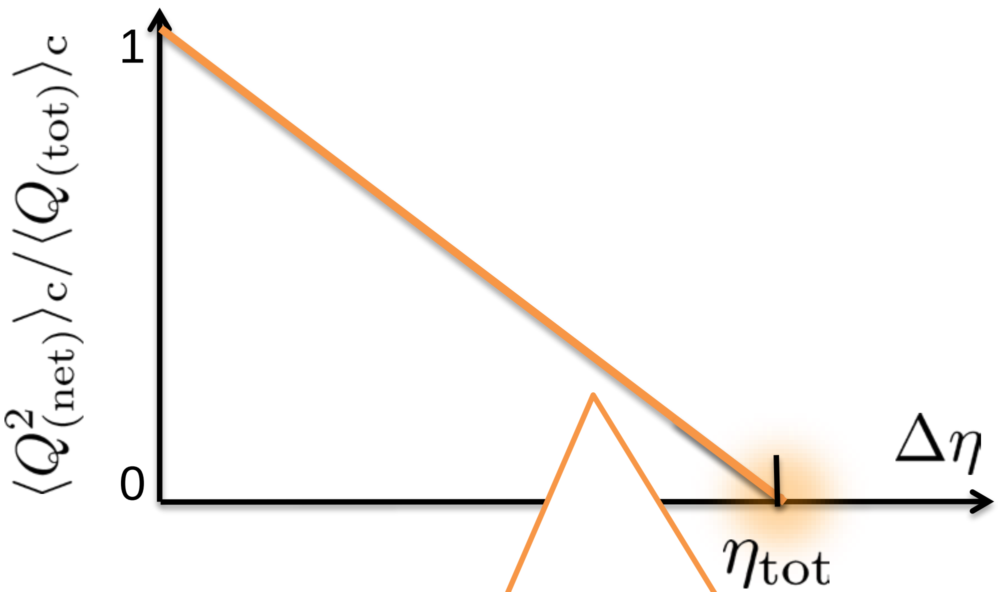
Global Charge Conservation

Conserved charges in the total system do no fluctuate!



Global Charge Conservation

Conserved charges in the total system do no fluctuate!



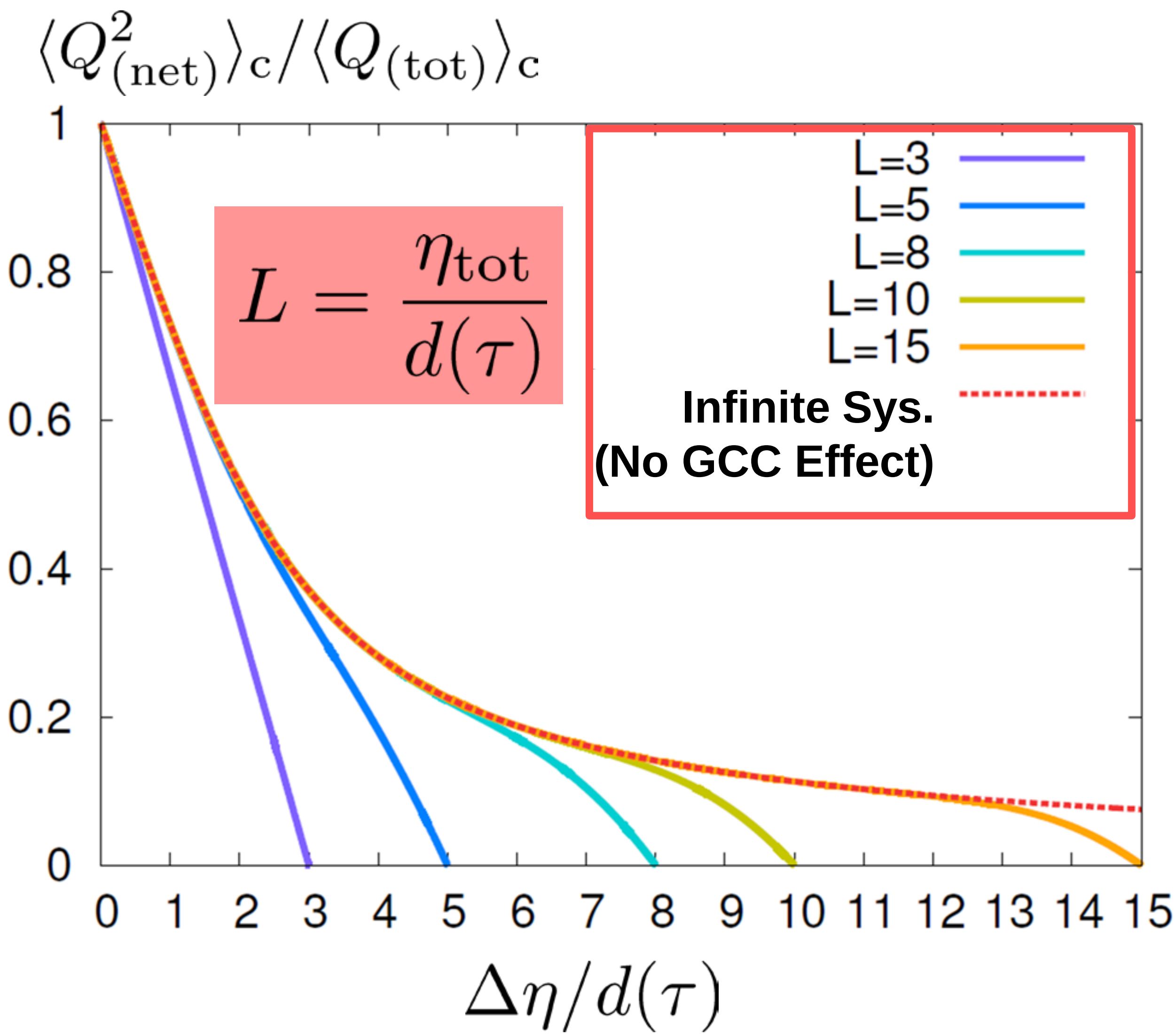
An Estimate of GCC Effect

$$\langle \delta N^2 \rangle_{\mathrm{GCC}} = \langle \delta N^2 \rangle_{\mathrm{inf}} \times \left(1 - \frac{\Delta\eta}{\eta_{\mathrm{tot}}} \right)$$

Jeon, Koch, PRL2000; Bleicher, Jeon, Koch (2000)

Diffusion in Finite Volume

Solve the diffusion master equation in finite volume



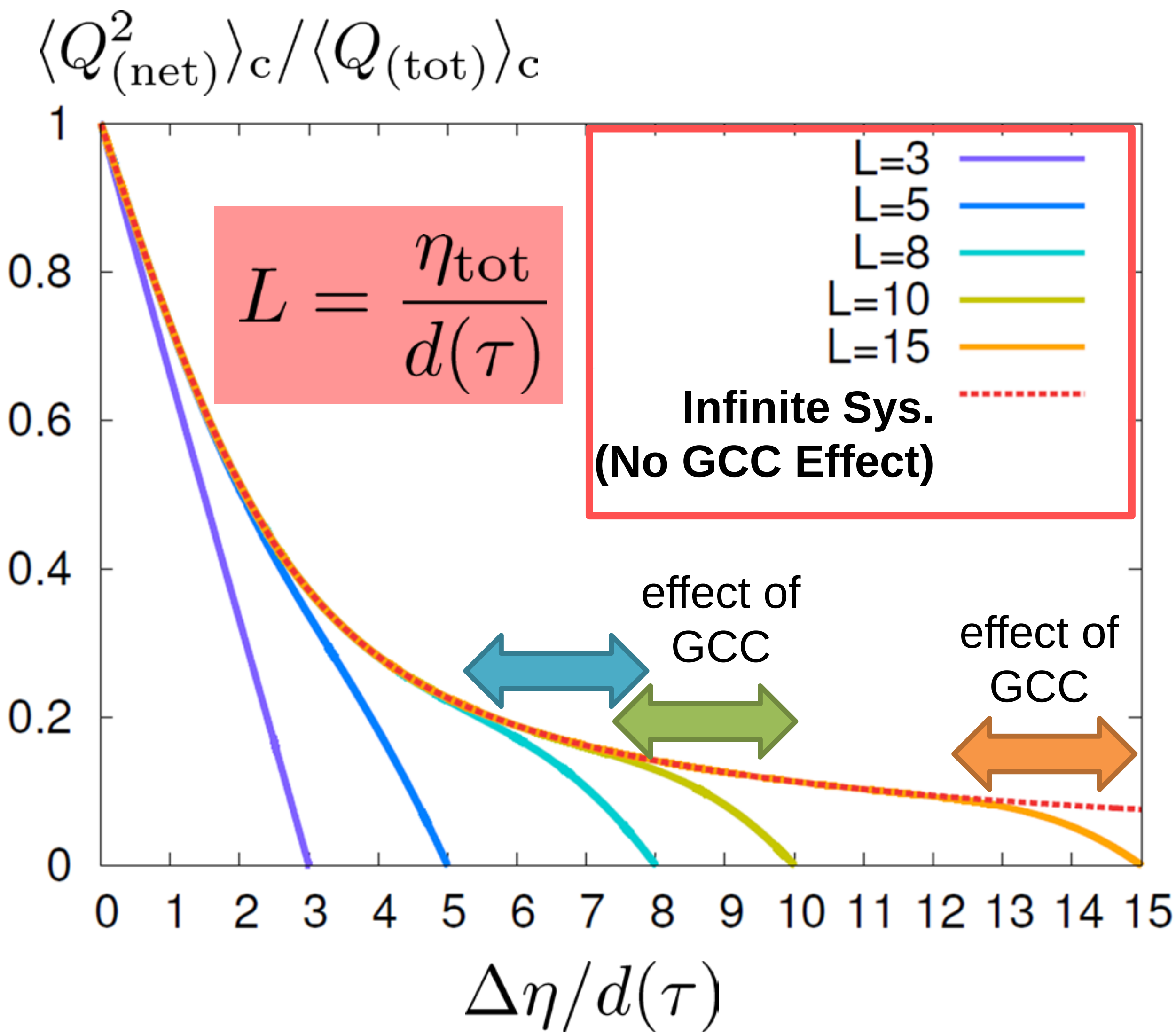
$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

: Average Diffusion Length

$D(\tau)$: Diffusion Coefficient

Diffusion in Finite Volume

Solve the diffusion master equation in finite volume



$$d(\tau) = \sqrt{2\int_{\tau_0}^{\tau} D(\tau')d\tau'}$$

: Average Diffusion Length

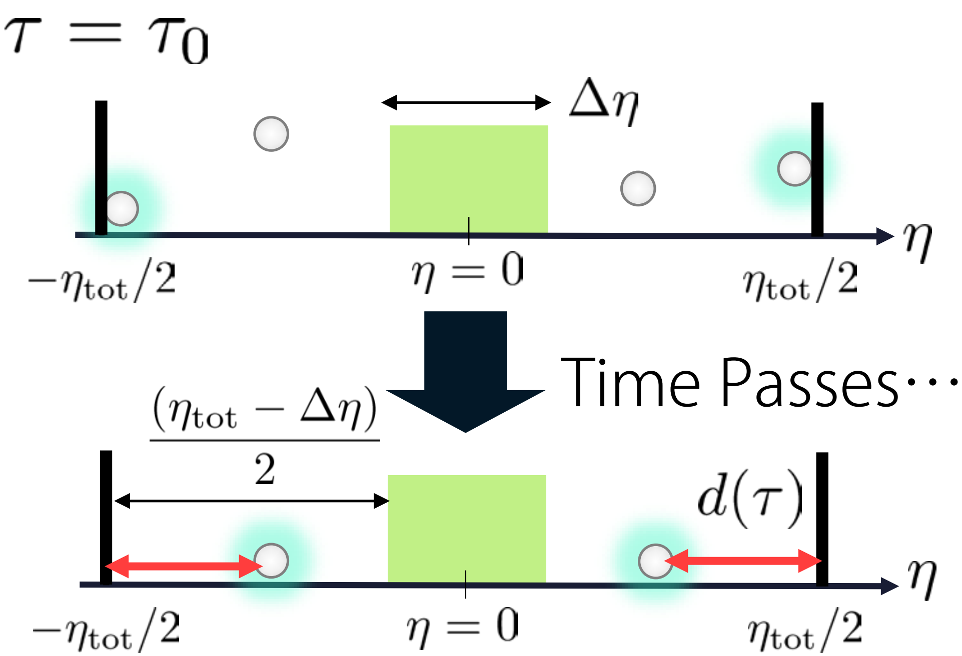
$D(\tau)$: Diffusion Coefficient

suppression only for

$$\Delta\eta/d \geq L-2$$

Physical Interpretation

slide by M. Sakaida



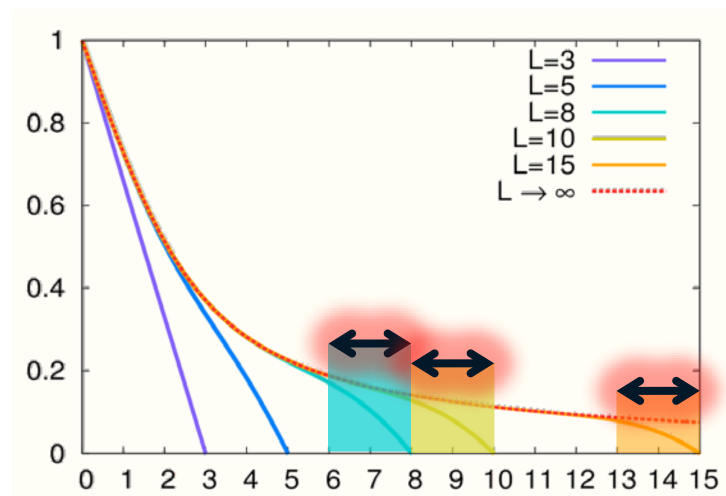
$d(\tau)$: Averaged Diffusion Distance

$D(\tau)$: Diffusion Coefficient

$\eta_{\rm tot}$: Total Length of Matter

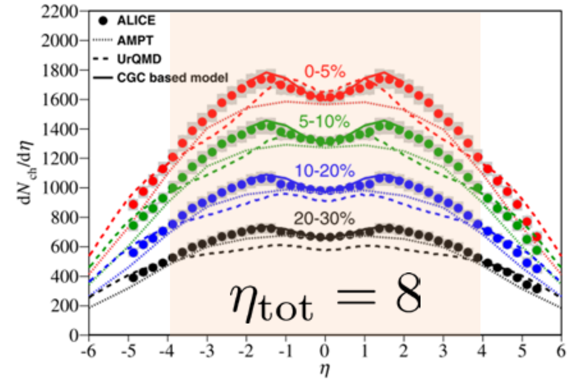
Condition for effects of the GCC

$$\Delta\eta/d \geq L - 2 \Leftrightarrow \frac{\eta_{\rm tot} - \Delta\eta}{2} \leq d$$



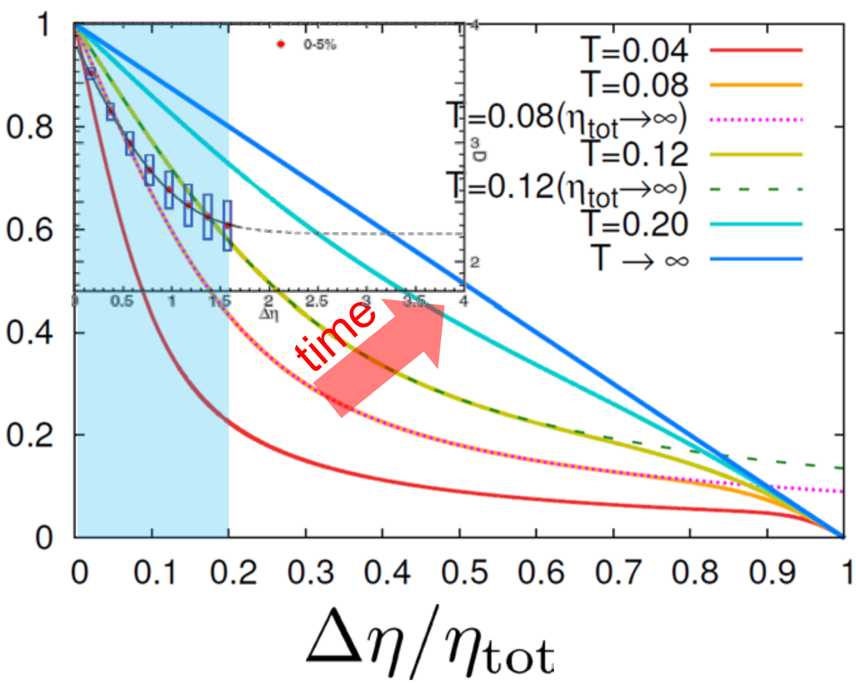
Effects of the GCC appear only near the boundaries.

Comparison with ALICE Result

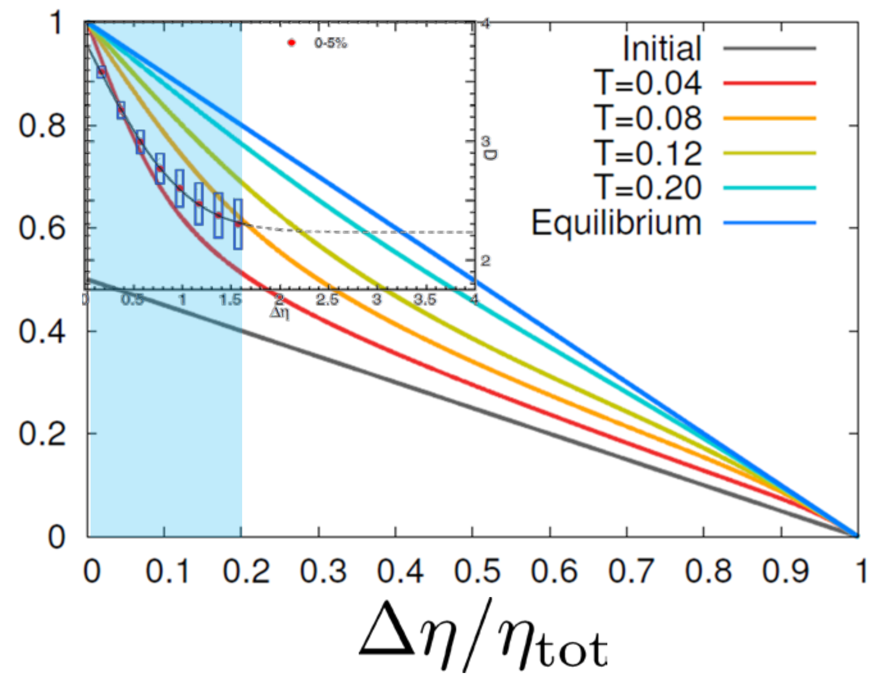


$$\langle Q^2_{(\rm net)}\rangle_{\rm c}/\langle Q_{(\rm tot)}\rangle_{\rm c}$$

without initial fluc.



with initial fluc.

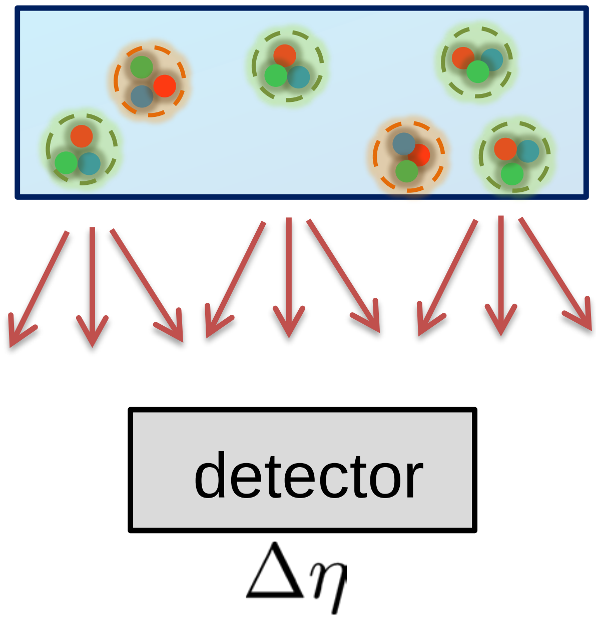


- No GCC effect in ALICE experiments!
- Same conclusion for higher order cumulants

$$T = \frac{d(\tau)}{\eta_{\rm tot}}$$

Very Low Energy Collisions

- Large contribution of global charge conservation
- Violation of Bjorken scaling



Careful treatment is required to interpret fluctuations at low beam energies!

Many information should be encoded in $\Delta\eta$ dep.

Summary

Plenty of information in $\Delta\eta$ dependences of various cumulants

$$\langle N_Q^2 \rangle_c,\ \langle N_Q^3 \rangle_c,\ \langle N_Q^4 \rangle_c,\ \langle N_B^2 \rangle_c,\ \langle N_B^3 \rangle_c,\ \langle N_B^4 \rangle_c,\ \langle N_S^2 \rangle_c,\ \cdots$$

and those of non-conserved charges, mixed cumulants…

With $\Delta\eta$ dep. we can explore

- primordial thermodynamics
- non-thermal and transport property
- effect of thermal blurring

Future Studies

- Experimental side:
  - rapidity window dependences
  - baryon number cumulants
  - consistency between RHIC and LHC

- Theoretical side:
  - rapidity window dependences in dynamical models
  - description of non-equilibrium non-Gaussianity
  - accurate measurements on the lattice

- Both sides:
  - Compare theory and experiment carefully
  - Do not use a fixed $\Delta\eta$ cumulant for comparison!!!