Progress in NLO SMEFT… and more! :)

Michael Trott, Niels Bohr Institute, Copenhagen, Denmark.

Based on:

1505.02646 Hartmann, Trott, and to appear

1502.02570 Berthier, Trott and to appear

Some NLO overlap with recent results in:

1505.03706 Ghezzi, Gomez-Ambrosio, Passarino, Uccirati

Why question NLO?

- Why are EFT’s good? Why do they succeed?

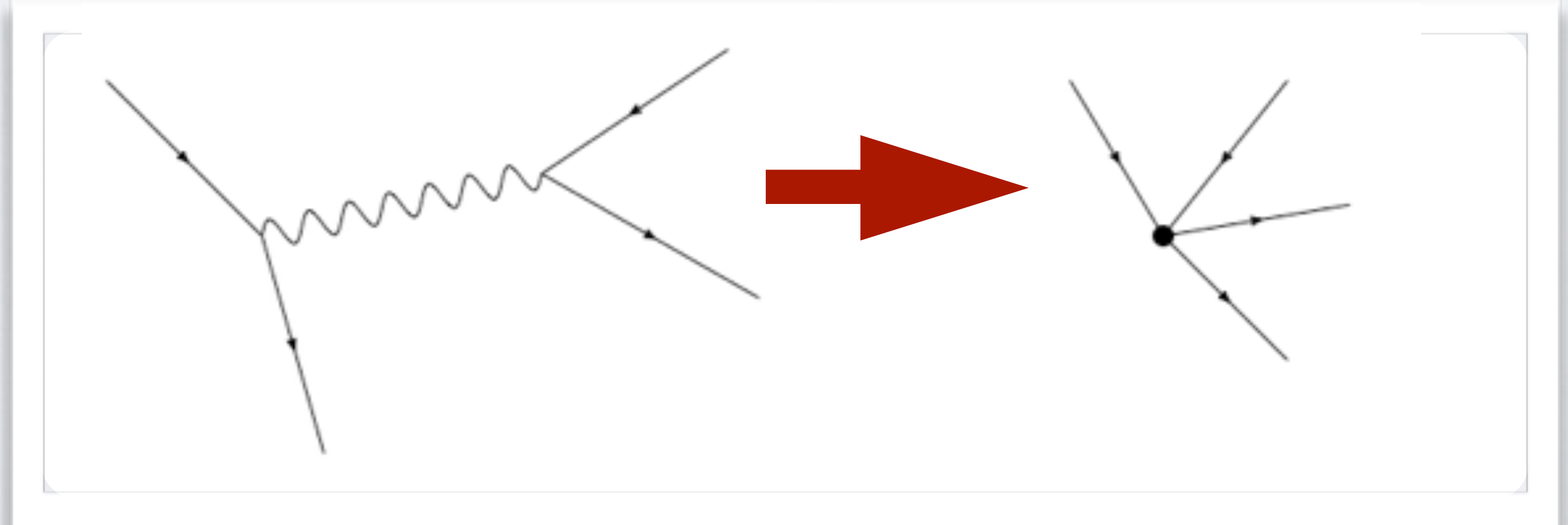
Fermi’s 4 fermion theory - still useful.

Cool Wiki knowledge:

Fermi first submitted his "tentative" theory of beta decay to Nature, which rejected it for being "too speculative." Published in Italian and German 1933, did not appear in English. Nature finally belatedly republished Fermi's report on beta decay in English on January 16, 1939. (Fermi temporarily quit theoretical physics over this.)

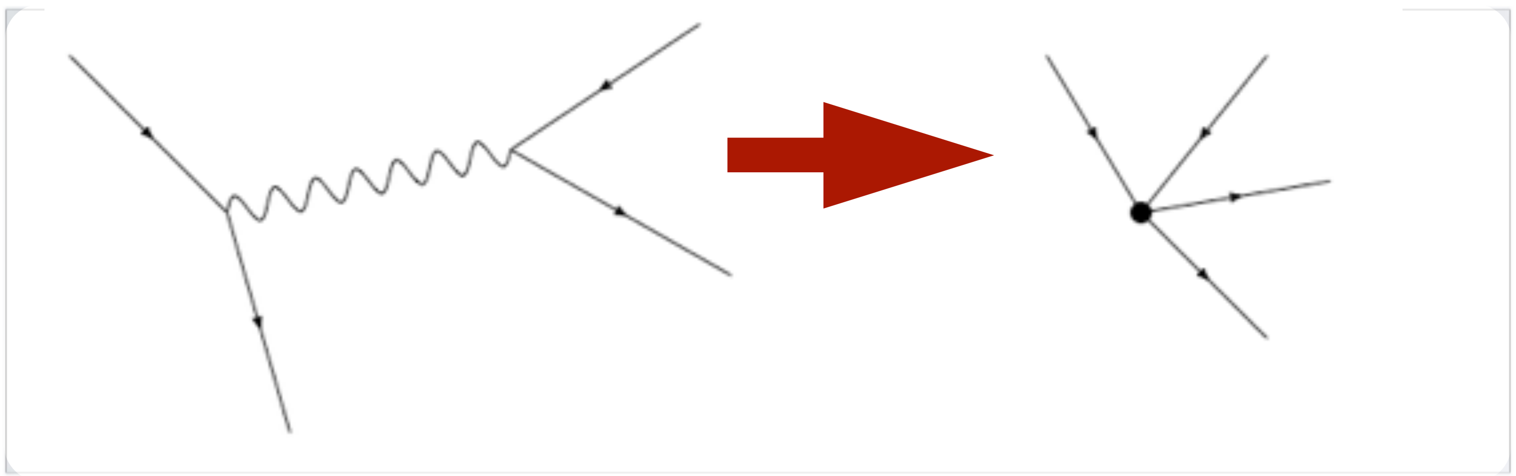
Why is Fermi’s theory still useful? Experimental precision has increased remarkably in beta decay.

Fermi’s theory is a real, no b.s., limit of the SM: $p^2 \ll M_W^2$



Why NLO?….Why not NLO?

- Why are EFT's good? Why do they succeed?



Operator level

$$\mathcal{L}_{G_F} = -\frac{4\mathcal{G}_F}{\sqrt{2}} \left(\bar{\nu}_\mu \gamma^\mu P_L \mu\right) \left(\bar{e}\, \gamma_\mu P_L \nu_e\right).$$

- The Fermi guess stuck around as this is SYSTEMATICALLY IMPROVABLE.
- It can be loop improved, you can add even higher D operators in the expansion, etc. You can improve this.
- Key EFT benefit:

Can increase theory precision to match experimental precision

How precise for NLO SMEFT?

- If you want to gain maximum benefit from a measurement, a constraint, or a pattern of deviations, the theory precision should be better than the experimental precision.

Consider LEP I observables:

Observable	Experimental Value	Ref.	SM Theoretical Value	Ref.
$\hat{m}_Z$[GeV]	91.1875 ± 0.0021	[38]	-	-
$\hat{m}_W$[GeV]	80.385 ± 0.015	[39]	80.365 ± 0.004	[40]
σ_h^0 [nb]	41.540 ± 0.037	[38]	41.488 ± 0.006	[41]
Γ_Z[GeV]	2.4952 ± 0.0023	[38]	2.4942 ± 0.0005	[41]
R_ℓ^0	20.767 ± 0.025	[38]	20.751 ± 0.005	[41]
R_b^0	0.21629 ± 0.00066	[38]	0.21580 ± 0.00015	[41]
R_c^0	0.1721 ± 0.0030	[38]	0.17223 ± 0.00005	[41]
A_{FB}^ℓ	0.0171 ± 0.0010	[38]	0.01616 ± 0.00008	[42]
A_{FB}^c	0.0707 ± 0.0035	[38]	0.0735 ± 0.0002	[42]
A_{FB}^b	0.0992 ± 0.0016	[38]	0.1029 ± 0.0003	[42]

arXiv:1311.3107. Chen et al.

arXiv:1501.0280. Petrov et al.

arXiv:1406.6070 Wells,Zhang

arXiv:1404.3667 Ellis et al.

1211.1320 Masso, Sanz

And Many others…

1308.2803 Pomarol, Riva. 1409.7605 Trott hep-ph/0412166] Han, Skiba

1411.0669 Falkowski, Riva. 1503.07872 Efrati et al. arXiv:1306.4644 Ciuchini et al.

What about LEP?

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per-mill

percent!

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Note that theorists worked hard in SM for this to be the case.

Many 2 loop SM calculations

What about LEP?

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Consider LEP I observables:

arXiv:1502.02570
Berthier, Trott

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If you go beyond % constraints, LO SMEFT alone inconsistent.

What is the theory error here in the SMEFT?
% level errors is the theory error.

How precise for NLO SMEFT?

- Why do i say this? arXiv:1502.02570 Berthier, Trott and to appear:

  The theory error is defined by the effects in the SMEFT that are neglected. Main things neglected:

  - All perturbative one loop corrections, LO → NLO
  - Higher order terms in the SMEFT

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda_{\delta L \neq 0}} \mathcal{L}_5 + \frac{1}{\Lambda^2_{\delta B = 0}} \mathcal{L}_6 + \frac{1}{\Lambda^2_{\delta B \neq 0}} \mathcal{L}'_6 + \frac{1}{\Lambda^3_{\delta L \neq 0}} \mathcal{L}_7 + \cdots$$

- (red circle, $\mathcal{L}_{SM}$) Glashow 1961, Weinberg 1967 (Salam 1967)
- (blue circle, $\mathcal{L}_5$) Weinberg 1977
- (green circle, $\mathcal{L}_6$) Leung, Love, Rao 1984, Buchmuller Wyler 1986, Grzadkowski, Iskrzynski, Misiak, Rosiek 2010
- (cyan circle, $\mathcal{L}'_6$) Weinberg 1979
- (black circle, $\mathcal{L}_7$) Lehman 2014 (student at Notre Dame) arXiv:1410.4193 L. Lehman

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$\bigcirc + \bigcirc$ Violate lepton number 1410.4193 Lehman

$\mathcal{L}^{(8)}$ on the way - see 1503.07537 Lehman, Martin, 1410.4193 Lehman

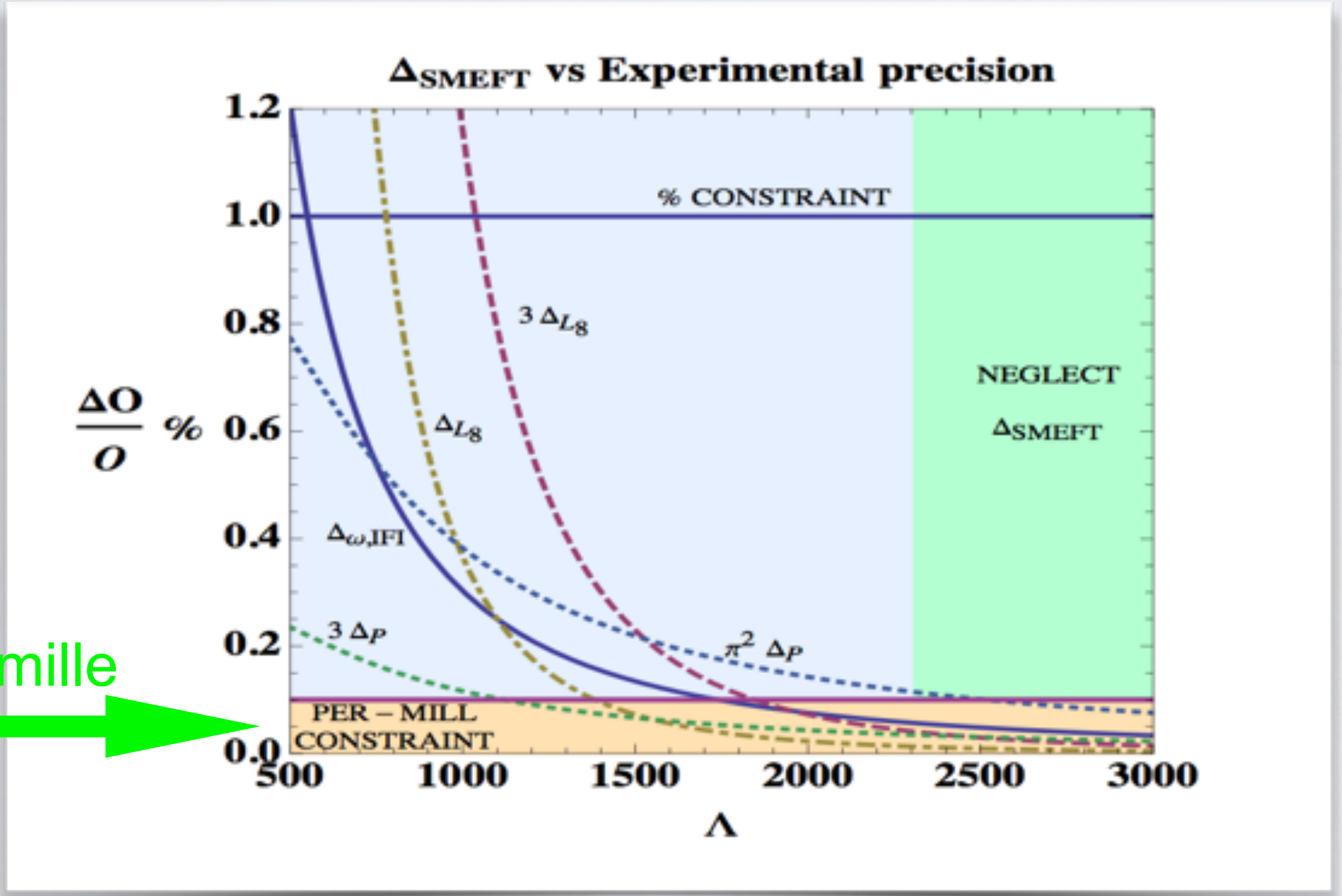
It is important this is on the way to motivate NLO.

How precise for NLO SMEFT?

percent

- Why do i say this? arXiv:1502.02570 Berthier, Trott and to appear:

per-mille



Conservative here. Assume no hierarchy of Wilson coefficients.

Constraints above the percent level are not consistently projected into the SMEFT for lower Λ

If analysis set $\Delta_{SMEFT} \to 0$ only valid for large cut off scale.

$\mathcal{L}^{(8)}$ Major issue, but it is on the way, a solvable problem, like NLO SMEFT

How precise for NLO SMEFT?

percent

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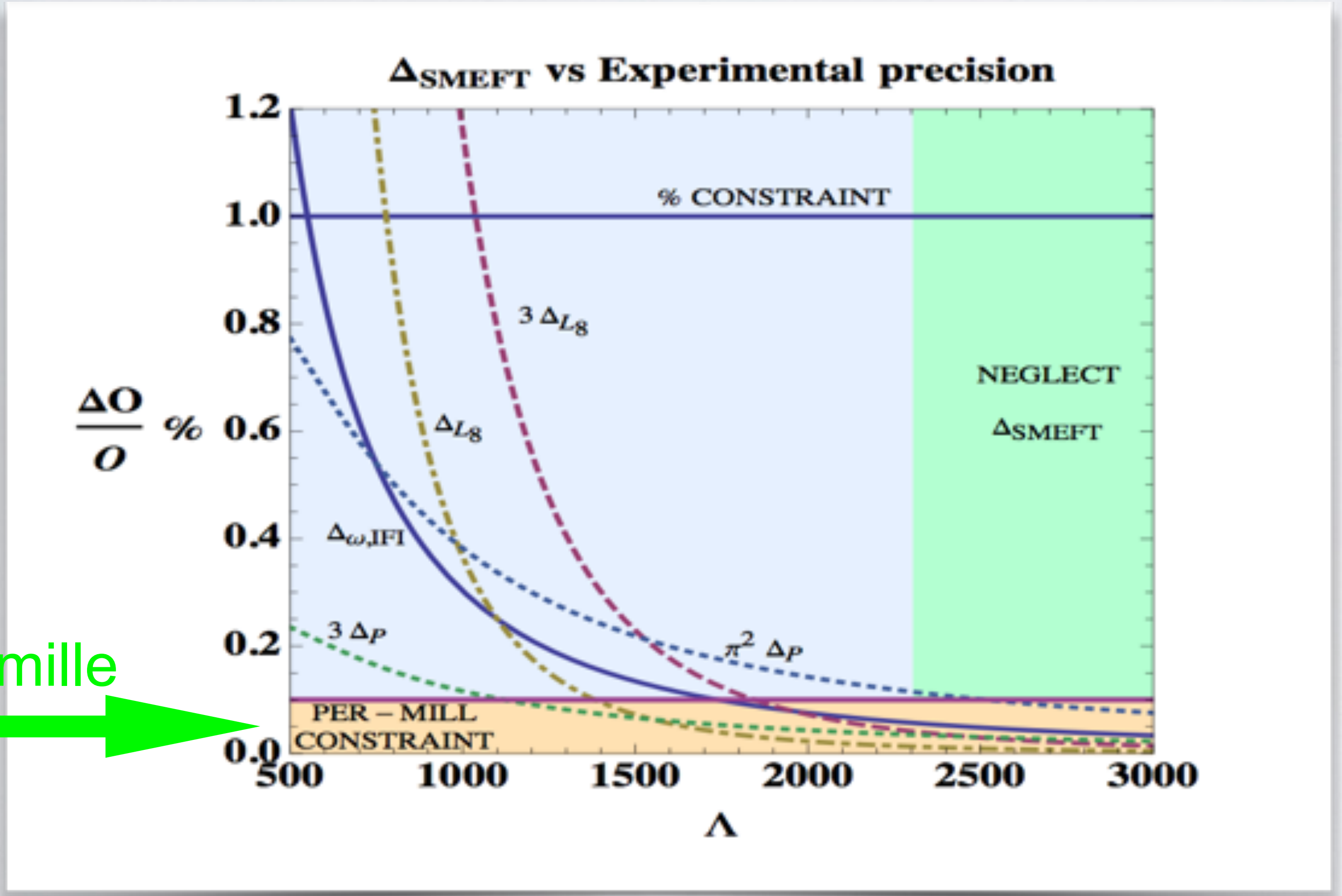
per-mille



Need of loops in SMEFT once measurements are 10% precise appears again and again in the literature

1209.5538 Passarino

1301.2588 Grojean, Jenkins, Manohar, Trott

1408.5147 Englert, Spannowsky

And many others..

When do we need NLO SMEFT?

- 

20 years ago!!!!

arXiv:1502.02570 Berthier, Trott and to appear:

- This is why it is important to not blindly project the per-mille - % constraint hierarchy EXPERIMENTALLY into the SMEFT parameters and set parameters to 0.

- <u>The Higgs basis does not help with this issue.</u>
  It confuses this issue - breaking of relations due to perturbative corrections and $\mathcal{L}^{(8)}$ essentially hidden in formalism.

How does "Higgs basis" obscure?

- For example

$$\begin{aligned}
\Delta\mathcal{L}^{D=6}_{\rm hvv} = \frac{h}{v}\Big[& 2\delta c_w m_W^2 W_\mu^+ W_\mu^- + \delta c_z m_Z^2 Z_\mu Z_\mu \\
& + c_{ww}\frac{g^2}{2} W^+_{\mu\nu}W^-_{\mu\nu} + \tilde{c}_{ww}\frac{g^2}{2} W^+_{\mu\nu}\tilde{W}^-_{\mu\nu} + c_{w\Box}g^2\left(W_\mu^- \partial_\nu W^+_{\mu\nu} + \text{h.c.}\right) \\
& + c_{gg}\frac{g_s^2}{4} G^a_{\mu\nu}G^a_{\mu\nu} + c_{\gamma\gamma}\frac{e^2}{4}A_{\mu\nu}A_{\mu\nu} + c_{z\gamma}\frac{eg}{2c_\theta}Z_{\mu\nu}A_{\mu\nu} + c_{zz}\frac{g^2}{4c_\theta^2}Z_{\mu\nu}Z_{\mu\nu} \\
& + c_{z\Box}g^2 Z_\mu \partial_\nu Z_{\mu\nu} + c_{\gamma\Box}gg' Z_\mu \partial_\nu A_{\mu\nu} \\
& + \tilde{c}_{gg}\frac{g_s^2}{4} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} + \tilde{c}_{\gamma\gamma}\frac{e^2}{4}A_{\mu\nu}\tilde{A}_{\mu\nu} + \tilde{c}_{z\gamma}\frac{eg}{2c_\theta}Z_{\mu\nu}\tilde{A}_{\mu\nu} + \tilde{c}_{zz}\frac{g^2}{4c_\theta^2}Z_{\mu\nu}\tilde{Z}_{\mu\nu}\Big].
\end{aligned} \quad (3.10)$$

Here $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$, and $\tilde{X}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}\partial_\rho X_\sigma$. The dependent couplings can be expressed by the independent couplings as[2]

$$\begin{aligned}
\delta c_w &= \delta c_z + 4\delta m, \\
c_{ww} &= c_{zz} + 2s_\theta^2 c_{z\gamma} + s_\theta^4 c_{\gamma\gamma}, \\
\tilde{c}_{ww} &= \tilde{c}_{zz} + 2s_\theta^2 \tilde{c}_{z\gamma} + s_\theta^4 \tilde{c}_{\gamma\gamma}, \\
c_{w\Box} &= \frac{1}{g^2 - g'^2}\left[g^2 c_{z\Box} + g'^2 c_{zz} - e^2 s_\theta^2 c_{\gamma\gamma} - (g^2 - g'^2)s_\theta^2 c_{z\gamma}\right], \\
c_{\gamma\Box} &= \frac{1}{g^2 - g'^2}\left[2g^2 c_{z\Box} + (g^2 + g'^2)c_{zz} - e^2 c_{\gamma\gamma} - (g^2 - g'^2)c_{z\gamma}\right].
\end{aligned} \quad (3.11)$$

This choice:

$$\mathcal{L}^{\rm SM} + \frac{1}{v^2}\sum_i c_i O_i,$$

Makes it hard to think the C cannot be arbitrarily small due to constraints.

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This choice:

$$\mathcal{L}^{\rm SM} + \frac{1}{v^2}\sum_i c_i O_i,$$

Makes it hard to think the C cannot be arbitrarily small due to constraints.

- Custodial relations broken by loops and $\mathcal{L}^{(8)}$ too.

- "Dependent-independent" coupling relations are not pure symmetry statements, so modified by these concerns at couple % accuracy.

NLO EFT - Step I

- (Probably) our lagrangian: $\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda_{\delta L \neq 0}} \mathcal{L}_5 + \frac{1}{\Lambda^2_{\delta B=0}} \mathcal{L}_6 + \frac{1}{\Lambda^2_{\delta B=0}} \mathcal{L}'_6 + \frac{1}{\Lambda^3_{\delta L \neq 0}} \mathcal{L}_7 + \cdots$

- Step 1: Define a basis, groups that deserve main credit on this.

Leung, Love, Rao 1984, Buchmuller Wyler 1986,

Hagiwara, Ishihara, Szalapski, Zeppenfeld 1993.

1008.4884 Grzadkowski, Iskrzynski, Misiak,Rosiek
operator basis FULLY reduced by SM EOM.

Once that is known, can use EOM to relate operator forms:

Contino, Ghezzi, Grojean, Muhlleitner, Spira 1303.3876

however in doing so be careful about flavour indicies

Alonso, Jenkins, Manohar, Trott 1312.2014

Here is ~~a~~ the well defined SMEFT basis

- Clearly specified, all flavour indicies, all operators.

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
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$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

Table 2: Dimension-six operators other than the four-fermion ones.

Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek JHEP 1010 (2010) 085, [arXiv:1008.4884]

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{pr\cdots}}{\Lambda^2} O_i^{pr\cdots}$$

6 gauge dual

28 non dual operators

25 four fermi ops

59 +h.c. operators

Notation:

$\widetilde X_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma} X^{\rho\sigma} \quad (\varepsilon_{0123} = +1)$

$\widetilde\varphi^j = \varepsilon_{jk}(\varphi^k)^\star \qquad \varepsilon_{12} = +1$

$\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi \equiv i\varphi^\dagger\left(D_\mu - \overleftarrow{D}_\mu\right)\varphi$

$\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi \equiv i\varphi^\dagger\left(\tau^I D_\mu - \overleftarrow{D}_\mu \tau^I\right)\varphi$

NLO EFT - Step I DONE

- Four fermion operators: 1008.4884 Grzadkowski, Iskrzynski, Misiak,Rosiek

$8:(\bar{L}L)(\bar{L}L)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$

$8:(\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$
Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$
Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$8:(\bar{L}L)(\bar{R}R)$	
Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$8:(\bar{L}R)(\bar{R}L)+\mathrm{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$

$8:(\bar{L}R)(\bar{L}R)+\mathrm{h.c.}$	
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

NLO EFT - Step 2 Renormalize

- Number of parameters is

$$\left[107 n_g^4 + 2 n_g^3 + 135 n_g^2 + 60\right]/4$$

$n_g = 1$ total parameters 76

$n_g = 3$ total parameters 2499

Alonso,Jenkins,Manohar Trott arXiv:1312.2014

We need on the order of hundreds of parameters, not thousands.

- We know the Warsaw basis is self consistent at one loop as it has been completely renormalized - DONE!

arXiv:1301.2588 Grojean, Jenkins, Manohar, Trott
arXiv:1308.2627, 1309.0819, 1310.4838 Jenkins, Manohar, Trott
arXiv: 1312.2014 Alonso, Jenkins, Manohar, Trott

Some partial results were also obtained in a "SILH basis"

arXiv:1302.5661, 1308.1879 Elias-Miro, Espinosa, Masso, Pomarol
1312.2928 Elias-Miro, Grojean, Gupta, Marzocca

If this is self consistent at one loop as a SMEFT it should be demonstrated.

NLO EFT - Step 2 Renormalize

- How was this renormalization done?

  Calculated in the unbroken phase of the theory, using the background field method.

  G. 't Hooft, Acta Universitatis Wratislaviensis No.368, Vol.1*, Wroclaw 1976, 345-369

  B. S. DeWitt, Phys.Rev. 162 (1967) 1195–1239

  L. Abbott, Acta Phys.Polon. B13 (1982) 33

EW App. →

  A. Denner, G. Weiglein, and S. Dittmaier, Nucl.Phys. B440 (1995) 95–128, hep-ph/9410338.

  M. B. Einhorn and J. Wudka, Phys.Rev. D39 (1989) 2758.

  A. Denner, Fortsch.Phys. 41 (1993) 307–420, [arXiv:0709.1075].

- Background field method not necessary, but a nice trick, and allowed US to succeed in avoiding gauge dependent results.
  (Some competition did not use the background field method.)

NLO EFT - Step 3 WAR.

- Somewhat unexpected step in this program was a total war that broke out over how big these effects in RGE mixing terms can be.

  Putting aside comments on unscientific behavior.
  Physics summary:

  - There is NO model independent EFT statement on some Wilson coefficients being big and others small

    arXiv:1305.0017 Jenkins,Manohar, Trott

  - Some weakly coupled renormalizable theories allow an operator classification scheme in terms of “tree” and “loop” generated operators.

    Artz, Einhorn Wudka [hep-ph/9405214](hep-ph/9405214)

  - Even in the later scheme “tree” and “loop” operators still mix

    arXiv:1308.2627 Jenkins,Manohar, Trott

  - There is no “special or better” operator basis.

NLO EFT - Step 4 Understanding.

		g^3X^3	H^6	H^4D^2	$g^2X^2H^2$	$y\psi^2H^3$	$gy\psi^2XH$	ψ^2H^2D	ψ^4
		1	2	3	4	5	6	7	8
g^3X^3	1	g^2	0	0	1	0	0	0	0
H^6	2	$g^6\lambda$	λ, g^2, y^2	$g^4, g^2\lambda, \lambda^2$	$g^6, g^4\lambda$	$\lambda y^2, y^4$	0	$\lambda y^2, y^4$	0
H^4D^2	3	g^6	0	g^2, λ, y^2	g^4	y^2	g^2y^2	g^2, y^2	0
$g^2X^2H^2$	4	g^4	0	1	g^2, λ, y^2	0	y^2	1	0
$y\psi^2H^3$	5	g^6	0	g^2, λ, y^2	g^4	g^2, λ, y^2	$g^2\lambda, g^4, g^2y^2$	g^2, λ, y^2	λ, y^2
$gy\psi^2XH$	6	g^4	0	0	g^2	1	g^2, y^2	1	1
ψ^2H^2D	7	g^6	0	g^2, y^2	g^4	y^2	g^2y^2	g^2, λ, y^2	y^2
ψ^4	8	g^6	0	0	0	0	g^2y^2	g^2, y^2	g^2, y^2

NDA explains some structure

Combined results. This pattern in an arbitrary EFT is now better understood.

Normalize ops using NDA: $f^2\Lambda^2 \left(\frac{\phi}{f}\right)^A \left(\frac{\psi}{f\sqrt{\Lambda}}\right)^B \left(\frac{gX}{\Lambda^2}\right)^C \left(\frac{D}{\Lambda}\right)^D$

Entries follow the rule: $\left(\frac{g^2}{16\pi^2}\right)^{n_g} \left(\frac{y^2}{16\pi^2}\right)^{n_y} \left(\frac{\lambda}{16\pi^2}\right)^{n_\lambda}, \qquad N = n_g + n_y + n_\lambda$

Where: $N = L + w - \sum_k w_k \equiv L + \Delta.$

Jenkins, Manohar, Trott arXiv: 1309.0819
(nice follow up) Buchalla et al. arXiv: 1312.5624

The ω is the power of f^2 in the operator normalization.

NLO EFT - Step 4 Understanding.

- Can check against full result now known:

	H^6	H^4D^2	$y\psi^2H^3$	ψ^2H^2D	ψ^4	$g^2X^2H^2$	$gy\psi^2XH$	g^3X^3
Class	2	3	5	7	8	4	6	1
NDA Weight	2	1	1	1	1	0	0	-1
H^6	λ, y^2, g^2	$\lambda^2, \lambda g^2, g^4$	$\lambda y^2, y^4$	$\lambda y^2, \lambda g^2, \cancel{y^4}$	0	$\lambda g^4, g^6$	0	$\cancel{\lambda g^6}$
H^4D^2	0	λ, y^2, g^2	$\cancel{y^2}$	y^2, g^2	0	$\cancel{g^4}$	$\cancel{y^2 g^2}$	$\cancel{g^6}$
$y\psi^2H^3$	0	λ, y^2, g^2	λ, y^2, g^2	λ, y^2, g^2	λ, y^2	g^4	$\cancel{g^2}\lambda, g^4, g^2y^2$	$\cancel{g^6}$
ψ^2H^2D	0	g^2, y^2	$\cancel{y^2}$	$g^2, \cancel{\lambda}, y^2$	g^2, y^2	$\cancel{g^4}$	$\cancel{g^2 y^2}$	$\cancel{g^6}$
ψ^4	0	0	0	g^2, y^2	g^2, y^2	0	g^2y^2	$\cancel{g^6}$
$g^2X^2H^2$	0	$\cancel{1}$	0	$\cancel{1}$	0	λ, y^2, g^2	y^2	g^4
$gy\psi^2XH$	0	0	$\cancel{1}$	$\cancel{1}$	1	g^2	g^2, y^2	g^4
g^3X^3	0	0	0	0	0	$\cancel{1}$	0	g^2

- Crossed hatched entries vanish despite naive degree of divergence, or through cancelations

Blue is explicit one loop "tree-loop" mixing even in weakly coupled renormalizable UV theories

“No tree loop mixing” is just wrong.

- ”No Tree-loop” mixing does not work to understand the anomalous dimension matrix. Here is the explicit example:

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}C_{\substack{eB\\pr}} = \frac{1}{16\pi^2}\left[4g_1 N_c\left(\mathsf{y}_u+\mathsf{y}_q\right)C^{(3)}_{\substack{lequ\\prst}}\left[Y_u\right]_{ts}\right]+\dots$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}C_{\substack{eW\\pr}} = \frac{1}{16\pi^2}\left[-2g_2 N_c C^{(3)}_{\substack{lequ\\prst}}\left[Y_u\right]_{ts}\right]+\dots$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}C_{\substack{uB\\pr}} = \frac{1}{16\pi^2}\left[4g_1\left(\mathsf{y}_e+\mathsf{y}_l\right)C^{(3)}_{\substack{lequ\\stpr}}\left[Y_e\right]_{ts}\right]+\dots$$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}C_{\substack{uW\\pr}} = \frac{1}{16\pi^2}\left[-2g_2 C^{(3)}_{\substack{lequ\\stpr}}\left[Y_e\right]_{ts}\right]+\dots,$$

Can be generated by (3,2,7/6) scalars. Even for weakly coupled renormalizable theories, this is the case at one loop. arXiv:1308.2627 Jenkins,Manohar, Trott

- Holomorphy is approximately respected at one loop. Alonso, Jenkins, Manohar hep/1409.0868
It is not exact, Yukawas violate this scheme at one loop as well.
Understood in terms of helicity and unitarity 1505.01844 in Cheung, Shen.
(This is a very nice generalization of the NDA approach in terms of defined operator weights.)

NLO EFT - Step 5 Full one loop

- In SMEFT the cut off scale is not TOO high. So RGE log terms not expected to be much bigger than remaining one loop “finite terms”
- Further, no reason to expect that structure of the divergences in mixing will have to be preserved in finite terms. So - lets calculate finite terms for $\Gamma(h \to \gamma\,\gamma)$
- Initial calc - mirror initial RGE work, just use operators

$$\mathcal{O}_{HB}^{(0)} = g_1^2\, H^\dagger\, H\, B_{\mu\nu}\, B^{\mu\nu}, \qquad \mathcal{O}_{HW}^{(0)} = g_2^2\, H^\dagger\, H\, W^a_{\mu\nu}\, W_a^{\mu\nu},$$

$$\mathcal{O}_{HWB}^{(0)} = g_1\, g_2\, H^\dagger\, \sigma^a H\, B_{\mu\nu}\, W_a^{\mu\nu}.$$

Hartmann, Trott 1505.02646.pdf

Calculation with all of these operators has been performed, and is being finalized.

$$\mathcal{O}_H^{(0)} = \lambda (H^\dagger H)^3, \qquad \mathcal{O}_{HW}^{(0)} = g_2^2\, H^\dagger\, H\, W^a_{\mu\nu}\, W_a^{\mu\nu}, \qquad \mathcal{O}_{HB}^{(0)} = g_1^2\, H^\dagger\, H\, B_{\mu\nu}\, B^{\mu\nu},$$

$$\mathcal{O}_{HD}^{(0)} = (H^\dagger D_\mu H)^\star (H^\dagger D^\mu H), \qquad \mathcal{O}_W^{(0)} = \epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}, \qquad \mathcal{O}_{HWB}^{(0)} = g_1\, g_2\, H^\dagger\, \sigma^a H\, B_{\mu\nu}\, W_a^{\mu\nu},$$

$$\mathcal{O}_{uH}^{(0)} = y_u H^\dagger H (\bar{q}_p u_r \tilde{H}), \qquad \mathcal{O}_{\substack{eB \\ rs}}^{(0)} = \bar{l}_{r,a} \sigma^{\mu\nu}\, e_s\, H_a\, B_{\mu\nu}, \qquad \mathcal{O}_{\substack{eW \\ rs}}^{(0)} = \bar{l}_{r,a} \sigma^{\mu\nu}\, e_s\, \tau^I_{ab}\, H_b\, W^I_{\mu\nu},$$

$$\mathcal{O}_{H\Box}^{(0)} = H^\dagger H \Box (H^\dagger H), \qquad \mathcal{O}_{\substack{uB \\ rs}}^{(0)} = \bar{q}_{r,a} \sigma^{\mu\nu}\, u_s\, \tilde{H}_a\, B_{\mu\nu}, \qquad \mathcal{O}_{\substack{uW \\ rs}}^{(0)} = \bar{q}_{r,a} \sigma^{\mu\nu}\, u_s\, \tau^I_{ab}\, \tilde{H}_b\, W^I_{\mu\nu},$$

$$\mathcal{O}_{eH}^{(0)} = y_e H^\dagger H (\bar{l}_p e_r H), \qquad \mathcal{O}_{\substack{dB \\ rs}}^{(0)} = \bar{q}_{r,a} \sigma^{\mu\nu}\, d_s\, H_a\, B_{\mu\nu}, \qquad \mathcal{O}_{dW}^{(0)} = \bar{q}_{r,a} \sigma^{\mu\nu}\, d_s\, \tau^I_{ab}\, H_b\, W^I_{\mu\nu},$$

$$\mathcal{O}_{dH}^{(0)} = y_d H^\dagger H (\bar{q}_p d_r H).$$

Hartmann, Trott to appear

NLO EFT - Step 5a subtract div.

Recent results:
Hartmann, Trott 1505.02646.pdf
Ghezzi et al. 1505.03706
Pruna, Signer 1408.3565 others..

- The Algorithm: Use RGE results to renormalize.

  Also use SM counter term subtractions.

  Define a scheme that fixes that asymptotic properties of states in the S matrix, this fixes the finite terms in renormalization conditions.

  Gauge fix, calculate, and then check gauge independence!

- Here is how this works in $\Gamma(h \to \gamma\,\gamma)$

$$\mathcal{O}_i^{(0)} = Z_{i,j}\,\mathcal{O}_j^{(r)}, \qquad \mathcal{Z}_{i,j} = \begin{pmatrix} \frac{g_1^2}{4} - \frac{9g_2^2}{4} + 6\lambda + Y & 0 & g_1^2 \\ 0 & -\frac{3g_1^2}{4} - \frac{5g_2^2}{4} + 6\lambda + Y & g_2^2 \\ \frac{3g_2^2}{2} & \frac{g_1^2}{2} & -\frac{g_1^2}{4} + \frac{9g_2^2}{4} + 2\lambda + Y \end{pmatrix},$$

$$\begin{aligned} \mathcal{L}_6^{(0)} &= Z_{SM}\,Z_{i,j}\,C_i\,\mathcal{O}_j^{(r)}, \\ &= Z_{SM}\,\mathcal{N}_{HB}\,\mathcal{O}_{HB}^{(r)} + Z_{SM}\,\mathcal{N}_{HW}\,\mathcal{O}_{HW}^{(r)} + Z_{SM}\,\mathcal{N}_{HWB}\,\mathcal{O}_{HWB}^{(r)}. \end{aligned}$$

$$\begin{aligned} \mathcal{N}_{HB} &= \frac{1}{16\pi^2\epsilon}\left[\left(16\pi^2\epsilon + \frac{g_1^2}{4} - \frac{9g_2^2}{4} + 6\lambda + Y\right) C_{HB}(\Lambda) + \frac{3\,g_2^2}{2} C_{HWB}(\Lambda)\right], \qquad (2.8) \\ \mathcal{N}_{HW} &= \frac{1}{16\pi^2\epsilon}\left[\left(16\pi^2\epsilon - \frac{3g_1^2}{4} - \frac{5g_2^2}{4} + 6\lambda + Y\right) C_{HW}(\Lambda) + \frac{g_1^2}{2} C_{HWB}(\Lambda)\right], \\ \mathcal{N}_{HWB} &= \frac{1}{16\pi^2\epsilon}\left[\left(16\pi^2\epsilon - \frac{g_1^2}{4} + \frac{9g_2^2}{4} + 2\lambda + Y\right) C_{HWB}(\Lambda) + g_1^2 C_{HB}(\Lambda) + g_2^2 C_{HW}(\Lambda)\right]. \end{aligned}$$

NLO EFT - Step 5a subtract div.

- To define the SM counter terms use background field , use R_ξ gauge

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} i \phi^+ \\ h + v + \delta v + i \phi_0 \end{pmatrix}$$

Background field method (with particular operator normalization) gives:

$$Z_A Z_e = 1, \qquad Z_h = Z_{\phi_\pm} = Z_{\phi_0}, \qquad Z_W Z_{g_2} = 1.$$

Also need the Higgs wavefunction and vev renorm

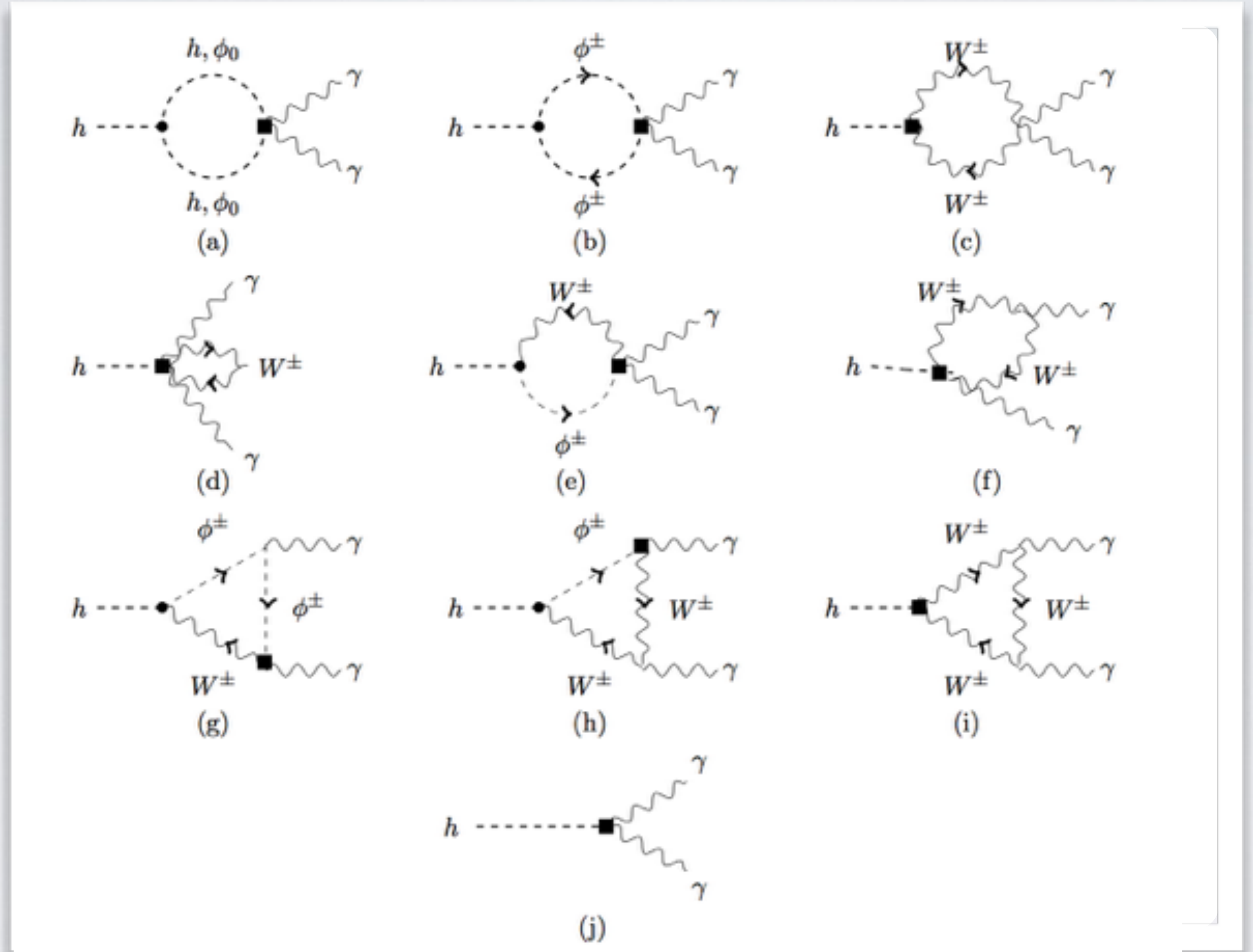
$$Z_h = 1 + \frac{(3+\xi)\,(g_1^2 + 3\,g_2^2)}{64\,\pi^2\,\epsilon} - \frac{Y}{16\,\pi^2\,\epsilon}.$$

$$(\sqrt{Z_v} + \frac{\delta v}{v})_{div} = 1 + \frac{(3+\xi)\,(g_1^2 + 3\,g_2^2)}{128\,\pi^2\,\epsilon} - \frac{Y}{32\,\pi^2\,\epsilon}.$$

We used a clever trick involving $h \to g\,g$ for the latter.

NLO EFT - Step 5a subtract div.

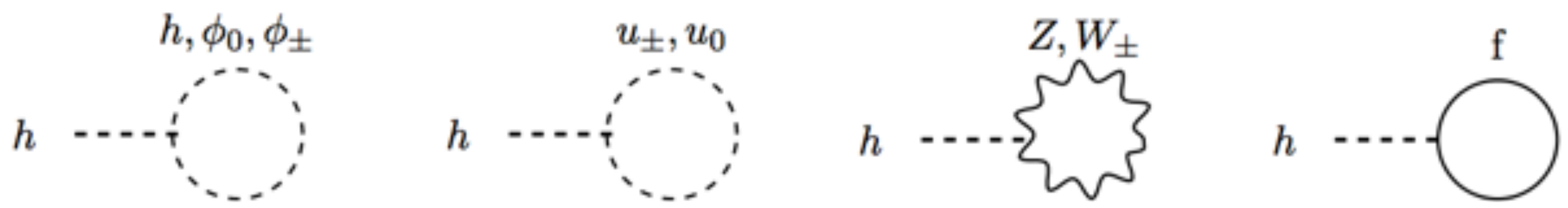
- Calculate in BF method, in R_ξ gauge



- Gauge dependence cancels ☑ remaining divergences cancel exactly ☑

NLO EFT - Step 5b fix finite terms

- Define vev of the theory as the one point function vanishing - fixes δv



$$T = m_h^2\, h\, v\, \frac{1}{16\pi^2}\left[-16\pi^2\,\frac{\delta v}{v} + 3\,\lambda\left(1+\log\left[\frac{\mu^2}{m_h^2}\right]\right) + \frac{m_W^2}{v^2}\xi\left(1+\log\left[\frac{\mu^2}{\xi\, m_W^2}\right]\right)\right., \quad (3.3)$$

$$+\frac{1}{2}\frac{m_Z^2}{v^2}\xi\left(1+\log\left[\frac{\mu^2}{\xi\, m_Z^2}\right]\right) - \frac{1}{2}\sum_i y_i^4\, N_c\frac{1}{\lambda}\left(1+\log\left[\frac{\mu^2}{m_i^2}\right]\right),$$

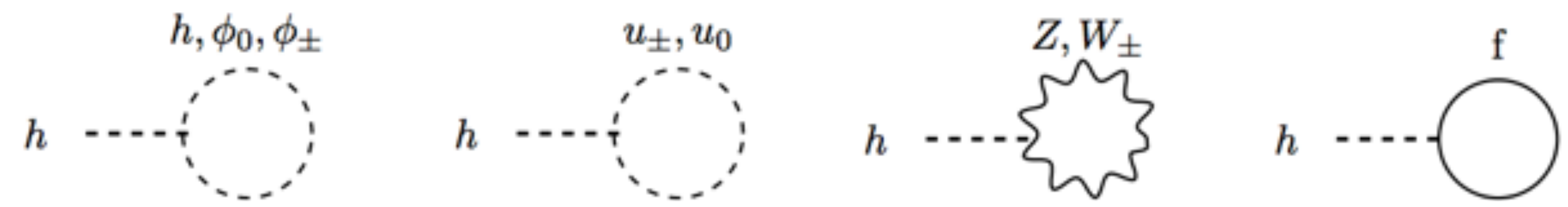
$$\left.+\frac{g_2^2}{2}\frac{m_W^2}{m_h^2}\left(1+3\log\left[\frac{\mu^2}{m_W^2}\right]\right) + \frac{1}{4}(g_1^2+g_2^2)\frac{m_Z^2}{m_h^2}\left(1+3\log\left[\frac{\mu^2}{m_Z^2}\right]\right)\right].$$

- The finite terms that are fixed by renormalization conditions (at one loop) in the theory enter as

$$\langle h(p_h)|S|\gamma(p_a,\alpha),\gamma(p_b,\beta)\rangle_{BSM} = (1+\frac{\delta R_h}{2})\,(1+\delta R_A)\,(1+\delta R_e)^2\, i \sum_{x=a..o} \mathcal{A}_x.$$

NLO EFT - Step 5b fix finite terms

- Define vev of the theory as the one point function vanishing - fixes δv



$$T = m_h^2\, h\, v\, \frac{1}{16\pi^2}\left[-16\pi^2 \frac{\delta v}{v} + 3\,\lambda\left(1 + \log\left[\frac{\mu^2}{m_h^2}\right]\right) + \frac{m_W^2}{v^2}\xi\left(1 + \log\left[\frac{\mu^2}{\xi\, m_W^2}\right]\right)\right., \quad (3.3)$$

$$+\frac{1}{2}\frac{m_Z^2}{v^2}\xi\left(1 + \log\left[\frac{\mu^2}{\xi\, m_Z^2}\right]\right) - \frac{1}{2}\sum_i y_i^4\, N_c \frac{1}{\lambda}\left(1 + \log\left[\frac{\mu^2}{m_i^2}\right]\right),$$

$$\left.+\frac{g_2^2}{2}\frac{m_W^2}{m_h^2}\left(1 + 3\log\left[\frac{\mu^2}{m_W^2}\right]\right) + \frac{1}{4}(g_1^2 + g_2^2)\frac{m_Z^2}{m_h^2}\left(1 + 3\log\left[\frac{\mu^2}{m_Z^2}\right]\right)\right].$$

- The finite terms that are fixed by renormalization conditions (at one loop) in the theory enter as

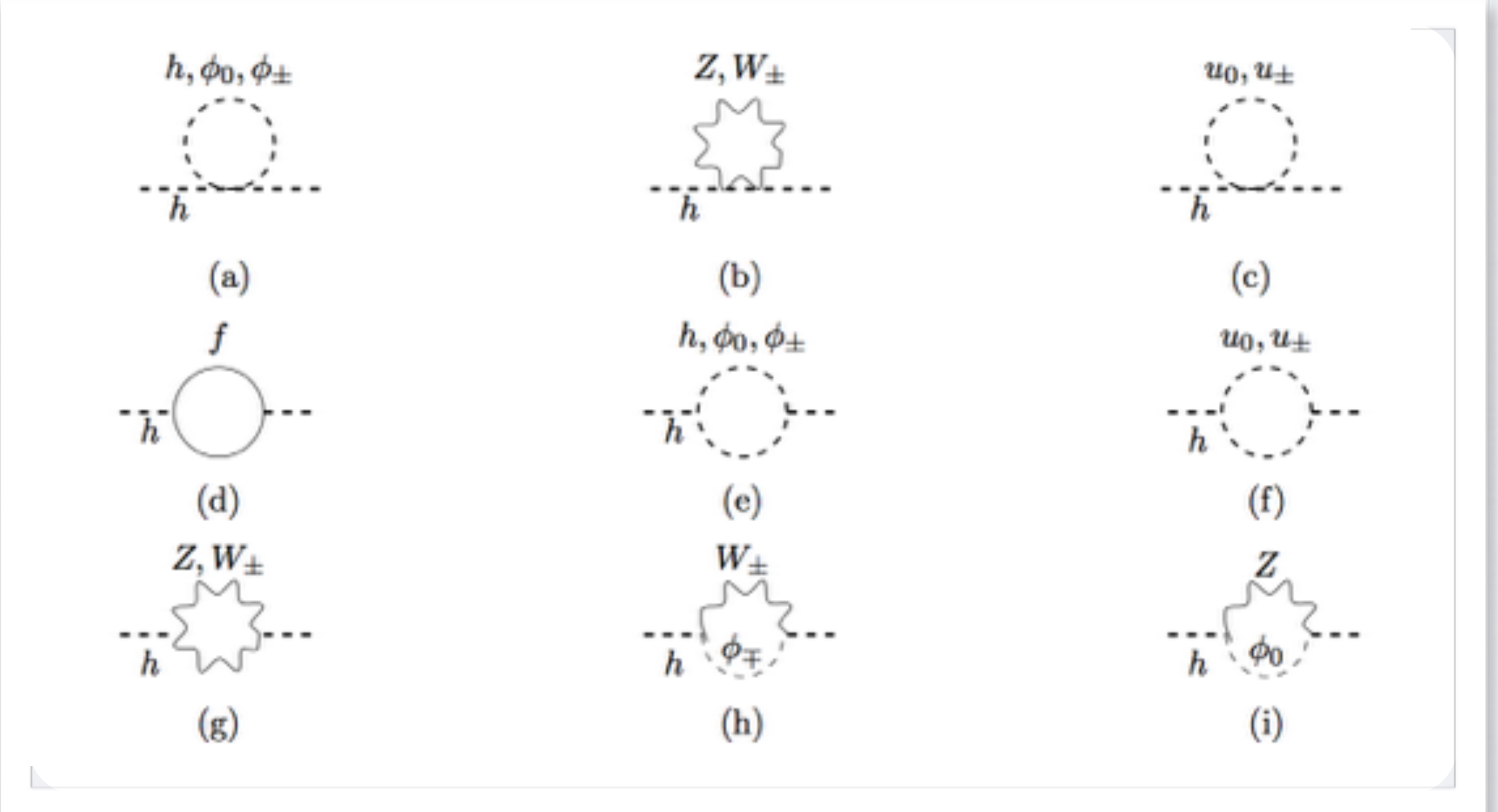
$$\langle h(p_h)|S|\gamma(p_a,\alpha),\gamma(p_b,\beta)\rangle_{BSM} = (1 + \frac{\delta R_h}{2})\,(1 + \delta R_A)\,(1 + \delta R_e)^2\, i \sum_{x=a..o} \mathcal{A}_x.$$

NLO EFT - Step 5b fix finite terms

- Remaining finite terms fixed by defining in renormalization conditions on the couplings and two point function residues and poles

$$\delta R_h = -\frac{\partial \Pi_{hh}(p^2)}{\partial p^2}\Big|_{p^2=m_h^2} \qquad \delta R_e = -\frac{1}{2}\delta R_A,$$

So also calc:



This result is pretty well known, but where is it ?! for finite terms in R_ξ gauge in BF method
We will supply it upon request for general xi.

NLO EFT - Final result

- The final result is of the form

1505.02646 Hartmann, Trott

$$\frac{i\,\mathcal{A}^{NP}_{total}}{i\,v\,e^2\,A^{h\gamma\gamma}_{\alpha\beta}} = C_{\gamma\gamma}\left(1+\frac{\delta R_h}{2}+\frac{\delta\,v}{v}\right),$$
$$+\left(\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\frac{g_1^2}{4}+\frac{3\,g_2^2}{4}+6\,\lambda\right)+\frac{C_{HWB}}{16\,\pi^2}\left(-3\,g_2^2+4\,\lambda\right)\right)\log\left(\frac{m_h^2}{\Lambda^2}\right),$$
$$+\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\left(\frac{g_1^2}{4}+\frac{g_2^2}{4}+\lambda\right)\mathcal{I}[m_Z^2]+\left(\frac{g_2^2}{2}+2\lambda\right)\mathcal{I}[m_W^2]+(\sqrt{3}\,\pi-6)\,\lambda\right),$$
$$+\frac{C_{HWB}}{16\,\pi^2}\left(2e^2\left(1+6\frac{m_W^2}{m_h^2}\right)-2\,g_2^2\left(1+\log\left(\frac{m_W^2}{m_h^2}\right)\right)+(4\,\lambda-g_2^2)\,\mathcal{I}[m_W^2],\right.$$
$$\left.+4\left(3e^2-g_2^2-6e^2\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right),$$
$$-\frac{g_2^2\,C_{HW}}{4\,\pi^2}\left(3\,\frac{m_W^2}{m_h^2}+\left(4-\frac{m_h^2}{m_W^2}-6\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right). \tag{3.6}$$

Where $C^{NP}_{\gamma\gamma} = C_{HB}+C_{HW}-C_{HWB}$

$$\mathcal{I}[m^2] \equiv \int_0^1 dx\,\log\left(\frac{m^2-m_h^2\,x\,(1-x)}{m_h^2}\right),\quad \mathcal{I}_y[m^2] \equiv \int_0^{1-x} dy\int_0^1 dx\,\frac{m^2}{m^2-m_h^2\,x\,(1-x-y)},$$

NLO EFT - Final result

- The final result is of the form

$$\frac{i\,\mathcal{A}^{NP}_{total}}{i\,v\,e^2\,A^{h\gamma\gamma}_{\alpha\beta}} = C_{\gamma\gamma}\left(1+\frac{\delta R_h}{2}+\frac{\delta v}{v}\right),$$
$$+\left(\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\frac{g_1^2}{4}+\frac{3\,g_2^2}{4}+6\,\lambda\right)+\frac{C_{HWB}}{16\,\pi^2}\left(-3\,g_2^2+4\,\lambda\right)\right)\log\left(\frac{m_h^2}{\Lambda^2}\right),$$
$$+\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\left(\frac{g_1^2}{4}+\frac{g_2^2}{4}+\lambda\right)\mathcal{I}[m_Z^2]+\left(\frac{g_2^2}{2}+2\lambda\right)\mathcal{I}[m_W^2]+(\sqrt{3}\,\pi-6)\,\lambda\right),$$
$$+\frac{C_{HWB}}{16\,\pi^2}\left(2e^2\left(1+6\frac{m_W^2}{m_h^2}\right)-2\,g_2^2\left(1+\log\left(\frac{m_W^2}{m_h^2}\right)\right)+(4\,\lambda-g_2^2)\,\mathcal{I}[m_W^2],\right.$$
$$\left.+4\left(3e^2-g_2^2-6e^2\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right),$$
$$-\frac{g_2^2\,C_{HW}}{4\,\pi^2}\left(3\,\frac{m_W^2}{m_h^2}+\left(4-\frac{m_h^2}{m_W^2}-6\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right). \qquad (3.6)$$

Fixed by renorm conditions

Where $C^{NP}_{\gamma\gamma} = C_{HB} + C_{HW} - C_{HWB}$

$$\mathcal{I}[m^2] \equiv \int_0^1 dx\,\log\left(\frac{m^2-m_h^2\,x\,(1-x)}{m_h^2}\right), \quad \mathcal{I}_y[m^2] \equiv \int_0^{1-x} dy \int_0^1 dx\,\frac{m^2}{m^2-m_h^2\,x\,(1-x-y)},$$

NLO EFT - Final result

- The final result is of the form

$$
\begin{aligned}
\frac{i\,\mathcal{A}^{NP}_{total}}{i\,v\,e^2\,A^{h\gamma\gamma}_{\alpha\beta}} = & \; C_{\gamma\gamma}\left(1+\frac{\delta R_h}{2}+\frac{\delta v}{v}\right), \\
& + \left(\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\frac{g_1^2}{4}+\frac{3\,g_2^2}{4}+6\,\lambda\right)+\frac{C_{HWB}}{16\,\pi^2}\left(-3\,g_2^2+4\,\lambda\right)\right)\log\left(\frac{m_h^2}{\Lambda^2}\right), \\
& + \frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\left(\frac{g_1^2}{4}+\frac{g_2^2}{4}+\lambda\right)\mathcal{I}[m_Z^2]+\left(\frac{g_2^2}{2}+2\lambda\right)\mathcal{I}[m_W^2]+(\sqrt{3}\,\pi-6)\,\lambda\right), \\
& + \frac{C_{HWB}}{16\,\pi^2}\left(2e^2\left(1+6\frac{m_W^2}{m_h^2}\right)-2\,g_2^2\left(1+\log\left(\frac{m_W^2}{m_h^2}\right)\right)+(4\,\lambda-g_2^2)\,\mathcal{I}[m_W^2],\right. \\
& \qquad \left. +4\left(3e^2-g_2^2-6e^2\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right), \\
& - \frac{g_2^2\,C_{HW}}{4\,\pi^2}\left(3\,\frac{m_W^2}{m_h^2}+\left(4-\frac{m_h^2}{m_W^2}-6\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right). \qquad (3.6)
\end{aligned}
$$

“(not so) Large” log terms consistent with RGE

Where $C^{NP}_{\gamma\gamma} = C_{HB} + C_{HW} - C_{HWB}$

$$
\mathcal{I}[m^2] \equiv \int_0^1 dx\,\log\left(\frac{m^2-m_h^2\,x\,(1-x)}{m_h^2}\right), \quad \mathcal{I}_y[m^2] \equiv \int_0^{1-x} dy \int_0^1 dx\,\frac{m^2}{m^2-m_h^2\,x\,(1-x-y)},
$$

NLO EFT - Final result

- The final result is of the form

$$
\begin{aligned}
\frac{i\,\mathcal{A}^{NP}_{total}}{i\,v\,e^2\,A^{h\gamma\gamma}_{\alpha\beta}} &= C_{\gamma\gamma}\left(1+\frac{\delta R_h}{2}+\frac{\delta v}{v}\right),\\
&+\left(\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\frac{g_1^2}{4}+\frac{3\,g_2^2}{4}+6\,\lambda\right)+\frac{C_{HWB}}{16\,\pi^2}\left(-3\,g_2^2+4\,\lambda\right)\right)\log\left(\frac{m_h^2}{\Lambda^2}\right),\\
&+\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\left(\frac{g_1^2}{4}+\frac{g_2^2}{4}+\lambda\right)\mathcal{I}[m_Z^2]+\left(\frac{g_2^2}{2}+2\lambda\right)\mathcal{I}[m_W^2]+(\sqrt{3}\,\pi-6)\,\lambda\right),\\
&+\frac{C_{HWB}}{16\,\pi^2}\left(2e^2\left(1+6\frac{m_W^2}{m_h^2}\right)-2\,g_2^2\left(1+\log\left(\frac{m_W^2}{m_h^2}\right)\right)+(4\,\lambda-g_2^2)\,\mathcal{I}[m_W^2],\right.\\
&\qquad\left.+4\left(3e^2-g_2^2-6e^2\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right),\\
&-\frac{g_2^2\,C_{HW}}{4\,\pi^2}\left(3\,\frac{m_W^2}{m_h^2}+\left(4-\frac{m_h^2}{m_W^2}-6\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right). \qquad (3.6)
\end{aligned}
$$

Finite terms with associated logs terms

Where $C^{NP}_{\gamma\gamma} = C_{HB}+C_{HW}-C_{HWB}$

$$
\mathcal{I}[m^2]\equiv\int_0^1 dx\,\log\left(\frac{m^2-m_h^2\,x\,(1-x)}{m_h^2}\right),\quad \mathcal{I}_y[m^2]\equiv\int_0^{1-x}dy\int_0^1 dx\,\frac{m^2}{m^2-m_h^2\,x\,(1-x-y)},
$$

NLO EFT - Final result

- The final result is of the form

$$
\begin{aligned}
\frac{i\,\mathcal{A}^{NP}_{total}}{i\,v\,e^2\,A^{h\gamma\gamma}_{\alpha\beta}} &= C_{\gamma\gamma}\left(1+\frac{\delta R_h}{2}+\frac{\delta v}{v}\right),\\
&+\left(\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\frac{g_1^2}{4}+\frac{3\,g_2^2}{4}+6\,\lambda\right)+\frac{C_{HWB}}{16\,\pi^2}\left(-3\,g_2^2+4\,\lambda\right)\right)\log\left(\frac{m_h^2}{\Lambda^2}\right),\\
&+\frac{C_{\gamma\gamma}}{16\,\pi^2}\left(\left(\frac{g_1^2}{4}+\frac{g_2^2}{4}+\lambda\right)\mathcal{I}[m_Z^2]+\left(\frac{g_2^2}{2}+2\lambda\right)\mathcal{I}[m_W^2]+(\sqrt{3}\,\pi-6)\,\lambda\right),\\
&+\frac{C_{HWB}}{16\,\pi^2}\left(2e^2\left(1+6\frac{m_W^2}{m_h^2}\right)-2\,g_2^2\left(1+\log\left(\frac{m_W^2}{m_h^2}\right)\right)+(4\,\lambda-g_2^2)\,\mathcal{I}[m_W^2],\right.\\
&\qquad\left.+4\left(3e^2-g_2^2-6e^2\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right),\\
&-\frac{g_2^2\,C_{HW}}{4\,\pi^2}\left(3\,\frac{m_W^2}{m_h^2}+\left(4-\frac{m_h^2}{m_W^2}-6\,\frac{m_W^2}{m_h^2}\right)\mathcal{I}_y[m_W^2]\right). \qquad (3.6)
\end{aligned}
$$

"Pure" finite terms not in $C_{\gamma\,\gamma}$ and no associated log

Where $C^{NP}_{\gamma\gamma} = C_{HB} + C_{HW} - C_{HWB}$

$$
\mathcal{I}[m^2] \equiv \int_0^1 dx\,\log\left(\frac{m^2-m_h^2\,x\,(1-x)}{m_h^2}\right),\quad \mathcal{I}_y[m^2] \equiv \int_0^{1-x} dy \int_0^1 dx\,\frac{m^2}{m^2-m_h^2\,x\,(1-x-y)},
$$

NLO EFT - Physics lessons

- Operators can contribute a "pure finite term" at NLO and not have a corresponding RGE log. This fact consistent with results in 1505.03706 Ghezzi et al.

- Finite terms are not small in general compared to the log terms

$$R_{C_{HWB}/C_{HW}} \simeq \frac{C_{HWB}}{C_{HW}} \left(0.5 + 0.7 \log \frac{m_h^2}{\Lambda^2}\right) \qquad R_{C_{HWB}} \simeq 1 + 0.7 \log^{-1} \frac{m_h^2}{\Lambda^2}$$

- Log mu dependence of RGE consistent with full one loop result, but important modification due to mass scales running (vev not 0)

- <u>The RGE is not a good proxy for the full one loop structure of the SMEFT.</u>

(0's in the rge do not mean 0's guaranteed at one loop for finite terms)

"Cool stuff" Addendum

- Gauge fixing in the SMEFT subtle compared to the SM. Consider:

$$\mathcal{L}_{GF} = -\frac{1}{2\,\xi_W}\sum_a \left[\partial_\mu W^{a,\mu} - g_2\,\epsilon^{abc}\hat{W}_{b,\mu}W_c^\mu + i\,g_2\,\frac{\xi}{2}\left(\hat{H}_i^\dagger \sigma_{ij}^a H_j - H_i^\dagger \sigma_{ij}^a \hat{H}_j\right)\right]^2,$$
$$-\frac{1}{2\,\xi_B}\left[\partial_\mu B^\mu + i\,g_1\,\frac{\xi}{2}\left(\hat{H}_i^\dagger H_i - H_i^\dagger \hat{H}_i\right)\right]^2.$$

$$\mathcal{L}_{FP} = -\bar{u}^\alpha\,\frac{\delta G^\alpha}{\delta\theta^\beta}\,u^\beta.$$

Some operators in $\mathcal{L}_6$ then source ghosts!

- The mismatch of the mass eigenstates in the SMEFT with the SM means gauge fixing in the former results in some interesting local contact operators

$$-\frac{c_w\,s_w}{\xi_B\,\xi_W}(\xi_B - \xi_W)\,(\partial^\mu A_\mu\,\partial^\nu Z_\nu) - \frac{C_{HWB}v^2(s_w^2 - c_w^2)(s_w^2\xi_B + c_w^2\xi_W)}{\xi_B\,\xi_W}\,(\partial^\mu A_\mu\,\partial^\nu Z_\nu)\,.$$