LHCHXSWG-INT-2015-001
June 14, 2015

LHC Higgs Cross Section Working Group 2 (Higgs Properties)

Higgs Basis: Proposal for an EFT basis choice for LHC HXSWG

We acknowledge contributions and feedback from: Hermès Belusca-Maïto, André David, Michael Dührssen, Adam Falkowski, Christophe Grojean, Florian Goertz, Gino Isidori, Ken Mimasu, Kin Mimouni, Marc Riembau, Tilman Plehn, Alex Pomarol, Francesco Riva, Veronica Sanz.

This document is currently a work in progress. Appearing in acknowledgements does not imply full or partial endorsement. Further contributions, comments, and criticism are welcome. Please send your remarks to lhc-higgs-properties-convener@cern.ch.

1 Introduction

The LHC Higgs Cross Section Working Group is focused on various steps of the analysis chain:

$$\textbf{Data} \rightarrow \textbf{Pseudo-observables} \rightarrow \textbf{Model-independent EFT} \rightarrow \textbf{BSM Models}\,.$$

This note concerns model-independent interpretations of the data in the framework of effective field theory (EFT) beyond the Standard Model (SM), which is a part of the scope of the Working Group 2. The purpose of this note is to propose a common EFT language and conventions that could be universally used in LHC Higgs analyses and be implemented in numerical tools.

In the EFT approach, the basic assumption is that the mass scale Λ of new particles in the UV theory beyond the SM is larger than the electroweak scale v, $\Lambda \gg v$. If this is the case, physics at energies $E \ll \Lambda$ can be parametrized by the SM Lagrangian supplemented by a set of higher-dimensional operators. These operators are constructed out of the SM fields, and respect the local $SU(3) \times SU(2) \times U(1)$ symmetry of the SM. The coefficients of $d > 4$-dimensional operators in the EFT Lagrangian are of order $1/\Lambda^{d-4}$, and their contribution to amplitudes of physical processes at the energy scale of order v scales[1] as $(v/\Lambda)^{d-4}$. The leading new physics effects are expected from operators with $d = 6$ whose effects scale as $(v/\Lambda)^2$ (all dimension-5 operators violate the lepton number; experimental constraints dictate that their coefficients must be suppressed at the level unobservable at the LHC). Since $(v/\Lambda)^2 < 1$ by construction, EFT is suitable to describe *small* deviations from the SM predictions, except for observables that vanish or are suppressed by small parameters in the SM.

[1] Apart from the scaling with Λ, the effects of higher-dimensional operators also scale with appropriate powers of couplings in the UV theory. The latter may be important to assess the validity range of the EFT description.

An *operator basis* is a complete, non-redundant set of dimension-6 operators. Complete means that any dimension-6 operator is either a part of the basis, or can be obtained from a combination of operators in the basis using equations of motion, integration by parts, field redefinitions, and Fierz transformations. Non-redundant means it is a minimal such set. Any basis leads to the same physical predictions concerning possible new physics effects. Several bases have been proposed in the literature, and they may be convenient for specific applications. In this note we propose a basis that is particularly convenient for LHC Higgs analyses.

Preparing this proposal, we have taken into account the following guidelines:

- The formulation should be simple enough that it can be used by people not acquainted with the nuts and bolts of EFTs.
- The relationship between parameters of the EFT and (pseudo)-observables should be transparent.
- The constraints on EFT parameters from electroweak precision observables should be easy to impose.
- The formalism should be easily implementable in Monte-Carlo codes.
- The formalism should be flexible enough, such that, in the future, the application scope may be extended beyond the original one. In particular, the formalism should be applicable outside Higgs physics and allow one to also combine non-LHC data.
- A connection to the pseudo-observables in the *extended kappa formalism* should be straightforward.
- Limits of the EFT validity range should be easy to define.
- The formalism should be well suited to include higher-order QCD and electroweak corrections.

The salient features of our proposal are the following:

- We restrict ourselves to EFT with dimension-6 operators in the *linear* formulation of electroweak symmetry breaking (in other words, the Higgs boson belongs to a doublet of the weak $SU(2)$ group).
- In the spirit of Ref. [1], we proceed with a classification of the operators that more easily map to independent interaction terms of the SM mass eigenstates, in particular the W, Z, and the Higgs boson. Such interaction terms are invariant under $SU(3) \times U(1)$ color and electromagnetic symmetry, but they do not necessarily correspond to $SU(2)$-invariant operators. However, they allow us to identify a set of *independent couplings* from which a complete basis of $SU(2)$-invariant terms is constructed. We denote the latter the *Higgs basis*. The advantage of this formulation is that the effective couplings are related in a simpler way to quantities observable in experiments, compared to other proposals.

- We choose the independent couplings such that the constraints from the Z and W partial decay widths (measured with a per-mille precision by the LEP experiment) can be easily incorporated. These are among the most stringent constraints on EFT parameters, and they have an important impact on possible signals in Higgs searches. It is unlikely that, at any point in the future, the precision of LHC Higgs searches will be such that the couplings constrained by LEP can be probed by the LHC with a comparable accuracy. Therefore it is recommended that the the electroweak constraints on Z and W boson couplings to fermions are always imposed when analyzing LHC data, especially on Higgs physics. Other precision observables, such as WW production or off-shell fermion scattering, lead to less stringent constraints that are not discussed in this note (see e.g. [2, 3, 4] for a recent discussion).

- The disadvantage of the Higgs basis is that the operator list is cumbersome, being defined by the identification of a set of independent interaction terms after electroweak symmetry breaking. For this reason, we also map the Higgs basis to a set of manifestly $SU(3) \times SU(2) \times U(1)$ invariant operators before electroweak symmetry breaking. For the latter, in this note we use operators in the *Warsaw basis* of Ref. [5], but it is straightforward to work out a map to any other basis used in the literature. Working with $SU(3) \times SU(2) \times U(1)$ invariant operators may be more convenient for certain calculations (for example, when renormalization group running of the Wilson coefficients needs to be calculated).

- We do not demand that the dimension-6 operators are flavor blind. While generic constraints on flavor violation are strong, it is plausible that there is a large hierarchy between the coefficients of dimension-6 operators corresponding to different fermion generations. In particular, many models predict the coefficients of operators involving the 3rd generation to be much larger than those involving the first two generations. Keeping the more general approach will allow us to obtain much more robust constraints on new physics.

- We allow CP violating operators to be present in our basis. In particular, we discuss the most general set of Higgs couplings to matter that include CP violating couplings.

- We assume that dimension-6 operators conserve the baryon and lepton number.

In Section 2, to define our notation and conventions, we write down the Standard Model (SM) Lagrangian. In Section 3 we define the Higgs basis, which is the basis we propose for LHC Higgs analyses. The dictionary between the independent couplings and Wilson coefficients of $SU(3) \times SU(2) \times U(1)$ invariant dimension-6 operators in the Warsaw basis is worked out in Section 4.

2 Standard Model Lagrangian

The SM Lagrangian in our notation takes the form

$$
\begin{aligned}
\mathcal{L}^{\rm SM} &= -\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^i_{\mu\nu} - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} + D_\mu H^\dagger D_\mu H + \mu_H^2 H^\dagger H - \lambda (H^\dagger H)^2 \\
&+ \sum_{f\in q,\ell} i\bar{f}_L\gamma_\mu D_\mu f_L + \sum_{f\in u,d,e} i\bar{f}_R\gamma_\mu D_\mu f_R \\
&- \left[\tilde{H}^\dagger \bar{u}_R y_u q_L + H^\dagger \bar{d}_R y_d V_{\rm CKM}^\dagger q_L + H^\dagger \bar{e}_R y_e \ell_L + {\rm h.c.}\right]. \qquad (2.1)
\end{aligned}
$$

Here, G^a_μ, W^i_μ, and B_μ denote the gauge fields of the $SU(3)\times SU(2)\times U(1)$ local symmetry. The corresponding gauge couplings are denoted by g_s, g, g'; we also define the electromagnetic coupling $e = gg'/\sqrt{g^2+g'^2}$, and the Weinberg angle $s_\theta = g'/\sqrt{g^2+g'^2}$. The field strength tensors are defined as $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$, $W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon^{ijk} W^j_\mu W^k_\nu$, $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. The Higgs doublet is denoted as H, and we also define $\tilde{H}_i = \epsilon_{ij} H^*_k$. It acquires the vacuum expectation value (VEV) $\langle H^\dagger H\rangle = v^2/2$. In the unitary gauge, $H = (0, (v+h)/\sqrt{2})$, where h is the Higgs boson field. After electroweak symmetry breaking, the electroweak gauge boson mass eigenstates are defined as $W^\pm = (W^1 \mp iW^2)/\sqrt{2}$, $Z = c_\theta W^3 - s_\theta B$, $A = s_\theta W^3 + c_\theta B$, where $c_\theta = \sqrt{1-s_\theta^2}$. The tree-level masses of W and Z bosons are given by $m_W = gv/2$, $m_Z = \sqrt{g^2+g'^2}v/2$. The left-handed Dirac fermions $q_L = (u_L, V_{\rm CKM} d_L)$ and $\ell_L = (\nu_L, e_L)$ are doublets of the SU(2) gauge group, and the right-handed Dirac fermions u_R, d_R, e_R are SU(2) singlets. All fermions are 3-component vectors in the generation space, and y_f are 3×3 matrices. We work in the basis where the fermion mass matrix is diagonal with real, positive entries. In this basis, y_f are diagonal, and the fermion masses are given by $m_{f_i} = v[y_f]_{ii}/\sqrt{2}$.

For later convenience, we explicitly write down the gauge boson mass terms:

$$
\mathcal{L}^{\rm SM}_{\rm mass} = \frac{g^2v^2}{4} W^+_\mu W^-_\mu + \frac{(g^2+g'^2)v^2}{8} Z_\mu Z_\mu, \qquad (2.2)
$$

the gauge boson couplings to fermions:

$$
\mathcal{L}^{\rm SM}_{aff} = eA_\mu \sum_{f\in u,d,e} Q_f \bar{f}\gamma_\mu f + g_s G^a_\mu \sum_{f\in u,d} \bar{f}\gamma_\mu T^a f, \qquad (2.3)
$$

$$
\begin{aligned}
\mathcal{L}^{\rm SM}_{vff} &= \frac{g}{\sqrt{2}}\left(W^+_\mu \bar{u}_L\gamma_\mu V_{\rm CKM} d_L + W^+_\mu \bar{\nu}_L \gamma_\mu e_L + {\rm h.c.}\right) \\
&+ \sqrt{g^2+g'^2} Z_\mu \sum_{f\in u,d,e,\nu} \left(T^3_f \bar{f}_L\gamma_\mu f_L - s_\theta^2 Q_f \bar{f}\gamma_\mu f\right), \qquad (2.4)
\end{aligned}
$$

the couplings of the Higgs boson to gauge bosons, fermions, and itself:

$$
\mathcal{L}^{\rm SM}_h = \left(\frac{h}{v} + \frac{h^2}{2v^2}\right)\left[\frac{g^2v^2}{2} W^+_\mu W^-_\mu + \frac{(g^2+g'^2)v^2}{4} Z_\mu Z_\mu\right] - \frac{h}{v}\sum_f m_f \bar{f} f - \frac{m_h^2}{2v}h^3 - \frac{m_h^2}{8v^2}h^4, \qquad (2.5)
$$

and the triple and quartic self-interactions of the vector bosons:

$$\begin{aligned}\mathcal{L}_{\rm tgc}^{\rm SM} &= ie\left[\left(W^+_{\mu\nu}W^-_\mu - W^-_{\mu\nu}W^+_\mu\right)A_\nu + A_{\mu\nu}\,W^+_\mu W^-_\nu\right] \\ &+ igc_\theta\left[\left(W^+_{\mu\nu}W^-_\mu - W^-_{\mu\nu}W^+_\mu\right)Z_\nu + Z_{\mu\nu}\,W^+_\mu W^-_\nu\right] \\ &- g_s f^{abc}\partial_\mu G^a_\nu G^b_\mu G^c_\nu. \end{aligned} \tag{2.6}$$

$$\begin{aligned}\mathcal{L}_{\rm qgc}^{\rm SM} &= \frac{g^2}{2}\left(W^+_\mu W^+_\mu W^-_\nu W^-_\nu - W^+_\mu W^-_\mu W^+_\nu W^-_\nu\right) + g^2 c_\theta^2\left(W^+_\mu Z_\mu W^-_\nu Z_\nu - W^+_\mu W^-_\mu Z_\nu Z_\nu\right) \\ &+ g^2 s_\theta^2\left(W^+_\mu A_\mu W^-_\nu A_\nu - W^+_\mu W^-_\mu A_\nu A_\nu\right) \\ &+ g^2 c_\theta s_\theta\left(W^+_\mu Z_\mu W^-_\nu A_\nu + W^+_\mu A_\mu W^-_\nu Z_\nu - 2W^+_\mu W^-_\mu Z_\nu A_\nu\right) \\ &- g_s^2 f^{abc}f^{ade}G^b_\mu G^c_\nu G^d_\mu G^e_\mu. \end{aligned} \tag{2.7}$$

These couplings depend on just 5 input parameters: g_s, g, g', m_h and v. The Higgs boson mass m_h has been precisely measured at the LHC, while the strong coupling constant is extracted from jet production data. The remaining 3 parameters are customarily derived from the observable Fermi constant G_F (more precisely, from the measured muon lifetime $\tau_\mu = 192\pi^3/G_F^2 m_\mu^5$), Z boson mass m_Z, and the low-energy electromagnetic coupling $\alpha(0)$. The tree-level relations between the input observables and the electroweak parameters are given by:

$$G_F = \frac{1}{\sqrt{2}v^2}, \qquad \alpha = \frac{g_L^2 g_Y^2}{4\pi(g_L^2 + g_Y^2)}, \qquad m_Z = \frac{\sqrt{g_L^2 + g_Y^2}\,v}{2}. \tag{2.8}$$

3 Higgs Basis

We present the effective dimension-6 Lagrangian in the linear realization of electroweak symmetry in a formalism inspired by (but not identical to) Ref. [1]. The goal is to choose a particular basis of operators that can be more easily connected (at least at tree-level) to observable quantities in Higgs physics. The basis, which we call the *Higgs basis*, is spanned by particular combinations of dimension-6 operators. Each of these combinations maps to a simple interaction term of the SM mass-eigenstate fields that can be probed by experiment. The coefficients multiplying these combinations in the Lagrangian are called the *independent couplings*. In order to make the Higgs basis convenient to study Higgs physics, the couplings of W and Z bosons to fermions and single Higgs couplings to the SM fermions and gauge bosons are chosen among the independent couplings.

We stress that the Higgs basis should be regarded as one of many possible bases of the dimension-6 Lagrangian beyond the SM. In particular, the independent couplings can be related by a linear transformation to parameters defining any other such basis in the literature, for example the Warsaw [5] or the SILH [6] basis. At the same time, the independent couplings can be easily connected to Higgs *pseudo-observables* at the amplitude level, as defined e.g. in Ref. [7].

By construction, our effective Lagrangian has the following features:

- All kinetic terms of SM mass eigenstates are canonically normalized. In particular, there is no kinetic mixing between the Z boson and the photon.

- Tree-level relations between the electroweak parameters and input observables are the same as the SM ones in Eq. (2.8). In particular, the photon and the gluon interact with fermions as in Eq. (2.3), and there is no correction to the Z boson mass term.

- Two-derivative self-interactions of the Higgs boson are absent.

In general, dimension-6 operators do induce corrections to the Lagrangian that do not respect these features. However, all 3 above features can always be achieved, *without any loss of generality*, by using equations of motion, integrating by parts, and redefining the fields and couplings.

In the complete effective Lagrangian each independent coupling multiplies an independent combination of $SU(3) \times SU(2) \times U(1)$ invariant operators (such combinations formally define the operator basis). However, we find it more transparent to define the independent couplings via the interaction terms of SM mass eigenstates in the Lagrangian after electroweak symmetry breaking: see the Section 4 for the expressions of the independent couplings in terms of Wilson coefficients of $SU(3) \times SU(2) \times U(1)$ invariant operators.

Several other Higgs couplings can be expressed by the independent couplings; we call them the *dependent* couplings. The relations between dependent and independent couplings displayed below *hold at the level of the dimension-6 Lagrangian*, and they are in general not respected by dimension-8 and higher operators. Of course, the choice which couplings are independent and which are dependent is subjective and dictated by convenience. In our case, the independent couplings are more easily mapped to observables constrained by electroweak precision tests and Higgs searches. However, other choices can be envisaged and may be more convenient for other applications.

3.1 Kinetic terms

In the Higgs basis, by construction, dimension-6 operators do not introduce corrections to kinetic of the SM mass eigenstates. The only exception is the (relative) shift of the W boson mass, which is an independent parameter in our formalism:

$$\textbf{Independent}: \quad \delta m. \tag{3.1}$$

It is defined as a correction to the SM W boson mass in the Lagrangian of Eq. (2.2):

$$\mathcal{L}_{\rm kinetic}^{D=6} = 2\delta m \frac{g^2 v^2}{4} W_\mu^+ W_\mu^- . \tag{3.2}$$

While δm is a free parameter from the EFT point of view, precision measurements of the W mass constrain it to be smaller than 10^{-3}.

3.2 Vertex corrections

We choose the following set of independent and dependent vertex corrections:

$$\begin{array}{rc} \textbf{Independent}: & \delta g_L^{Ze},\ \delta g_R^{Ze},\ \delta g_L^{W\ell},\ \delta g_L^{Zu},\ \delta g_R^{Zu},\ \delta g_L^{Zd},\ \delta g_R^{Zd},\ \delta g_R^{Wq}, \\ \textbf{Dependent}: & \delta g_L^{Z\nu},\ \delta g_L^{Wq}, \end{array} \tag{3.3}$$

where all the δg are 3×3 Hermitian matrices in the generation space, except for δg_R^{Wq} who is a general 3×3 complex matrix. These parameters are defined via corrections of the SM W and Z couplings to fermions in the Lagrangian of Eq. (2.3).

$$\begin{aligned}\mathcal{L}_{\rm vertex}^{D=6} &= \frac{g}{\sqrt{2}}\left(W_\mu^+\bar{\nu}_L\gamma_\mu\delta g_L^{W\ell}e_L + W_\mu^+\bar{u}\gamma_\mu\delta g_L^{Wq}V_{\rm CKM}d_L + W_\mu^+\bar{u}_R\gamma_\mu\delta g_R^{Wq}d_R + \text{h.c.}\right)\\ &+ \sqrt{g^2+g'^2}Z_\mu\left[\sum_{f\in u,d,e,\nu}\bar{f}_L\gamma_\mu\delta g_L^{Zf}f_L + \sum_{f\in u,d,e}\bar{f}_R\gamma_\mu\delta g_R^{Zf}f_R\right] \end{aligned}\tag{3.4}$$

where the dependent couplings $\delta g_L^{Z\nu}$, δg_L^{Wq} can be expressed by the independent couplings as:

$$\delta g_L^{Z\nu} = \delta g_L^{Ze} + \delta g_L^{W\ell}, \qquad \delta g_L^{Wq} = \delta g_L^{Zu} - \delta g_L^{Zd}. \tag{3.5}$$

Note that we choose the W couplings to leptons (rather than the Z couplings to neutrinos) as our independent couplings, because in the flavor non-universal case the former are more directly constrained by experiment (in particular, in leptonic W decays measured at LEP).

The parameters in Eq. (3.3) form a complete set to describe all single on-shell Z and W decay and production processes within an EFT with linear realization of electroweak symmetry. They are free parameters from the effective field theory viewpoint but, as we argue in more detail near the end of this section, they are typically strongly constrained by precision measurements of Z and W production and decays at LEP.

3.3 Dipole moments

At the dimension-6 level the dipole-type interactions are described by the following independent and dependent couplings:

$$\begin{aligned}\textbf{Independent}: &\quad d_{Gu},\ d_{Gd},\ d_{Ae},\ d_{Au},\ d_{Ad},\ d_{Ze},\ d_{Zu},\ d_{Zd},\\ &\quad \tilde{d}_{Gu},\ \tilde{d}_{Gd},\ \tilde{d}_{Ae},\ \tilde{d}_{Au},\ \tilde{d}_{Ad},\ \tilde{d}_{Ze},\ \tilde{d}_{Zu},\ \tilde{d}_{Zd};\\ \textbf{Dependent}: &\quad d_{Wq},\ \tilde{d}_{Wq},\end{aligned}\tag{3.6}$$

where all the d_{Vf} and $\widetilde{d}_{Vf}$ are Hermitian matrices. They are defined by the following interactions between the gauge boson and fermions:

$$\begin{aligned}\mathcal{L}_{\rm dipole}^{D=6} &= -\frac{1}{4v}\left[g_s\sum_{f\in u,d}\bar{f}\sigma_{\mu\nu}T^a d_{Gf}fG^a_{\mu\nu} + e\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}d_{Af}fA_{\mu\nu}\right.\\ &\left.+\sqrt{g^2+g'^2}\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}d_{Zf}fZ_{\mu\nu} + \sqrt{2}g\left(\bar{d}\sigma_{\mu\nu}d_{Wq}uW^-_{\mu\nu} + \text{h.c.}\right)\right]\\ &-\frac{1}{4v}\left[g_s\sum_{f\in u,d}\bar{f}\sigma_{\mu\nu}T^a\tilde{d}_{Gf}f\widetilde{G}^a_{\mu\nu} + e\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}\tilde{d}_{Af}f\widetilde{A}_{\mu\nu}\right.\\ &\left.+\sqrt{g^2+g'^2}\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}\tilde{d}_{Zf}f\widetilde{Z}_{\mu\nu} + \sqrt{2}g\left(\bar{d}\sigma_{\mu\nu}\tilde{d}_{Wq}u\widetilde{W}^-_{\mu\nu} + \text{h.c.}\right)\right],\end{aligned}\tag{3.7}$$

where $\sigma_{\mu\nu} = i[\gamma_\mu,\gamma_\nu]/2$. The dependent coupling are related to the independent ones by

$$c_\theta^2 d_{Wq} = d_{Wu} - d_{Wd}, \quad c_\theta^2\tilde{d}_{Wq} = \tilde{d}_{Wu} - \tilde{d}_{Wd}. \tag{3.8}$$

3.4 Single Higgs couplings to gauge bosons

Interactions of the Higgs bosons with the SM gauge boson are described by the following independent and dependent couplings:

$$\begin{array}{rl} \textbf{Independent}: & c_{gg},\ \delta c_z,\ c_{\gamma\gamma},\ c_{z\gamma},\ c_{zz},\ c_{z\Box},\ \tilde{c}_{gg},\ \tilde{c}_{\gamma\gamma},\ \tilde{c}_{z\gamma},\ \tilde{c}_{zz}; \\ \textbf{Dependent}: & \delta c_w,\ c_{ww},\ \tilde{c}_{ww},\ c_{w\Box},\ c_{\gamma\Box}. \end{array} \tag{3.9}$$

These couplings do not affect the precision W and Z observables at tree-level, therefore they are only weakly constrained. Typically, the strongest limits on the independent couplings in Eq. (3.9) come from Higgs studies at the LHC.

The couplings listed in Eq. (3.9) are defined via the Higgs boson couplings to the SM gauge bosons:

$$\begin{aligned} \Delta\mathcal{L}_{\rm hvv}^{D=6} = \frac{h}{v}\Big[& 2\delta c_w m_W^2 W_\mu^+ W_\mu^- + \delta c_z m_Z^2 Z_\mu Z_\mu \\ &+ c_{ww}\frac{g^2}{2}W_{\mu\nu}^+ W_{\mu\nu}^- + \tilde{c}_{ww}\frac{g^2}{2}W_{\mu\nu}^+ \tilde{W}_{\mu\nu}^- + c_{w\Box} g^2\left(W_\mu^-\partial_\nu W_{\mu\nu}^+ + \text{h.c.}\right) \\ &+ c_{gg}\frac{g_s^2}{4}G_{\mu\nu}^a G_{\mu\nu}^a + c_{\gamma\gamma}\frac{e^2}{4}A_{\mu\nu}A_{\mu\nu} + c_{z\gamma}\frac{eg}{2c_\theta}Z_{\mu\nu}A_{\mu\nu} + c_{zz}\frac{g^2}{4c_\theta^2}Z_{\mu\nu}Z_{\mu\nu} \\ &+ c_{z\Box} g^2 Z_\mu \partial_\nu Z_{\mu\nu} + c_{\gamma\Box} g g' Z_\mu \partial_\nu A_{\mu\nu} \\ &+ \tilde{c}_{gg}\frac{g_s^2}{4}G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a + \tilde{c}_{\gamma\gamma}\frac{e^2}{4}A_{\mu\nu}\tilde{A}_{\mu\nu} + \tilde{c}_{z\gamma}\frac{eg}{2c_\theta}Z_{\mu\nu}\tilde{A}_{\mu\nu} + \tilde{c}_{zz}\frac{g^2}{4c_\theta^2}Z_{\mu\nu}\tilde{Z}_{\mu\nu}\Big]. \end{aligned} \tag{3.10}$$

Here $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$, and $\tilde{X}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}\partial_\rho X_\sigma$. The dependent couplings can be expressed by the independent couplings as[2]

$$\begin{aligned} \delta c_w &= \delta c_z + 4\delta m, \\ c_{ww} &= c_{zz} + 2s_\theta^2 c_{z\gamma} + s_\theta^4 c_{\gamma\gamma}, \\ \tilde{c}_{ww} &= \tilde{c}_{zz} + 2s_\theta^2 \tilde{c}_{z\gamma} + s_\theta^4 \tilde{c}_{\gamma\gamma}, \\ c_{w\Box} &= \frac{1}{g^2 - g'^2}\left[g^2 c_{z\Box} + g'^2 c_{zz} - e^2 s_\theta^2 c_{\gamma\gamma} - (g^2 - g'^2) s_\theta^2 c_{z\gamma}\right], \\ c_{\gamma\Box} &= \frac{1}{g^2 - g'^2}\left[2g^2 c_{z\Box} + (g^2 + g'^2) c_{zz} - e^2 c_{\gamma\gamma} - (g^2 - g'^2) c_{z\gamma}\right]. \end{aligned} \tag{3.11}$$

Note that, using equations of motion, we could get rid of certain 2-derivative interactions between the Higgs and gauge bosons: $hZ_\mu\partial_\nu Z_{\nu\mu}$, $hZ_\mu\partial_\nu A_{\nu\mu}$, and $hW_\mu^\pm\partial_\nu W_{\nu\mu}^\mp$. These interactions would then be traded for additional contact interactions of the Higgs, gauge bosons and fermions Eq. (3.17), which would change the relation between the coefficients of these contact interactions c^{Vf} and independent couplings. We find the current representation more convenient in practice. Namely, in the presence of the box couplings satisfying the relations in Eq. (3.11), one has $c^{Vf} = \delta g^{Vf}$. Since vertex corrections strongly constrained by precision observables, they can be set to zero in LHC analyses. If that is done, *all* the contact interaction terms are consequently also set to zero.

[2]The relation between c_{ww}, $\tilde{c}_{ww}$ and other parameters can also be viewed as a consequence of the accidental custodial symmetry at the level of the dimension-6 operators [8].

3.5 Single Higgs couplings to fermions

The single Higgs couplings to the SM fermions are described by the following set of independent couplings:

$$\textbf{Independent}: \quad \delta y_u,\ \delta y_d,\ \delta y_e,\ \sin\phi_u,\ \sin\phi_d,\ \sin\phi_\ell. \tag{3.12}$$

where δy_f and $\sin\phi_f$ are 3×3 real matrices. They are defined via the corrections Higgs boson couplings to the SM fermions:

$$\mathcal{L}^{D=6}_{\rm hff} = -\frac{h}{v}\sum_{f\in u,d,e}\sum_{ij}\sqrt{m_{f_i}m_{f_j}}[\delta y_f]_{ij}\left[Os\phi^f_{ij}\bar{f}_i f_j - i\sin\phi^f_{ij}\bar{f}_i\gamma_5 f_j\right]. \tag{3.13}$$

As in the case of the Higgs boson couplings to gauge boson, these couplings do not affect the precision W and Z observables at tree-level. Limits on some of the independent couplings in Eq. (3.12) come from Higgs studies at the LHC.

3.6 Higgs contact interactions with fermions and gauge bosons

At the dimension-6 level there arise contact interactions between the Higgs boson, one gauge boson, and two fermions, which are not present in the SM. To describe these, we need the following set of dependent couplings:

$$\begin{aligned}\textbf{Dependent}: \quad & d_{hGu},\ d_{hGd},\ d_{hAe},\ d_{hAu},\ d_{hAd},\ d_{hZe},\ d_{hZu},\ d_{hZd},\ d_{hWq},\\ & \tilde{d}_{hGu},\ \tilde{d}_{hGd},\ \tilde{d}_{hAe},\ \tilde{d}_{hAu},\ \tilde{d}_{hAd},\ \tilde{d}_{hZe},\ \tilde{d}_{hZu},\ \tilde{d}_{hZd}, \tilde{d}_{hWq},\\ & c^{Ze}_L,\ c^{Ze}_R,\ c^{Z\nu}_L,\ c^{W\ell}_L,\ c^{Zu}_L,\ c^{Zu}_R,\ c^{Zd}_L,\ c^{Zd}_R,\ ,c^{Wq}_L,\ c^{Wq}_R.\end{aligned} \tag{3.14}$$

These coupling are 3×3 Hermitian matrices, except for c^{Wq}_R who is a general 3×3 complex matrix. The couplings in the first two lines are defined by the following dipole-type contact interactions of the Higgs boson:

$$\begin{aligned}\mathcal{L}^{D=6}_{\rm hdvff} = & -\frac{h}{4v^2}\Bigg[g_s\sum_{f\in u,d}\bar{f}\sigma_{\mu\nu}T^a d_{hGf} f G^a_{\mu\nu} + e\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}d_{hAf} f A_{\mu\nu}\\ & + \sqrt{g_L^2+g_Y^2}\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}d_{hZf} f Z_{\mu\nu} + \sqrt{2}g_L\left(\bar{d}\sigma_{\mu\nu}d_{hWq}uW^-_{\mu\nu} + \text{h.c.}\right)\Bigg]\\ & -\frac{h}{4v^2}\Bigg[\sum_{f\in u,d}\bar{f}\sigma_{\mu\nu}T^a\tilde{d}_{hGf} f\widetilde{G}^a_{\mu\nu} + e\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}\tilde{d}_{hAf} f\widetilde{A}_{\mu\nu}\\ & + \sqrt{g_L^2+g_Y^2}\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}\tilde{d}_{hZf} f\widetilde{Z}_{\mu\nu} + \sqrt{2}g_L\left(\bar{d}\sigma_{\mu\nu}\tilde{d}_{hWq}u\widetilde{W}^-_{\mu\nu} + \text{h.c.}\right)\Bigg]\end{aligned} \tag{3.15}$$

The coefficient above are simply related to the independent couplings describing dipole interactions in Eq. (3.6):

$$d_{hVf} = d_{Vf}, \qquad \tilde{d}_{hVf} = \tilde{d}_{Vf}. \tag{3.16}$$

The couplings in the last line of Eq. (3.14) are defined via the vertex-like contact interactions between the Higgs, electroweak gauge bosons, and fermions:

$$\begin{aligned}\mathcal{L}_{hvff}^{D=6} &= \sqrt{2}g\frac{h}{v}W_\mu^+\left(\bar{u}_L\gamma_\mu c_L^{Wq}V_{\rm CKM}d_L+\bar{u}_R\gamma_\mu c_R^{Wq}d_R+\bar{\nu}_L\gamma_\mu c_L^{W\ell}e_L\right)+\text{h.c.}\\ &+ 2\frac{h}{v}\sqrt{g^2+g'^2}Z_\mu\left[\sum_{f=u,d,e,\nu}\bar{f}_L\gamma_\mu c_L^{Zf}f_L+\sum_{f=u,d,e}\bar{f}_R\gamma_\mu c_R^{Zf}f_R\right], \qquad (3.17)\end{aligned}$$

The coefficients of these interactions are simply related to the vertex correction introduced in Eq. (3.3):

$$c^{Zf}=\delta g^{Zf}, \qquad c^{Wf}=\delta g^{Wf}. \qquad (3.18)$$

3.7 Triple and quartic gauge couplings

To describe the triple gauge couplings we need the following independent and dependent couplings:

$$\begin{aligned}\textbf{Independent}: &\quad \lambda_z,\ \tilde{\lambda}_z,\ c_{3G},\ \tilde{c}_{3G};\\ \textbf{Dependent}: &\quad \delta g_{1,z},\ \delta\kappa_\gamma,\ \delta\kappa_z,\ \lambda_\gamma,\ \tilde{\kappa}_\gamma,\ \tilde{\kappa}_z,\ \tilde{\lambda}_\gamma. \qquad (3.19)\end{aligned}$$

These couplings are defined via cubic interactions of gauge bosons, in addition to the SM ones in Eq. (2.6):

$$\begin{aligned}\mathcal{L}_{\rm v^3}^{\rm D=6} &= ie\left[\delta\kappa_\gamma A_{\mu\nu}\,W_\mu^+W_\nu^-+\tilde{\kappa}_\gamma\tilde{A}_{\mu\nu}\,W_\mu^+W_\nu^-\right]\\ &+ igc_\theta\left[\delta g_{1,z}\left(W_{\mu\nu}^+W_\mu^--W_{\mu\nu}^-W_\mu^+\right)Z_\nu+\delta\kappa_z\,Z_{\mu\nu}\,W_\mu^+W_\nu^-+\tilde{\kappa}_z\,\tilde{Z}_{\mu\nu}\,W_\mu^+W_\nu^-\right]\\ &+ i\frac{e}{m_W^2}\left[\lambda_\gamma W_{\mu\nu}^+W_{\nu\rho}^-A_{\rho\mu}+\tilde{\lambda}_\gamma W_{\mu\nu}^+W_{\nu\rho}^-\tilde{A}_{\rho\mu}\right]+i\frac{gc_\theta}{m_W^2}\left[\lambda_z W_{\mu\nu}^+W_{\nu\rho}^-Z_{\rho\mu}+\tilde{\lambda}_z W_{\mu\nu}^+W_{\nu\rho}^-\tilde{Z}_{\rho\mu}\right]\\ &+ \frac{c_{3G}}{v^2}g_s^3f^{abc}G_{\mu\nu}^aG_{\nu\rho}^bG_{\rho\mu}^c+\frac{\tilde{c}_{3G}}{v^2}g_s^3f^{abc}\widetilde{G}_{\mu\nu}^aG_{\nu\rho}^bG_{\rho\mu}^c, \qquad (3.20)\end{aligned}$$

where the dependent couplings can be expressed by the independent couplings as

$$\begin{aligned}\delta g_{1,z} &= \frac{1}{2(g^2-g'^2)}\left[c_{\gamma\gamma}e^2g'^2+c_{z\gamma}(g^2-g'^2)g'^2-c_{zz}(g^2+g'^2)g'^2-c_{z\Box}(g^2+g'^2)g^2\right]\\ \delta\kappa_\gamma &= -\frac{g^2}{2}\left(c_{\gamma\gamma}\frac{e^2}{g^2+g'^2}+c_{z\gamma}\frac{g^2-g'^2}{g^2+g'^2}-c_{zz}\right),\\ \tilde{\kappa}_\gamma &= -\frac{g^2}{2}\left(\tilde{c}_{\gamma\gamma}\frac{e^2}{g^2+g'^2}+\tilde{c}_{z\gamma}\frac{g^2-g'^2}{g^2+g'^2}-\tilde{c}_{zz}\right),\\ \delta\kappa_z &= \delta g_{1,z}-t_\theta^2\delta\kappa_\gamma, \qquad \tilde{\kappa}_z=-t_\theta^2\tilde{\kappa}_\gamma,\\ \lambda_\gamma &= \lambda_z, \quad \tilde{\lambda}_\gamma=\tilde{\lambda}_z. \qquad (3.21)\end{aligned}$$

The couplings of electroweak gauge bosons follow the customary parametrization of Ref. [9]: Other possible cubic gauge interactions do not appear at the dimension-6 level. Similarly, cubic gauge interactions with only neutral electroweak gauge bosons do not appear at the dimension-6 level.

Note that $\delta g_{1,z}$, $\delta\kappa_\gamma$, and $\tilde\kappa_\gamma$ are *dependent* couplings here, unlike in Ref. [1]. Our motivation is that the Higgs basis should be parametrized such that the connection with Higgs observables is the simplest. However, for the sake of studying WW and WZ production a different set of independent couplings would be more convenient. For example, one could choose the independent couplings as $\delta g_{1,z}$, $\delta\kappa_\gamma$, λ_z, $\tilde\kappa_\gamma$, $\tilde\lambda_z$, and consider $c_{z\Box}$, c_{zz}, and $\tilde c_{zz}$ as dependent couplings expressed by this set.

At the level of the $D=6$ Lagrangian, the corrections to the zero-derivative quartic gauge couplings in Eq. (2.6) are fixed by $\delta g_{1,z}$:

$$\begin{aligned}\mathcal{L}_{\mathrm{v}^4}^{\mathrm{D=6}} &= \delta g_{W^4}\frac{g^2}{2}\left(W_\mu^+W_\mu^+W_\nu^-W_\nu^- - W_\mu^+W_\mu^-W_\nu^+W_\nu^-\right)\\ &+ \delta g_{W^2Z^2}g^2c_\theta^2\left(W_\mu^+Z_\mu W_\nu^-Z_\nu - W_\mu^+W_\mu^-Z_\nu Z_\nu\right)\\ &+ \delta g_{W^2Z\gamma}g^2c_\theta s_\theta\left(W_\mu^+Z_\mu W_\nu^-A_\nu + W_\mu^+A_\mu W_\nu^-Z_\nu - 2W_\mu^+W_\mu^-Z_\nu A_\nu\right), \end{aligned}\tag{3.22}$$

$$\delta g_{W^4} = 2c_\theta^2\delta g_{1,z}, \quad \delta g_{W^2Z^2} = 2\delta g_{1,z}, \quad \delta g_{W^2Z\gamma} = \delta g_{1,z}. \tag{3.23}$$

On top of that, two-derivative quartic gauge couplings appear with the coefficient related to λ_z and c_{3G}:

$$\begin{aligned}\mathcal{L}_{\mathrm{d}^2\mathrm{v}^4}^{\mathrm{D=6}} &= -\frac{g^2}{2}\frac{\lambda_z}{m_W^2}\left(W_{\mu\nu}^+W_{\nu\rho}^- - W_{\mu\nu}^-W_{\nu\rho}^+\right)\left(W_\mu^+W_\rho^- - W_\mu^-W_\rho^+\right)\\ &- g^2c_\theta^2\frac{\lambda_z}{m_W^2}\left[W_\mu^+\left(Z_{\mu\nu}W_{\nu\rho}^- - W_{\mu\nu}^-Z_{\nu\rho}\right)Z_\rho + W_\mu^-\left(Z_{\mu\nu}W_{\nu\rho}^+ - W_{\mu\nu}^+Z_{\nu\rho}\right)Z_\rho\right]\\ &- e^2\frac{\lambda_z}{m_W^2}\left[W_\mu^+\left(A_{\mu\nu}W_{\nu\rho}^- - W_{\mu\nu}^-A_{\nu\rho}\right)A_\rho + W_\mu^-\left(A_{\mu\nu}W_{\nu\rho}^+ - W_{\mu\nu}^+A_{\nu\rho}\right)A_\rho\right]\\ &- egc_\theta\frac{\lambda_z}{m_W^2}\left[W_\mu^+\left(A_{\mu\nu}W_{\nu\rho}^- - W_{\mu\nu}^-A_{\nu\rho}\right)Z_\rho + W_\mu^-\left(A_{\mu\nu}W_{\nu\rho}^+ - W_{\mu\nu}^+A_{\nu\rho}\right)Z_\rho\right]\\ &- egc_\theta\frac{\lambda_z}{m_W^2}\left[W_\mu^+\left(Z_{\mu\nu}W_{\nu\rho}^- - W_{\mu\nu}^-Z_{\nu\rho}\right)A_\rho + W_\mu^-\left(Z_{\mu\nu}W_{\nu\rho}^+ - W_{\mu\nu}^+Z_{\nu\rho}\right)A_\rho\right]\\ &+ 3g_s^3\frac{c_{3G}}{v^2}f^{abc}f^{cde}G_{\mu\nu}^aG_{\nu\rho}^bG_\rho^dG_\mu^e + \mathrm{CP\,odd},\end{aligned}\tag{3.24}$$

where CP odd stands for analogous terms with $\lambda_z \to \tilde\lambda_z$, $c_{3G} \to \tilde c_{3G}$, and one of the field strength tensor replaced by the dual one.

3.8 Couplings of two Higgs bosons

To describe double Higgs production process $gg \to hh$ at the LHC we need, apart from the single Higgs couplings introduced in Section 3.6, the following independent and dependent couplings

$$\begin{aligned}\textbf{Independent}: &\quad \delta\lambda_3,\\ \textbf{Dependent}: &\quad c_{gg}^{(2)},\ \tilde c_{gg}^{(2)},\ y_u^{(2)},\ y_d^{(2)},\ y_e^{(2)}.\end{aligned}\tag{3.25}$$

The independent coupling is defined via the correction to the triple Higgs boson coupling in Eq. (2.5)

$$\mathcal{L}_{h^3}^{D=6} = -\delta\lambda_3 v h^3. \tag{3.26}$$

The dependent couplings are defined via double Higgs interaction with fermions and gluons (which are not present in the SM):

$$\mathcal{L}_{hhff}^{D=6} = \frac{h^2}{v^2}\frac{g_s^2}{8}\left(c_{gg}^{(2)} G_{\mu\nu}^a G_{\mu\nu}^a + \tilde{c}_{gg}^{(2)} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a\right) - \frac{h^2}{2v^2}\sum_{f;ij}\sqrt{m_{f_i} m_{f_j}}\left[\bar{f}_{i,R}[y_f^{(2)}]_{ij} f_{j,L} + \text{h.c.}\right]. \tag{3.27}$$

They are related to the independent couplings by

$$\begin{aligned} c_{gg}^{(2)} &= c_{gg}, \qquad \tilde{c}_{gg}^{(2)} = \tilde{c}_{gg}, \\ [y_f^{(2)}]_{ij} &= 3[\delta y_f]_{ij} e^{i\phi_{ij}} - \delta c_z\, \delta_{ij}, \end{aligned} \tag{3.28}$$

Besides the couplings to fermions, other dependent couplings with two Higgs bosons arise at the dimension-6 level. Specifically, these are the couplings h^2VV to the SM electroweak gauge bosons, and $h^2 ffV$ contact interactions. As these do not play the role in the double Higgs production processes currently studied at the LHC, we do not display them here.

3.9 Four-fermion terms

In order to promote our framework to a complete $D = 6$ basis it is necessary to include 4-fermion terms. These are not relevant for Higgs searches at the LHC at tree level, therefore we discuss them in less detail than the interactions listed in the previous section. The 4-fermion Lagrangian is given by

$$\mathcal{L}_{4f}^{D=6} = \sum_i c_{4f,i} O_{4f,i}. \tag{3.29}$$

We choose the set of 4-fermion operators $O_{4f,i}$ to coincide with those in the Warsaw basis, see the bottom columns of Table 1. There is only one subtlety that needs to be taken into account. The basic premise of the Higgs basis is that the tree-level relation between the SM electroweak parameters and input observables is not affected by new physics. On the other hand, one of the four-fermion couplings in the Lagrangian,

$$\mathcal{L}_{4f}^{D=6} \supset [c_{\ell\ell}]_{1221}(\bar{\ell}_{1,L}\gamma_\rho \ell_{2,L})(\bar{\ell}_{2,L}\gamma_\rho \ell_{1,L}) \tag{3.30}$$

does affect the relation between the parameter v and the muon decay width from which $G_F = 1/\sqrt{2}v^2$ is determined:

$$\frac{\Gamma(\mu \to e\nu\nu)}{\Gamma(\mu \to e\nu\nu)_{\text{SM}}} \approx 1 + 2[\delta g_L^{We}]_{11} + 2[\delta g_L^{We}]_{22} - 4\delta m - [c_{\ell\ell}]_{1221}. \tag{3.31}$$

Therefore, to keep the muon width unchanged, $[c_{\ell\ell}]_{12;21}$ has to be a *dependent* coupling related to the independent parameters δm and δg as

$$[c_{\ell\ell}]_{1221} = 2\delta[g_L^{We}]_{11} + 2[\delta g_L^{We}]_{22} - 4\delta m. \tag{3.32}$$

Hence, in the Higgs basis the coefficient of one 4-lepton operators defined in the Warsaw basis is a dependent coupling; coefficients of all the remaining 4-fermion operators are independent couplings.

3.10 Summary of the Higgs basis Lagrangian

In summary, the Higgs basis is parametrized by the independent couplings in Eqs. (3.1), (3.3), (3.6), (3.9), (3.12), (3.19). In total, the Higgs basis, much as any complete basis at the dimension-6 level, is parametrized by 2499 independent real couplings [10]. One should not, however, be intimidated by this number. The point is that a much smaller subset in Eq. (3.9) is adequate for EFT analyses of Higgs data at the leading order in new physics parameters. For example, to describe single Higgs production and decay processes in full generality one needs 10 bosonic and $2 \times 3 \times 3 \times 3 = 54$ fermionic couplings. Furthermore, 31 of these couplings are CP-odd, therefore they affect the Higgs signal strength measurement only at the quadratic level, while flavor off-diagonal Yukawa couplings only affect exotic Higgs decays. In the limit where fermionic couplings are flavor blind, 9 parameters are enough to describe leading order EFT corrections to the current Higgs signal strength measurements at the LHC.

The full Lagrangian in the Higgs basis is given by

$$\begin{aligned}\mathcal{L}_{\rm Higgs\,Basis} &= \mathcal{L}^{\rm SM} + \mathcal{L}^{D=6}_{\rm kinetic} + \mathcal{L}^{D=6}_{\rm vertex} + \mathcal{L}^{D=6}_{\rm dipole} + \mathcal{L}^{D=6}_{\rm hvv} + \mathcal{L}^{D=6}_{\rm hff} \\ &+ \mathcal{L}^{D=6}_{hdvff} + \mathcal{L}^{D=6}_{hvff} + \mathcal{L}^{D=6}_{v^3} + \mathcal{L}^{D=6}_{\rm v^4} + \mathcal{L}^{\rm D=6}_{\rm d^2v^4} + \mathcal{L}^{D=6}_{h^3} + \mathcal{L}^{D=6}_{hhff} + \mathcal{L}^{D=6}_{4f} \\ &+ \mathcal{L}_{\rm other}. \end{aligned} \tag{3.33}$$

Here, $\mathcal{L}_{\rm other}$ contains additional interactions terms: quartic and higher Higgs boson self-interactions, interactions of 3 Higgs bosons with fermion fields, couplings of a single Higgs boson to 3 or more gauge bosons, etc. These are not listed in this note because they are currently relevant neither for electroweak precision tests nor for single and double Higgs production and decay. If necessity or interest arises, these additional terms can be easily calculated and added to this note.

We conclude with a number of comments.

- The relations between independent and dependent couplings in Eq. (3.5), Eq. (3.11), Eq. (3.18), Eq. (3.28) are consequences of the *linear* realization of electroweak symmetry breaking at the level of dimension-6 EFT operators. *They are an essential part of the definition of the Higgs basis.* If the independent and dependent couplings were unrelated, then $\mathcal{L}_{\rm Higgs\,Basis}$ would not be a dimension-6 basis but would belong to a more general class of theories. Such theories are outside of the scope of this note, however they will be discussed in the framework of the extended kappa formalism.

- The independent couplings in Eq. (3.3) are probed by precision measurements of Z and W production and decays at LEP. In particular, assuming vertex corrections are flavor blind, all the independent couplings in Eq. (3.3) are constrained to be smaller than $O(10^{-3})$ (for the leptonic vertex corrections and $\delta m \equiv \delta m_W/m_W$), or $O(10^{-2})$ (for the quark vertex corrections) [2, 4, 11]. Dropping the assumption of flavor blindness, all the leptonic, bottom and charm quark vertex corrections are still constrained, in a model-independent way, at the level of $O(10^{-2})$ or better [12]. These constraints imply these couplings are too small to have any measurable effects at the LHC, therefore we recommend to impose the electroweak bounds on such constraints before analyzing LHC data. The 1st generation quark vertex corrections are less constrained in a model-independent way, though one combination of them is tightly constrained by measurements of the hadronic Z decays at LEP. Furthermore, the top quark vertex corrections are poorly constrained (at the $O(1)$ level) by experiment, especially the right-handed top couplings to Z. If feasible, the light quark and top couplings should be considered as free parameters in experimental analyses at the LHC, as this may provide new valuable information to constrain these couplings.

- The Higgs basis is convenient for extracting constraints on dimension-6 operators from Higgs and electroweak precision data. However, it may not be the optimal basis for some other applications. In particular, computing renormalization group running of the couplings or matching to concrete BSM model may be more straightforward in the language of $SU(3) \times SU(2) \times U(1)$ invariant operators.

- Customarily, the SM electroweak parameters are extracted from $\alpha(0)$, m_Z and G_F. One could also use m_W instead of G_F, as suggested in Ref. [2]. This formalism leads to the same relations between the independent and dependent couplings as written down here, except that $\delta m = 0$ by definition, and that $c'_{\ell\ell}$ defined in Eq. (3.30) becomes an independent couplings. The downside of this formalism is that the SM predictions for all observables would have to be recalculated, as all existing high-precision calculations use G_F as an input.

- The number of independent couplings in Eq. (3.9) relevant for Higgs observables is still large. At the early stages of the LHC run-2 it may be reasonable to employ simplified analyses with a smaller number of parameters. There are several motivated assumptions about the underlying UV theory that reduce the number of parameters:

  - *Flavor universality*, in which case the matrices $m_f \delta y_f$ and $\sin\phi_f$ reduce to a single number for each $f = u, d, e$.
  - *Minimal flavor violation*, in which case the dominant entries in δy_f are $[\delta y_u]_{33}$ and $[\delta y_d]_{33}$, while other diagonal entries are suppressed by the respective mass square ratio.
  - *CP conservation*, in which case all CP-odd couplings vanish: $\tilde{c}_i = 0 = \sin\phi_f$.
  - *Custodial symmetry*, in which case $\delta m = 0$.[3]

  We stress that independent couplings should not be arbitrarily set to zero without an underlying symmetry assumption. Furthermore, the relations between the dependent and independent couplings should be consistently imposed, so as to preserve the weak $SU(2)$ local symmetry.

- The independent couplings are formally of order v^2/Λ^2, where Λ is the scale of new physics. For completeness, it is important to define the range of independent couplings such that the EFT description is valid. The rule of thumb is that this is

[3] Custodial symmetry implies several relations between Higgs couplings to gauge bosons: $\delta c_w = \delta c_z$, $c_{w\Box} = c_\theta^2 c_{z\Box} + s_\theta^2 c_{\gamma\Box}$, $c_{ww} = c_{zz} + 2 s_\theta^2 c_{z\gamma} + s_\theta^4 c_\gamma$, and $\tilde{c}_{ww} = \tilde{c}_{zz} + 2 s_\theta^2 \tilde{c}_{z\gamma} + s_\theta^4 \tilde{c}_\gamma$. The last three are satisfied automatically at the level of dimension-6 Lagrangian, while the first one is true for $\delta m = 0$, see Eq. (3.11).

the case for $|c_i| \lesssim 1$; more sophisticated criteria will be worked out in the future when specific Higgs processes are discussed.

4 Map to Warsaw Basis of Dimension-6 Operators

We turn to discussing the map between the independent couplings introduced in Section 3 and coefficients of dimension-6 operators in the electroweak basis before electroweak symmetry breaking. The complete set of dimension-6 operators can be written in many different equivalent bases which are related by the use of equations of motion and integration by parts. Here we work with the so-called *Warsaw basis* of Ref. [5, 10], which is distinguished by the simplest tensor structure of the higher-dimensional operators. The analogous procedure can be applied to other bases.

The Lagrangian in the Warsaw basis is given by[4]

$$\mathcal{L}_{\rm warsaw} = \mathcal{L}^{\rm SM} + \frac{1}{v^2}\sum_i c_i O_i, \tag{4.1}$$

where the SM Lagrangian $\mathcal{L}^{\rm SM}$ was introduced in Section 2, and the dimension-6 operators O_i are summarized in Table 1.

To map the coefficients of dimension-6 operators into the independent couplings in Eq. (3.3) and Eq. (3.9) , we need first to bring $\mathcal{L}_{\rm warsaw}$ into the same form as $\mathcal{L}_{\rm Higgs\ Basis}$ in Eq. (3.33). This can be achieved by a series of transformations using equations of motion, integration by parts, and rescaling of the fields and couplings. To begin with, the operator O_{WB} leads to a kinetic mixing between the hypercharge and SU(2) gauge bosons, $O_{WB} \to -1/2 g g' W^3_{\mu\nu} B_{\mu\nu}$. To get rid of it, we use the equations of motion:

$$\begin{aligned} \partial_\nu B_{\nu\mu} &= g' \frac{(v+h)^2}{4}\left(g W^3_\mu - g' B_\mu\right) - g' j^Y_\mu, \\ \partial_\nu W^3_{\nu\mu} &= -g \frac{(v+h)^2}{4}\left(g W^3_\mu - g' B_\mu\right) - g j^3_\mu - g\epsilon^{3jk} W^j_\nu W^k_{\nu\mu}, \end{aligned} \tag{4.2}$$

where $j^Y_\mu = \sum_f Y_f \bar{f}\gamma_\mu f$, and $j^3_\mu = \bar{q}\gamma_\mu T^3 P_L q + \bar{\ell}\gamma_\mu T^3 P_L \ell$. Using this,

$$\begin{aligned} -c_{WB}\frac{g g'}{2} W^3_{\mu\nu} B_{\mu\nu} \;\to\; & c_{WB} e^2 \Big[\frac{(v+h)^2}{4}\left(g W^3_\mu - g' B_\mu\right)^2 - g W^3_\mu j^Y_\mu - g' B_\mu j^3_\mu \\ & - \frac{g^2}{2g'}\epsilon^{3jk} W^j_\mu W^k_\nu B_{\mu\nu} - g'\epsilon^{3jk} B_\mu W^j_\nu W^k_{\nu\mu} \Big] \\ = \;& c_{WB} e^2 \left[\frac{(g^2+g'^2)(v+h)^2}{4} Z^2_\mu - e A_\mu j^{\rm em}_\mu + \sqrt{g^2+g'^2}\, Z_\mu \left(j^3_\mu - c^2_\theta j^{\rm em}_\mu\right)\right] \\ + \;& i c_{WB}\frac{g^2 g'}{(g^2+g'^2)^{3/2}} \left[g^2 (g A_{\mu\nu} - g' Z_{\mu\nu}) W^+_\mu W^-_\nu \right. \\ & \left. - g'^2 (g A_\mu - g' Z_\mu)(W^+_{\mu\nu} W^-_\nu - W^-_{\mu\nu} W^+_\nu)\right], \end{aligned} \tag{4.3}$$

[4]We use a different notation than the original reference. We also replaced the operator $|H^\dagger D_\mu H|^2$ by $(H^\dagger D_\mu H - D_\mu H^\dagger H)^2$. For Yukawa-type operators O_f we subtracted v^2 so that these operators do not contribute to off-diagonal mass terms. This way we avoid tedious rotations of the fermion fields to bring them back to the mass eigenstate basis. Starting with the Yukawa couplings $-H\bar{f}'_R(Y'_f + c'_f H^\dagger H/v^2) f'_L$ we can bring them to the form in Eq. (2.1) and Table 1 by defining $f'_{L,R} = U_{L,R} f_{L,R}$, $c_f = U^\dagger_R c'_f U_L$, $Y_f = U^\dagger_R (Y'_f + c'_f/2) U_L$, where $U_{L,R}$ are unitary rotations to the mass eigenstate basis.

where $j^{\rm em}_\mu = j^3_\mu + j^Y_\mu$ is the electromagnetic current. Next, the operators O_{BB}, O_{WW}, and O_{GG} change the normalization of the kinetic terms of the gauge bosons. To recover the canonical normalization we redefine the gauge fields as

$$B_\mu \to B_\mu \left(1 + \frac{c_{BB} g'^2}{4}\right), \; W^i_\mu \to W^i_\mu \left(1 + \frac{c_{WW} g^2}{4}\right), \; G^a_\mu \to G^a_\mu \left(1 + \frac{c_{GG} g_s^2}{4}\right). \quad (4.4)$$

We ignore here the contribution of the operator $\tilde{O}_{GG}$ to the QCD θ-term (we can always assume it cancels agains the θ-term in the SM Lagrangian, or is dynamically removed by an axion field). The operator O_H changes the normalization of the Higgs boson kinetic term, and also induces Higgs boson self-interactions that contain two derivatives. To recover the canonical normalization and remove the 2-derivative self-interactions we redefine the Higgs field as

$$h \to h \left(1 - c_H - \frac{h}{v} c_H - \frac{h^2}{3v^2} c_H\right). \quad (4.5)$$

The relation between the Higgs VEV v_0 and the mass parameter in the SM Lagrangian is affected by the O_{6H} operator:

$$v_0^2 = \frac{\mu_H^2}{\lambda}\left(1 + \frac{3}{4\lambda} c_{6H}\right), \quad (4.6)$$

while the relation between Higgs boson mass and the quartic coupling in the SM Lagrangian is affected by both O_{6H} and O_H:

$$m_h^2 = 2 v_0^2 \left(\lambda - 2 c_H \lambda - \frac{3}{2} c_{6H}\right). \quad (4.7)$$

We have to make sure that the gauge couplings and the Higgs VEV have the same meaning as in the SM. This is a non-trivial requirement, because dimension-6 operators affect the observables used to extract these parameters. We have seen that the operator O_{WB} shifts the electric charge and the Z boson mass. Similarly, the operator O_T shifts the Z boson mass term. Furthermore, one of the $O_{\ell\ell}$ operators leads to the 4-fermion coupling $v^{-2}[c_{\ell\ell}]_{1221}(\bar{\nu}_{\mu,L}\gamma_\rho\nu_{e,L})(\bar{e}_L\gamma_\rho\mu_L)$ that contributes to the muon decay at the linear level and thus shifts the Fermi constant. Finally, the leptonic vertex operator $O_{H\ell}$ also shifts the Fermi constant. To undo these effects, we need to ensure that the photon and the gluon couple to the electromagnetic and strong currents as in Eq. (2.3). Furthermore, the Z boson mass term in the Lagrangian should be as in Eq. (2.2), and the tree-level $\mu \to e\bar{\nu}_e\nu_\mu$ decay width should be given by $\Gamma = \frac{m_\mu^5}{384\pi^3 v^4}$. This is achieved by the following redefinition of the coupling constants and the VEV:

$$\begin{aligned}
g_s &\to g_s \left(1 - c_{GG}\frac{g_s^2}{4}\right), \\
g &\to g \left(1 - c_{WW}\frac{g^2}{4} - c_{WB}\frac{g^2 g'^2}{g^2 - g'^2} + (c_T - \delta v)\frac{g^2}{g^2 - g'^2}\right), \\
g' &\to g' \left(1 - c_{BB}\frac{g'^2}{4} + c_{WB}\frac{g^2 g'^2}{g^2 - g'^2} - (c_T - \delta v)\frac{g'^2}{g^2 - g'^2}\right), \\
v_0 &\to v\,(1 + \delta v), \quad (4.8)
\end{aligned}$$

where $\delta v = ([c'_{H\ell}]_{11} + [c'_{H\ell}]_{22})/2 - [c_{\ell\ell}]_{1221}/4$.

One last transformation is needed to match the Higgs basis. At this point, the coefficients of the contact interactions in Eq. (3.17) differ from the vertex corrections by flavor universal terms depending only on the electric charge and the isospin of the fermions. It is possible to get rid of the latter using equations of motion for the gauge bosons, so as to traded them into zero- and two-derivative Higgs boson interactions with gauge bosons of the form $hV_\mu V_\mu$ and $hV_\mu \partial_\nu V_{\mu\nu}$.

After all these transformations the Lagrangian takes the same form as $\mathcal{L}_{\text{Higgs Basis}}$. The dictionary between the coefficients of dimension-6 operators and the independent and dependent couplings in $\mathcal{L}_{\text{Higgs Basis}}$ goes as follows. The shift of the W boson mass is given by

$$\delta m = \frac{1}{g^2 - g'^2}\left[-g^2 g'^2 c_{WB} + g^2 c_T - g'^2 \delta v\right]. \tag{4.9}$$

The shift of W and Z boson couplings to leptons are given by

$$\begin{aligned}
\delta g_L^{W\ell} &= c'_{H\ell} + f(1/2, 0) - f(-1/2, -1), \\
\delta g_L^{Z\nu} &= \frac{1}{2}c'_{H\ell} - \frac{1}{2}c_{H\ell} + f(1/2, 0), \\
\delta g_L^{Ze} &= -\frac{1}{2}c'_{H\ell} - \frac{1}{2}c_{H\ell} + f(-1/2, -1), \\
\delta g_R^{Ze} &= -\frac{1}{2}c_{He} + f(0, -1),
\end{aligned} \tag{4.10}$$

where

$$f(T^3, Q) = I_3\left[-Q c_{WB}\frac{g^2 g'^2}{g^2 - g'^2} + (c_T - \delta v)\left(T^3 + Q\frac{g'^2}{g^2 - g'^2}\right)\right], \tag{4.11}$$

and I_3 is the 3×3 identity matrix. Vertex corrections to W and Z boson couplings to quarks are given by

$$\begin{aligned}
\delta g_L^{Wq} &= c'_{Hq} + f(1/2, 2/3) - f(-1/2, -1/3), \\
\delta g_R^{Wq} &= -\frac{1}{2}c_{Hud}, \\
\delta g_L^{Zu} &= \frac{1}{2}c'_{Hq} - \frac{1}{2}c_{Hq} + f(1/2, 2/3), \\
\delta g_L^{Zd} &= -\frac{1}{2}c'_{Hq} - \frac{1}{2}c_{Hq} + f(-1/2, -1/3), \\
\delta g_R^{Zu} &= -\frac{1}{2}c_{Hu} + f(0, 2/3), \\
\delta g_R^{Zd} &= -\frac{1}{2}c_{Hd} + f(0, -1/3).
\end{aligned} \tag{4.12}$$

The coefficients of vertex-like contact interactions between the Higgs boson, W or Z boson, and two fermions in Eq. (3.17) are given by

$$c^{Vf} = \delta g^{Vf}. \tag{4.13}$$

The shifts of the Higgs couplings to W and Z are given by

$$\begin{aligned}
\delta c_w &= -c_H - c_{WB}\frac{4g^2 g'^2}{g^2 - g'^2} + 4c_T\frac{g^2}{g^2 - g'^2} - \delta v\frac{3g^2 + g'^2}{g^2 - g'^2}, \\
\delta c_z &= -c_H - 3\delta v.
\end{aligned} \tag{4.14}$$

The two-derivative Higgs couplings to gauge bosons are given by

$$
\begin{aligned}
c_{gg} &= c_{GG}, \qquad c^{(2)}_{gg} = c_{GG}, \\
c_{\gamma\gamma} &= c_{WW} + c_{BB} - 4c_{WB}, \\
c_{zz} &= \frac{g^4 c_{WW} + g'^4 c_{BB} + 4g^2 g'^2 c_{WB}}{(g^2+g'^2)^2}, \\
c_{z\Box} &= -\frac{2}{g^2}\left(c_T - \delta v\right), \\
c_{z\gamma} &= \frac{g^2 c_{WW} - g'^2 c_{BB} - 2(g^2-g'^2)c_{WB}}{g^2+g'^2}, \\
c_{\gamma\Box} &= \frac{2}{g^2-g'^2}\left((g^2+g'^2)c_{WB} - 2c_T + 2\delta v\right), \\
c_{ww} &= c_{WW}, \\
c_{w\Box} &= \frac{2}{g^2-g'^2}\left(g'^2 c_{WB} - c_T + \delta v\right).
\end{aligned}
\tag{4.15}
$$

and the same for the CP-odd couplings $\tilde{c}_{gg}$, $\tilde{c}_{\gamma\gamma}$, $\tilde{c}_{z\gamma}$, $\tilde{c}_{zz}$, $\tilde{c}_{ww}$, with $c \to \tilde{c}$ on the right hand side. The Yukawa interactions are given by

$$
\begin{aligned}
[\delta y_f]_{ij}\cos\phi^f_{ij} &= \frac{v\mathrm{Re}[c_f]_{ij}}{\sqrt{2m_{f_i}m_{f_j}}} - \delta_{ij}\left(c_H + \delta v\right), \\
[\delta y_f]_{ij}\sin\phi^f_{ij} &= \frac{v\mathrm{Im}[c_f]_{ij}}{\sqrt{2m_{f_i}m_{f_j}}}.
\end{aligned}
\tag{4.16}
$$

The coefficients of Yukawa-type interactions of two Higgs bosons with fermions in Eq. (3.27) are given by

$$
[y^{(2)}_f]_{ij} = 3[\delta y_f]_{ij}e^{i\phi_{ij}} + (c_H + 3\delta v)\delta_{ij}. \tag{4.17}
$$

The anomalous triple gauge couplings of electroweak gauge bosons are given by

$$
\begin{aligned}
\delta g_{1,z} &= \frac{g^2+g'^2}{g^2-g'^2}\left(-g'^2 c_{WB} + c_T - \delta v\right), \\
\delta\kappa_\gamma &= g^2 c_{WB}, \\
\delta\kappa_z &= -2c_{WB}\frac{g^2 g'^2}{g^2-g'^2} + \frac{g^2+g'^2}{g^2-g'^2}\left(c_T - \delta v\right), \\
\lambda_\gamma &= -\frac{3}{2}g^4 c_{3W}, \\
\lambda_z &= -\frac{3}{2}g^4 c_{3W}, \\
\tilde{\kappa}_\gamma &= g^2 \tilde{c}_{WB}, \\
\tilde{\kappa}_z &= -g'^2 \tilde{c}_{WB}, \\
\tilde{\lambda}_\gamma &= -\frac{3}{2}g^4 \tilde{c}_{3W}, \\
\tilde{\lambda}_z &= -\frac{3}{2}g^4 \tilde{c}_{3W}.
\end{aligned}
\tag{4.18}
$$

The Higgs cubic interaction is given by

$$\delta\lambda_3 = -\lambda\left(3c_H + \delta v\right) - c_{6H}. \tag{4.19}$$

From these expressions one can derive the relations between dependent and independent couplings listed in Section 3.

To summarize, in the Warsaw basis the parameters affecting electroweak precision tests, Higgs production (single or double) and Higgs decay are the following

$$\begin{array}{c} c_H,\ c_T,\ c_{GG},\ c_{WW},\ c_{BB},\ c_{WB},\ \tilde{c}_{GG},\ \tilde{c}_{WW},\ \tilde{c}_{BB},\ \tilde{c}_{WB},\ \ c_{3W},\ \tilde{c}_{3W},\ c_{6H}, \\ c'_{H\ell},\ c_{H\ell},\ c_{He},\ c'_{Hq},\ c_{Hq},\ c_{Hu},\ c_{Hd},\ c_{Hud} \\ c_u,\ c_d,\ c_e, \\ [c_{\ell\ell}]_{12;21}. \end{array} \tag{4.20}$$

The linear transformation between these parameters and the independent couplings in Eq. (3.3), Eq. (3.9), Eq. (3.19), and Eq. (3.25) is given in Eqs. (4.9)-(4.18). In principle, one can also perform the LHC analyses in the Warsaw (or any other) basis. One difficulty is that the electroweak precision constraints, which are transparent in the Higgs basis, constrain rather complicated combinations of the parameters in Eq. (4.20). Alternatively, the constraints derived in the Higgs basis can be easily recast into constraints in the Warsaw basis using the map Eqs. (4.9)-(4.18), provided that the former are given with the full correlation matrix.

5 Map to SILH Basis of Dimension-6 Operators

In this section we present the translation between the couplings in the Higgs basis and Wilson coefficients of dimension-6 operators in the SILH basis [6, 8]. The Lagrangian is written as

$$\mathcal{L}_{\rm SILH} = \mathcal{L}^{\rm SM} + \frac{1}{v^2}\sum_i s_i O_i. \tag{5.1}$$

Compared to the Warsaw basis defined in Section 4, the SILH basis of dimension-6 operators introduces the following nine new operators:

$$\begin{aligned} O_W &= \frac{ig}{2}\left(H^\dagger\sigma^i \overleftrightarrow{D_\mu} H\right) D_\nu W^i_{\mu\nu}, \\ O_B &= \frac{ig'}{2}\left(H^\dagger \overleftrightarrow{D_\mu} H\right) \partial_\nu B_{\mu\nu}, \\ O_{HW} &= ig\left(D_\mu H^\dagger \sigma^i D_\nu H\right) W^i_{\mu\nu}, \\ O_{HB} &= ig'\left(D_\mu H^\dagger D_\nu H\right) B_{\mu\nu}, \\ O_{\widetilde{HW}} &= ig\left(D_\mu H^\dagger \sigma^i D_\nu H\right) \widetilde{W}^i_{\mu\nu}, \\ O_{\widetilde{HB}} &= ig'\left(D_\mu H^\dagger D_\nu H\right) \widetilde{B}_{\mu\nu}, \\ O_{2W} &= D_\mu W^i_{\mu\nu} D_\rho W^i_{\rho\nu}, \\ O_{2B} &= \partial_\mu B_{\mu\nu} \partial_\rho B_{\rho\nu}, \\ O_{2G} &= D_\mu G^a_{\mu\nu} D_\rho G^a_{\rho\nu}. \end{aligned} \tag{5.2}$$

Consequently, in order to have a non-redundant set of operators, 9 operators present in the Warsaw basis must be absent in the SILH basis. The absent ones are 4 bosonic operators O_{WW}, $O_{\widetilde{WW}}$, O_{WB}, $O_{\widetilde{WB}}$, 2 vertex operators $[O_{H\ell}]_{11}$, $[O'_{H\ell}]_{11}$, and 3 four-fermion operators $[O_{\ell\ell}]_{12;21}$, $[O_{\ell\ell}]_{11;22}$, $[O'_{uu}]_{33;33}$. The remaining operators are the same as in the Warsaw basis, and we use the normalizations in Table 1, which are often different than in Refs. [6, 8].[5]

One way to derive the translation is to first transform the operators in Eq. (5.2) to the Warsaw basis using integration by parts, Fierz transformations, and the equations of motion:

$$\begin{aligned}
\partial_\nu B_{\mu\nu} &= \frac{ig'}{2} H^\dagger \overleftrightarrow{D}_\mu H + g' \sum_{f=q,\ell} Y_f \bar{f}_L \gamma_\mu f_L + g' \sum_{f=u,d,e} Y_f \bar{f}_R \gamma_\mu f_R, \\
D_\nu W^i_{\mu\nu} &= \frac{ig}{2} H^\dagger \sigma^i \overleftrightarrow{D}_\mu H + \frac{g}{2} \sum_{f=q,\ell} \bar{f}_L \sigma^i \gamma_\mu f_L, \\
D_\nu G^a_{\mu\nu} &= g_s \bar{q}_L T^a \gamma_\mu q_L + g_s \sum_{f\in u,d} \bar{q}_R T^a \gamma_\mu q_R.
\end{aligned} \tag{5.3}$$

[5] The original references do not discuss the flavor structure explicitly, and the flavor indices of the absent operators are not specified. Here, for concreteness, we made a particular though somewhat arbitrary choice of these indices.

Using these, one can obtain:

$$
\begin{aligned}
O_{HB} &= O_B - \frac{1}{4}O_{WB} - O_{BB}, \\
O_{HW} &= O_W - \frac{1}{4}O_{WB} - O_{WW}, \\
O_{\widetilde{HB}} &= -\frac{1}{4}O_{\widetilde{WB}} - O_{\widetilde{BB}}, \\
O_{\widetilde{HW}} &= -\frac{1}{4}O_{\widetilde{WB}} - O_{\widetilde{WW}}, \\
O_B &= g'^2 \left[-\frac{1}{4}O_T + \frac{1}{2}\sum_{f\in q,u,d,\ell,e} Y_f \sum_i [O_{Hf}]_{ii} \right], \\
O_W &= g^2 \left[-\frac{1}{4}O_H + O_{HD} + \frac{1}{4}\sum_{f\in q,\ell}\sum_i [O'_{Hf}]_{ii} \right], \\
O_{2B} &= g'^2 \left[-\frac{1}{4}O_T + \sum_{f\in q,u,d,\ell,e} Y_f \sum_i [O_{Hf}]_{ii} + \sum_{f_1 f_2\in q,u,d,\ell,e} Y_{f_1} Y_{f_2} \sum_{i,j} [O_{f_1 f_2}]_{ii;jj} \right], \\
O_{2W} &= g^2 \left[-\frac{1}{4}O_H + O_{HD} + \frac{1}{2}\sum_{f\in q,\ell}\sum_i [O'_{Hf}]_{ii} \right. \\
&+ \left. \sum_{ij} \left(\frac{1}{2}[O_{\ell\ell}]_{ij;ji} - \frac{1}{4}[O_{\ell\ell}]_{ii;jj} + \frac{1}{2}[O_{\ell q}]_{ii;jj} + \frac{1}{4}[O_{qq}]_{ii;jj} \right) \right], \\
O_{2G} &= g_s^2 \sum_{i,j} \left[\frac{1}{4}[O'_{qq}]_{ij;ji} + \frac{1}{4}[O_{qq}]_{ij;ji} - \frac{1}{6}[O_{qq}]_{ii;jj} + 2[O'_{qu}]_{ii;jj} + 2[O'_{qd}]_{ii;jj} \right. \\
&+ \left. 2[O'_{ud}]_{ii;jj} + \frac{1}{2}[O'_{uu}]_{ij;ji} - \frac{1}{6}[O'_{uu}]_{ii;jj} + \frac{1}{2}[O'_{dd}]_{ij;ji} - \frac{1}{6}[O'_{dd}]_{ii;jj} \right].
\end{aligned} \tag{5.4}
$$

The operator $O_{HD} = |H|^2|D_\mu H|^2$ appearing above is present neither in the Warsaw nor in the SILH basis. One can remove it from the Lagrangian by rescaling the Higgs field and the Yukawa couplings as $H \to H(1+\epsilon|H|^2/v^2)$, $y_f \to y_f(1-\epsilon/2)$. To lowest order in ϵ, this rescaling generates the following terms in the Lagrangian

$$
\Delta\mathcal{L} = \epsilon \left(2O_{HD} + O_H - 4\lambda O_{6H} + \sum_{f\in u,d,e}\sum_i [y_f]_{ii}[O_f]_{ii} \right). \tag{5.5}
$$

Thus, to get rid of the O_{HD} operator generated by the transformation from the SILH to the Warsaw basis we need to choose $\epsilon = -g^2(s_W + s_{HW} + s_{2W})/2$. Effectively, this amount to replacing in Eq. (5.4):

$$
O_{HD} \to -\frac{1}{2}O_H + 2\lambda O_{6H} - \frac{1}{2}\sum_{f\in u,de}\sum_i [y_f]_{ii}[O_f]_{ii}. \tag{5.6}
$$

We are ready to give the translation between the Wilson coefficient in the SILH and

Warsaw basis:

$$
\begin{aligned}
c_H &= s_H - \frac{3g^2}{4}\left(s_W + s_{HW} + s_{2W}\right), \\
c_T &= s_T - \frac{g'^2}{4}\left(s_B + s_{HB} + s_{2B}\right), \\
c_{6H} &= s_{6H} + 2\lambda g^2\left(s_W + s_{HW} + s_{2W}\right), \\
c_{WB} &= -\frac{1}{4}\left(s_{HB} + s_{HW}\right), \\
c_{BB} &= s_{BB} - s_{HB}, \\
c_{WW} &= -s_{HW}, \\
\tilde{c}_{WB} &= -\frac{1}{4}\left(\tilde{s}_{HB} + \tilde{s}_{HW}\right), \\
\tilde{c}_{BB} &= \tilde{s}_{BB} - \tilde{s}_{HB}, \\
\tilde{c}_{WW} &= -\tilde{s}_{HW},
\end{aligned}
\tag{5.7}
$$

$$
\begin{aligned}
[c_{Hf}]_{ij} &= [s_{Hf}]_{ij} + \frac{g'^2 Y_f}{2}\left(s_B + s_{HB} + 2s_{2B}\right)\delta_{ij}, \\
[c'_{Hf}]_{ij} &= [s'_{Hf}]_{ij} + \frac{g^2}{4}\left(s_W + s_{HW} + 2s_{2W}\right)\delta_{ij},
\end{aligned}
\tag{5.8}
$$

$$
[c_f]_{ij} = [s_f]_{ij} - \delta_{ij} g^2 [y_f]_{ii} \frac{s_W + s_{HW} + s_{2W}}{2}, \tag{5.9}
$$

$$
\begin{aligned}
[c_{\ell\ell}]_{ii;ii} &= [s_{\ell\ell}]_{ii;ii} + \frac{1}{4}\left(g'^2 s_{2B} + g^2 s_{2W}\right), \\
[c_{\ell\ell}]_{ii;jj} &= [s_{\ell\ell}]_{ii;jj} + \frac{1}{2}\left(g'^2 s_{2B} - g^2 s_{2W}\right), \qquad i < j, \\
[c_{\ell\ell}]_{ij;ji} &= [s_{\ell\ell}]_{ij;ji} + g^2 s_{2W}, \qquad i < j,
\end{aligned}
\tag{5.10}
$$

where it is implicit that $[s_{H\ell}]_{11} = [s'_{H\ell}]_{11} = [s_{\ell\ell}]_{12;21} = [s_{\ell\ell}]_{11;22} = 0$. For the 4-lepton operators one should take into account that $[O_{\ell\ell}]_{ji;ij} \equiv [O_{\ell\ell}]_{ij;ji}$ and $[O_{\ell\ell}]_{jj;ii} \equiv [O_{\ell\ell}]_{ii;jj}$. The translation of other 4-fermion Wilson coefficients apart from the one in Eq. (5.10) can be easily derived from Eq. (5.4), but it will not be needed in the following. For the Wilson coefficients not listed above the translation is trivial: $c_i = s_i$.

Given these relations between the Warsaw and SILH basis Wilson coefficients and using the results of Section 4, we can derive the translation between the Higgs basis couplings and the SILH basis Wilson coefficients:

$$
\delta m = -\frac{g^2 g'^2}{4(g^2 - g'^2)}\left(s_W + s_B + s_{2W} + s_{2B} - \frac{4}{g'^2}s_T + \frac{2}{g^2}[s'_{H\ell}]_{22}\right), \tag{5.11}
$$

$$
\begin{aligned}
\hat{f}(T^3, Q) \equiv\ & \frac{1}{4}\left[g^2 s_{2W} + g'^2 s_{2B} + 4s_T - 2[s'_{H\ell}]_{22}\right] T^3 \\
+\ & \frac{g'^2}{4(g^2 - g'^2)}\left[-(2g^2 - g'^2)s_{2B} - g^2(s_{2W} + s_W + s_B) + 4s_T - 2[s'_{H\ell}]_{22}\right] Q,
\end{aligned}
\tag{5.12}
$$

$$
\begin{aligned}
\delta g_L^{Z\nu} &= \frac{1}{2}s'_{H\ell} - \frac{1}{2}s_{H\ell} + \hat{f}(1/2,0),\\
\delta g_L^{Ze} &= -\frac{1}{2}s'_{H\ell} - \frac{1}{2}s_{H\ell} + \hat{f}(-1/2,-1),\\
\delta g_R^{Ze} &= -\frac{1}{2}s_{He} + \hat{f}(0,-1),\\
\delta g_L^{Zu} &= \frac{1}{2}s'_{Hq} - \frac{1}{2}s_{Hq} + \hat{f}(1/2,2/3),\\
\delta g_L^{Zd} &= -\frac{1}{2}s'_{Hq} - \frac{1}{2}s_{Hq} + \hat{f}(-1/2,-1/3),\\
\delta g_R^{Zu} &= -\frac{1}{2}s_{Hu} + \hat{f}(0,2/3),\\
\delta g_R^{Zd} &= -\frac{1}{2}s_{Hd} + \hat{f}(0,-1/3),\\
\delta g_L^{W\ell} &= s'_{H\ell} + \hat{f}(1/2,0) - \hat{f}(-1/2,-1),\\
\delta g_L^{Wq} &= s'_{Hq} + \hat{f}(1/2,2/3) - \hat{f}(-1/2,-1/3),\\
\delta g_R^{Wq} &= -\frac{1}{2}s_{Hud},
\end{aligned}
\tag{5.13}
$$

$$
c^{Vf} = \delta g^{Vf}, \tag{5.14}
$$

$$
\begin{aligned}
\delta c_w &= -s_H - \frac{g^2 g'^2}{g^2 - g'^2}\left[s_W + s_B + s_{2W} + s_{2B} - \frac{4}{g'^2}s_T + \frac{3g^2 + g'^2}{2g^2 g'^2}[s'_{H\ell}]_{22}\right],\\
\delta c_z &= -s_H - \frac{3}{2}[s'_{H\ell}]_{22},\\
c_{gg} &= s_{GG},\\
c_{\gamma\gamma} &= s_{BB},\\
c_{zz} &= -\frac{1}{g^2 + g'^2}\left[g^2 s_{HW} + g'^2 s_{HB} - g'^2 s_\theta^2 s_{BB}\right],\\
c_{z\Box} &= \frac{1}{2g^2}\left[g^2(s_W + s_{HW} + s_{2W}) + g'^2(s_B + s_{HB} + s_{2B}) - 4s_T + 2[s'_{H\ell}]_{22}\right],\\
c_{z\gamma} &= \frac{s_{HB} - s_{HW}}{2} - s_\theta^2 s_{BB},\\
c_{\gamma\Box} &= \frac{s_{HW} - s_{HB}}{2} + \frac{1}{g^2 - g'^2}\left[g^2(s_W + s_{2W}) + g'^2(s_B + s_{2B}) - 4s_T + 2[s'_{H\ell}]_{22}\right],\\
c_{ww} &= -s_{HW},\\
c_{w\Box} &= \frac{s_{HW}}{2} + \frac{1}{2(g^2 - g'^2)}\left[g^2(s_W + s_{2W}) + g'^2(s_B + s_{2B}) - 4s_T + 2[s'_{H\ell}]_{22}\right],
\end{aligned}
\tag{5.15}
$$

$$
\begin{aligned}
[\delta y_f]_{ij}\cos\phi_{ij}^f &= \frac{v\mathrm{Re}[c_f]_{ij}}{\sqrt{2m_{f_i}m_{f_j}}} - \delta_{ij}\left[s_H + \frac{3g^2}{4}\left(s_W + s_{HW} + s_{2W}\right) + \frac{1}{2}[s'_{H\ell}]_{22}\right],\\
[\delta y_f]_{ij}\sin\phi_{ij}^f &= \frac{v\mathrm{Im}[s_f]_{ij}}{\sqrt{2m_{f_i}m_{f_j}}}.
\end{aligned}
\tag{5.16}
$$

$$\delta\lambda_3 = -\lambda\left(3s_H + \frac{1}{2}[s'_{H\ell}]_{22}\right) - s_{6H}, \tag{5.17}$$

$$\begin{aligned}
\delta g_{1z} &= -\frac{g^2+g'^2}{4(g^2-g'^2)}\left[(g^2-g'^2)s_{HW} + g^2(s_W+s_{2W}) + g'^2(s_B+s_{2B}) - 4s_T + 2[s'_{H\ell}]_{22}\right],\\
\delta\kappa_\gamma &= -\frac{g^2}{4}\left[s_{HW}+s_{HB}\right],\\
\delta\kappa_z &= -\frac{1}{4}\left(g^2 s_{HW} - g'^2 s_{HB}\right) - \frac{g^2+g'^2}{4(g^2-g'^2)}\left[g^2(s_W+s_{2W}) + g'^2(s_B+s_{2B}) - 4s_T + 2[s'_{H\ell}]_{22}\right],\\
\lambda_z &= -\frac{3}{2}g^4 s_{3W}, \qquad \lambda_\gamma = \lambda_z,\\
\delta\tilde\kappa_\gamma &= -\frac{g^2}{4}\left[\tilde s_{HW}+\tilde s_{HB}\right],\\
\delta\tilde\kappa_z &= \frac{g'^2}{4}\left[\tilde s_{HW}+\tilde s_{HB}\right],\\
\tilde\lambda_z &= -\frac{3}{2}g^4 \tilde s_{3W}, \qquad \tilde\lambda_\gamma = \tilde\lambda_z.
\end{aligned} \tag{5.18}$$

A Dictionary

In this section we give a translation between the Higgs basis parameters and other EFT formalisms used in the literature, keeping all the normalization and conventions as in the original references. On request, translation to other formalisms may be added in the future.

A.1 HISZ basis

To describe the di-boson production, Ref. [13] proposes to use the following 5 operators:

$$\begin{aligned}
\hat O_{WW} &= \mathrm{Tr}\left[W_{\mu\nu}W_{\nu\rho}W_{\rho\mu}\right],\\
\hat O_W &= D_\mu H^\dagger W_{\mu\nu} D_\mu H,\\
\hat O_B &= D_\mu H^\dagger B_{\mu\nu} D_\mu H,\\
\hat O_{\widetilde{WW}} &= \mathrm{Tr}\left[W_{\mu\nu}W_{\nu\rho}\widetilde W_{\rho\mu}\right],\\
\hat O_{\widetilde W} &= D_\mu H^\dagger \widetilde W_{\mu\nu} D_\mu H.
\end{aligned} \tag{A.1}$$

This is a subset of operators considered by Hagiwara et al. (HISZ) in Ref. [9]. The dimension-6 Lagrangian contains

$$\mathcal{L}^{\mathrm{D}=6} \supset \frac{1}{\Lambda^2}\left(d_{WW}\hat O_{WW} + d_W \hat O_W + d_B \hat O_B + \tilde d_{WW}\hat O_{\widetilde{WW}} + \tilde d_W \hat O_{\tilde W}\right). \tag{A.2}$$

These 5 operators contribute to the TGCs and Higgs couplings, but they do not contribute to oblique or vertex corrections. Thus, they are not strongly constrained by electroweak precision tests, and therefore represent a perfectly fine parameterization of EFT new physics in di-boson production.

One should remember that the covariant derivatives in Refs. [9, 13] are defined with the opposite sign than here. This amounts to rescaling the gauge fields as $W_\mu \to -W_\mu$, $B_\mu \to -B_\mu$ in the translation. Then the electroweak field strength tensors defined in Ref. [13] are related to the ones used here by

$$B_{\mu\nu} \to -\frac{i}{2} g' B_{\mu\nu}, \qquad W_{\mu\nu} \to -\frac{i}{2} g \sigma^i W^i_{\mu\nu}. \tag{A.3}$$

This allows us to relate

$$\begin{aligned} \hat{O}_{WW} &= -\frac{1}{4} O_{3W}, \quad \hat{O}_W = -\frac{1}{2} O_{HW}, \quad \hat{O}_B = -\frac{1}{2} O_{HB}, \\ \hat{O}_{\widetilde{WW}} &= -\frac{1}{4} O_{3\widetilde{W}}, \quad \hat{O}_{\widetilde{W}} = -\frac{1}{2} O_{H\widetilde{W}}. \end{aligned} \tag{A.4}$$

where O_i on the right-hand side are operators in the SILH basis in the normalization of Section 5. Thus, the map between the HISZ and SILH coefficients is the following:

$$\begin{aligned} s_{3W} &= -\frac{1}{4} \frac{v^2}{\Lambda^2} d_{WW}, \quad s_{HW} = -\frac{1}{2} \frac{v^2}{\Lambda^2} d_W, \quad s_{HB} = -\frac{1}{2} \frac{v^2}{\Lambda^2} d_B, \\ \tilde{s}_{3W} &= -\frac{1}{4} \frac{v^2}{\Lambda^2} \tilde{d}_{WW}, \quad \tilde{s}_{HW} = -\frac{1}{2} \frac{v^2}{\Lambda^2} \tilde{d}_W. \end{aligned} \tag{A.5}$$

The anomalous TGCs and the HISZ basis Wilson coefficients are related by:

$$\begin{aligned} \delta g_{1z} &= \frac{g^2 + g'^2}{8} \frac{v^2}{\Lambda^2} d_W \\ \delta \kappa_\gamma &= \frac{g^2}{8} \frac{v^2}{\Lambda^2} \left(d_W + d_B \right), \qquad \delta \tilde{\kappa}_\gamma = \frac{g^2}{8} \frac{v^2}{\Lambda^2} \tilde{d}_W \\ \lambda_z &= \frac{3g^4}{8} \frac{v^2}{\Lambda^2} d_{WW}, \qquad \tilde{\lambda}_z = \frac{3g^4}{8} \frac{v^2}{\Lambda^2} \tilde{d}_{WW}. \end{aligned} \tag{A.6}$$

Inverting these formulas, the relation between the Wilson coefficients in the HISZ basis and the Higgs basis parameters reads

$$\begin{aligned} d_{WW} &= \frac{8\Lambda^2}{3g^4 v^2} \lambda_z, \\ d_W &= -\frac{4\Lambda^2}{(g^2 - g'^2) v^2} \left[g^2 c_{z\Box} + g'^2 c_{zz} - s_\theta^2 e^2 c_{\gamma\gamma} - s_\theta^2 (g^2 - g'^2) c_{z\gamma} \right], \\ d_B &= \frac{4\Lambda^2}{(g^2 - g'^2) v^2} \left[g^2 c_{z\Box} + g^2 c_{zz} - c_\theta^2 e^2 c_{\gamma\gamma} - c_\theta^2 (g^2 - g'^2) c_{z\gamma} \right], \\ \tilde{d}_{WW} &= \frac{8\Lambda^2}{3g^4 v^2} \tilde{\lambda}_z, \\ \tilde{d}_W &= \frac{8\Lambda^2}{g^2 v^2} \delta \tilde{\kappa}_\gamma. \end{aligned} \tag{A.7}$$

References

[1] R. S. Gupta, A. Pomarol, and F. Riva, *BSM Primary Effects*, arXiv:1405.0181.

[2] A. Pomarol and F. Riva, *Towards the Ultimate SM Fit to Close in on Higgs Physics*, *JHEP* **1401** (2014) 151, [arXiv:1308.2803].

[3] M. Trott, *On the consistent use of Constructed Observables*, arXiv:1409.7605.

[4] A. Falkowski and F. Riva, *Model-independent precision constraints on dimension-6 operators*, *JHEP* **1502** (2015) 039, [arXiv:1411.0669].

[5] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, *Dimension-Six Terms in the Standard Model Lagrangian*, *JHEP* **1010** (2010) 085, [arXiv:1008.4884].

[6] G. Giudice, C. Grojean, A. Pomarol, and R. Rattazzi, *The Strongly-Interacting Light Higgs*, *JHEP* **0706** (2007) 045, [hep-ph/0703164].

[7] M. Gonzalez-Alonso, A. Greljo, G. Isidori, and D. Marzocca, *Pseudo-observables in Higgs decays*, arXiv:1412.6038.

[8] R. Contino, M. Ghezzi, C. Grojean, M. Muhlleitner, and M. Spira, *Effective Lagrangian for a light Higgs-like scalar*, *JHEP* **1307** (2013) 035, [arXiv:1303.3876].

[9] K. Hagiwara, S. Ishihara, R. Szalapski, and D. Zeppenfeld, *Low-energy effects of new interactions in the electroweak boson sector*, *Phys.Rev.* **D48** (1993) 2182–2203.

[10] R. Alonso, E. E. Jenkins, A. V. Manohar, and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators III: Gauge Coupling Dependence and Phenomenology*, *JHEP* **1404** (2014) 159, [arXiv:1312.2014].

[11] J. Ellis, V. Sanz, and T. You, *The Effective Standard Model after LHC Run I*, arXiv:1410.7703.

[12] A. Efrati, A. Falkowski, and Y. Soreq, *Electroweak constraints on flavorful effective theories*, arXiv:1503.07872.

[13] C. Degrande, N. Greiner, W. Kilian, O. Mattelaer, H. Mebane, et al., *Effective Field Theory: A Modern Approach to Anomalous Couplings*, *Annals Phys.* **335** (2013) 21–32, [arXiv:1205.4231].

H^4D^2 and H^6

O_H	$\left[\partial_\mu(H^\dagger H)\right]^2$
O_T	$\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$
O_{6H}	$(H^\dagger H)^3$

f^2H^3

O_e	$-(H^\dagger H - \frac{v^2}{2})\bar{e}H^\dagger \ell$
O_u	$-(H^\dagger H - \frac{v^2}{2})\bar{u}\widetilde{H}^\dagger q$
O_d	$-(H^\dagger H - \frac{v^2}{2})\bar{d}H^\dagger q$

V^3D^3

O_{3G}	$g_s^3 f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$
$O_{3\widetilde{G}}$	$g_s^3 f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$
O_{3W}	$g^3 \epsilon^{ijk} W^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$
$O_{3\widetilde{W}}$	$g^3 \epsilon^{ijk} \widetilde{W}^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$

V^2H^2

O_{GG}	$\frac{g_s^2}{4} H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu}$
$O_{\widetilde{GG}}$	$\frac{g_s^2}{4} H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu}$
O_{WW}	$\frac{g^2}{4} H^\dagger H\, W^i_{\mu\nu} W^i_{\mu\nu}$
$O_{\widetilde{WW}}$	$\frac{g^2}{4} H^\dagger H\, \widetilde{W}^i_{\mu\nu} W^i_{\mu\nu}$
O_{BB}	$\frac{g'^2}{4} H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$
$O_{\widetilde{BB}}$	$\frac{g'^2}{4} H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu}$
O_{WB}	$g g' H^\dagger \sigma^i H\, W^i_{\mu\nu} B_{\mu\nu}$
$O_{\widetilde{WB}}$	$g g' H^\dagger \sigma^i H\, \widetilde{W}^i_{\mu\nu} B_{\mu\nu}$

f^2H^2D

$O_{H\ell}$	$i\bar{\ell}\gamma_\mu \ell H^\dagger \overleftrightarrow{D}_\mu H$
$O'_{H\ell}$	$i\bar{\ell}\sigma^i\gamma_\mu \ell H^\dagger \sigma^i \overleftrightarrow{D}_\mu H$
O_{He}	$i\bar{e}\gamma_\mu e H^\dagger \overleftrightarrow{D}_\mu H$
O_{Hq}	$i\bar{q}\gamma_\mu q H^\dagger \overleftrightarrow{D}_\mu H$
O'_{Hq}	$i\bar{q}\sigma^i\gamma_\mu q H^\dagger \sigma^i \overleftrightarrow{D}_\mu H$
O_{Hu}	$i\bar{u}\gamma_\mu u H^\dagger \overleftrightarrow{D}_\mu H$
O_{Hd}	$i\bar{d}\gamma_\mu d H^\dagger \overleftrightarrow{D}_\mu H$
O_{Hud}	$i\bar{u}\gamma_\mu d \tilde{H}^\dagger D_\mu H$

f^2VHD

O_{eW}	$g\bar{\ell}\sigma_{\mu\nu} e \sigma^i H W^i_{\mu\nu}$
O_{eB}	$g'\bar{\ell}\sigma_{\mu\nu} e H B_{\mu\nu}$
O_{uG}	$g_s \bar{q}\sigma_{\mu\nu} T^a u \widetilde{H}\, G^a_{\mu\nu}$
O_{uW}	$g\bar{q}\sigma_{\mu\nu} u \sigma^i \widetilde{H}\, W^i_{\mu\nu}$
O_{uB}	$g'\bar{q}\sigma_{\mu\nu} u \widetilde{H}\, B_{\mu\nu}$
O_{dG}	$g_s \bar{q}\sigma_{\mu\nu} T^a d H\, G^a_{\mu\nu}$
O_{dW}	$g\bar{q}\sigma_{\mu\nu} d \sigma^i H\, W^i_{\mu\nu}$
O_{dB}	$g'\bar{q}\sigma_{\mu\nu} d H\, B_{\mu\nu}$

$(\bar{L}L)(\bar{L}L)$ and $(\bar{L}R)(\bar{L}R)$

$O_{\ell\ell}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{\ell}\gamma_\mu \ell)$
O_{qq}	$(\bar{q}\gamma_\mu q)(\bar{q}\gamma_\mu q)$
O'_{qq}	$(\bar{q}\gamma_\mu \sigma^i q)(\bar{q}\gamma_\mu \sigma^i q)$
$O_{\ell q}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{q}\gamma_\mu q)$
$O'_{\ell q}$	$(\bar{\ell}\gamma_\mu \sigma^i \ell)(\bar{q}\gamma_\mu \sigma^i q)$
O_{quqd}	$(\bar{q}^j u)\epsilon_{jk}(\bar{q}^k d)$
O'_{quqd}	$(\bar{q}^j T^a u)\epsilon_{jk}(\bar{q}^k T^a d)$
$O_{\ell equ}$	$(\bar{\ell}^j e)\epsilon_{jk}(\bar{q}^k u)$
$O'_{\ell equ}$	$(\bar{\ell}^j \sigma_{\mu\nu} e)\epsilon_{jk}(\bar{q}^k \sigma^{\mu\nu} u)$
$O_{\ell edq}$	$(\bar{\ell}^j e)(\bar{d} q^j)$

$(\bar{R}R)(\bar{R}R)$

O_{ee}	$(\bar{e}\gamma_\mu e)(\bar{e}\gamma_\mu e)$
O_{uu}	$(\bar{u}\gamma_\mu u)(\bar{u}\gamma_\mu u)$
O_{dd}	$(\bar{d}\gamma_\mu d)(\bar{d}\gamma_\mu d)$
O_{eu}	$(\bar{e}\gamma_\mu e)(\bar{u}\gamma_\mu u)$
O_{ed}	$(\bar{e}\gamma_\mu e)(\bar{d}\gamma_\mu d)$
O_{ud}	$(\bar{u}\gamma_\mu u)(\bar{d}\gamma_\mu d)$
O'_{ud}	$(\bar{u}\gamma_\mu T^a u)(\bar{d}\gamma_\mu T^a d)$

$(\bar{L}L)(\bar{R}R)$

$O_{\ell e}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{e}\gamma_\mu e)$
$O_{\ell u}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{u}\gamma_\mu u)$
$O_{\ell d}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{d}\gamma_\mu d)$
O_{qe}	$(\bar{q}\gamma_\mu q)(\bar{e}\gamma_\mu e)$
O_{qu}	$(\bar{q}\gamma_\mu q)(\bar{u}\gamma_\mu u)$
O'_{qu}	$(\bar{q}\gamma_\mu T^a q)(\bar{u}\gamma_\mu T^a u)$
O_{qd}	$(\bar{q}\gamma_\mu q)(\bar{d}\gamma_\mu d)$
O'_{qd}	$(\bar{q}\gamma_\mu T^a q)(\bar{d}\gamma_\mu T^a d)$

Table 1: A complete, non-redundant set of baryon-and-lepton-number-conserving dimension-6 operators built from SM fields [5]. In this table, e, u, d are always right-handed fermions, while ℓ and q are left-handed. A flavor index is implicit for each fermion field. For complex operators the complex conjugate operator is implicit. Including the flavor structure and complex conjugates, this table contains 2499 distinct operators [10].