FCC-ee:

Parameters, Beam-Beam, Luminosity

Dmitry Shatilov

BINP, Novosibirsk

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Luminosity

For flat beams (both head-on and crossing angle collision):

$$L = \frac{\gamma}{2er_e} \cdot \frac{I\xi_y}{\beta_y^*} \cdot R_H$$

I – total beam current (defined by SR power of 50 MW)

ξ_y – vertical betatron tune shift, its limit depends on the collision scheme

R_H – hour-glass factor: $R_H \approx$ [0.86, 0.71, 0.6] for L_i/β_y^* = [1, 2, 3]

L_i – length of the interaction area:

$$L_i = \frac{\sigma_z}{\sqrt{1+\phi^2}} \qquad \phi = \frac{\sigma_z}{\sigma_x} tg\left(\frac{\theta}{2}\right) \text{ – Piwinski angle}$$

β_y^* should be minimized as much as possible, but there are restrictions:

- beta-function at the final quads raises as $1/\beta_y^*$ that affects dynamic aperture and can create problems with chromaticity corrections
- L_i should be squeezed to $L_i \sim \beta_y^*$, for head-on it means $\sigma_z \sim \beta_y^*$

Beamstrahlung

At high energies (tt, H) the luminosity is limited by the beamstrahlung lifetime, which is proportional to:

$$\tau_{bs} \sim \exp\left(\frac{2\eta\alpha\rho}{3r_e\gamma^2}\right)\cdot\frac{\rho^{3/2}}{L_i\gamma^2}$$

α – fine structure constant
η – energy acceptance
ρ – average bending radius of a particle's trajectory at IP

Obviously, the major tool for reducing beamstrahlung is making ρ larger. For flat beams, ρ is inversely proportional to the surface charge density of the opposing beam:

$$\frac{1}{\rho} \sim \frac{N_p}{\sigma_x\sigma_z} \sim \frac{\xi_y}{L_i}\sqrt{\frac{\varepsilon_y}{\beta_y^*}}$$

It follows that the vertical emittance should be minimized, and L_i should be not small. As a consequence, β_y^* also should be not too small, the optimum value is about 2 mm (at high energies only!).

Baseline vs. crab waist, what is the difference ?

The “standard” requirements for crab waist, which sometimes are difficult to achieve:

- Very small emittances.
- Very small (preferably sub-millimeter) vertical beta-function.

At high energies FCC-ee:

- Very small emittances for all collision schemes.
- Optimum β_y^* is limited by beamstrahlung, 2mm is enough.
- Very large energy acceptance $\eta \sim 0.02$ (the larger, the better).

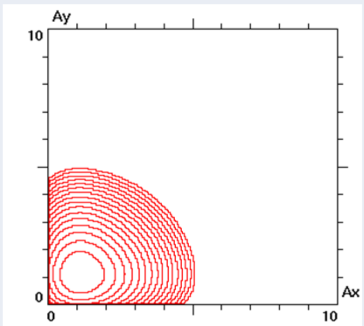
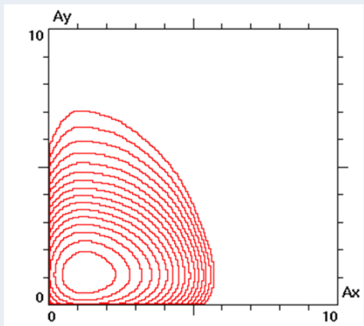
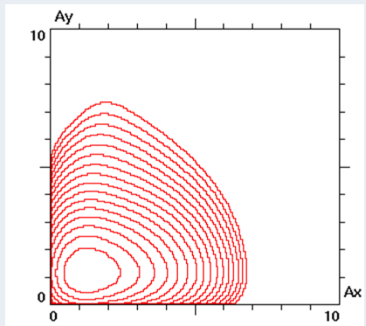
These are common requirements not related to crab waist!

Therefore, the major part of lattice studies (small emittances, large DA and η) are common. The only differences are the crossing angle and CW sextupoles.

- If the crossing angle exists it should be not too small, otherwise we need to either increase L^* or make one-aperture final quads (with some negative consequences).
- Probably, $\theta = 30$ mrad is a good choice. Additional bonus: longitudinal polarization at 45.5 GeV.
- CW sextupoles may limit DA, this is a known problem, we hope it can be solved or softened. But this can be done only *after* the lattice is optimized, DA and η are large enough, etc.
- **At 120 GeV, crab waist can raise the luminosity by ~50%.**

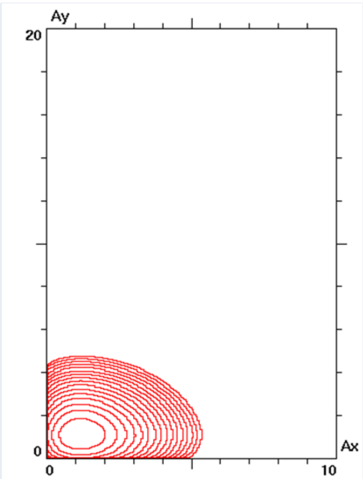
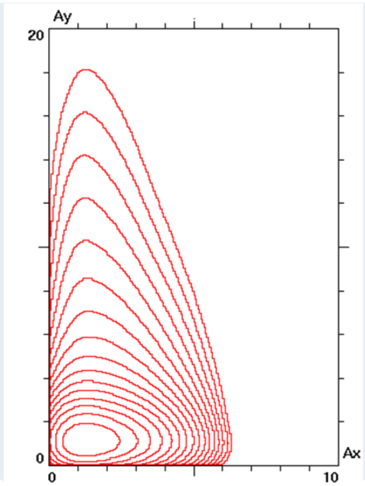
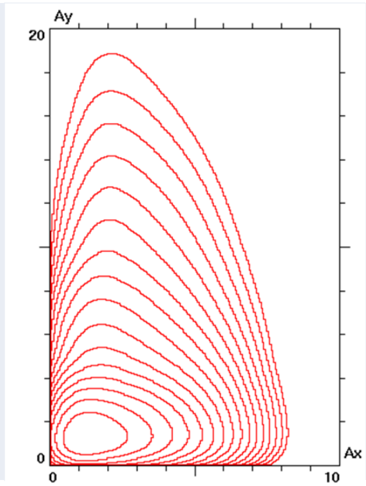
175 GeV, β_y= 2 mm

(Simulation Results of December 2014, to be updated)

Collision scheme	Crab Waist	Head-on	Crossing (11 mrad)
RF voltage [GV]	9.5	11	11
RF frequency [MHz]	400	400	400
Tunes $\nu_x / \nu_y / \nu_s$	0.54 / 0.57 / 0.0132	0.54 / 0.61 / 0.0172	0.52 / 0.57 / 0.0172
Bunch length [mm]	2.75 / 3.74	2.11 / 2.56	2.11 / 2.68
Bunch population	$2.0 \cdot 10^{11}$	$1.1 \cdot 10^{11}$	$1.2 \cdot 10^{11}$
Footprint size $\Delta\nu_x / \Delta\nu_y$	0.023 / 0.079	0.071 / 0.137	0.047 / 0.106
Lifetime τ_{bs} [min]	18	35	25
Luminosity [$cm^{-2}s^{-1}$]	$1.15 \cdot 10^{34}$	$1.3 \cdot 10^{34}$	$1.2 \cdot 10^{34}$
Luminosity (β_y = 1 mm)	$1.25 \cdot 10^{34}$	$1.3 \cdot 10^{34}$ (800 MHz)	$1.25 \cdot 10^{34}$ (800 MHz)
Density contour plots $10\sigma_x \times 10\sigma_y$			

120 GeV, β_y= 2 mm

(Simulation Results of December 2014, to be updated)

Collision scheme	Crab Waist	Head-on	Crossing (11 mrad)
RF voltage [GV]	2.3	5.5	5.5
RF frequency [MHz]	400	800	800
Tunes $\nu_x / \nu_y / \nu_s$	0.54 / 0.57 / 0.009	0.54 / 0.61 / 0.0255	0.52 / 0.57 / 0.0255
Bunch length [mm]	2.76 / 6.77	0.98 / 1.47	0.98 / 1.62
Bunch population	$3.5 \cdot 10^{11}$	$5 \cdot 10^{10}$	$6 \cdot 10^{10}$
Footprint size $\Delta\nu_x / \Delta\nu_y$	0.019 / 0.126	0.087 / 0.128	0.063 / 0.104
Lifetime bb+bs [min]	17	120	200
Luminosity [$cm^{-2}s^{-1}$]	$8.3 \cdot 10^{34}$	$6.8 \cdot 10^{34}$	$5.0 \cdot 10^{34}$
Luminosity (β_y = 1 mm)	$9.8 \cdot 10^{34}$	$7.2 \cdot 10^{34}$	$5.8 \cdot 10^{34}$
Density contour plots $10\sigma_x \times 20\sigma_y$			

Beamstrahlung at Low Energies

- At low energies, bunch lengthening due to beamstrahlung can be significant.
- For collisions with small Piwinski angle (head-on, small crossing angle), where $L_i \sim \sigma_z$, it leads to hour-glass amplification. The consequences are: increase of the actual betatron tune shift and excitement of synchro-betatron resonances.
- In respect that the damping is weak, this may result in the vertical blow-up which limits the maximum attainable tune shifts and the luminosity.
- In crab waist scheme with large Piwinski angle hour-glass is not affected, since L_i does not depend on σ_z. However, too strong bunch lengthening due to beamstrahlung may result in the longitudinal flip-flop instability, which in turn leads to the transverse blow-up.
- To reduce beamstrahlung, we need small emittances again. This is in good agreement with the general crab waist requirements.

Resistive wall power loss

- At the energy of Z (45.5 GeV) the total bunch current limited by the SR power loss of 50 MW is large: about 1.5 A. This may cause a significant resistive wall power loss.
- Given that the number of bunches N_b is inversely proportional to the number of particles per bunch N_p, the total power loss is proportional to:

$$P_{rw} \sim \frac{N_p}{\sigma_z^{3/2}}$$

- To make P_{rw} smaller, N_p should be minimized. If we want to keep ξ_y unchanged, emittances should be small too. This requirement is in good agreement with minimization of beamstrahlung.
- Crab waist scheme requires longer bunches, that is beneficial to reduce P_{rw}.
- Baseline scenario assumes much shorter bunches, otherwise hour-glass, etc.
- Some optimization of baseline parameters can be achieved by decreasing N_p and emittances, increasing the number of bunches N_b. Now, with RF 400 MHz and perimeter 100 km, we have ~133400 buckets and only ~13000 bunches.

Low Energy, β_y= 1 mm

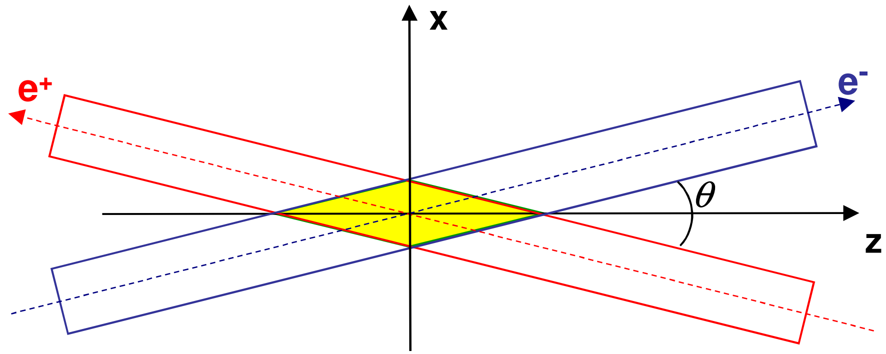
(Simulation Results of October 2014, to be updated)

Energy [GeV]	45.5		80	
Collision scheme	Head-on	Crab Waist	Head-on	Crab Waist
N_p $[10^{11}]$	1.8	1.0	0.7	4.0
θ [mrad]	0	30	0	30
σ_z [mm]	1.64 / 3.0	2.77 / 7.63	1.01 / 1.76	4.13 / 11.6
ε_x [nm]	29.2	0.14	3.3	0.44
ε_y [pm]	60.0	1.0	7.0	1.0
ξ_x / ξ_y [nominal]	0.03 / 0.03	0.02 / 0.14	0.06 / 0.06	0.02 / 0.20
L $[10^{34}\ cm^{-2}s^{-1}]$	17	180	13	45

At 45.5 GeV ξ_y can be raised up to 0.2. If we try to achieve this by 50% increase of N_p, additional bunch lengthening will occur due to beamstrahlung, so ξ_y will increase by 15% only. Decrease of ε_y would be much more efficient, since for flat bunches beamstrahlung does not depend on the vertical beam size.

What is the optimal β_x ?

In collisions with large ϕ the length of interaction area L_i is proportional to the horizontal beam size σ_x, given that the crossing angle θ is fixed.



If L_i > β_y hour-glass is amplified.

If L_i < β_y beam-beam resonances are enhanced since the vertical betatron phase averaging over the interaction area decreases.

The optimum ratio is: $L_i \sim \beta_y$, but some variations (± 30%) are acceptable.

In crab waist collision with θ = 30 mrad, β_x = 50 cm, ε_x = 1.7 nm at 175 GeV and $\varepsilon_x \sim E^2$, we have L_i [mm] ≈ {1.7, 1.3, 0.9, 0.5} for E [GeV] = {175, 120, 80, 45.5}.

Taking into account that β_y [mm] ≈ {2, 2, 1, 1}, **we can safely increase σ_x and L_i by ~ 40%.**

The main benefit comes from reducing the strength of beamstrahlung.

Increase of σ_x by ε_x is not desirable, as ε_y will increase in the same proportion due to betatron coupling.

Good solution could be the β_x increase from 50 to 100 cm. Also, this would simplify the horizontal chromaticity correction at the IR.

Current activities

The comparisons of head-on and crab waist schemes performed earlier were not perfect, since the main lattice parameters and some restrictions were different.

For better understanding we perform optimization and comparison of 4 collision schemes (head-on, θ = 11 mrad, θ = 30 mrad without CW, θ = 30 mrad with CW) for all 4 energies. In total, 4×4 = 16 independent cases.

What is fixed for all schemes at a given energy:

- Energy spread, energy loss per turn
- Energy acceptance
- Damping decrements
- Minimal emittances and momentum compaction
- Total beam current, transverse apertures

What is optimized:

- RF voltage (i.e. bunch length)
- Betatron tunes
- Beta functions at IP
- Bunch population

What is the goal:

- Maximum luminosity
- Lifetime > 10 min
- Tolerance to 10% difference in bunch population (critical due to beamstrahlung!)
- Tolerance to asymmetry of ~ 0.02 in betatron tune advances between IPs

Summary

- There is much synergy between baseline and crab waist lattice parameters and requirements, especially at high energies.
- At high energies β_y^* can be relaxed to 2 mm, and we need large $\eta \sim 0.02$.
- At low energies β_y^* should be ~ 1 mm, but η can be smaller (~ 0.01 at 45.5 Gev.)
- β_x^* should be increased from 0.5 to 1 m.
- At the top energy (175 GeV) the performances of baseline and crab waist schemes are almost equal.
- At 120 GeV crab waist has an advantage ~ 50% which growth with lowering the energy. At 45.5 GeV the advantage may reach factor of 10.
- Parameters optimization and comparative analysis of different collision schemes is now performing with the use of more accurate models and techniques. Work in progress.
- When the real nonlinear lattice is ready, more reliable beam-beam simulations will be possible.