Unfolding Procedure

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on behalf of the CMS Collaboration

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Issues

Problems and issues specific to the unfolding method in the Higgs measurements:

- **Signal Extraction**
  - Large background to the analysis (e.g. in $H\rightarrow\gamma\gamma$, the $\gamma\gamma$ continuum)
  - Mass is being profiled
  - A parametric fit is necessary

- **Categorization**
  - enhancement of the signal in different categories
  - simultaneous fit across categories
  - correct dealing with statistical uncertainties from the combination in the unfolding

- **Why unfolding**
  - undo detector effects
  - quoting fiducial and differential cross sections
  - correctly model statistical migrations

Literature

In literature there are different methods to perform unfolding.
The most widespread are based on a χ^2 minimization

- $$\|r\|^2 = \left\| \frac{\hat{\mathbf{A}}\vec{x}_T - \vec{y}}{\Delta\vec{y}} \right\|^2$$

Regularization:

- artificially changes the “Confidence Intervals”
- Tikhonov (SVD):
  - Penalization term in the minimization procedure $\|r\|^2 = \left\| \frac{\hat{\mathbf{A}}\vec{x}_T - \vec{y}}{\Delta\vec{y}} \right\|^2 + \delta \|\mathbf{L}\vec{x}_T\|^2$
- Iterative d’Agostini (Bayes):

$$p(x_T^i|x_M^j) = \frac{p(x_M^j|x_T^i)\,p_0(x_T^i)}{\sum_{l\in\text{truth}} p(x_M^j|x_T^l)\,p_0(x_T^l)}$$

  - d’Agostini regularization is given in terms of n. of iterations

$\vec{x}_T$ truth distribution

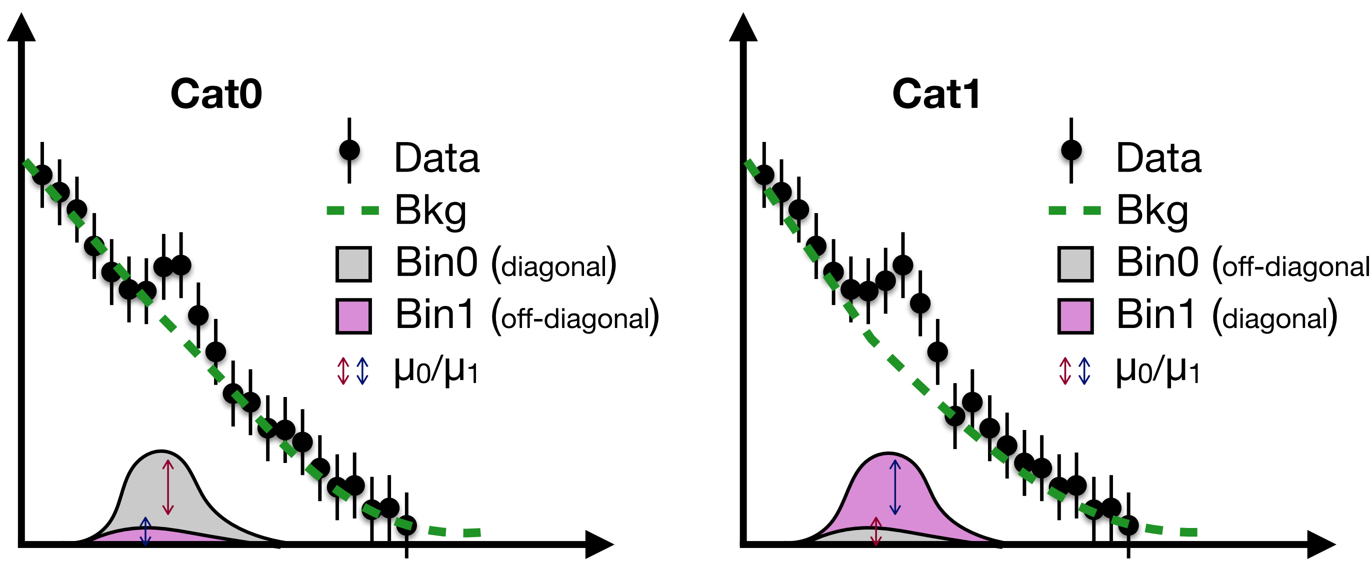
$\hat{\mathbf{A}}$ response matrix

$\vec{y}$ measured distribution

Proposed method

- H→γγ and H→ZZ differential and fiducial cross-section measurement (coming out soon)
  - similar to the coupling strength extraction

- **Signal extraction** and category **combination** is performed **simultaneously**
- Each generator bin has its corresponding detector shape (parametric)
- A simultaneous fit across all categories and all bins is performed
- Relative strength are extracted
- The cross-section is normalized back



fit simultaneously in cat0/cat1 to get the Bin strength modifiers μ=(μ0,μ1)

Likelihood function

- contains the "parametric" response matrix as a collection of shapes

$$\mathscr{F} = -2\log\mathscr{L}(\mathbf{A}\vec{\mu}|\vec{y})$$

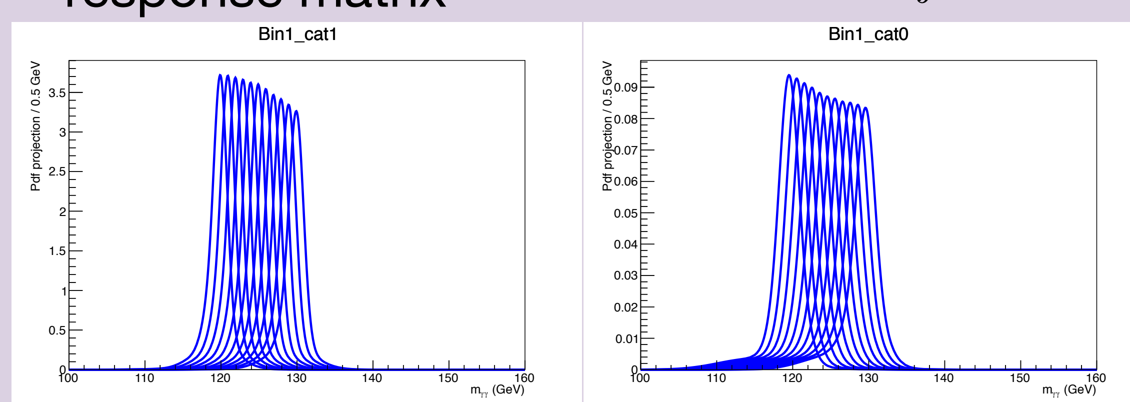
$$\Delta\hat{\sigma}_i = \frac{x^{\mathrm{MC}}_{T,i}\hat{\mu}_i}{L}$$

Cross section normalization is similar to the SVD

$$A_{ij} = \hat{A}_{ij}x^{\mathrm{MC}}_{T,i}$$

parametric response matrix

$$\mathbf{A}(m_{\mathrm{H}},\theta) = \int \mathrm{d}m_{\gamma\gamma}\,\mathbf{a}(m_{\gamma\gamma}|m_{\mathrm{H}},\theta)$$



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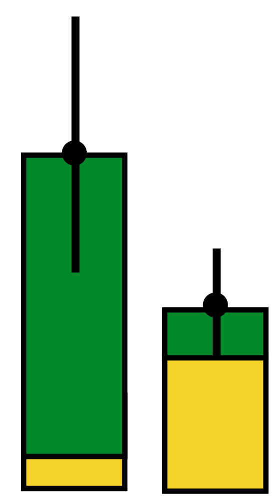
Importance of Unfolding

- Bin-by-Bin is a biased estimation (smaller uncertainties).

- **Out-of-acceptance:**
  - A out-of-acceptance shape should be subtracted from the fiducial results



- **Bin Migration** can be important:
  - change the best fit values
  - change the confidence intervals!

  - p_T differences in the statistical uncertainties are small (up to few percent)
  - N_{jets} differences in the statistical uncertainties can be big (up to 30%)
    - jet resolution induces important migrations
  - data can pull the best-fit values in the different bins

Conclusions

- We show how to unfold distributions in the Higgs framework
- Unfolding is important to set-up theory comparisons
- … and to give legacy in the presented results.

Bibliography

- COWAN, G. A survey of unfolding methods for particle physics.
- D'AGOSTINI, G. A multidimensional unfolding method based on Bayes' theorem.
- HOCKER, A., AND KARTVELISHVILI, V. SVD approach to data unfolding.

Backup

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Adding regularization

- Adding Tickhonov regularization to the likelihood

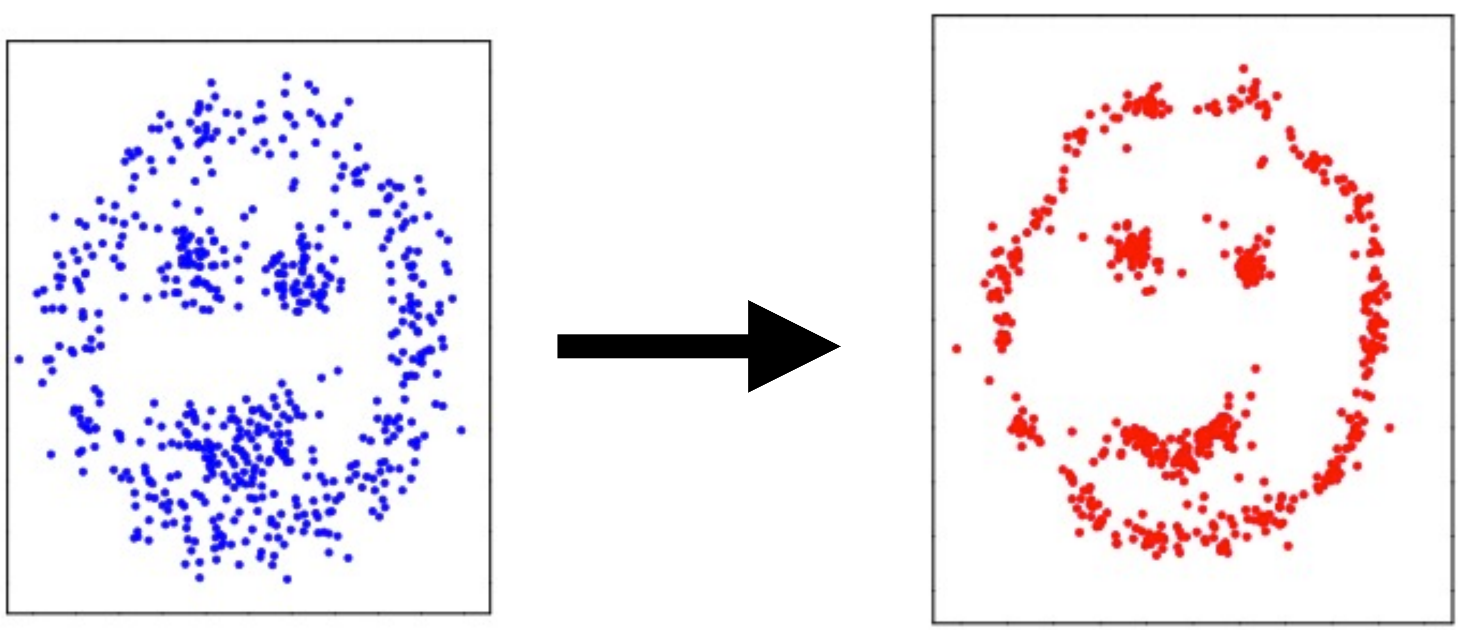
$$\mathscr{F} = -2\log\mathscr{L}(\mathbf{A}\vec{\mu}|\vec{y}) + \delta||\mathbf{L}\vec{\mu}||^2$$

A certain number of choices (L, delta) …

- it's not trivial to keep under control these parameters with the current statistics.

The goal of the regularization is to give a not distorted spectrum

- use the additional fact that distributions are continuous



Why not literature methods?

- **Categories** (SVD):
  - SVD can be extended with categories

$$\vec{y}_{\text{reg}} = \underline{0} \qquad \mathbf{B} = \left(\hat{\mathbf{A}}^{\text{T}}\Sigma^{-1}\hat{\mathbf{A}}\right)^{+}\hat{\mathbf{A}}^{\text{T}}\Sigma^{-1}$$

$$\hat{\mathbf{A}}_{\text{reg}} = \sqrt{\delta}\mathbf{L} \qquad \vec{x}_T = \mathbf{B}\vec{y}$$

$$\Delta\vec{y}_{\text{reg}} = \underline{1} \qquad \Sigma' = \mathbf{B}\Sigma\mathbf{B}^{\text{T}}$$

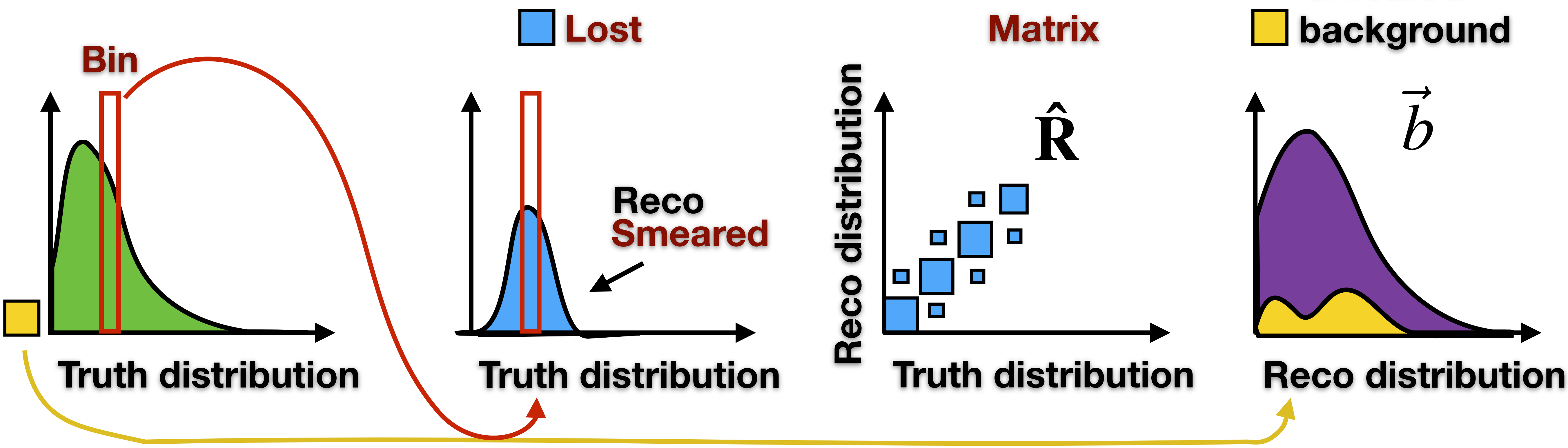
  but signal extraction must be performed before.
  - Bayes:
    - cannot use the “built-in” categories due to the very non-poissonian errors of the mgg continuum:
    - Each category should be unfolded separately and results re-combined later

Signal Extraction:

- These methods wants that signal extraction is performed before
- Systematics and nuisances (eg, m_H) will be just approximations
- Covariance matrix approximation for low yields

Unfolding I

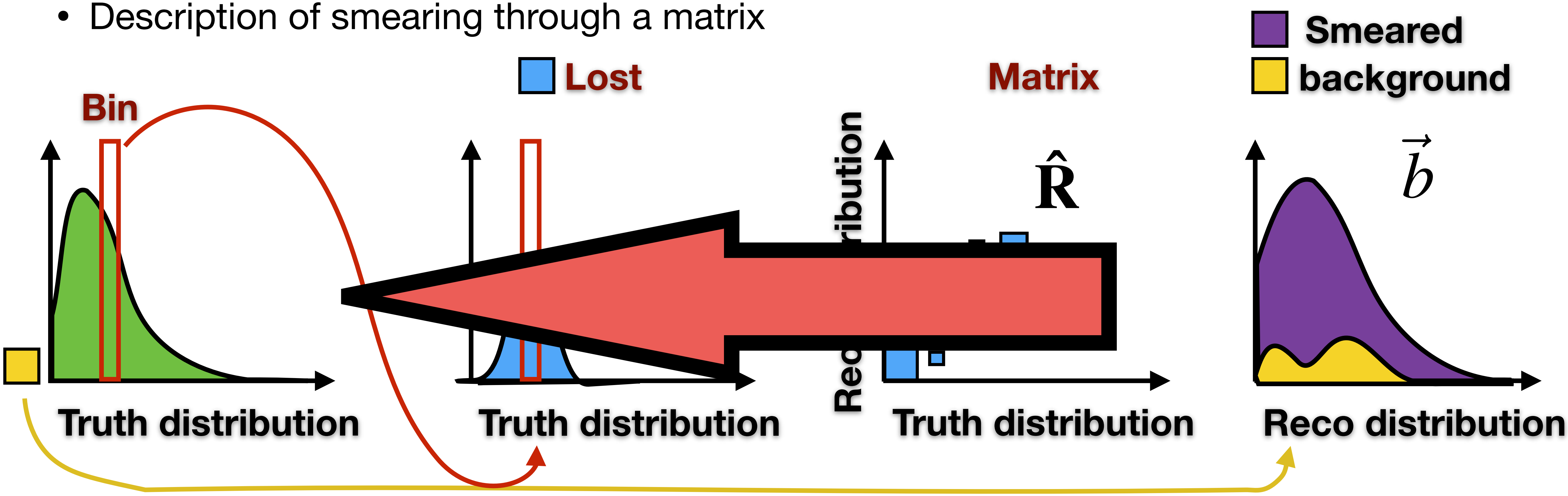
- Undo detector effects
- based on linearity assumption
  - Description of smearing through a matrix



$$x_M^i = \hat{R}^{ij} x_T^j + b^i$$

Unfolding 1

- Undo detector effects
- based on linearity assumption
  - Description of smearing through a matrix



$$x_M^i = \hat{R}^{ij} x_T^j + b^i$$

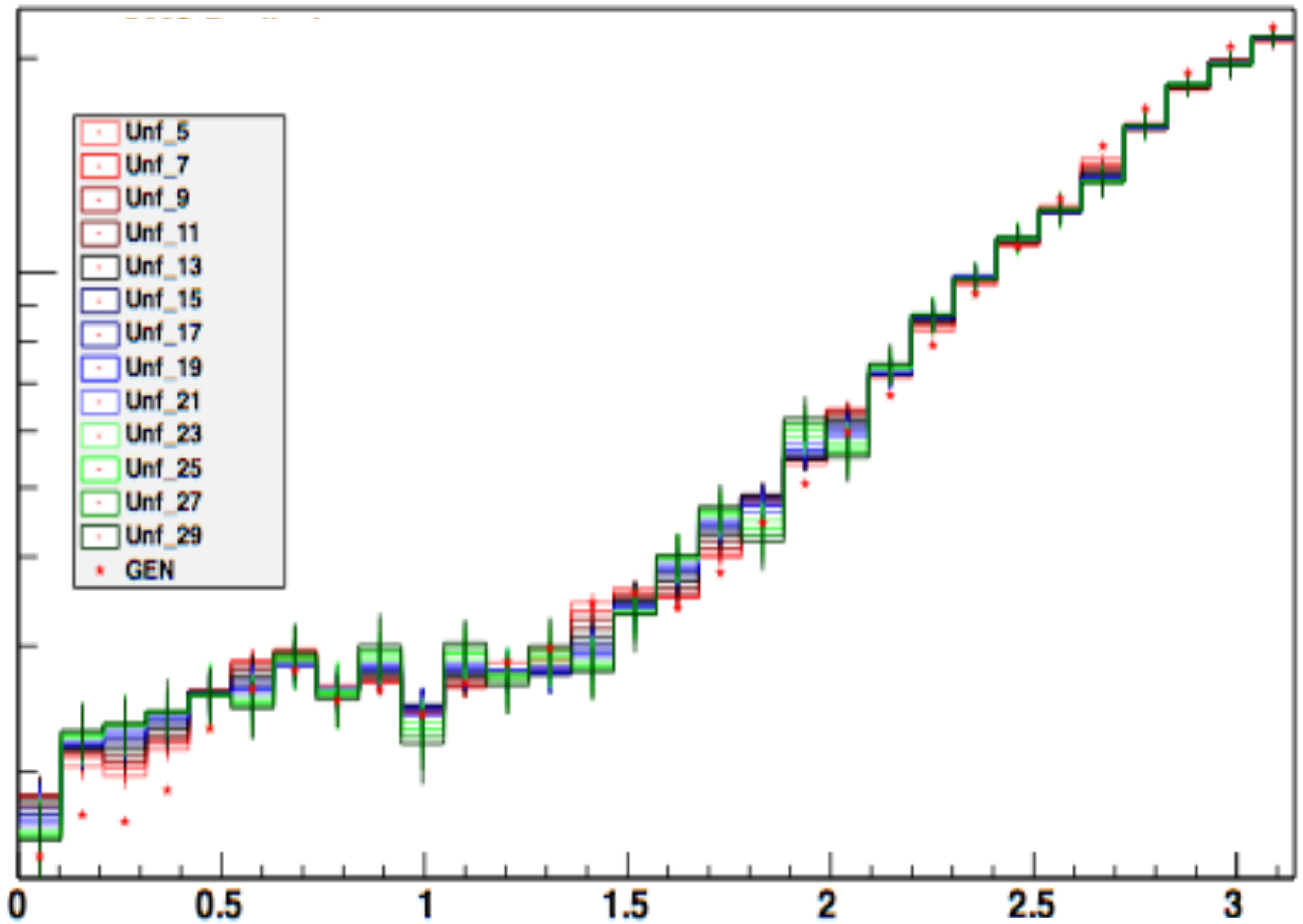
Regularization & Unfolding

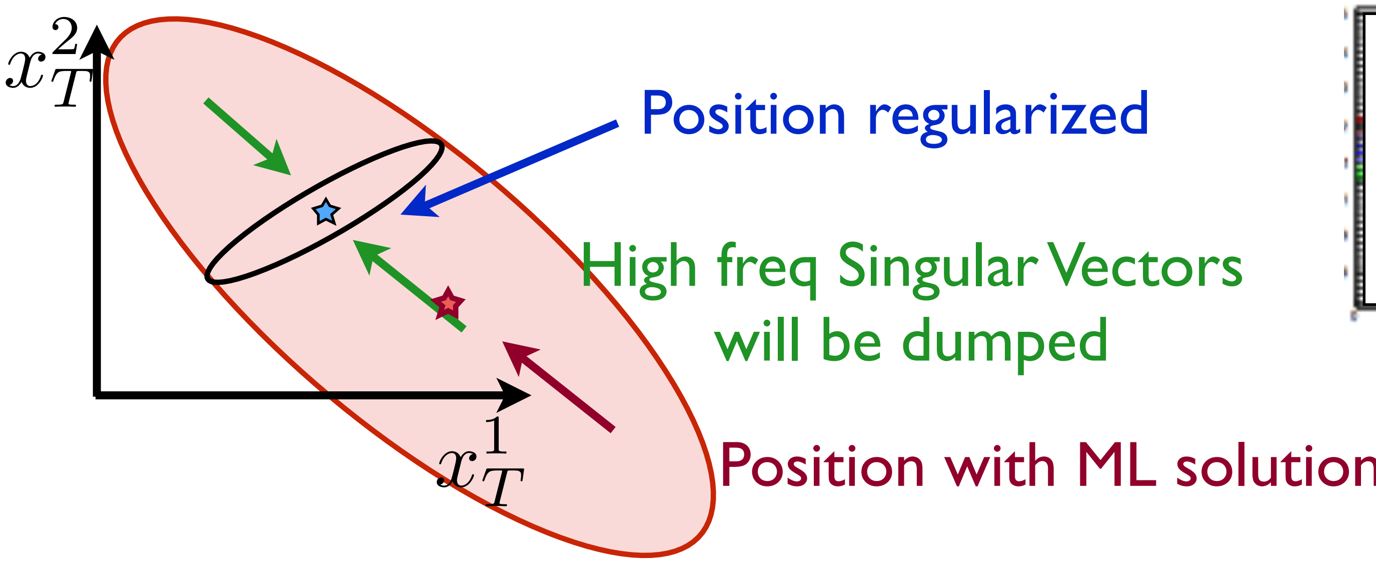
- What is regularization doing ?
- Penalize high fluctuating solutions
  - bias in the “minimum search”

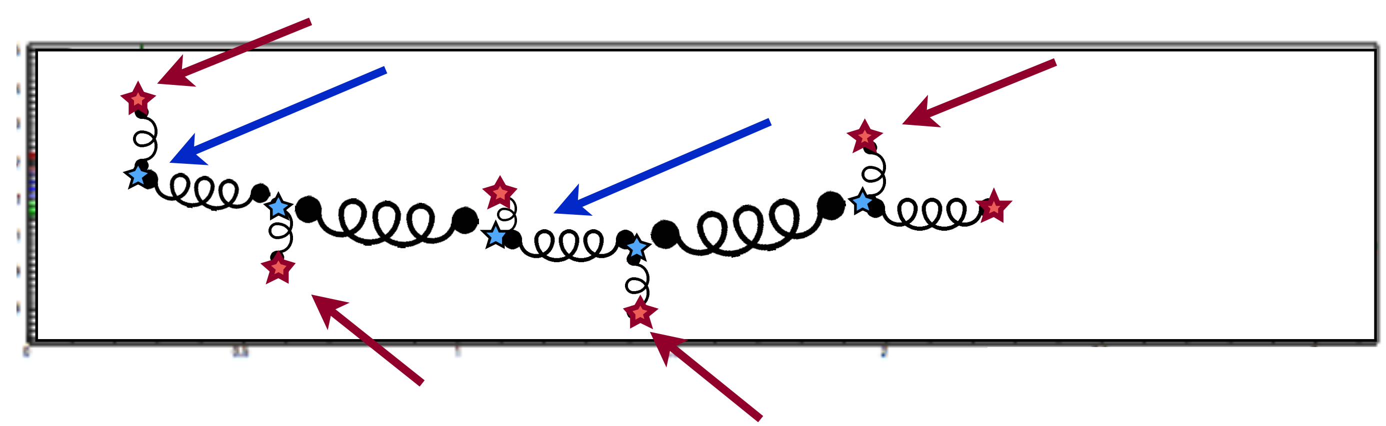
$$\min_{\mu} \|\vec{x}_M - \vec{b} - \mathbf{R}\cdot\vec{\mu}\|^2 + \delta\|\mathbf{L}\cdot\vec{\mu}\|^2$$

$$\vec{x}_T = \hat{\mathbf{R}}^{-1}(\vec{x}_M - \vec{b})$$

- Reduce variance of the final distribution







- Binning is an other way of “regularize”