Introduction to Reliability Engineering

e-learning course, CERN.
Learning Objectives

When you have read through and understood this material, you should be able to:

- **Know the definition of reliability and the factors associated with it.**
- **Understand the concepts of Reliability, Availability and Maintainability Engineering.**
- **Know the techniques for Reliability analysis.**
- **Calculate the failure rate under different conditions.**
- **Understand the failure and reliability curves as a factor of time.**
Reliability Definition

- Generally defined as the ability of a product to perform, as expected, over certain time.
- Formally defined as the **probability** that an item, a product, piece of equipment, or system will **perform** its intended function for a stated period of **time** under specified **operating conditions**.
- **In the simplest sense, reliability means how long an item (such as a machine) will perform its intended function without a breakdown.**

Reliability is performance over time, probability that something will work when you want it to.
The Reliability definition has four important elements:

- **Probability** (A value between 0 and 1, number of times that an event occurs (success) divided by total number trials)
  
  e.g. probability of 0.91 means that 91 of 100 items will still be working at stated time under stated conditions

- **Performance** (Some criteria to define when and how product fails, which also describes what is considered to be satisfactory system operation)

  e.g. amount of beam collisions, etc

- **Time** (system working until time (t), used to predict probability of an item surviving without failure for a designated period of time)

- **Operating conditions**

  These describe the operating conditions (environmental factors, humidity, vibration, shock, temperature cycle, operational profile, etc.) that correspond to the stated product life.
Conflicts with real world.

There are “Real World” conflicts with this definition that we need to keep in mind...

- **Probability** – Customers expect a probability of 1, “It Works”
- **Intended Function** – The product may be used in unintended ways and still be expected to work
- **Under Stated Conditions** – The product may be operated outside of the stated conditions and still be expected to work
- **Prescribed Procedures** – Customers may not have the required tools or skill level and may not follow procedures and still expect the product to work

Customers are looking for Quality over Time
WHY RELIABILITY ENGINEERING?

- Reliability, Availability, Maintainability, Safety and Quality are what the Customer says they are, not what the Engineers or the Designers say they are.
- Companies who control the Reliability of their products can only survive in the business in future as today's consumer is more “intelligent” and product aware.
- Liability for unreliable products can be very high.
- Complexity of products is ever increasing and thus challenge to Reliability Engineering is also increasing.
- Products are being advertised by their Reliability Ratings.

“PRIDE = Put Reliability In Daily Efforts”
When Should Reliability Be Applied?

“From the cradle to the grave.”

i.e. The entire life cycle of the product.
Basic Reliability Terms

Failure - A failure is an event when an item is not available to perform its function at specified conditions when scheduled or is not capable of performing functions to specification.

Failure Rate - The number of failures per unit of gross operating period in terms of time, events, cycles.

MTBF - Mean Time Between Failures - The average time between failure occurrences. The number of items and their operating time divided by the total number of failures. For Repairable Items

MTTF - Mean Time To Failure - The average time to failure occurrence. The number of items and their operating time divided by the total number of failures. For Repairable Items and Non-repairable Items

Hazard - The potential to cause harm. Harm including ill health and injury, damage to property, plant, products or the environment, production losses or increased liabilities.

Risk - The likelihood that a specified undesired event will occur due to the realisation of a hazard by, or during work activities or by the products and services created by work activities.
Basic Reliability Terms

**Maintainability** - A characteristic of design, installation and operation, usually expressed as the probability that an item can be retained in, or restored to, specified operable condition within a specified interval of time when maintenance is performed in accordance with prescribed procedures.

**MTTR - Mean Time To Repair** - The average time to restore the item to specified conditions.

**Maintenance Load** - The repair time per operating time for an item.

**Availability** - A measure of the time that a system is actually operating versus the time that the system was planned to operate.

It is the probability that the system is operational at any random time t.

**Supportability** - The ability of a service supplier to maintain the Plant inbuilt reliability and to perform scheduled and unscheduled maintenance according to the Plant inbuilt maintainability with minimum costs.
Reliability: an introspection

Try to formulate the answers for these questions,

- What are the intended functions of your system/product?
- What are the specified operating conditions?
- What is time $t$ at which you want to estimate reliability?
- What is the reliability? Do you know?
- What is expected by the users?
Quality, Reliability and Safety

• Reliability can be considered as “Quality over time”. Customers frequently use the terms “quality” and “reliability”. We need to understand what they expect.

• Measurement of reliability is related to failure rates, number of failures, warranty cost etc. Thus, reliability is experienced by the customers when they use the product.

• Quality Level is measured in terms of defect levels (such as ppm) when the product is received as new.

• Quality and reliability both can have significant impact on Safety.
Quality, Reliability and Safety

• Quality defects and failures both can adversely affect safety of user, bystanders and equipment.
• Some quality defects can lead to unreliable and/or unsafe product.
• Some examples of how unreliably can affect safety:
  – Failure of automobile steering system, brake system, axles etc, can result in serious accidents.
  – Short circuit in electrical equipment can result in a shock or death.
  – Failure of safety valve in a pressure cooker, leakage of regulator of an LPG cylinder can result in an explosion.
  – Poor reliability of a bridge can result in an accident and disaster
• However, all failures are not safety issues and all safety issues are not due to failures.
• As Reliability Engineering is concerned with analyzing failures and providing feedback to design and production to prevent future failures, it is only natural that a rigorous classification of failure types must be agreed upon.

• Reliability engineers usually speaks of

**Failures Causes**

**Failure Modes**

**Failure Mechanisms**

• **Reliability measurement** is based on the failure rate

\[
\text{Failure rate} = \frac{\text{Items Failed}}{\text{Total Operating Time}}
\]

• Some products (Non-repairable) are scrapped when they fail e.g. bulb

• Other products (Repairable) are repaired e.g. washing machine.
How Do Products Really Fail

- DESIGNED TO FAIL
- MANUFACTURED TO FAIL
- ASSEMBLED TO FAIL
- SCREENED TO FAIL
- STORED TO FAIL
- TRANSPORTED TO FAIL
- OPERATED TO FAIL

Two common types of failures:
1. Sudden failure (no indicators): Stress exceeds strength ....
2. Degradation (gradual wear out): degradation indicator such as crack growth, change of resistance, corrosion, ...
   This is ideal for Condition-Based Maintenance

Other failures may occur because of human errors.
Failure rate over the life of a product

The failure rate is expected to vary over the life of a product – *’Bathtub Curve’*
Bathtub Curve.

A-B Early Failure / Infant mortality / Debugging / Break-in

- ‘Teething’ problems. Caused by design/material flaws
  *Eg: Joints, Welds, Contamination, Misuse, Misassembly*

B-C Constant Failure / Useful life.

- Lower than initial failure rate and more or less constant until end of life

C-D End of life failure / Wear out phase.

- Failure rate rises again due to components reaching end of life
  *Eg: Corrosion, Cracking, Wear, Friction, Fatigue, Erosion, Lack of PM*
<table>
<thead>
<tr>
<th>Phase</th>
<th>Failure Rate</th>
<th>Possible Causes</th>
<th>Possible improvement actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burn-in (A-B)</td>
<td>Decreasing (DFR)</td>
<td>Manufacturing defects, welding, soldering, assembly errors, part defects, poor QC, poor workmanship, etc</td>
<td>Better QC, Acceptance testing, Burn-in testing, screening, Highly Accelerated Stress Screening, etc.</td>
</tr>
<tr>
<td>Useful Life (B-C)</td>
<td>Constant (CFR)</td>
<td>Environment, random loads, Human errors, chance events, ‘Acts of God’, etc</td>
<td>Excess Strength, redundancy, robust design, etc</td>
</tr>
<tr>
<td>Wear-out (C-D)</td>
<td>Increasing (IFR)</td>
<td>Fatigue, Corrosion, Aging, Friction, etc.</td>
<td>Derating, preventive maintenance, parts replacement, better material, improved designs, technology, etc.</td>
</tr>
</tbody>
</table>
Managing Reliability

Reliability management is concerned with performance and conformance over the **expected life** of the product.

A systems approach to planning for, designing in, verifying, and tracking the reliability of products throughout their life to achieve reliability goals.

- Reliability of a system is often specified by the failure rate \( \lambda \).
- \( \lambda = \text{failures per time unit (in a collection of systems)} \)
- For most technical products (incl. embedded systems), \( \lambda(t) \) is a "bath-tub curve":

"Bathtub Curve"

![Bathtub Curve Diagram]
Reliability Characteristics

Non-Repairable Systems:
Reliability = Availability
Failure Rate
MTTF
Time to First Failure
MRL (Mean Residual or remaining Life)

Repairable Systems:
Availability .... (Function of Reliability and Maintainability)
Failure Rate and Repair Rate
MTBF
MRL (Economic Justification)
Failure Rate for Repairable and Non-repairable systems

MTBF \[ \theta = \frac{\text{Total time}}{\text{Total Number of failures}} \]
Average Failure Rate \[ \lambda = \frac{1}{\theta} \Rightarrow \lambda \theta = 1 \]

**Example:**
300 cars have accumulated 45000 hours, 10 failures are observed. What is the MTBF? What is the failure rate?

**Note:** considering Car as repairable system, Use MTBF

MTBF = \( \frac{45000}{10} = 4500 \) hours.

Average Failure rate \( \lambda = \frac{10}{45000} = 0.00022 \) per hour.

Five oil pumps were tested with failure hours of 45, 33, 62, 94 and 105. What is the MTTF and failure rate?

**Note:** considering pumps as non repairable systems, Use MTTF.

MTTF = \( \frac{45+33+62+94+105}{5} = 67.8 \) hours

Failure rate \( \lambda = \frac{5}{45+33+62+94+105} = 0.0147 \) per hour.

*Note that MTTF is a reciprocal of failure rate.*
Example

10 components were tested. The components (not repairable) failed as follows:
Component 1,2,3,4,5 failed after 75, 125, 130, 325, 525 hours. Find the failure rate and mean time till failure.

Solution:
No. of failures = 5
Total operating time = 75 + 125 + 130 + 325 + 525 + 5*525 = 3805

**Failure rate** \( \lambda = \frac{5}{3805} = 0.001314 \)

**Mean time till failure** = \( \frac{1}{\lambda} \)

=\( \frac{1}{0.001314} \)

= 761.04 hours.
**Example:**

50 components are tested for two weeks. 20 of them fail in this time, with an average failure time of 1.2 weeks.

What is the mean time till failure assuming a constant failure rate?

**Answer:**

No. Of failures = 20

Total time = 20*1.2 + 30*2 = 84 weeks

Failure rate = 20/84 = 0.238/week

Mean time till failure is estimated to be = (1/failure rate)

= 1/0.238 = 4.2 weeks.
Calculating failure rate from historical information.

- We can take in use, known historical information from records at site to estimate the failure rates of various components.

- For example, if you had 5 hydraulic pumps in standby mode and each ran for 2000 hours in standby and 3 failed during the standby time

The failure rate during standby mode is:

Total standby hours = 5(2000 hours) = 10000 hours
Failure rate in standby mode = 3 / 10,000
= 0.0003 failures per hour
Important Analytical Functions In Reliability Engineering

1. FAILURE PROBABILITY DENSITY FUNCTION
2. FAILURE RATE FUNCTION
3. RELIABILITY FUNCTION
4. CONDITIONAL RELIABILITY FUNCTION
5. MEAN LIFE FUNCTION
Reliability function and failure rate

For a pdf $f(x)$ for the time till failure, define:

**Reliability function**
Probability of surviving at least till age $t$, i.e., that failure time is later than $t$

$$R(t) = P(T > t) = \int_{t}^{\infty} f(t)dt = 1 - \int_{t}^{\infty} f(t)dt = 1 - F(t)$$

$$F(t) = \int_{0}^{t} f(t)dt$$ is the cumulative distribution function.

**Failure rate**
This is failure rate at time $t$ given that it survived until time $t$: 

$$\phi(t) = \frac{f(t)}{R(t)}$$
Relationship between failure density and reliability

\[ f(t) = -\frac{d}{dt} R(t) \]

Relationship Between \( h(t), f(t), F(t) \) and \( R(t) \), \( h(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{1 - F(t)} \)

Remark: The failure rate \( h(t) \) is a measure of proneness to failure as a function of age, \( t \).

Relationship Between \( MTBF/MTTF \) and Reliability

\[ MTBF = MTTF = \int_{0}^{\infty} R(t) dt \]
Trial data shows that 105 items failed during a test with a total operating time of 1 million hours. (For all items i.e. both failed and passed). Also, find the reliability of the product after 1000 hours i.e. \( t = 1000 \)

**The failure rate** \[ \lambda = \frac{105}{1000000} = 1.05 \times 10^{-4} \text{ per hour} \]

**Reliability at 1000 hours** \[ R(1000) = e^{-\lambda t} \]

\[ R(1000) = e^{-(1.05 \times 10^4 \times 1000)} = 0.9 \]

Therefore the item has a 90\% chance of surviving for 1000 hours.
**Example**

The chart below shows operating time and breakdown time of a machine.

<table>
<thead>
<tr>
<th>Operating time</th>
<th>Down time</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.2</td>
<td>2.5</td>
</tr>
<tr>
<td>6.1</td>
<td>6.1</td>
</tr>
<tr>
<td>7.1</td>
<td>7.1</td>
</tr>
<tr>
<td>24.4</td>
<td>4.2</td>
</tr>
<tr>
<td>4.2</td>
<td>35.3</td>
</tr>
<tr>
<td>35.3</td>
<td>1.8</td>
</tr>
<tr>
<td>46.7</td>
<td>46.7</td>
</tr>
</tbody>
</table>

a) Determine the MTBF.

Solution:

Total operating time = 20.2 + 6.1 + 24.4 + 4.2 + 35.3 + 46.7

= 136.9 hours
\[ \lambda = \frac{4}{136.9} = 0.02922 \]

Therefore:
\[ \theta = MTBF = \frac{1}{\lambda} = 34.22 \text{ hours} \]

b) What is the system reliability for a mission time of 20 hours?

\[ R = e^{-\lambda t} \quad t = 20 \text{ hours} \]

\[ R = e^{-(0.02922)(20)} \]

\[ R = 55.74\% \]
Maintainability

- *Maintainability* is the measure of the ability of a system or item to be retained or restored to a specified condition when maintenance is performed by qualified personnel using specified procedure and resources.

- *Maintainability* can be measured with Mean Time To Repair (*MTTR*), *MTTR* is average repair time and is given by

  \[
  MTTR = \frac{\text{Total Maintenance Down Time}}{\text{Total Number of Maintenance Actions}}.
  \]

- **MTBMA** is Mean Time Between Maintenance Actions including preventive and corrective maintenance tasks.
OBJECTIVES OF MAINTAINABILITY

1. To influence design to achieve case of maintenance thus reducing maintenance time & cost.

2. To estimate the downtime for maintenance which, when compared with allowable downtime, determines whether redundancy is required to provide acceptable continuity of a critical function.

3. To estimate system availability by combining maintainability data with reliability data.

4. To estimate the man-hours and other resources required for performing maintenance, which are useful for determining the costs of maintenance and for maintenance planning.
ADVANTAGES OF MAINTAINABILITY PREDICTION

1. It highlights areas of poor maintainability which require product improvement, modification or change of design.

2. It permits user to make an early assessment of whether the predicted downtime, the quality, quantity of personnel, tools and test equipment are adequate and consistent with the needs of system operational requirements.
Maintainability is a function of the design of the system, the personnel available at the necessary skill levels, the procedures available to perform the maintenance, the test equipment available, and the environment in which the maintenance must be performed.

Maintainability is measured by Mean-Time-To-Repair (MTTTR).

\[
\text{MTTR} = \frac{\sum_{i=1}^{n} \lambda_i R_{p_i}}{\sum_{i=1}^{n} \lambda_i}
\]

- \( n \) = number of subsystems
- \( \lambda_i \) = Failure rate of the \( i \)th system
- \( R_{p_i} \) = Repair Time for the \( i \)th unit
Probability of Repair Within the Allowable Downtime

To calculate the probability of performing a maintenance action within an allowable time interval use:

\[ M(t) = 1 - e^{-\frac{t}{MTTR}} \]

Where:

- \( t \) = Allowable downtime
- \( MTTR \) = Expected downtime (MTTR)
Example: What is the probability of completing an action within 5 hours if the MTTR = 7 hours?

Solution: \( M(t) = 1 - e^{-t/MTTR} = 1 - e^{5/7} \)

\[ = 1 - 0.4895 = 0.5105 \]
There is approximately a 51% probability of completion.

Mean-Time-To-Repair

The total corrective maintenance time divided by the total number of corrective maintenance actions during a given period of time.
Availability

- A measure of the degree to which an item is in the operable and committable state at the start of a mission, when the mission is called for at an unknown time.

- Availability is thus defined as the probability that an item will be available when required or as the proportion of total time that an item is available for use.

The three common measures of availability are:

1. Inherent Availability ($A_i$)
2. Achieved Availability ($A_A$)
3. Operational Availability ($A_o$)
Inherent Availability \((A_I)\)

This is the ideal state for analyzing availability. The only considerations are the MTBF and the MTTR. This measure does not take into account the time for preventive maintenance and assumes repair begins immediately upon failure of the system.

This can also be defined as steady – state availability.

The measure for inherent (potential) availability \((A_I)\) is:

\[
A_I = \frac{\mu}{\lambda + \mu} = \frac{MTBF}{MTTR + MTBF}
\]

Where:
- \(\lambda\) = Failure rate = \(1/MTBF\)
- \(\mu\) = Repair rate = \(1/MTTR\)

Example: A system has an MTBF of 2080 hours and a MTTR of 10 Hours. What is the inherent availability of the system?

Solution:
\[
A_I = \frac{2080}{10 + 2080} = 0.9952 \text{ or } 99.52\%
\]
Achieved Availability ($A_A$)

Achieved availability is somewhat more realistic in that it takes preventive maintenance into account as well as corrective maintenance. The assumption here is that, as in $A_I$, there is no loss of time waiting for the maintenance action to begin.

The measure for achieved (final) availability ($A_A$) is:

$$A_A = \frac{MTBMA}{MTBMA + MMT}$$

Where: $MTBMA$ is the mean time between maintenance actions both preventive and corrective.

$MTM$ is the mean Maintenance Action Time, and $MMT$ is further decomposed into the effects of preventive and corrective maintenance and is given as:
\[
M_{\text{MT}} = \frac{F_c M_{\text{ct}} + F_p M_{\text{pt}}}{F_c + F_p}
\]

Where: \( F_c \) is the number of corrective maintenance actions per 1000 hours

\( F_p \) is the number of preventive maintenance actions per 1000 hours

\( M_{\text{ct}} \) is the mean active time for corrective maintenance (MTTR)

\( M_{\text{pt}} \) is the mean active time for preventive maintenance
**Example** : A system has a MTBMA of 110 hours, a $F_c$ of $\frac{1}{2}$, a $F_p$ of 1, and $M_{CT}$ of 2 hours, and $M_{PT}$ of 1 hour. What is $A_A$?

**Solution** :

First calculate MMT as :

$$\text{MMT} = \frac{(1/2) \times 2 + (1) \times 1}{1 + \frac{1}{2}} = 1.33$$

Then determine $A_A$ :

$$A_A = \frac{110}{110 + 1.33} = 0.988 \text{ or } 98.8\%$$
**Operational Availability (A₀)**

This is what generally occurs in practice. Operational availability takes into account that the maintenance response is not instantaneous, repair parts may not be in stock as well as other logistics issues.

The measure of **operational (actual) availability** $A_0$ is:

$$A_0 = \frac{MTBMA}{MTBMA + MDT}$$

Where: MDT is mean down time.

**Example**: A system has a MTBMA of 168 hours and a MDT of 4 hours. What is $A_0$?

**Solution**: 

$$A_0 = \frac{168}{168 + 4} = 0.977 \text{ or } 97.7\%$$
Availability for Constant Failures Rates and Mean Repair Rates

The steady – state availability was given earlier as:

\[ A = \frac{\mu}{\lambda + \mu} = \frac{\text{MTBF}}{\text{MTTR} + \text{MTBF}} \]

The instantaneous availability, (probability) that an item will be available at time \( T \) is:

\[ A = \frac{\mu}{\lambda + \mu} + \frac{\mu}{\lambda + \mu} \ast \text{Exp} \left[ - (\lambda + \mu) t \right] \]

When \( t \) is large, the expression reduces to:

\[ A = \frac{\mu}{\lambda + \mu} \]
Example: Given exponential failure rates and repair rates of $\lambda = 5$, $\mu = 3$, determine the instantaneous availability at 0.2 hours:

Solution:

$$A = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} \exp \left[ - (\lambda + \mu) t \right]$$

$$= \left\{ \frac{3}{8} + \left( \frac{5}{8} \right) e^{(-1.6)} \right\} = .501$$

Example: The instantaneous unavailability for the previous example is:

Solution: \[ A_\mu = 1 - \left\{ \frac{\mu}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu} \exp \left[ - (\lambda + \mu) t \right] \right\} \]

$$= 1 - \left\{ \frac{3}{8} - \left( \frac{5}{8} \right) e^{(-1.6)} \right\} = .499$$
Other configurations to consider:

Series Configuration

\[
A = \prod_{i} A_i = \prod_{i=1}^{n} \frac{\mu_i}{\lambda_i + \mu_i}
\]

Parallel Configuration

\[
A = 1 - \prod_{i=1}^{n} \frac{\lambda_i}{\lambda_i + \mu_i}
\]
Example: Given exponential failure rates and repair times of $\lambda = 5, \mu = 3$, if two identical units are in series, determine the steady state (A): 

Solution:
\[
A = \prod \frac{\mu_i}{\lambda_i + \mu_i} = \left( \frac{3}{5 + 3} \right) \left( \frac{3}{5 + 3} \right) = \frac{9}{64} = 0.14063
\]

Example: If the same two units in the prior example are in parallel:

Solution:
\[
A = 1 - \prod_{i=1}^{n} \frac{\lambda_i}{\lambda_i + \mu_i} = 1 - \left( \frac{\lambda_1}{\lambda_1 + \mu_1} \right) \left( \frac{\lambda_2}{\lambda_2 + \mu_2} \right) = 1 - \left( \frac{5}{8} \right) \left( \frac{5}{8} \right) = 1 - \frac{25}{64} = 0.6094
\]
Distribution Analysis

*Distribution Analysis* involves identifying the statistical distribution that the field data follows and estimates of reliability and confidence intervals based on the identified distribution.

- The most basic types of Distribution Analysis involve having a complete set of data for the population or sample population and knowing all failure times (uncensored) or failure times and times at which the field study was ended (censored).

- Specialized software can be used to perform this analysis.

*You will study this topic in detail, in the RAMS training courses.*
Data Collection

Minimum Data Needs for each item

- Design Configuration
- When Produced
- How Many Produced
  - Total Population
  - Or Sample Population being studied
- When Entered Service
- When Failed
- How Many Failed

Additional Data Needed to better identify Failure Modes and Corrective/Preventive Action

- Usage Application and Environment
- How Failed – Failure Mode
- Immediate Action Taken to Repair or Replace
- Time to Repair or Replace
- Root Cause of Failure
Suggested Reading. (REFERENCES).


