Application of an MT method to CALET e/p separation

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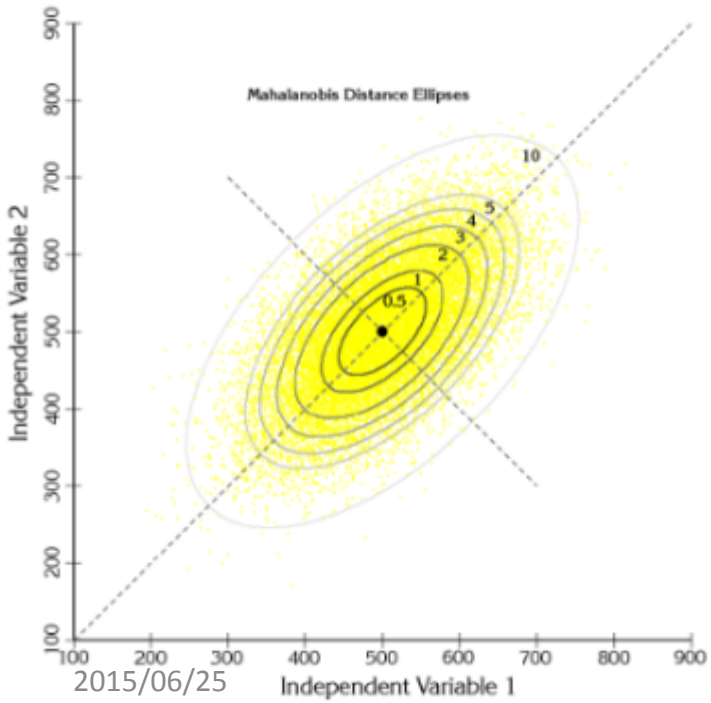
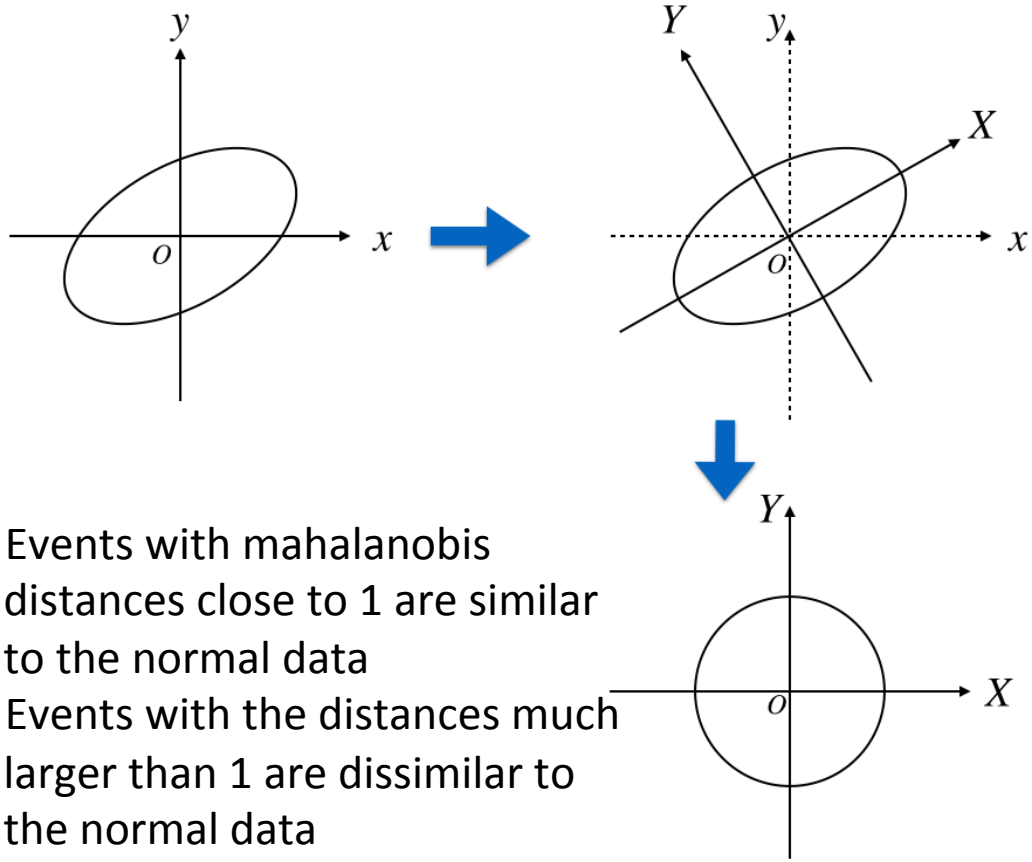
Event classification and background rejection for cosmic-ray observations

- There are various data mining techniques:
  - Artificial Neural Network, Boosted Decision Tree, etc.
- A Mahalanobis Taguchi (MT) method is an alternative to these data mining techniques.

Mahalanobis Taguchi (MT) method

- Constructing one standard multidimensional unit space from characteristic variables of normal data (e.g., electron data)
- Applying test data to the unit space
- Calculating each Mahalanobis distance of test events on the unit space





- Events with mahalanobis distances close to 1 are similar to the normal data
- Events with the distances much larger than 1 are dissimilar to the normal data

How to calculate the Mahalanobis Distance

Mahalanobis distance

$$D^2 = (\frac{1}{k}) Z_i^T C^{-1} Z_i$$

where Z_i = standardized vector obtained by values of $X_i (i = 1, \ldots, k)$

$Z_i = (X_i - m_i)/s_i$

X_i =value of i-th characteristic

m_i =mean of i-th characteristic

s_i =standard deviation of i-th characteristic

k =the number of characteristics / variables

T =transpose of the vector

C^{-1} =inverse of the correlation matrix

Procedure

- In the application of an MT method to CALET e/p separation, electron data are used to construct the unit space as normal data.
- The normal data of electrons correspond to “training data” in machine learning algorithm.
  - We can “train” without proton data.
- In addition, by using proton data, it is possible to select significant characteristic variables.

Selection of characteristic variables

- Using an orthogonal array and SN ratio

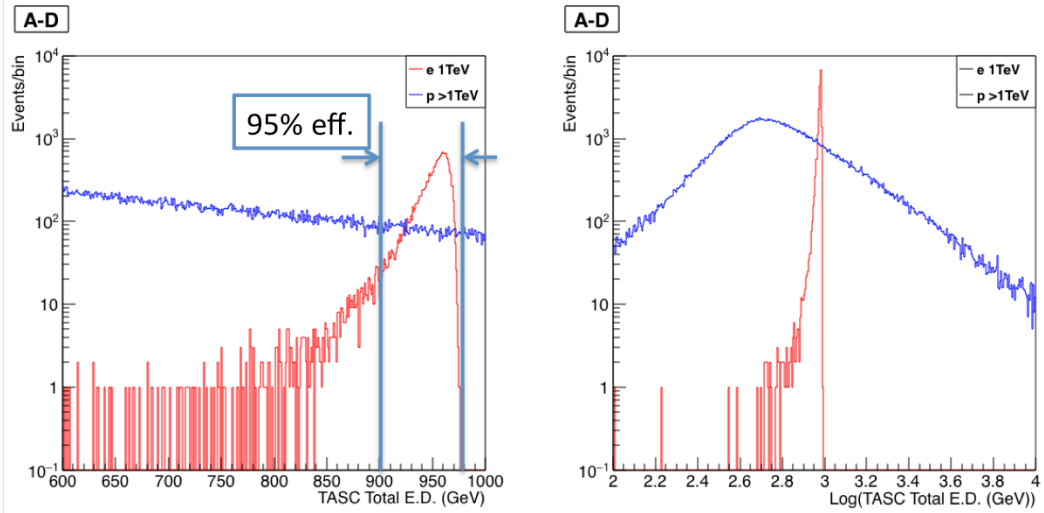
SN ratio: $$\eta(dB) = -10\log\sigma^2$$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{D_i^2}$$

- Evaluating the contribution of each attribute and selecting the characteristic variables
  - Proton events are used for the selection.

Simulation Data and Preselection

- CALET CAD model Rev.15 Epics simulation data
- Electron 1TeV, Proton $E^{-2.7}$ (>1TeV) (dpmjet3)
- Preselection:
  - High Energy Shower Trigger
  - Particle incident direction inside acceptance (Geometric conditions A-D)
  - TASC Energy Deposit inside energy bin (903-975GeV) to be 95% efficiency



Characteristic variables in this study

- Characteristic variables:
  - Lateral Spreads of CHD, IMC, and TASC
  - Energy Deposit of each CHD, IMC layer
  - E.D. Fraction of each TASC layer
  - Lateral Spread of each TASC layer
  - TASC_rms^2*E.D.F. of each TASC layer
  - IMC longitudinal profile fitting => p0, p1, red-χ^2
  - TASC longitudinal shower profile fitting => t_{max}, b, red-χ^2

$$EDF_i = \frac{\sum_j \Delta E_{i,j}}{\sum_i \sum_j \Delta E_{i,j}}$$

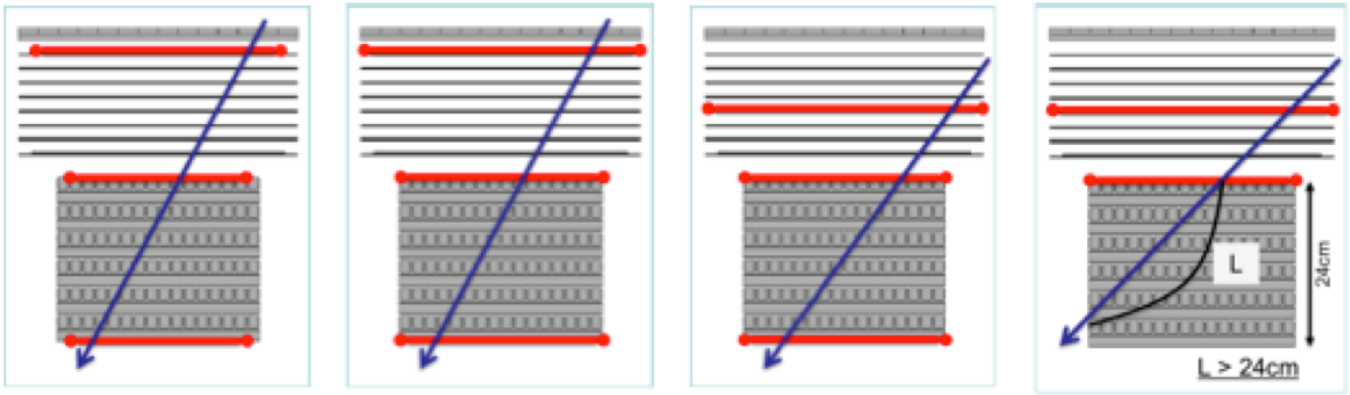
$$RMS_i = \sqrt{\frac{\sum_j \left(\Delta E_{i,j} \times (x_{i,j} - x_{i,c})^2\right)}{\sum_j \Delta E_{i,j}}}$$

$$\frac{dE}{dt} = p_0 t^2 + p_1$$

$$\frac{dE}{dt} = \frac{E_0 b}{\Gamma\left(bt_{\max} + 1\right)} (bt)^{bt_{\max}} e^{-bt}$$

Geometric Conditions: A-D

Applying the MT method for each geometric condition



A B C D

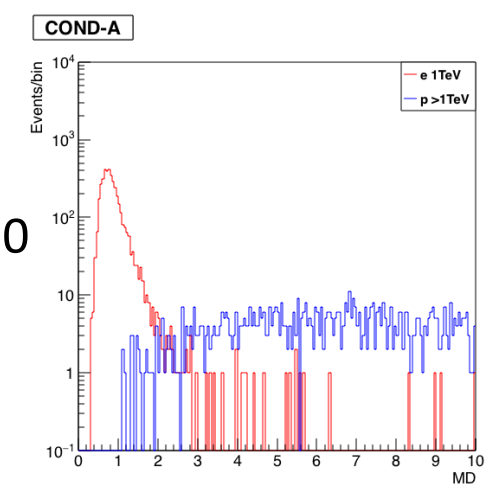
e/p separation: Geometric Condition A

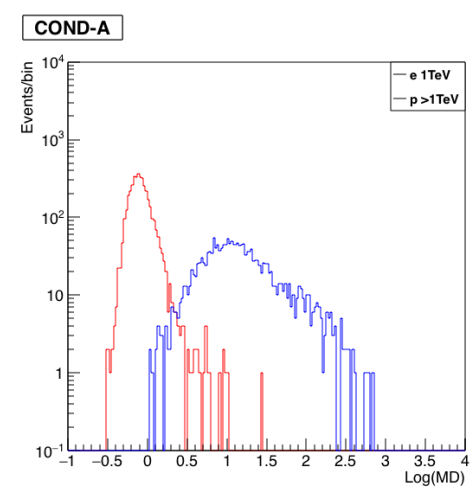
- For constructing the Unit Space:
  2386 electron events
- For selecting variables:
  336 proton events

- Test sample:
  3970 electron events
  1801 proton events

- In the case of the threshold D = 1.10
  Survived elctron events: 3419
  Survived proton events: 0

Selected variables	SN-<SN>
TASC12_E.D.Fraction	0.693
TASC12_rms^2*E.D.F.	0.663
TASC11_rm2^2*E.D.F.	0.304
TASC11_E.D.Fraction	0.251
TASC9_E.D.Fraction	0.185
TASC1_rms^2	0.170
…	
(18 variables in total)	





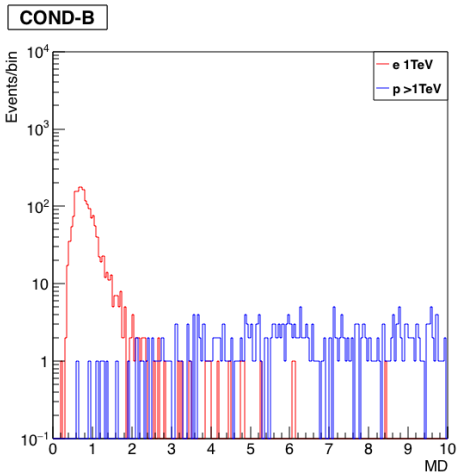
e/p separation: Geometric Condition B

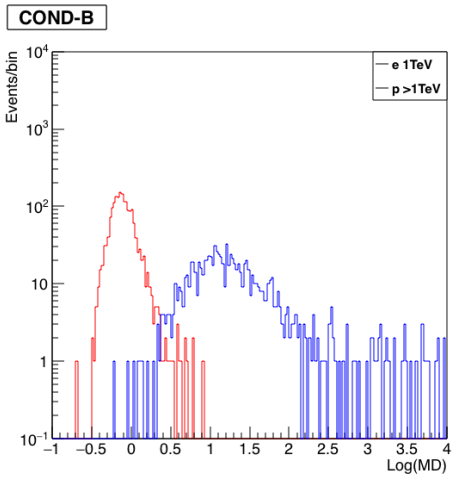
- For constructing the Unit Space:
  1681 electron events
- For selecting variables:
  313 proton events

- Test sample:
  1763 electron events
  918 proton events

- In the case of the threshold D = 1.23
  Survived elctron events: 1607
  Survived proton events: 3

Selected variables	SN-<SN>
TASC12_E.D.Fraction	0.834
TASC12_rms^2*E.D.F.	0.547
TASC11_rm2^2*E.D.F.	0.363
TASC11_E.D.Fraction	0.260
IMC_fit_p0	0.253
CHD_E.D.	0.179
…	
(11 variables in total)	





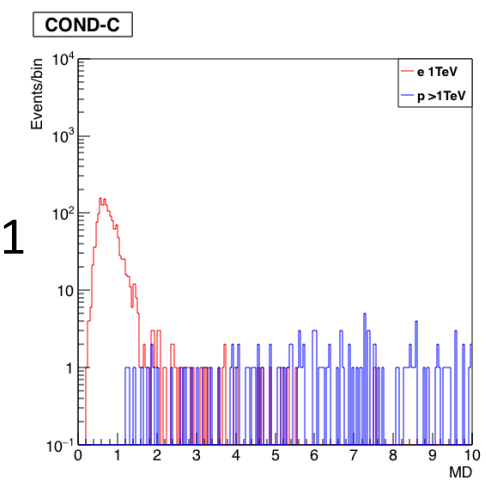
e/p separation: Geometric Condition C

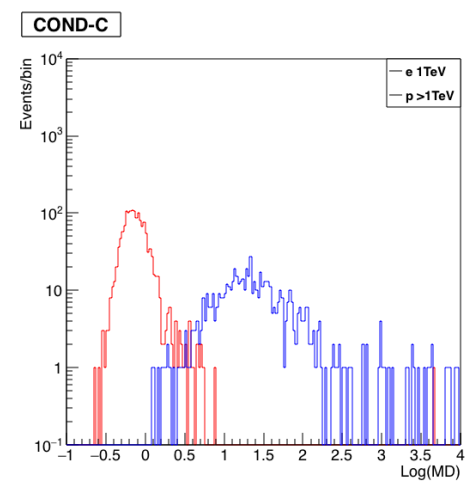
- For constructing the Unit Space:
  1302 electron events
- For selecting variables:
  299 proton events

- Test sample:
  1460 electron events
  614 proton events

- In the case of the threshold D = 1.21
  Survived elctron events: 1341
  Survived proton events: 0

Selected variables	SN-<SN>
TASC12_E.D.Fraction	0.891
TASC12_rms^2*E.D.F.	0.717
TASC11_rms^2*E.D.F.	0.408
TASC11_E.D.Fraction	0.370
TASC10_rms^2*E.D.F.	0.223
TASC9_rms^2*E.D.F.	0.197
…	
(12 variables in total)	





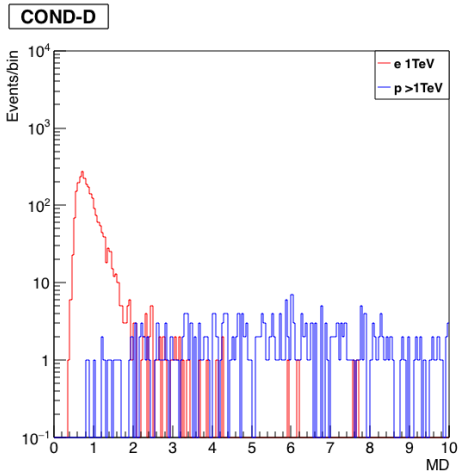
e/p separation: Geometric Condition D

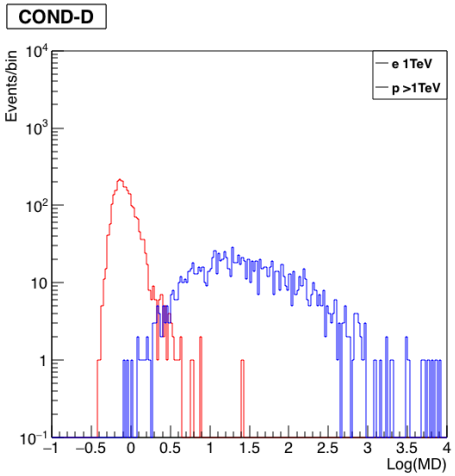
- For constructing the Unit Space:
  1875 electron events
- For selecting variables:
  345 proton events

- Test Sample:
  2358 electron events
  1172 proton events

- In the case of the threshold D = 1.20
  Survived elctron events: 2079
  Survived proton events: 3

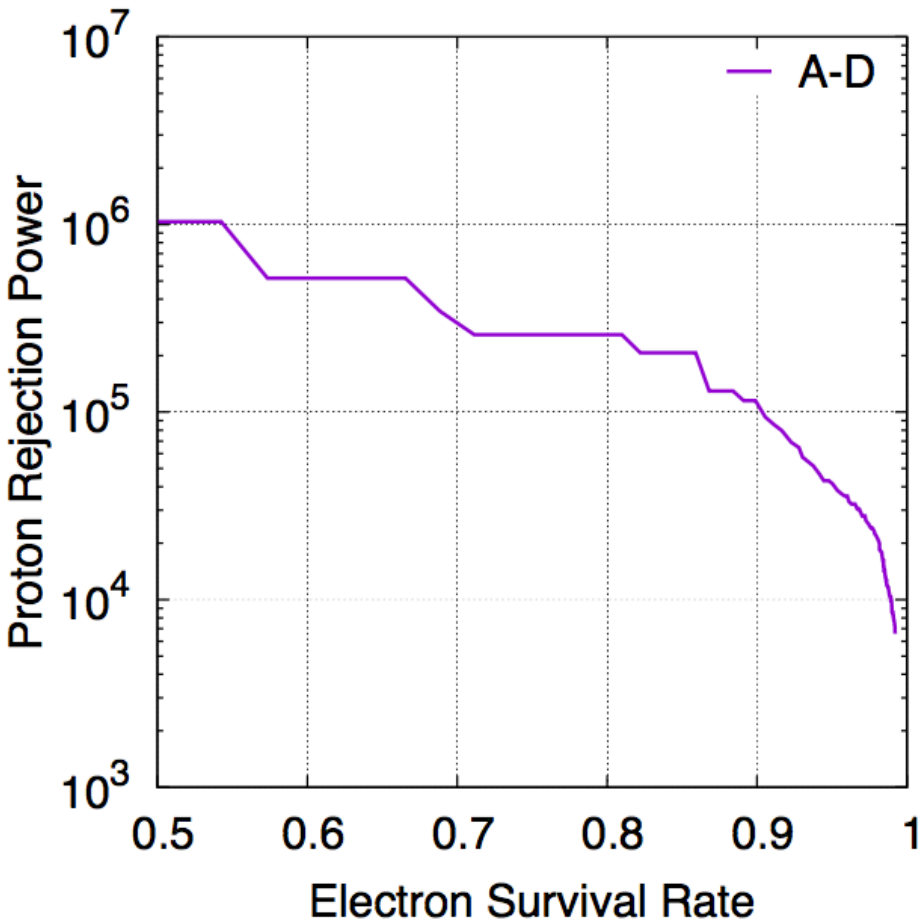
Selected variables	SN-<SN>
TASC11_rms^2*E.D.F.	0.523
TASC12_rms^2*E.D.F.	0.473
TASC12_E.D.Fraction	0.447
TASC11_E.D.Fraction	0.278
TASC9_rms^2*E.D.F.	0.226
TASC10_E.D.Fraction	0.212
…	
(25 variables in total)	





Proton Rejection Power

Proton Rejection Power = Incident protons 1.03×10^6/Survived protons



Electron Survival Rate = Survived electrons divided by electrons 9.55×10^3 in TASC energy bin 903-975GeV

The same threshold distance is used in geometric conditions A-D

Summary

- Application of an MT method to CALET e/p separation:
  - For the training, the number of electron events is relatively small, possibly without proton events.
  - e.g. Proton Rejection Power is 1×10^5 at the electron survival rate of 90%.
- Refining the merit variables for the MT method application should be in progress.