Transverse emittance growth due to CC RF noise, and requirements for the CC RF

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- Analytical derivations
- Simulations
- Sources of RF noise
- Resulting specifications for the LLRF
- Can noise be beneficial?
- Conclusions

Analytical Derivations

- Emittance growth caused by CC phase and amplitude noise

Transverse motion

- We follow the approach used by V. Lededev in SSCL,1993 and applied to the effect of vibrations and magnetic field fluctuations, for the SSC [1]
- We have adapted it to the CC RF noise
- We use classic normalized transverse (*x*,*p*) coordinates [2]

$$x = \frac{X}{\sqrt{\beta}} \quad \text{and} \quad p = \sqrt{\beta}\frac{dX}{ds} + \frac{\alpha}{\sqrt{\beta}}X$$

- The particle motion follows a circle in normalized (x,p) space
- Consider a particle ξ of transverse tune $2\pi\nu_b$, receiving a small momentum kick Δp_n at each turn *n*. In normalized coordinates, its position at turn *n* is given by

$$x_n = \overset{\substack{\text{Unperturbed trajectory}\\ \downarrow}}{x_0 \cos(\nu_b n + \theta_0)} + \sum_{k=-\infty}^{n} \overset{\substack{\text{Cumulative effect of the kicks}\\ \downarrow}}{\Delta p_k \sin(\nu_b (n-k))}$$

Statistical approach

- The transverse momentum kicks is a sequence of random numbers (a statistical process) with zero mean. It is assumed stationary so that it is characterized by its autocorrelation function (or Power Spectral Density)
- The bunch population is characterized by a random variable vector with density function

$$f(\hat{x},\nu_b,\theta,\hat{\phi},\nu_s,\psi)$$

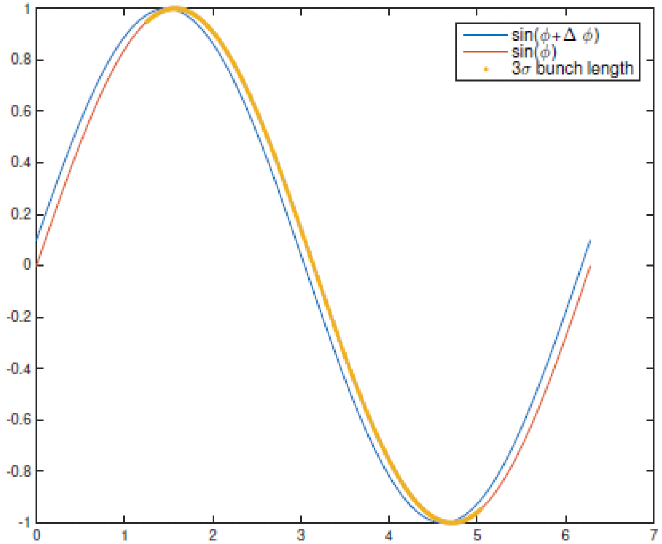
- The first three variables describe the betatron motion, the last three the synchrotron motion
- The density function does not change with time (valid for slow growth)
- We consider one transverse plane only

Momentum kicks

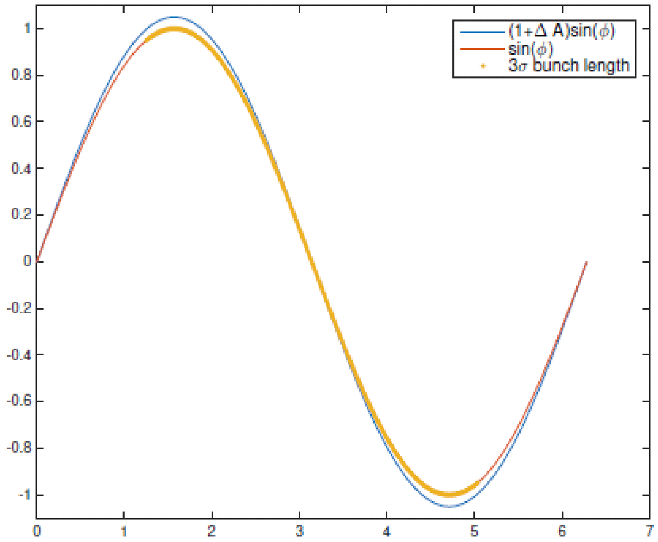
- with ϕ_n the particle's phase with respect to the synchronous particle, V_o the desired crab cavity voltage, ΔA_n the relative amplitude noise, and $\Delta\,\phi_n$ the phase noise

$$\Delta p_n = \sqrt{\beta_{cc}}\,\frac{e}{E}\,\Delta V_n = \sqrt{\beta_{cc}}\,\frac{eV_0}{E_b}\left[\sin(\phi_n+\Delta\phi_n)-\sin(\phi_n)\right]+\sqrt{\beta_{cc}}\,\frac{eV_0}{E_b}\,\Delta A_n\sin(\phi_n+\Delta\phi_n)$$

$$\approx \sqrt{\beta_{cc}}\,\frac{eV_0}{E_b}\cos(\phi_n)\Delta\phi_n+\sqrt{\beta_{cc}}\,\frac{eV_0}{E_b}\sin(\phi_n)\Delta A_n$$



Phase noise: acts strongly on the core of the bunch



Amplitude noise: acts on the head and tail in opposite directions, does not act on the core

- The effect of the noise on the motion of particle ξ is

$$\tilde{x}_n = \sum_{k=-\infty}^{n} \Delta p_k \sin(\nu_b(n-k))$$

$$= \sqrt{\beta_{cc}}\,\frac{eV_0}{E_b} \sum_{k=-\infty}^{n} \sin(\nu_b(n-k))\cos(\phi_k)\Delta\phi_k + \sqrt{\beta_{cc}}\,\frac{eV_0}{E_b} \sum_{k=-\infty}^{n} \sin(\nu_b(n-k))\sin(\phi_k)\Delta A_k$$

- The emittance growth is the variance of x_n, taken over the bunch distribution
- We assume that transverse filamentation is much faster than emittance growth, so that the mean of x_n will be zero at all time
- Then the emittance growth is given by

$$E\left[\tilde{x}_n^2\right] = \beta_{cc}\left(\frac{eV_0}{E_b}\right)^2 E\left[\begin{array}{l} \left\{\sum_{k=-\infty}^{n} \sin(\nu_b(n-k))\cos(\phi_k)\Delta\phi_k + \sum_{k=-\infty}^{n} \sin(\nu_b(n-k))\sin(\phi_k)\Delta A_k\right. \\ \left\{\sum_{l=-\infty}^{n} \sin(\nu_b(n-l))\cos(\phi_l)\Delta\phi_l + \sum_{l=-\infty}^{n} \sin(\nu_b(n-l))\sin(\phi_l)\Delta A_l\right\} \end{array}\right.$$

where E[.] stands for expected value taken over the particle distribution and noise process.

- For a non-accelerating bucket and a CC phase adjusted for zero at the bunch center, the angle of the longitudinal phase space distribution is uniformly distributed and the above equation can be solved exactly [3]
- We will further assume that amplitude and phase noise are uncorrelated. Then we get the growth rate of the absolute emittance, for one cavity

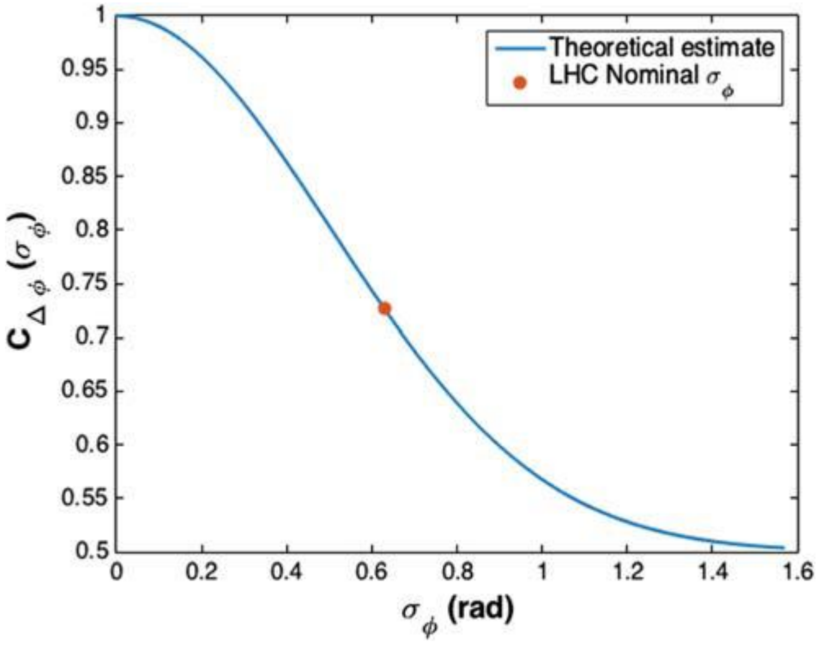
$$\frac{d\varepsilon_x}{dt} = \frac{E\left[\tilde{x}_n^2\right] - E\left[\tilde{x}_{n-1}^2\right]}{T_{rev}} = \beta_{cc}\left(\frac{eV_0 f_{rev}}{2E_b}\right)^2 C_{\Delta\phi}\left(\sigma_\phi\right)\sum_{k=-\infty}^{\infty}\int_0^\infty S_{\Delta\phi}\left[\left(k \pm \nu\right) f_{rev}\right]\rho\left(\nu\right)d\nu +$$

$$2\beta_{cc}\left(\frac{eV_0 f_{rev}}{2E_b}\right)^2 C_{\Delta A}\left(\sigma_\phi\right)\sum_{k=-\infty}^{\infty}\int_0^\infty S_{\Delta A}\left[\left(k \pm \nu\right) f_{rev}\right]\rho\left(\nu\right)d\nu$$

where $\rho(\nu)$ is the betatron tune distribution, $S_{\Delta\phi}(f)$ and $S_{\Delta A}(f)$ are the Power Spectral Density of the phase and amplitude noise process.

Phase noise

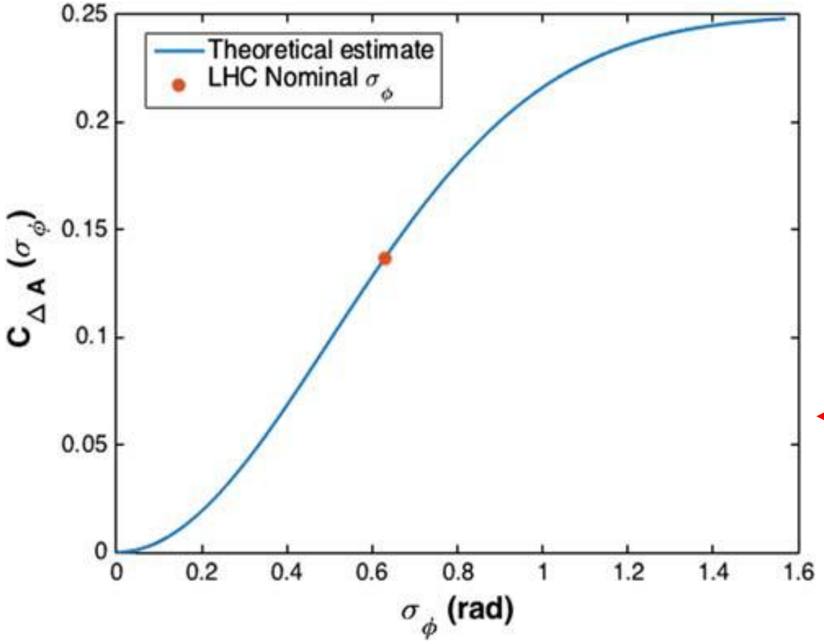
$$\frac{d\varepsilon_x}{dt} = \frac{E\left[\tilde{x}_n^2\right]-E\left[\tilde{x}_{n-1}^2\right]}{T_{rev}} = \beta_{cc}\left(\frac{eV_0 f_{rev}}{2E_b}\right)^2 C_{\Delta\phi}\left(\sigma_\phi\right)\sum_{k=-\infty}^{\infty}\boxed{\int_0^\infty S_{\Delta\phi}\left[\left(k\pm\nu\right)f_{rev}\right]\rho\left(\nu\right)d\nu}$$



- Depends on the overlap between phase noise spectrum and betatron tune distribution
- If the noise spectrum does not change inside the tune distribution, the growth rate is independent of tune distribution
- Phase noise spectrum is aliased at frev
- The “geometric factor” decreases with bunch length

Amplitude noise

$$\frac{d\varepsilon_x}{dt} = \frac{E\left[\tilde{x}_n^2\right] - E\left[\tilde{x}_{n-1}^2\right]}{T_{rev}} = 2\beta_{cc}\left(\frac{eV_0 f_{rev}}{2E_b}\right)^2 C_{\Delta A}\left(\sigma_\phi\right) \sum_{k=-\infty}^{\infty} \boxed{\int_0^\infty S_{\Delta A}\left[\left(k \pm \nu\right) f_{rev}\right] \rho\left(\nu\right) d\nu}$$



- Depends on the overlap between phase noise spectrum and betatron tune distribution
- If noise spectrum does not change within tune distribution, the later has no effect on growth rate
- Amplitude noise spectrum is aliased at frev
- The “geometric factor” increases with bunch length
- Additional factor of two that comes from the summation on the two synchrotron sidebands
- For the LHC (without damper), phase noise effect is 2.65 times larger than amplitude noise

Transverse damper

- The LHC transverse damper acts bunch per bunch: it measures the transverse position averaged over all particles in one bunch and generates a correcting kick
- It cannot correct for the effect of CC amplitude noise (as head and tails move in opposite direction)
- It can correct part of the phase noise effect
- Its effectiveness is a race against filamentation. It will depend on the ratio of damping time over filamentation time
- As it measures the ensemble bunch response, its effect will depend somewhat on the Beam Transfer Function
- Analytical formulas can be derived assuming a perfect damper (constant gain, zero delay, 90 degree phase shift) [3]

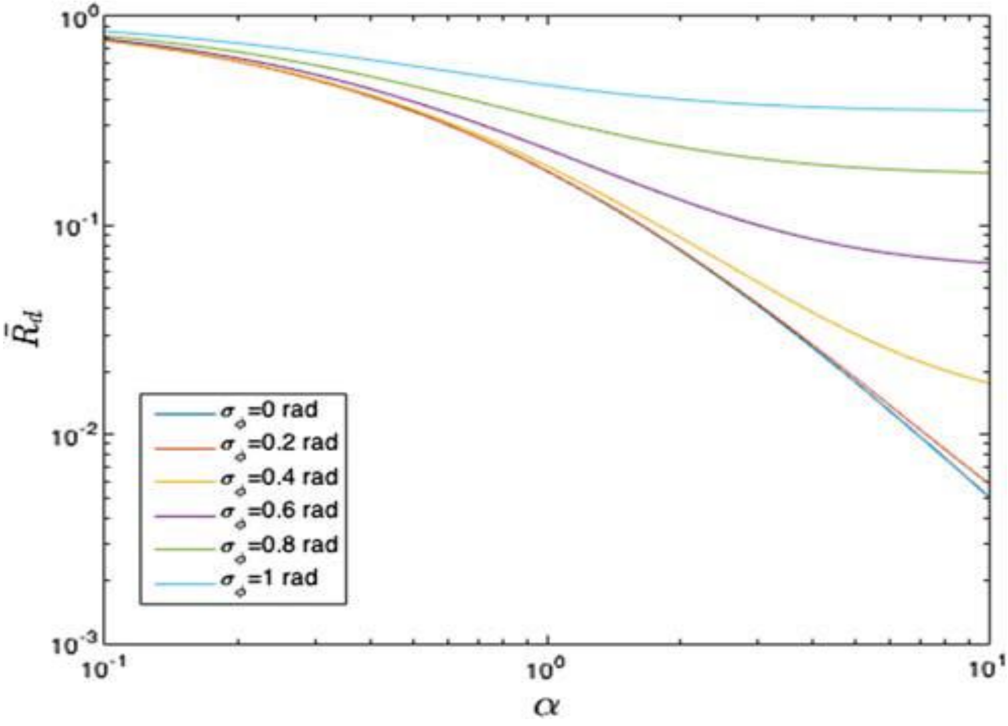
Damper reduction factor (phase noise)

$$\bar{R}_d\left(\sigma_\phi,\alpha\right)=\frac{1}{\pi}\int_{-\infty}^{\infty} g\left(u\right)\left\{1-\frac{e^{-\sigma_\phi^2}}{C_{\Delta\phi}\left(\sigma_\phi\right)}\frac{\alpha^2\left[g\left(u\right)^2+f\left(u\right)^2\right]+2\alpha g\left(u\right)}{\left[1+\alpha g\left(u\right)\right]^2+\left[\alpha f\left(u\right)\right]^2}\right\}du$$

With $\alpha=\frac{G}{2\pi\sigma_v}$, $g(u)=\pi\,\sigma_v\,\rho(\bar{v}-\sigma_v u)$, $f\left(u\right)=\sigma_v\ \mathrm{PV}\int_{-\infty}^{\infty}\frac{\rho(x)}{x-\bar{v}+\sigma_v u}dx$

f(u),g(u) are scaled versions of the real and imaginary parts of the Beam Transfer Function.

The damper efficiency drops quickly as bunch length increases. The damper kick is uniform along the batch while the effect of phase noise is strong in the core but week in the tails. Increasing the damper kicks therefore excites the tails

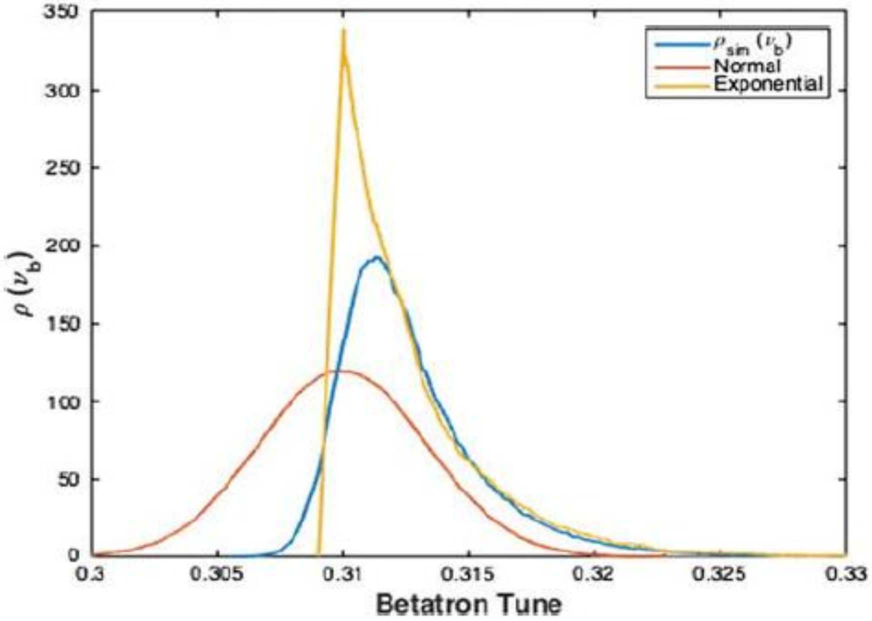


Simulation results

- HEADTAIL
  BeamBeam3D

HEADTAIL

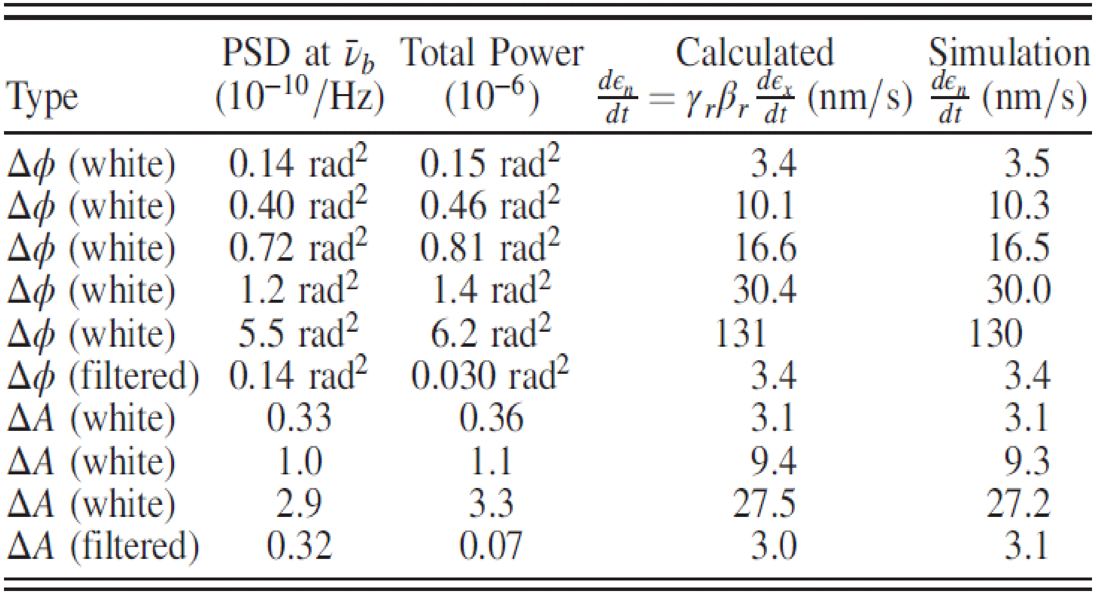
- HEADTAIL is a software package developed at CERN for simulation of multiparticle beam dynamics[4]
- Single-beam simulations were run to validate the results
- Betatron tune distribution was created using octopoles and a small chromaticity



- Betatron tune distributions $\rho(\nu_b)$ with the same 0.003 rms tune spread.
- $\rho_{sim}(\nu_b)$ is the distribution used in the simulations.

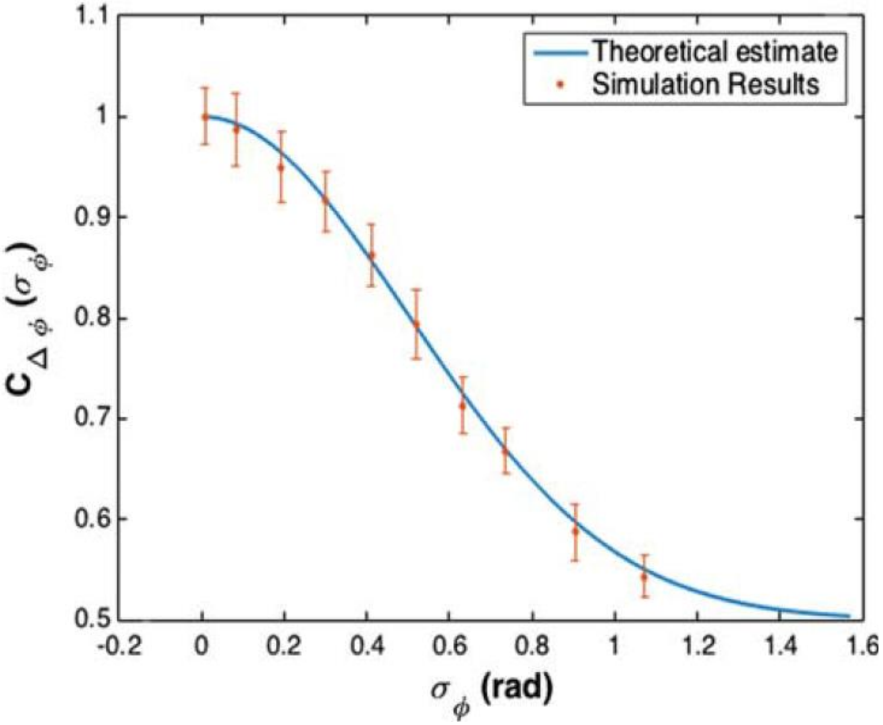
Dependance on noise PSD

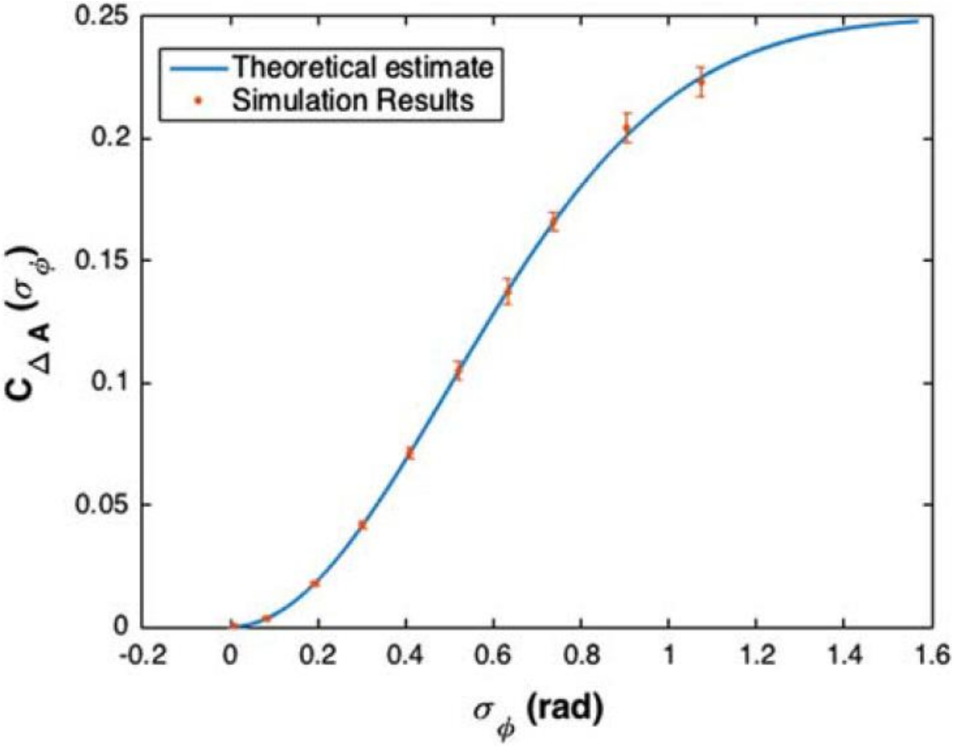
- White noise of varying PSD was injected
- Filtered noise centered around the betatron frequency was also used to show the emittance growth rate dependence on PSD, not on total noise power
- Also checked was the (in)dependence on tune distribution



Type	PSD at $\bar{\nu}_b$ (10^{-10}/Hz)	Total Power (10^{-6})	Calculated $\frac{d\epsilon_n}{dt} = \gamma_r \beta_r \frac{d\epsilon_x}{dt}$ (nm/s)	Simulation $\frac{d\epsilon_n}{dt}$ (nm/s)
$\Delta\phi$ (white)	0.14 rad^2	0.15 rad^2	3.4	3.5
$\Delta\phi$ (white)	0.40 rad^2	0.46 rad^2	10.1	10.3
$\Delta\phi$ (white)	0.72 rad^2	0.81 rad^2	16.6	16.5
$\Delta\phi$ (white)	1.2 rad^2	1.4 rad^2	30.4	30.0
$\Delta\phi$ (white)	5.5 rad^2	6.2 rad^2	131	130
$\Delta\phi$ (filtered)	0.14 rad^2	0.030 rad^2	3.4	3.4
ΔA (white)	0.33	0.36	3.1	3.1
ΔA (white)	1.0	1.1	9.4	9.3
ΔA (white)	2.9	3.3	27.5	27.2
ΔA (filtered)	0.32	0.07	3.0	3.1

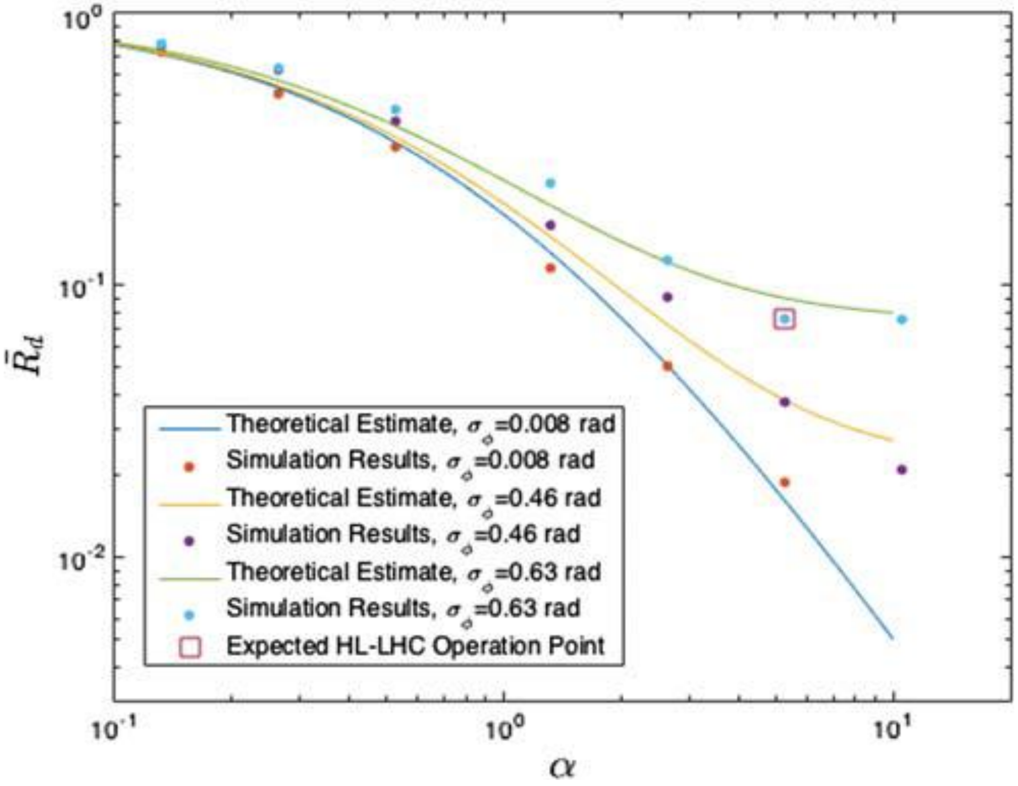
Dependence on bunch length





- Validation of the geometric factor with phase noise (left) and amplitude noise.

Dependence on damper gain

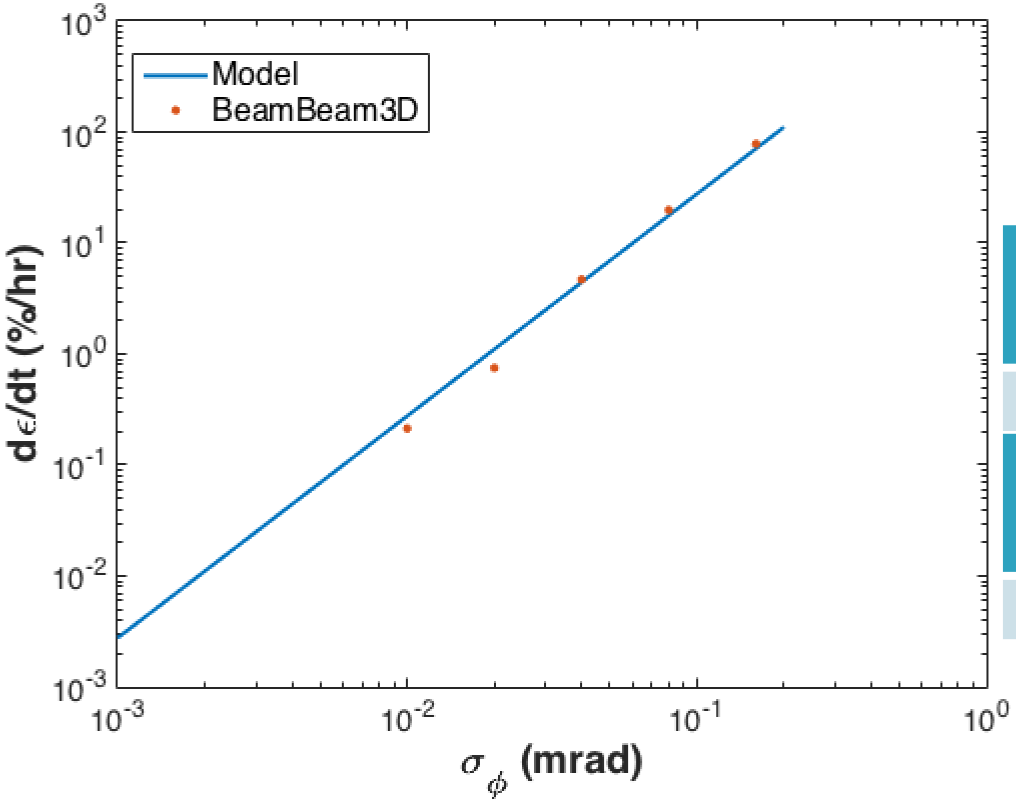


$$\alpha = \frac{G}{2\pi\sigma_\nu}$$

$$\sigma_\nu = 0.003$$

- Reduction factor function of damper gain for various bunch lengths

BeamBeam3D. Phase noise



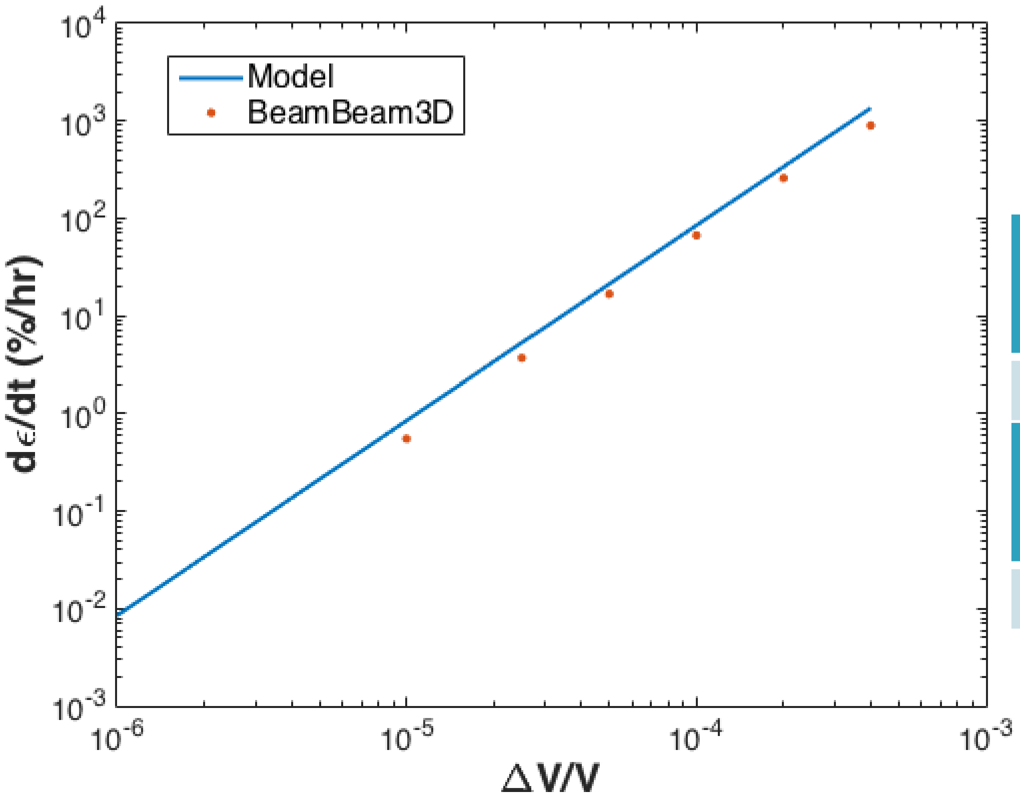
- Simulation data, courtesy of Ji Qiang, LBNL

β^* (m)	β_{cc} (m)	$\theta/2$ (μrad)	V_0 (MV)	Nbr cavities per plane
0.49	4000	295	5.6	2

G	ξ sim	σ_ν analytic	ε_n (μm. rad)	E_b (TeV)	σ_ϕ (rad)
0.1	0.011	0.003	2.5	7	0.63

- Emittance growth in %/hour versus phase noise $\sigma_{\Delta\phi}$ (rad)
- The red dots are results of BeamBeam3D simulations
- The blue line is the analytical formula

BeamBeam3D. Amplitude noise



- Simulation data, courtesy of Ji Qiang, LBNL

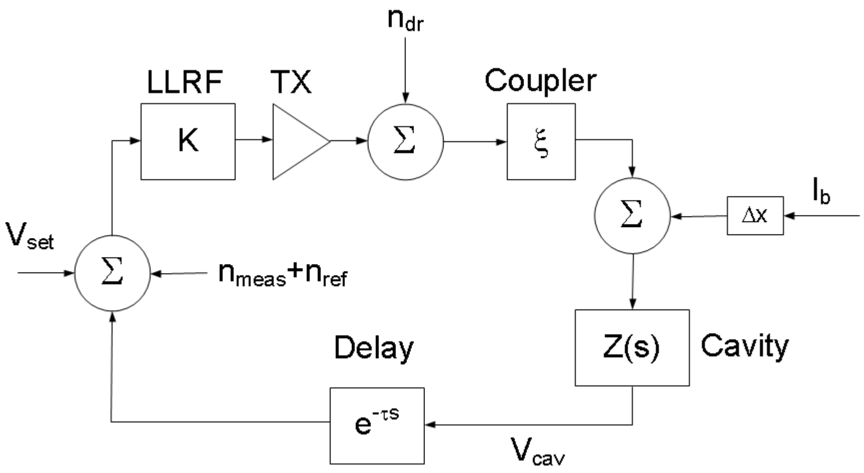
β^* (m)	β_{cc} (m)	$\theta/2$ (μrad)	V_0 (MV)	Nbr cavities per plane
0.49	4000	295	5.6	2

G	ξ sim	σ_v analytic	ε_n (μm. rad)	E_b (TeV)	σ_ϕ (rad)
0.1	0.011	0.003	2.5	7	0.63

- Emittance growth in %/hour versus relative amplitude noise $\sigma_{\Delta A}$ (dimensionless)
- The red dots are results of BeamBeam3D simulations
- The blue line is the analytical formula

Sources of RF noise

Cavity RF Noise



- RF feedback noise sources:
  - The RF reference noise n_{ref}
  - The demodulator noise (measurement noise) n_{meas}
  - The TX (driver) noise n_{dr}. It includes also the LLRF noise not related to the demodulator
  - The Beam Loading $I_b\ \Delta x$
- We get

$$Z(s) = \frac{\frac{R}{Q} Q_L}{1 + 2Q_L \frac{s}{\omega_0}}$$

with $s = j\Delta\omega$

Main coupler

$$V_{cav} = \frac{K\ G\ e^{-\tau s} Z(s)}{1 + K\ G\ e^{-\tau s} Z(s)} \left[V_{set} + n_{ref} + n_{meas}\right] + \frac{Z(s)}{1 + K\ G\ e^{-\tau s} Z(s)} \left[\Delta x\ I_b + \sqrt{\frac{Z_0}{\frac{R}{Q} Q_L}}\, n_{dr}\right]$$

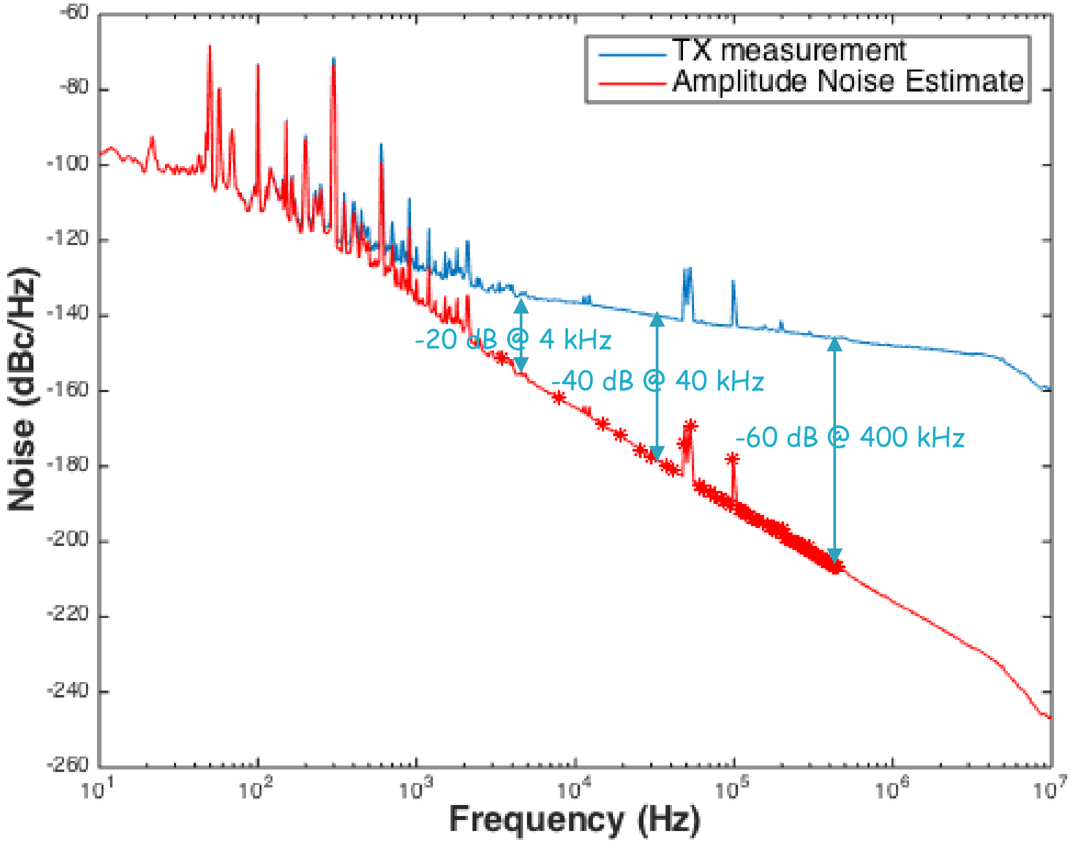
Closed Loop response CL(s)

- Equal to ~1 in the CL BW
- Increase of K increases the BW
- Within the BW, reference noise and measurement noise are reproduced in the cavity field

Beam Loading response = effective cavity impedance Zeff(s)

- Equal to ~1/KG in the CL BW
- Increase of K decreases Zeff within the CL BW
- Within the CL BW, TX noise and beam loading are reduced by the Open Loop gain KG

TX noise seen by the beam



- The TX noise seen by the beam is very small with high Q_L cavities, as it is filtered by the cavity response.
- The first betatron line is at –150 dBc/Hz
- The resulting emittance growth would be 0.1 %/hour…

- SPS 800 MHz 240 kW IOT SSB amplitude noise filtered by a cavity with Q_L=500000.

- The TX noise has little influence on emittance growth in the presence of strong RF feedback

Reference and measurement noise

$$\tilde{V}_{cav} = \frac{K\ G\ e^{-\tau s} Z(s)}{1 + K\ G\ e^{-\tau s} Z(s)} \left[n_{ref} + n_{meas} \right] + \frac{Z(s)}{1 + K\ G\ e^{-\tau s} Z(s)} \left[\sqrt{\frac{Z_0}{\frac{R}{Q} Q_L}}\ n_{dr} \right]$$

- The reference noise can be reduced to an arbitrary level at and above first betatron band by PLL on the reference RF
- The main noise source is therefore the measurement noise. Within the RF feedback BW it is reproduced in the cavity field

- Conclusion:
  - The performance is dominated by the RF demodulator (receiver) noise

Parameters – Cavities

- With the given β^* , half-crossing angle and β_{cc}, the total crabbing voltage is 10 MV
- As we have 4 cavities/plane/IP side, the voltage per cavity V_0 is 2.5 MV
- The emittance growth is produced by 8 cavities per plane

β^* (m)	β_{cc} (m)	$\theta/2$ (μrad)	Total Crabbing voltage (MV)	Voltage/ cavity V_0 (MV)	Nbr cavities per plane	f_{RF} (MHz)
0.15	4000	295	10	2.5	8	400.8

Parameters – Beam and damper

- We consider a 1 ns long bunch (4σ) with 0.003 betatron tune spread and 2.5 μm.rad normalized transverse emittance
- We assume a damping time of 10 turns (damper gain = 0.2). The phase noise reduction factor (from damper) is $R_d=0.1$, ($\alpha=5.3$)
- The scaling factors coming from bunch size are $C_{\Delta f}=0.726$ (phase noise) and $C_{\Delta A}=0.137$ (amplitude)

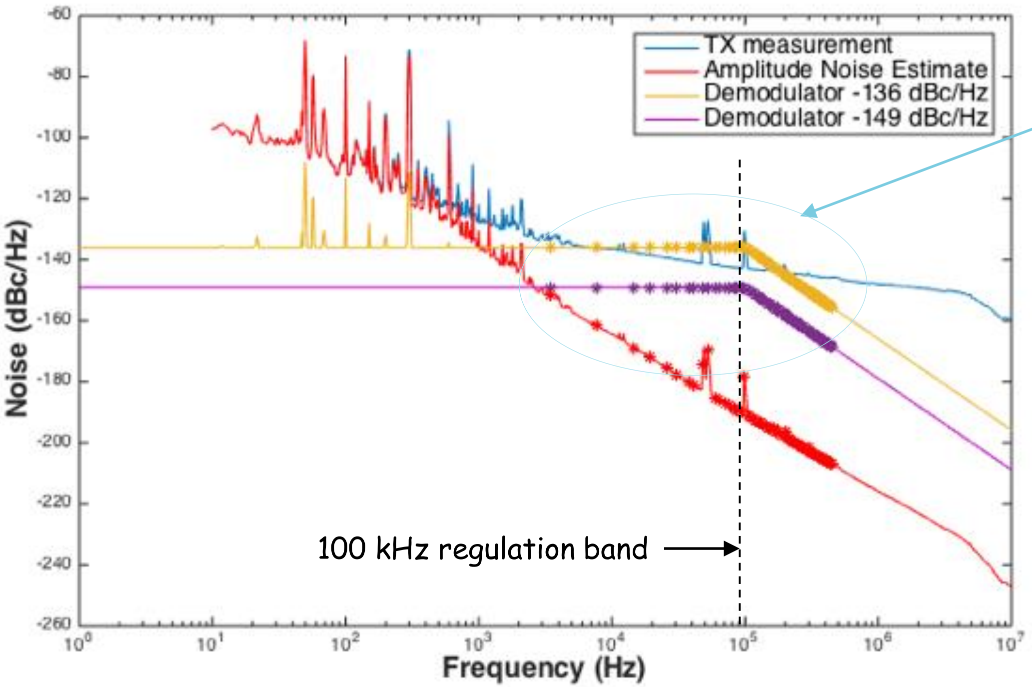
ν_x	σ_ν	ε_x (pm. rad)	ε_n (μm. rad)	E_b (TeV)	σ_ϕ (rad)	G	α
62.31	0.003	335	2.5	7	0.63	0.2	5.3

R_d	$C_{\Delta f}$	$C_{\Delta A}$
0.097	0.726	0.137

Growth rate

- Using the analytical formulas for 8 cavities we then get
  - 0.85 %/hour for $\sigma^2_{\Delta\phi} = 5\ 10^{-10}$ rad^2
  - 3.3 %/hour for $\sigma^2_{\Delta A} = 5\ 10^{-10}$
- To keep the growth around 4%/hour, the RF demodulator (*Receiver*) must be designed so that the noise power (phase or amplitude), summing over all aliased bands, remains below $5\ 10^{-10}$
- Assuming a 100 kHz regulation BW, we have ~10 aliased bands, so that the noise power per f_{rev} band must be below $5\ 10^{-11}$
- This translate in a noise PSD of $5\ 10^{-15}$/Hz, or a **SSB noise floor of -146 dBc/Hz**

Target demodulator noise level



For all betatron band, the demodulator noise dominates the TX noise (filtered by cavity).

- Amplitude noise using an IOT type TX spectrum
  - The yellow curve: regulation with 100 kHz BW and -136 dBc/Hz demodulator noise gives 33 %/hour
  - The violet curve (100 kHz BW, -149 dBc/Hz) gives **1.6 %/hour**. **Very good.**

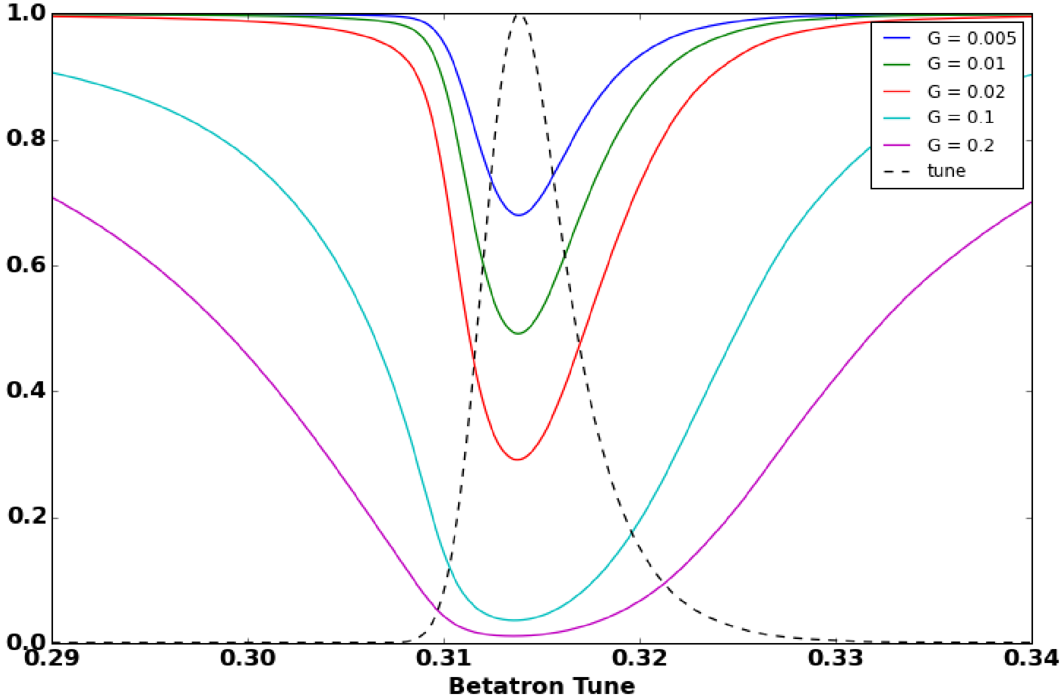
- The receiver design will aim at -150 dBc/Hz noise floor. Machine performances will be acceptable up to -146 dBc/Hz (4%/hour emittance growth).

Mitigations?

- The transverse damper considered so far gives one kick per bunch. It can therefore do nothing to mitigate Crab Cavity amplitude noise (symmetric Head-Tail motion) and is limited for correcting the effect of phase noise with long bunches (uneven excitation along the bunch caused by RF curvature). Increasing its BW up to the inverse bunch length would help. But the limitation will come from measurement noise (as with the CC feedback). Such Wideband damper is being tested in the SPS.
- Another idea is to measure the bunch head-tail motion (variance of the transverse displacement over the bunch) and use it to feedback on the Crab Cavity amplitude. This feedback will be simulated.

Phase noise *shaping* by damper

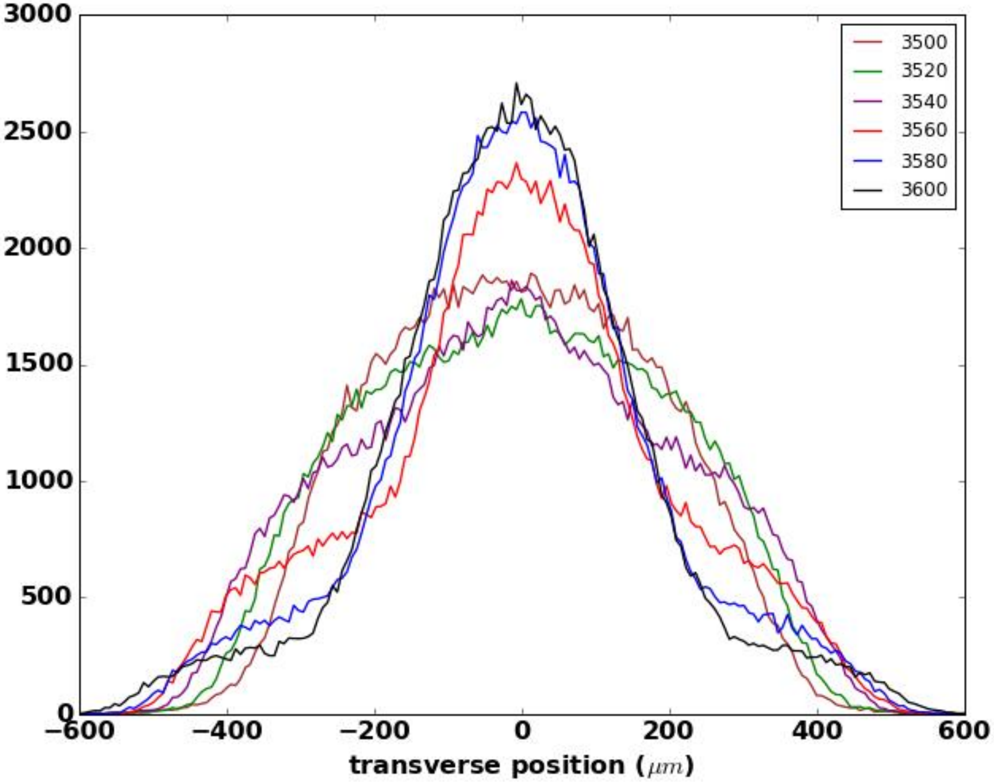
- The damper reduction factor R_d is an average taken over the tune distribution
- Actually the effect of the damper is tune-dependent: It has a large effect where the tune distribution is large (core of the bunch)
- This is easily understood: the damper measures the particle motion averaged over the bunch and therefore "ignores" the tails
- It has little effect in the tails of the tune distribution
- The tails should therefore diffuse out of the bunch



- The dotted line is the tune distribution
- The solid lines are the noise reduction factors for several gains
- The damper has the largest effect where there are the most particles.

Active noise *shaping*

- We could also inject noise whose spectrum overlaps with the tails of the betatron distribution, and does not affect the core
- If the tune distribution is dominated by beam-beam, this noise will excite the tails of transverse distribution and make these particles diffuse out of the bunch
- Such a transverse distribution is beneficial for Machine Protection as it would limit the losses following a CC trip
- Being studied…



- Distribution of particles after 100,000 turns in the presence of phase? noise with a 6 Hz BW, centred at various frequencies within the tune
- Bunch core at 3500 Hz
- Damper off

- We have analytical formulas for the effect of phase and amplitude noise, with finite bunch length and damper
- These formulas were confirmed by HEADTAIL and BeamBeam3D simulations
- The main cause of emittance growth is the Demodulator noise
  - For 4% growth/hour the SSB phase and amplitude noise must be –146 dBc/Hz at a 3 kHz offset from carrier
  - Challenging but reachable. LCLS-II developments @ 1.3 GHz require -160 dBc/Hz @ 10 kHz offset (B. Chase FNAL)
  - Design of the Demodulator should start asap
- Alternative mitigations of the effect of amplitude noise are possible
  - Using a Wideband transverse damper (being tested in the SPS)
  - Using a dedicated head-tail feedback on the CC voltage (being studied by simulations)
- With the damper ON, the phase noise should deplete the tails of the betatron distribution. This effect can be enhanced by injecting noise with a chosen narrow spectrum. Can help protect the machine. Being studied.

Thank you for your attention
Questions? Comments?

References

[1] V. Lebedev et al., Emittance Growth Due to Noise and its Suppression with the Feedback System in Large Hadron Colliders, SSCL-Preprint-188, 1993

[2] S.Y.Lee, Accelerators Physics

[3] P. Baudrenghien, T. Mastoridis, Transverse emittance growth due to RF noise in the high-luminosity LHC Crab Cavities, PRST AB, 18, 101001 (2015)

[4] G. Rumolo and F. Zimmermann, PRST AB, 5, 121002 (2002).