Theory input for FCC-ee precision measurements

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First FCC-ee Workshop on Higgs Physics,
24-25 September 2015

1. **Summary of the First FCC-ee mini-workshop on Precision Observables and Radiative Corrections (13-14 July 2015)**

2. **Open questions after the workshop**

3. **Theory predictions for Higgs precision observables**

Summary of 1st FCC-ee mini-workshop on POs and RCs

Phenomenology WG2, conveners: S. Heinemeyer and A. Freitas

Main topics

- SM corrections to electroweak precision observables and related processes
  - Fixed-order multi-loop calculations
  - Soft QED exponentiation
  - Monte-Carlo tools
- SM input parameters
  - Robust theoretical definition of top-quark masses
  - Prospects for experimental determination of m_t, m_b, α_s, $\Delta\alpha_{\rm rad}$, ...
- Higgs observables

  $\longrightarrow$ Section 3 of this talk
- Implications for BSM physics
  - (N)MSSM
  - Effective field theory

Electroweak SM corrections to EWPOs:

G. Degrassi, A. Freitas, S. Martin

- Current state of the art (2-loop) not sufficient for ILC/FCC-ee
- Theory error depends on chosen method
- 3-loop and partial higher orders needed for FCC-ee precision

	ILC	FCC-ee	perturb. error with 3-loop†	Param. error ILC*	Param. error FCC-ee**
M_{W} [MeV]	3–5	~ 1	1	2.6	1
Γ_Z [MeV]	~ 1	~ 0.1	$\lesssim 0.2$	0.5	0.06
R_b $[10^{-5}]$	15	$\lesssim 5$	5–10	< 1	< 1
$\sin^2\theta^{\ell}_{\mathrm{eff}}$ $[10^{-5}]$	1.3	0.3	1.5	2	2

† **Theory scenario:** $\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N_f^2\alpha^2\alpha_s)$

(N_f^n = at least n closed fermion loops)

Parametric inputs:

* **ILC:** $\delta m_t = 100$ MeV, $\delta\alpha_s = 0.001$, $\delta M_Z = 2.1$ MeV

****FCC-ee:** $\delta m_t \lesssim 50$ MeV, $\delta\alpha_s = 0.0001$, $\delta M_Z = 0.1$ MeV

also: $\delta(\Delta\alpha) = 5 \times 10^{-5}$

A. Freitas

SM corrections to electroweak precision observables

QCD corrections to electroweak and Higgs observables: J.H. Kühn

- Very advanced (up to 4-loop in some cases)
- Mostly not the leading error source
- Ambiguity in defining flavored final states (may also affect Higgs physics)

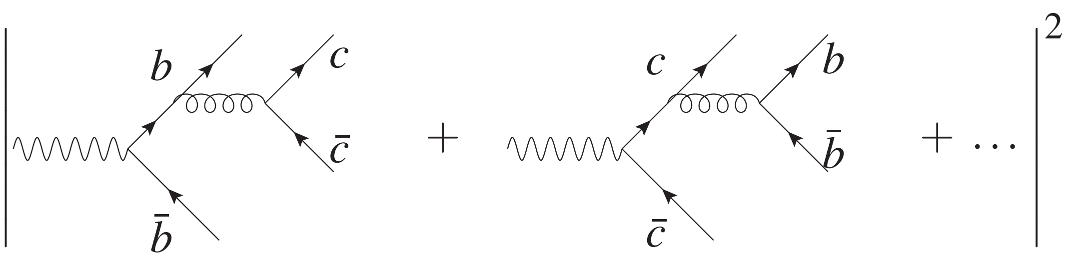
Tera Z: $\Gamma_b(Z \to b\bar{b})$

Can we isolate the $Zb\bar{b}$-vertex?

$R_b = 0.21629 \pm 0.00066$ (LEP); $3\% \widehat{=} 1.65$ MeV

TLEP: $2-5 \times 10^{-5} \widehat{=} 50-120$ keV

conceptual problem: singlet-terms



J. Kühn

Monte-Carlo tools for $e^+e^- \rightarrow f\bar{f}$: Z. Wąs, F. Piccinini

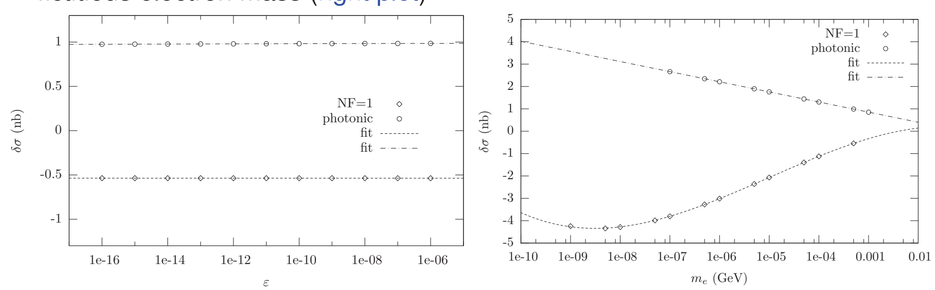
- State of the art: `KKMC, BabaYaga`
- YFS exponentiation for QED radiation, approximate NNLO QED
- currently $\mathcal{O}(0.1\%)$ precision, $\mathcal{O}(0.01\%)$ feasible in (near) future, but more may be needed for FCC-ee

Comparison with (a subset of) NNLO

G. Balossini et al., NPB758 (2006) 227

Comparison of $\sigma_{\mathrm{SV}}^{\alpha^2}$ calculation of BabaYaga@NLO with

- Penin (photonic): switching off the vacuum polarisation contribution in BabaYaga@NLO, as a function of the logarithm of the soft photon cut–off (left plot) and of a fictitious electron mass (right plot)



- ⋆ differences are infrared safe, as expected
- ⋆ $\delta\sigma(\mathrm{photonic})/\sigma_0 \propto \alpha^2 L$, as expected
- Numerically, for various selection criteria at the Φ and B factories

$$\sigma_{\mathrm{SV}}^{\alpha^2}(\mathrm{Penin}) - \sigma_{\mathrm{SV}}^{\alpha^2}(\mathrm{BabaYaga@NLO}) < 0.02\% \times \sigma_0$$

F. Piccinini

SM input parameters

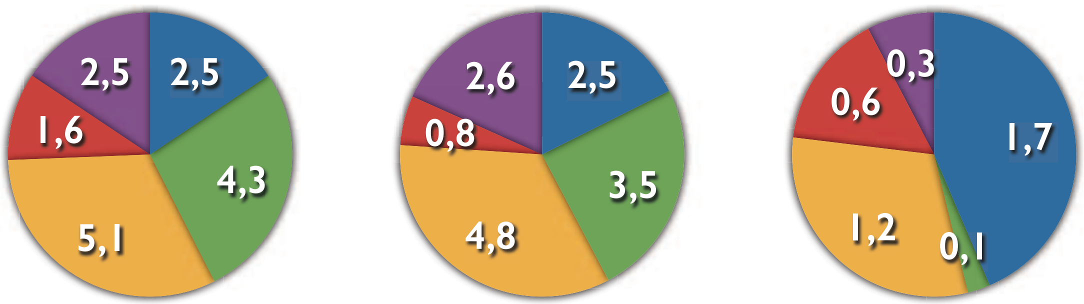
Parametric uncertainties from SM inputs: A. Hoang, S. Weinzierl, N. Garron

- Most important parameters: m_t, m_b, α_s, $\Delta\alpha_{\mathrm{had}}$, M_Z
- Can become limiting factor in global fits

R. Kogler

Today		LHC-300		ILC/GigaZ	
δ_{meas} =	15 MeV	δ_{meas} =	8 MeV	δ_{meas} =	5 MeV
δ_{fit} =	8 MeV	δ_{fit} =	6 MeV	δ_{fit} =	2 MeV
$\delta_{\mathrm{fit}}^{\mathrm{theo}}$ =	5 MeV	$\delta_{\mathrm{fit}}^{\mathrm{theo}}$ =	2 MeV	$\delta_{\mathrm{fit}}^{\mathrm{theo}}$ =	1 MeV

δM_Z δm_{top} $\delta \sin^2(\theta^l_{\mathrm{eff}})$ $\delta\Delta\alpha_{\mathrm{had}}$ $\delta\alpha_s$



Impact of individual uncertainties on δM_W in fit (numbers in MeV)

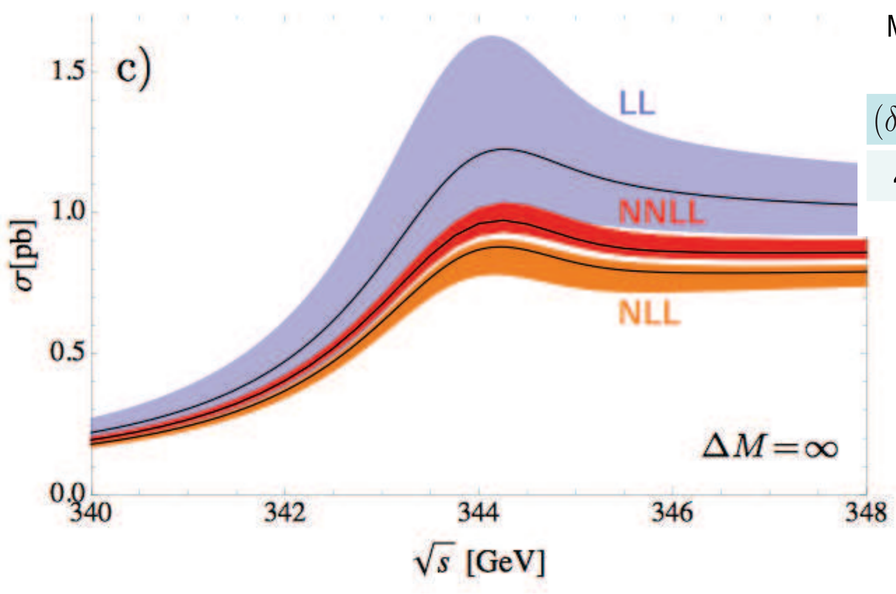
Improved theoretical precision needed already for the LHC-300!

R. Kogler

Top-quark mass: A. Hoang, S. Weinzierl

- From $\sigma(e^+e^- \to t\bar{t})$, NNNLO and NNLL corrections known
- OS top mass not well-defined ($\gtrsim$ 200 MeV renormalon contributions)

  $\longrightarrow$ Need short-distance definition: $m_t^{\overline{\text{MS}}}$, m_t^{1S}, m_t^{MSR}
- Theory errors not negligible, $\mathcal{O}(100\ \text{MeV})$



Msbar mass error budget (from threshold scan)

$(\delta M_t^{\text{SD-low}})^{\text{exp}}$	$(\delta M_t^{\text{SD-low}})^{\text{theo}}$	$(\delta \overline{m}_t(\overline{m}_t))^{\text{conversion}}$	$(\delta \overline{m}_t(\overline{m}_t))^{\alpha_s}$
40 MeV	50 MeV	7 – 23 MeV	70 MeV

$\delta\alpha_s(M_z) = 0.001$

A. Hoang

SM input parameters

Bottom-quark mass:

N. Garron

- Lattice calculations for controlling non-perturbative effects
- m_b too large for ab-initio LQCD calculations, need combination with PQCD or EFT
- New ideas being developed, but progress is challenging

$n_f = 3$ flavours HPQCD, McNeile et al '10

$$m_b^{\overline{\rm MS}}(m_b, n_f = 5) = 4.164(23)\ {\rm GeV}$$

New result $n_f = 4$ HPQCD, Chakraborty et al '15

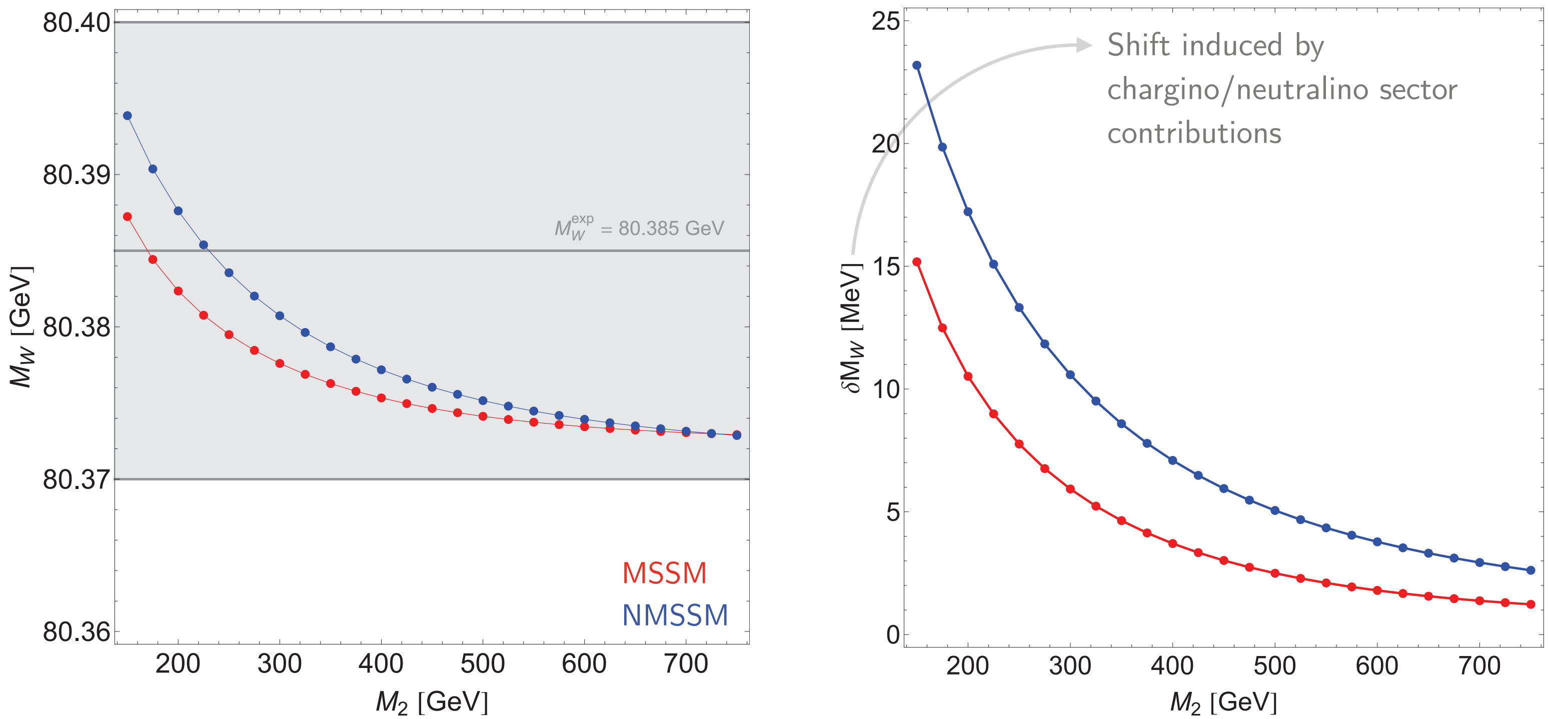
$$m_b^{\overline{\rm MS}}(m_b, n_f = 5) = 4.162(48)\ {\rm GeV}$$

N. Garron

Implications for BSM physics

SUSY precision calculations: L. Zeune, G. Villadoro

- MSSM is not everything, NMSSM can give important corrections w.r.t. MSSM
- More complete 2-loop corrections for M_h prediction important



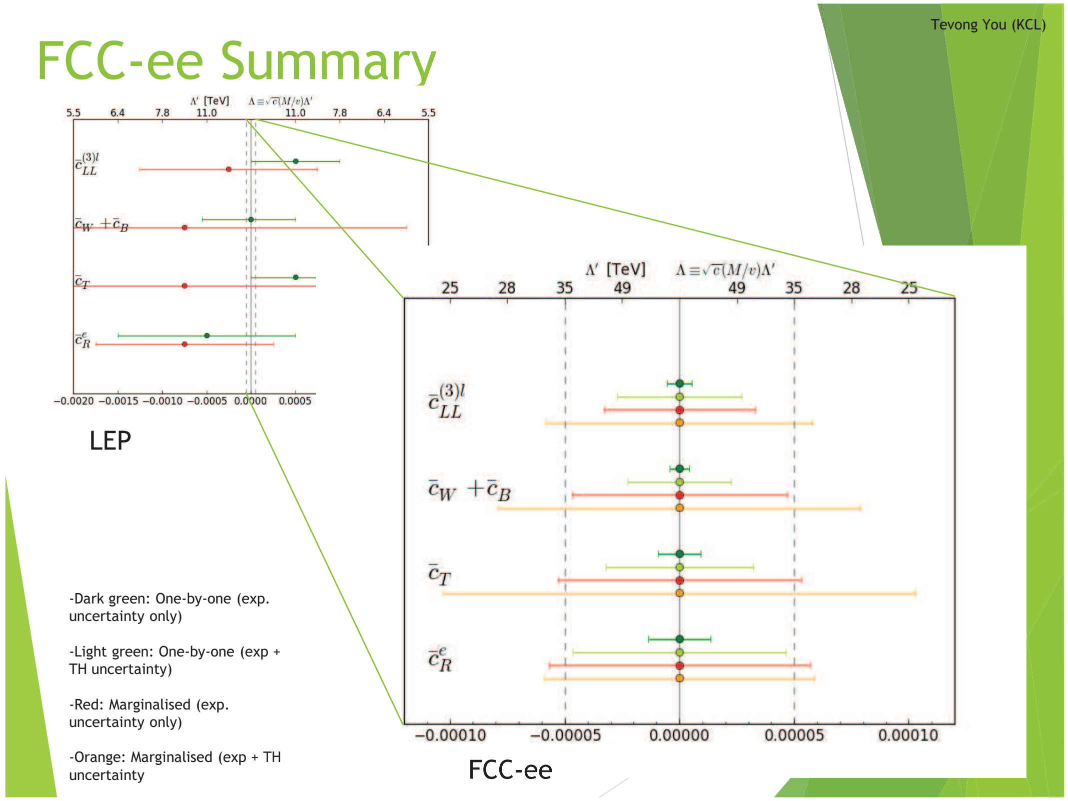
= SU(2) gaugino mass parameter

L. Zeune

Implications for BSM physics

EFT description (dimension-6 operators): A. Falkowski, V. Sanz, T. You

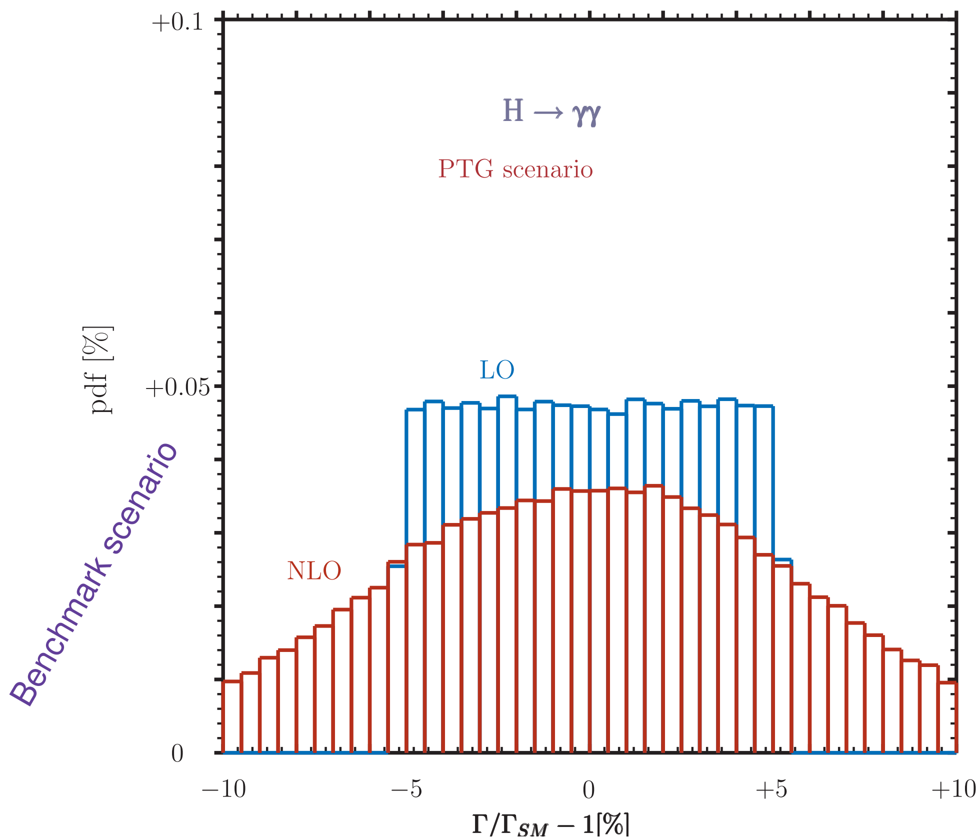
- To break degeneracies/flat directions, combine inputs from M_W, Z-pole, W decays, WW production, Higgs physics
- EFT may break down in tails of distributions at LHC
- Projections for FCC-ee to assess new physics reach (S/T parameter are not adequate for FCC-ee precision)



T. You

EFT at NLO: G. Passarino

- EFT operators in SM loops can generate qualitatively new effects
- Much work remains to be done!



G. Passarino

Open questions after the workshop

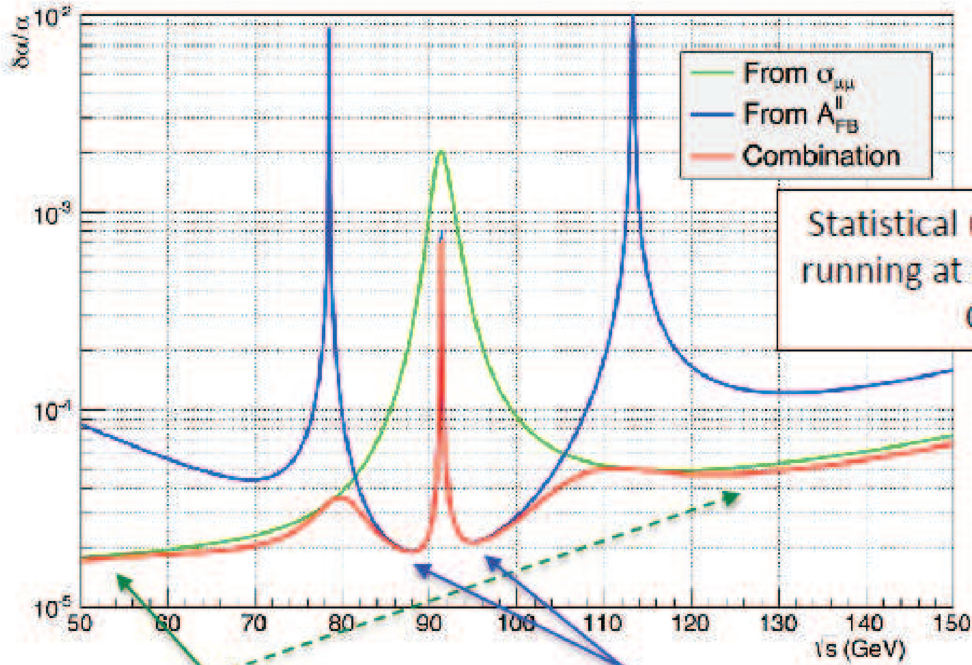
- Many methods for evaluating **theory error** and no clear statistical interpretation
  $\longrightarrow$ Ideally we want $\Delta_{\text{th}} \ll \Delta_{\text{exp}}$
  But: Theory error may be limiting factor for ILC/FCC-ee precision

- Electroweak 3-loop calculations for **EWPO** necessary + leading 4-loop (5-loop?)
  [leading $O(\alpha\alpha_{\text{s}}^3)$ already known]

- Need consistent use of **short-distance definition** of m_{t} across all sectors

- Control of **parametric uncertainties** crucial:
  - m_{t}: Several error sources, extracted value of m_{t} depends itself on α_s
  - m_b: No agreement between groups on error estimate, path forward unclear
  - M_{W}: May need 2-loop corrections for $e^+e^- \to WW \to 4f$ near threshold to achieve $\delta M_{\text{W}} \sim 1$ MeV
  - α_s: Big improvement expected from FCC-ee, but discrenpancy to event shapes and DIS must be understood
  - $\Delta\alpha_{\text{had}}$: Could be limiting factor, but possibly determined directly at FCC-ee

Potential of $\alpha_{QED}(m_Z)$ measurement (2)



Statistical uncertainty for one year of running at any centre-of-mass energy. Crab-waiste, 4 IP.

From $\sigma_{\mu\mu}$ measurement
- Sensitivity best "far" away from Z peak, particularly at the low side
- Systematics (normalisation) probably a killer

From $A_{FB}^{\mu\mu}$ measurement
- Sensitivity best at 88 and 95 GeV
- Experimental systs. looks controlable; further studies needed
- Theoretical systs. to large degree cancel by "averaging" over 88 and 95 GeV point

By running six months at each of 88 and 95 GeV points:
- ➢ Could potentially reach a precision of : $\delta\alpha/\alpha = 2 \times 10^{-5}$

A. Blondel, CEPC workshop August 2015

Open questions after the workshop

- **EFT:** Tool of choice for presenting new-physics bounds from data?
  (S/T parameter not adequate for FCC-ee precision)
  $\longrightarrow$ Need to combine EFT and NLO calculations (and higher-order QCD where applicable)

- **MC Tools:** Control of QED effects to sub-permille level is challenge,
  but progress is underway
  $\longrightarrow$ Implementation of EFT into MC tools?

Theory predictions for Higgs precision observables

Measurable properties of $h(125)$:

- **Spin, CP:** already underway at LHC
- **Mass:** direct LHC measurement more precision than SM/MSSM prediction
- **BRs, couplings:** currently $\mathcal{O}(20\%)$, improvement will greatly enhance sensitivity to higher new-physics scales

Englert et al. '14

Target precision of future e^+e^- colliders:

	ILC500	HL-ILC500	FCC-ee
hbb	1%	0.6%	0.4%
htt	2.5%	1.3%	0.7%
$h\tau\tau$	2%	1%	0.5%
hWW	0.4%	0.2%	0.1%
hZZ	0.5%	0.25%	0.05%
$h\gamma\gamma$	8%	4.5%	1.5%
hgg	2%	1%	0.8%

Snowmass Higgs WG '13

SM predictions for Higgs decays

Review: Lepage, Mackenzie, Peskin '14, see also LHC HXSWG '13

hbb:

- $\mathcal{O}(\alpha_s^4)$ QCD corrections — Baikov, Chetyrkin, Kühn '05
- $\mathcal{O}(\alpha)$ QED+EW — Dabelstein, Hollik '92; Kniehl '92
- leading $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha\alpha_s)$ for large m_t — Kwiatkowski, Steinhauser '94; Butenschoen, Fugel, Kniehl '07
  $\longrightarrow$ Use for error estimate

Theory error: $\Delta_{th} < 0.4\%$

Parametric error:

$$\left.\begin{aligned} m_b &= 4.169 \pm 0.009 \text{ GeV} \\ \alpha_s &= 0.118 \pm 0.001 \end{aligned}\right\} \rightarrow \Delta_{par} \approx 0.8\%$$

Penin, Zerf '14; Bethke '12

$$\left.\begin{aligned} \delta m_b &= 0.005 \text{ GeV} \\ \delta\alpha_s &= 0.0001 \end{aligned}\right\} \rightarrow \Delta_{par} \approx 0.25\%$$

Note: m_b value from Penin & Zerf from perturbative sum rule calculation, very small non-perturbative error (< 1 MeV) $\longrightarrow$ Not generally accepted

SM predictions for Higgs decays

h$\tau\tau$:

$\mathcal{O}(\alpha)$ QED+EW, leading $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha\alpha_s)$

Theory error: $\Delta_{th} < 0.4\%$, parametric error negligible

hWW*/hZZ*:

- complete $\mathcal{O}(\alpha) + \mathcal{O}(\alpha_s)$ for $h \to 4f$ Bredenstein, Denner, Dittmaier, Weber '06
- leading $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha\alpha_s^2)$ for large m_t Djouadi, Gambino, Kniehl '97
  Kniehl, Spira '95; Kniehl, Steinhauser '95
  $\longrightarrow$ Small (0.2%) effect Kniehl, Veretin '12

Theory error: $\Delta_{th,EW} \approx 0.1\%$, $\Delta_{th,QCD} \approx 0.5\%$
$\longrightarrow$ Higher-order QCD necessary (doable if color connections neglected)

Parametric error:
$\delta M_H = 30$ MeV $\longrightarrow \Delta_{par} \approx 0.2\%$
$\longrightarrow$ What is the ultimate precision for M_H?

Note: Distributions affected by corrections $\longrightarrow$ implementation into MC tools

SM predictions for Higgs decays

hgg:

- $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s^3)$ (in large m_t-limit) QCD corrections — Baikov, Chetyrkin '06; Schreck, Steinhauser '07
- $\mathcal{O}(\alpha)$ EW — Aglietti, Bonciani, Degrassi, Vicini '04; Degrassi, Maltoni '04

Theory error: $\Delta_{th} \approx 3\%$ (dominated by QCD)
$\longrightarrow$ Improvement necessary but very challenging!

Parametric error:
$\delta\alpha_s = 0.001 \quad \rightarrow \Delta_{par} \approx 1\%$ (already sub-dominant now)

hγγ:

- $\mathcal{O}(\alpha_s^2)$ QCD corrections — Zheng, Wu '90; Djouadi, Spira, v.d.Bij, Zerwas '91; Dawson, Kauffman '93; Maierhöfer, Marquard '12
- $\mathcal{O}(\alpha)$ EW — Aglietti, Bonciani, Degrassi, Vicini '04; Degrassi, Maltoni '04; Actis, Passarino, Sturm, Uccirati '08

Theory error: $\Delta_{th} < 1\%$

Parametric error negligible

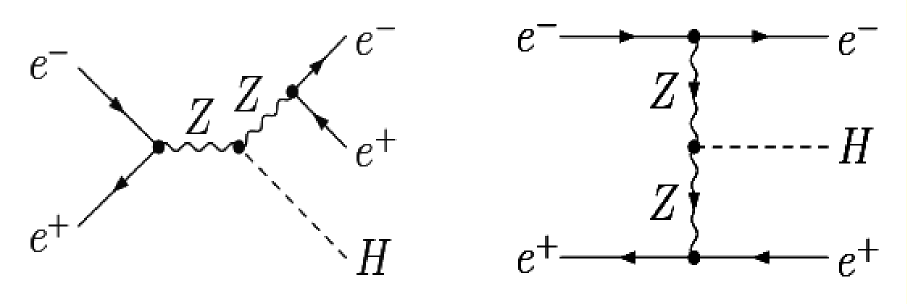
SM predictions for Higgs production

hZ production:

- $\mathcal{O}(\alpha)$ corr. to hZ production and Z decay — Kniehl '92; Denner, Küblbeck, Mertig, Böhm '92; Consoli, Lo Presti, Maiani '83; Jegerlehner '86; Akhundov, Bardin, Riemann '86
- Technology for $\mathcal{O}(\alpha)$ corr. with off-shell Z-boson available, but no phenomenological results (except for $Z \to \nu\nu$, see below) — Boudjema et al. '04
- Can be combined with h.o. ISR QED radiation — Skrzypek '92

Theory error: $\Delta_{\mathrm{th}} \sim O(1\%)$?

Parametric error:
negligible if $\delta M_{\mathrm{H}} < 100$ MeV

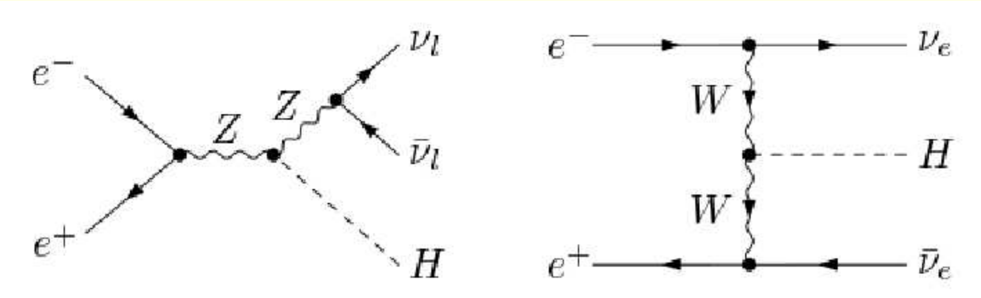


WW fusion:

- $\mathcal{O}(\alpha)$ corrections with h.o. ISR — Belanger et al. '02; Denner, Dittmaier, Roth, Weber '03

Theory error: $\Delta_{\text{th}} \sim O(1\%)$?

Parametric error: negligible



SM predictions for Higgs production

Improved calculations with 2-loop corrections necessary to meet $\mathcal{O}(0.1\%)$ precision target

- Full $\mathcal{O}(\alpha^2)$ for $2 \to 3$ process extremely challenging!
  $\longrightarrow$ Factorized form (for on-shell W/Z) maybe sufficient?
- Also need $O(\alpha)$ (or better?) corrections for backgrounds: $e^+e^-b\bar{b}$, $\nu\bar{\nu}b\bar{b}$, etc.
  $\longrightarrow$ Technology exists, but work needed Denner, Dittmaier, Roth, Wieders '05

Other issues:

- Implementation of radiative correction into MC tools with parton shower, etc. for evaluation of experimental acceptances
- Include new-physics effects thourgh $d{=}6$ effective operators and non-standard decay channels in consistent framework

Conclusions

EWPOs:

- FCC-ee will reduce exp. error by factor $\gtrsim 10$ compared to LEP/SLC
  $\longrightarrow$ Current SM theory calculations not sufficient
  $\longrightarrow$ 3-loop and partial 4-loop (5-loop?) corrections needed!

- Good control over input parameters m_t, M_W, α_s and $\Delta\alpha_\text{rad}$ is crucial
  $\longrightarrow$ Can possibly be all determined at FCC-ee itself (cross-checks?)
  $\longrightarrow$ Probably limited by theory uncertainties!

Higgs observables:

- Theory predictions for Higgs decays under good control
  $\longrightarrow$ Improvements for $h \to WW^*, ZZ^*$ needed but probably manageable
  $\longrightarrow$ Improvements for $h \to gg$ challenging (5-loop QCD)

- Significant work needed for Higgs production in e^+e^- (2-loop electroweak!)

- Good control over input parameters m_b and α_s crucial
  $\longrightarrow$ Improvements of lattice techniques for m_b?

Backup slides

Current status of SM loop results

Known corrections to Δr, $\sin^2\theta^f_{\rm eff}$, g_{Vf}, g_{Af}:

- Complete NNLO corrections (Δr, $\sin^2\theta^\ell_{\rm eff}$) Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02; Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07; Degrassi, Gambino, Giardino '14
- "Fermionic" NNLO corrections (g_{Vf}, g_{Af}) Czarnecki, Kühn '96
  Harlander, Seidensticker, Steinhauser '98
  Freitas '13,14
- Partial 3/4-loop corrections to ρ/T-parameter
  $\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^2)$, $\mathcal{O}(\alpha_{\rm t}^2\alpha_{\rm s})$, $\mathcal{O}(\alpha_{\rm t}\alpha_{\rm s}^3)$ Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05; Chetyrkin et al. '06
  $(\alpha_{\rm t} \equiv \frac{y_{\rm t}^2}{4\pi})$ Boughezal, Czakon '06

Current uncertainties

	Experiment	Theory error	Main source
M_{W}	80.385 ± 0.015 MeV	4 MeV	α^3, $\alpha^2\alpha_{\mathrm{s}}$
Γ_Z	2495.2 ± 2.3 MeV	0.5 MeV	α^2_{bos}, α^3, $\alpha^2\alpha_{\mathrm{s}}$, $\alpha\alpha_{\mathrm{s}}^2$
σ^0_{had}	41540 ± 37 pb	6 pb	α^2_{bos}, α^3, $\alpha^2\alpha_{\mathrm{s}}$
$R_b \equiv \Gamma^b_{\mathrm{Z}}/\Gamma^{\mathrm{had}}_{\mathrm{Z}}$	0.21629 ± 0.00066	0.00015	α^2_{bos}, α^3, $\alpha^2\alpha_{\mathrm{s}}$
$\sin^2\theta^{\ell}_{\mathrm{eff}}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_{\mathrm{s}}$

Methods for theory error estimates:

- Parametric factors, $i.e.$ factors of α, N_c, N_f, ...
- Geometric progression, $e.g.$ $\frac{\mathcal{O}(\alpha^3)}{\mathcal{O}(\alpha^2)} \sim \frac{\mathcal{O}(\alpha^2)}{\mathcal{O}(\alpha)}$
- Renormalization scale dependence (often underestimates error)
- Renormalization scheme dependence (may underestimate error)

Future projections

ILC: $\sqrt{s} \approx M_Z$ with 30 fb^{-1}

FCC-ee: $\sqrt{s} \approx M_Z$ with 4×3000 fb^{-1}

	Current exp.	ILC	FCC-ee	Current perturb.
M_W [MeV]	15	3–5	~ 1	4
Γ_Z [MeV]	2.3	~ 1	~ 0.1	0.5
R_b [10^{-5}]	66	15	$\lesssim 5$	15
$\sin^2\theta^\ell_{\rm eff}$ [10^{-5}]	16	1.3	0.3	4.5

$\longrightarrow$ Existing theoretical calculations adequate for LEP/SLC/LHC, but not ILC/FCC-ee!

Theory and parametric uncertainties

	ILC	FCC-ee	perturb. error with 3-loop†	Param. error ILC*	Param. error FCC-ee**
M_W [MeV]	3–5	~ 1	1	2.6	1
Γ_Z [MeV]	~ 1	~ 0.1	$\lesssim 0.2$	0.5	0.06
R_b $[10^{-5}]$	15	$\lesssim 5$	5–10	< 1	< 1
$\sin^2\theta^{\ell}_{\rm eff}$ $[10^{-5}]$	1.3	0.3	1.5	2	2

† **Theory scenario:** $\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N_f^2\alpha^2\alpha_s)$

(N_f^n = at least n closed fermion loops)

Parametric inputs:

*** ILC:** $\delta m_t = 100$ MeV, $\delta\alpha_s = 0.001$, $\delta M_Z = 2.1$ MeV

****FCC-ee:** $\delta m_t \lesssim 50$ MeV, $\delta\alpha_s = 0.0001$, $\delta M_Z = 0.1$ MeV

also: $\delta(\Delta\alpha) = 5 \times 10^{-5}$

Theory uncertainties in extraction of pseudo-observables

- Subtraction of QED radiation contributions

  $\longrightarrow$ Known to $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha^3 L^3)$ for **ISR**, $\mathcal{O}(\alpha^2)$ for **FSR** and $\mathcal{O}(\alpha^2 L^2)$ for **A$_{\mathrm{FB}}$**

  $(L = \log \frac{s}{m_e^2})$

  Berends, Burgers, v.Neerven '88
  Kniehl, Krawczyk, Kühn, Stuart '88
  Beenakker, Berends, v.Neerven '89
  Skrzypek '92; Montagna, Nicrosini, Piccinini '97

  $\longrightarrow$ $\mathcal{O}(0.1\%)$ uncertainty on σ_Z, A_{FB}

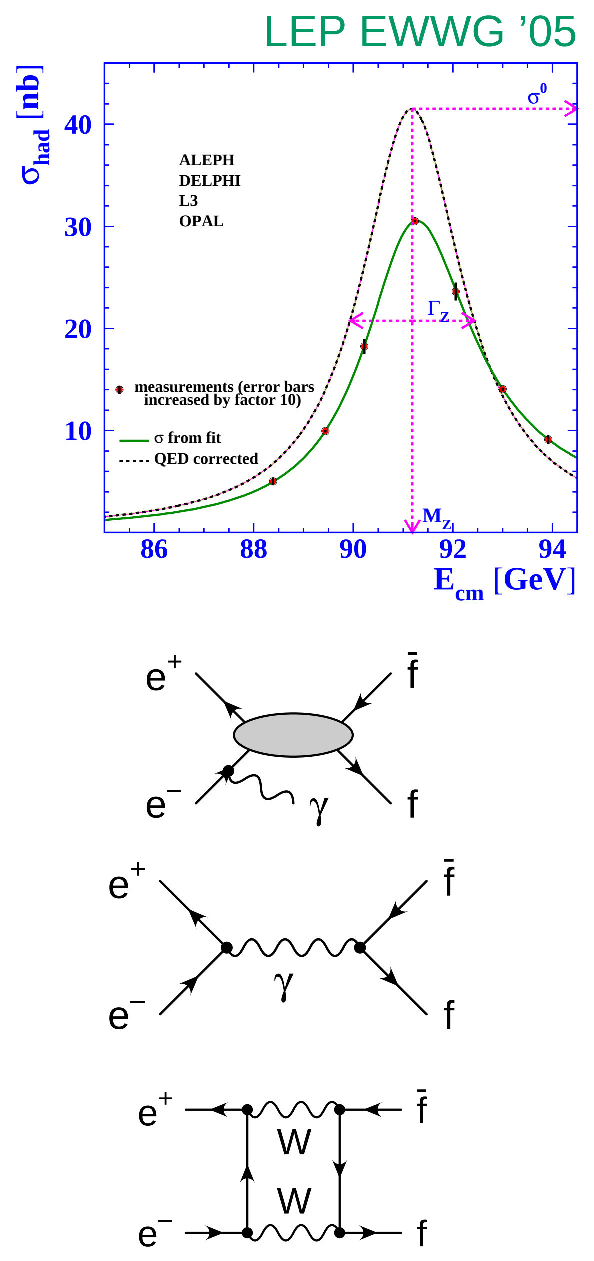
  $\longrightarrow$ Improvement needed for ILC/FCC-ee

- Subtaction of non-resonant γ-exchange, γ–Z interf., box contributions, Bhabha scattering

  see, e.g., Bardin, Grünewald, Passarino '99

  $\longrightarrow$ $\mathcal{O}(0.01\%)$ uncertainty within SM (improvements may be needed)

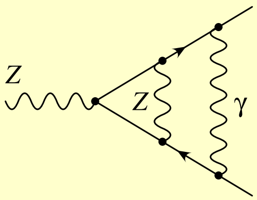
  $\longrightarrow$ Sensitivity to some NP beyond EWPO



Non-factorizable contributions

Factorization of massive and QED/QCD FSR:

$$\overline{\Gamma}_f \approx \frac{N_c \overline{M}_Z}{12\pi} \left[\left(\mathcal{R}_V^f |g_V^f|^2 + \mathcal{R}_A^f |g_A^f|^2 \right) \frac{1}{1 + \mathrm{Re}\, \Sigma_Z'} \right]_{s=\overline{M}_Z^2}$$



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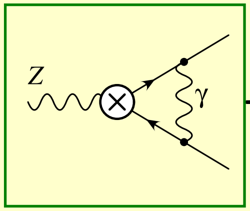
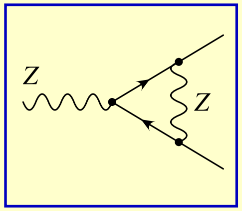
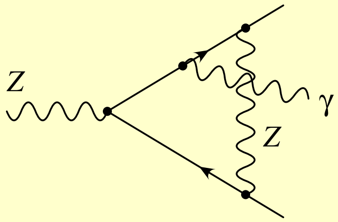


$+$ finite, with $\otimes =$


Additional non-factorizable contributions, e.g.



$\longrightarrow$ Known at $\mathcal{O}(\alpha\alpha_s)$ Czarnecki, Kühn '96
Harlander, Seidensticker, Steinhauser '98

$\longrightarrow$ Currently not known at $\mathcal{O}(\alpha^2)$ and beyond

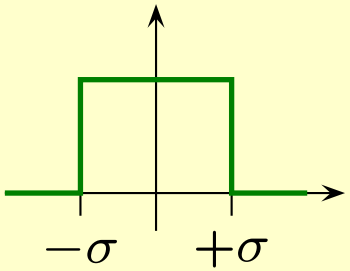
$\longrightarrow$ $\mathcal{O}(0.01\%)$ uncertainty on Γ_Z, σ_Z, maybe larger for A_b
(improvements may be needed)

Combination of theory errors

- Add theory errors from each source **linearly**:

  Idea: each value within error range in equally likely

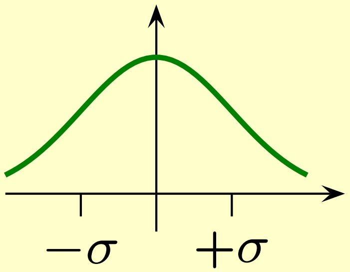
  $\longrightarrow$ Use flat prior in global fits



- Add theory errors from each source **quadratically**:

  Idea: different error sources are uncorrelated

  $\longrightarrow$ Use Gaussian prior in global fits (central limit theorm)



Parametrization of new physics

More general setup: Use pseudo-observables

M_W, Γ_Z, $\sigma^0_{\rm had}$, R_b, R_ℓ, A_ℓ, A_b, A_{lq} $(\ell = e, \mu, \tau)$ $\longrightarrow$ 12 quantities

Effective field theory: $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i + \mathcal{O}(\Lambda^{-3})$ $\quad (\Lambda \gg M_Z)$

$$\mathcal{O}_{\phi 1} = (D_\mu\Phi)^\dagger\Phi\,\Phi^\dagger(D^\mu\Phi) \qquad \alpha\Delta T = -\frac{v^2}{2}\frac{c_{\phi 1}}{\Lambda^2}$$

$$\mathcal{O}_{\rm BW} = \Phi^\dagger B_{\mu\nu}W^{\mu\nu}\Phi \qquad \alpha\Delta S = -e^2v^2\frac{c_{\rm BW}}{\Lambda^2}$$

$$O^{(3)e}_{\rm LL} = (\bar{L}^e_{\rm L}\sigma^a\gamma_\mu L^e_{\rm L})(\bar{L}^e_{\rm L}\sigma^a\gamma^\mu L^e_{\rm L}) \qquad \Delta G_F = -\sqrt{2}\frac{c^{(3)e}_{\rm LL}}{\Lambda^2}$$

$$O^f_{\rm R} = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{f}_{\rm R}\gamma^\mu f_{\rm R}) \qquad f = e, \mu\,\tau, b, lq$$

$$O^F_{\rm L} = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{F}_{\rm L}\gamma^\mu F_{\rm L}) \qquad F = \binom{\nu_e}{e}, \binom{\nu_\mu}{\mu}, \binom{\nu_\tau}{\tau}, \binom{u, c}{d, s}, \binom{t}{b}$$

$$O^{(3)F}_{\rm L} = i(\Phi^\dagger \overleftrightarrow{D}^a_\mu \Phi)(\bar{F}_{\rm L}\sigma_a\gamma^\mu F_{\rm L})$$

More operators than EWPOs

$\longrightarrow$ Some can be constrained by $W \to \ell\nu$, had., $e^+e^- \to W^+W^-$