Determination of $\alpha_{QED}(m_Z)$ @ FCC-ee

- **Outline**
  - Basics
    - Why measure $\alpha_{QED}(m_Z)$?
    - The FCC-ee potential
  - The $e^+e^- \to \mu^+\mu^-$ cross section
    - Expected integrated luminosity
    - Luminosity measurement
    - Uncertainty on α_{QED}
  - The $\mu^+\mu^-$ forward-backward asymmetry
    - Sensitivity to α_{QED}
    - Uncertainty on α_{QED}
  - Summary
    - Combination
    - Running
  - Conclusion and Outlook

Why measure $\alpha_{QED}(m_Z)$?

- **Uncertainty dominant in the interpretation of precision measurements**
  - **Limits severely the potential for new physics exploration at the FCC-ee**
    - **Would require the current uncertainty to be reduced by at least a factor 5**
  - **Current uncertainty dominated by extrapolation from $\alpha_{QED}(0)$ to $\alpha_{QED}(m_Z)$:**

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha_{\rm L}(s) - \Delta\alpha_{\rm HAD}(s)} \qquad \text{with} \qquad \Delta\alpha_{\rm HAD}(M_Z^2) = \frac{\alpha\, M_Z^2}{3\,\pi} P \int_{4m_\pi^2}^{\infty} \frac{R(s)}{s(M_Z^2 - s)} ds$$

    - **Where R(s) is the (experimental) normalized $e^+e^- \to$ hadrons cross section**
      - ➨ **Affected by the many resonances at low energies**
  - **Latest estimate**

$$\Delta\alpha_{\rm had}(M_Z^2) = (275.0 \pm 1.0) \cdot 10^{-4} \quad \Longrightarrow \quad \boxed{\alpha_{QED}^{-1}(m_Z) = 128.952 \pm 0.014}$$

arXiv:1010.4180, Davier et al.

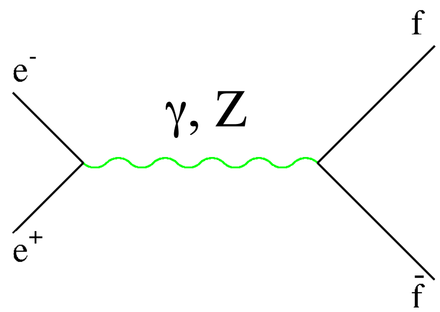
    - **Corresponds to a relative uncertainty of 1.1×10^{-4} on $\alpha_{QED}(m_Z)$**
      - ➨ **To be reduced by a factor at least 5 , i.e., to 2×10^{-5} or better**

arXiv:1308.6176, First look at TLEP

The FCC-ee potential for $\alpha_{QED}(m_Z)$

- **Is the large luminosity of FCC-ee sufficient to improve ?**

  - Could use the FCC-ee to measure $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ and $A_{FB}^{\mu\mu}$ at (a) judicious $\sqrt{s}$



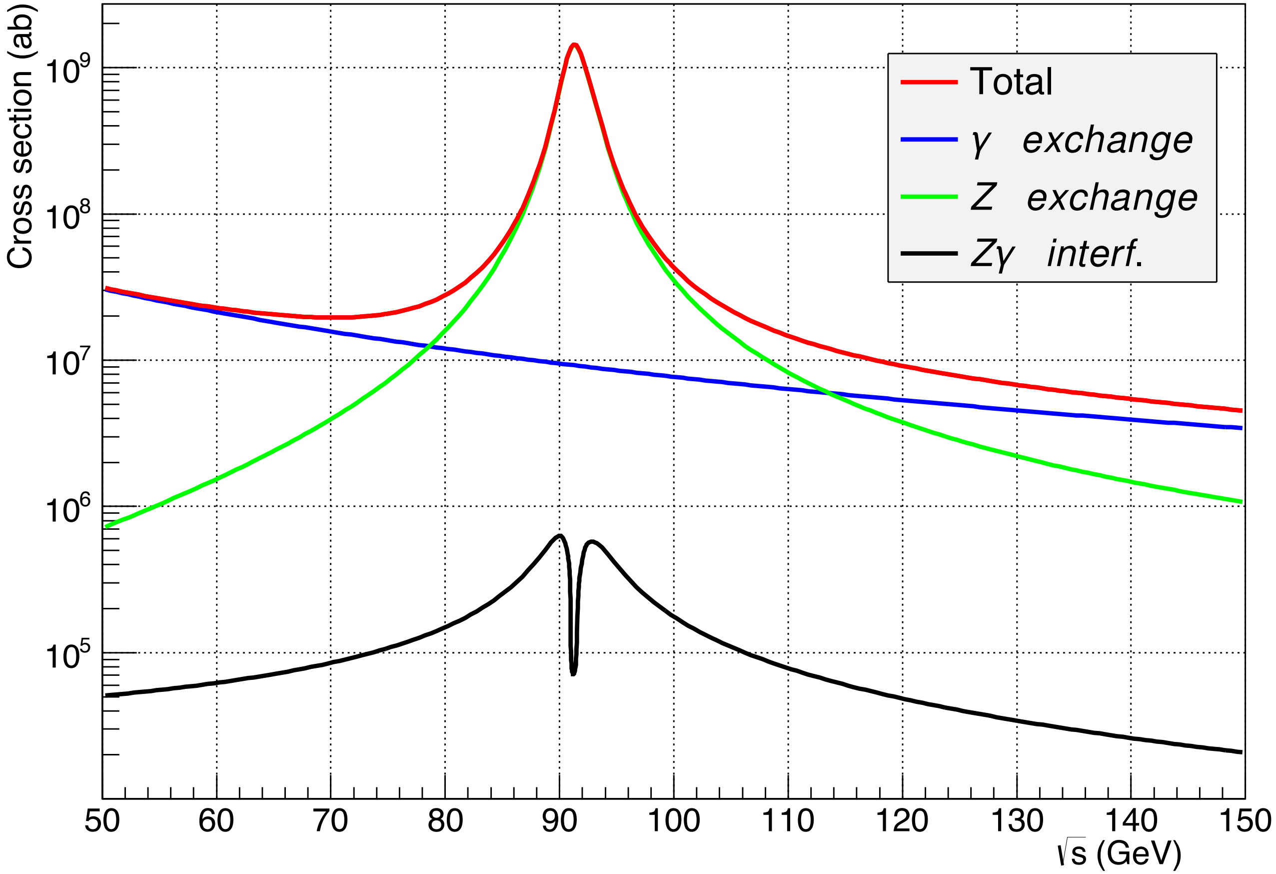
See for example:
- Leike, Riemann, hep-ph/9508390
- L. Berthier, M. Trott, arXiV:1502.0257

    - The γ exchange term is proportional to $\alpha^2_{QED}(\sqrt{s})$
    - The Z exchange term is proportional to G^2_F, hence independent of α_{QED}
    - The γZ interference is proportional to $\alpha_{QED}(\sqrt{s}) \times G_F$
      - ➨ The run at the Z pole is of course not well suited to the $\alpha_{QED}(m_Z)$ measurement

  - If the chosen $\sqrt{s}$ is close to m_Z (say, between 50 and 150 GeV)
    - The extrapolation to m_Z is not affected by e^+e^- resonances at small energies
      - ➨ The theoretical uncertainty from the limited running becomes negligible

The $e^+e^- \to \mu^+\mu^-$ cross section

- With $\sqrt{s}$ from 50 to 150 GeV, ISR corrected, with $s'/s > 0.95$, in ab



- A priori calls for as small a $\sqrt{s}$ as possible (provided that FCC-ee can run at this energy)
  - Largest photon-exchange cross section, smallest Z contamination

Uncertainty on α_{QED} from $\sigma_{\mu\mu}$ measurement (1)

- **Assume we can measure $\sigma_{\mu\mu}$ with a precision $\delta\sigma_{\mu\mu}$ at a given $\sqrt{s}$**
  - The cross section can be parameterized as a function of α_{QED} as follows

$$\sigma_{\mu\mu}(\alpha) = z + \frac{\alpha}{\alpha_0} I(\alpha_0) + \frac{\alpha^2}{\alpha_0^2}\gamma(\alpha_0)$$

    - Where I, z, and γ are the interference and the Z/γ exchange terms ($\alpha = \alpha_0$)

  - Hence $$\delta\sigma_{\mu\mu} = \frac{\delta\alpha}{\alpha} I(\alpha_0) + \frac{2\alpha\delta\alpha}{\alpha^2}\gamma(\alpha_0)$$

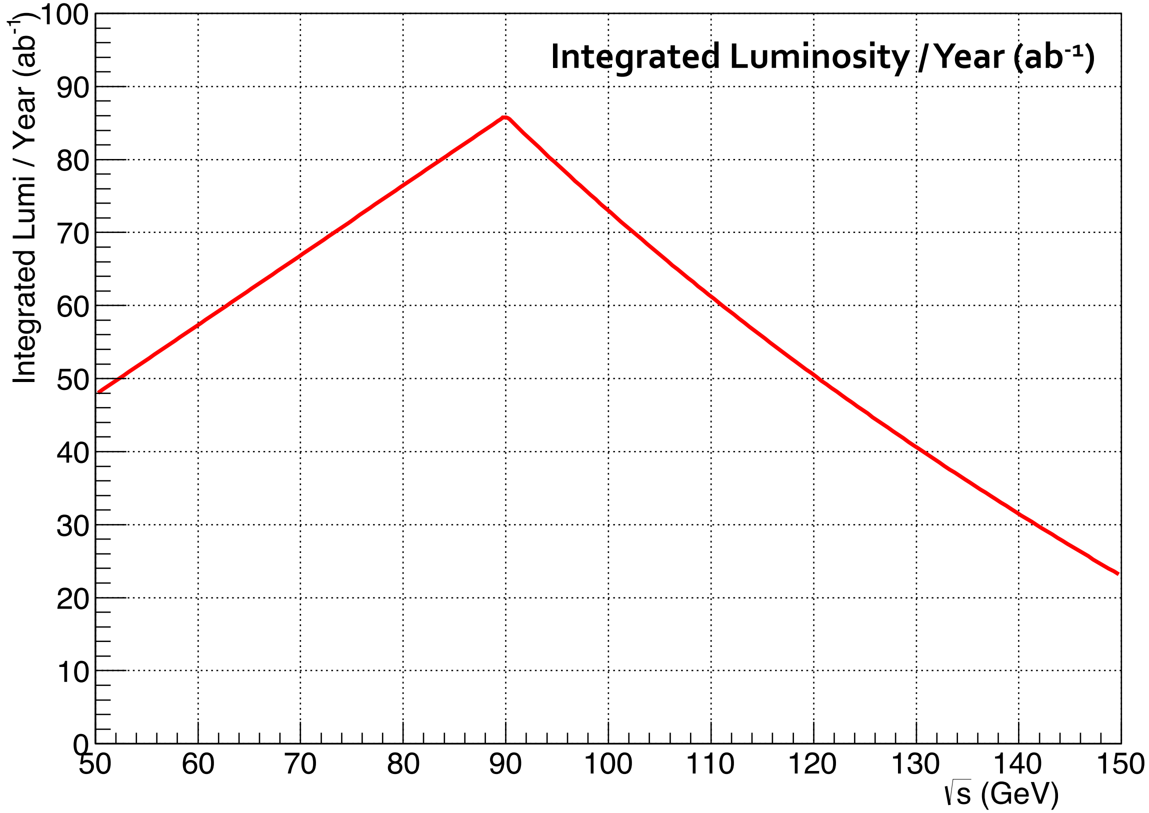
  - Or $$\frac{\delta\alpha}{\alpha} = \frac{\delta\sigma_{\mu\mu}}{I(\alpha_0) + 2\gamma(\alpha_0)} \approx \frac{\delta\sigma_{\mu\mu}}{2\gamma(\alpha_0)}$$

  - $\delta\sigma_{\mu\mu}$ depends on the number of μμ events and on the error on the luminosity: $\sigma_{\mu\mu} = N_{\mu\mu}/L$

$$\left(\frac{\delta\sigma_{\mu\mu}}{\sigma_{\mu\mu}}\right)^2 = \left(\frac{\delta N_{\mu\mu}}{N_{\mu\mu}}\right)^2 + \left(\frac{\delta L}{L}\right)^2$$

Expected luminosity in ab^{-1} / year

- **With the current crabbed-waist option (and 4 IPs)**
  - **Almost 60,000 bunches at the Z pole: all RF buckets are used**
    - **Expect the luminosity to linearly decrease with $\sqrt{s}$ below m_Z**
      - **Use Frank Zimmermann's (@ Washington) numbers above m_Z**

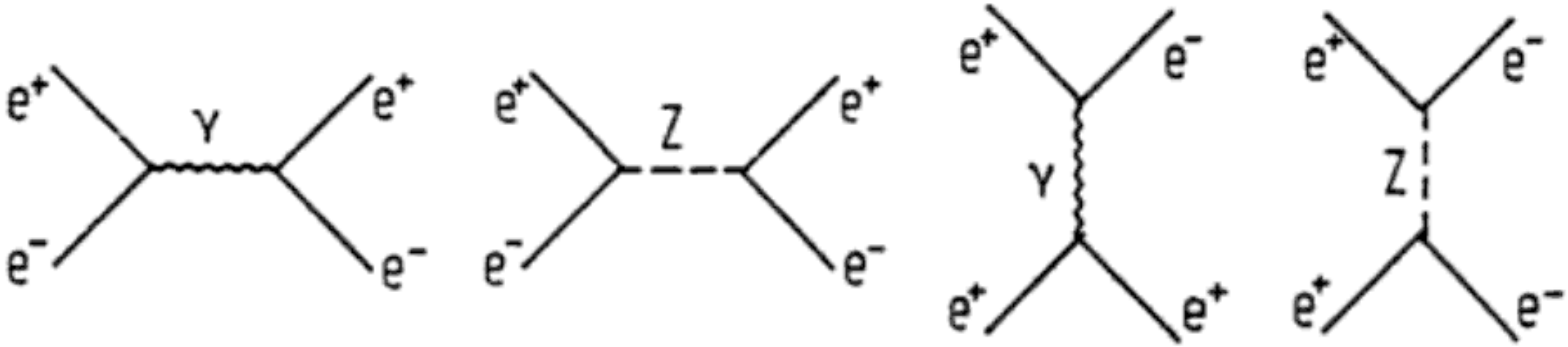


Luminosity measurement (1)

- Historically done with Bhabha scattering at small angle
  - Might prove difficult at FCC-ee
    - Very sensitive to fiducial acceptance definition
    - Small angle region busy with other things
    - Future theoretical uncertainty (NNLO) limited to about 2×10^{-4} S. Jadach, hep-ph/0306083
      - ➨ Part of the uncertainty (~10^{-4}) come from $\Delta\alpha_{had}$!

See also Z. Was, RadCor Workshop, 13-14 July 2015

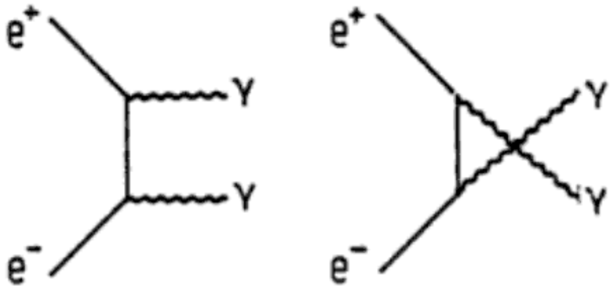
- Quite a few graphs even at lowest order



- ➨ Depends on Z parameters (especially if angle not as small as at LEP)
- ➨ Lots of radiative corrections involved between initial and final legs

Luminosity measurement (2)

- **How about large-angle photon-pair production instead ?**
  - Only "one" graph at lowest order – very poor literature at NLO and beyond



H.-U. Martyn, Adv.Ser.Direct.High Energy Phys. 7 (1990) 92-161

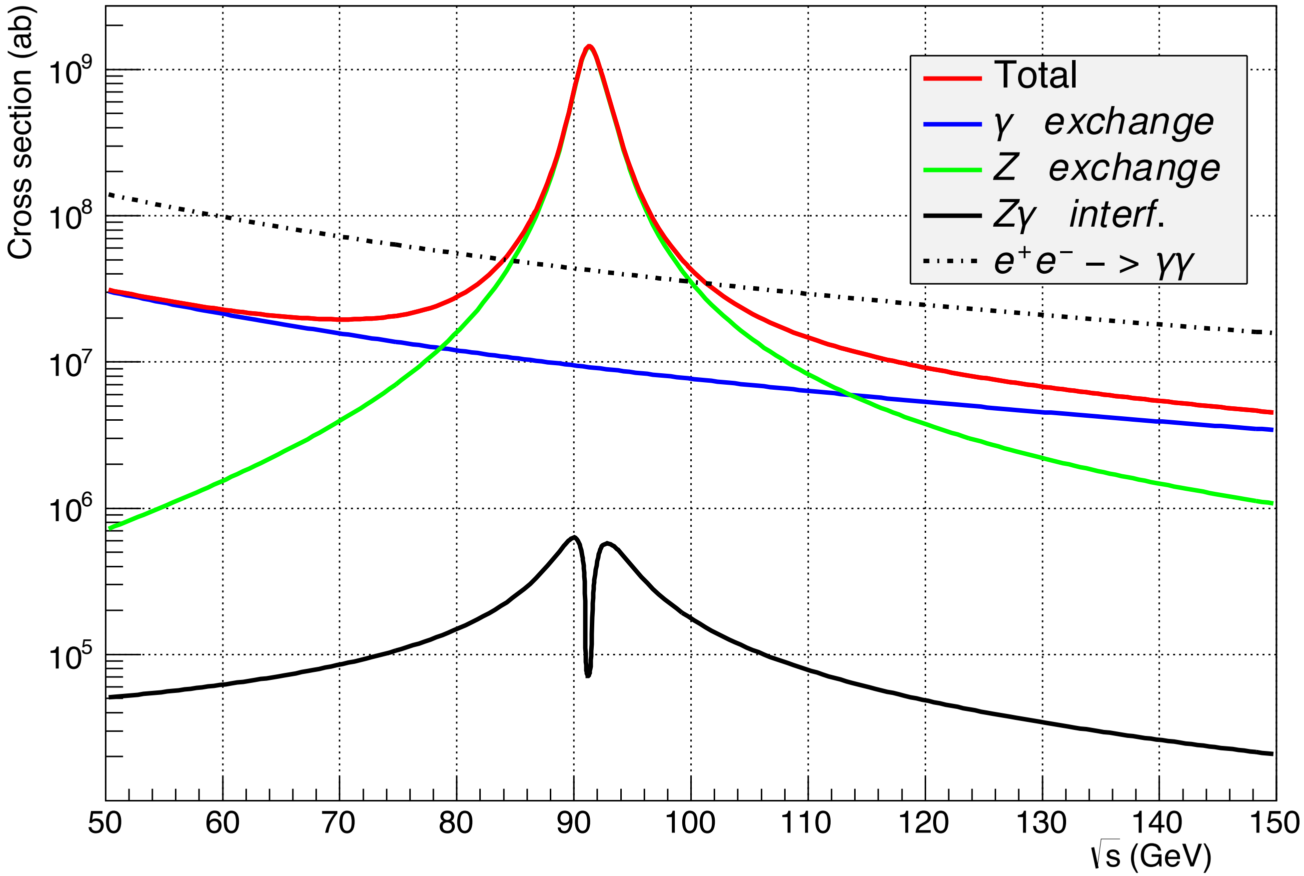
$$\sigma(e^+e^- \to \gamma\gamma) = \frac{2\pi\alpha^2}{s}\left\{\ln\frac{1+\cos\theta_{min}}{1-\cos\theta_{min}} - \cos\theta_{min}\right\}$$

(θ_{min} defines the ECAL acceptance)

- Pure QED process, with few radiative corrections between initial legs and propagator
  - NLO QED correction to σ_{tot} and three- / four-photon LO diff. cross sections known
    - ➨ Work needed for NLO + NNLO corrections at large angle
    - ➨ Reduce theory errors by a factor 10 with respect to low-angle Bhabha ?
- Cross section proportional to $\alpha^2_{QED}(0)$, not to $\alpha^2_{QED}(\sqrt{s})$
  - L. Trentadue: "This process has no dependence on $\Delta\alpha_{had}$ at least up to NNLO"
- Cross section is much smaller than low angle Bhabha, but not that small
  - Example on next slide, for $\theta_\gamma > 20°$ with respect to the beam axis

Luminosity measurement (3)

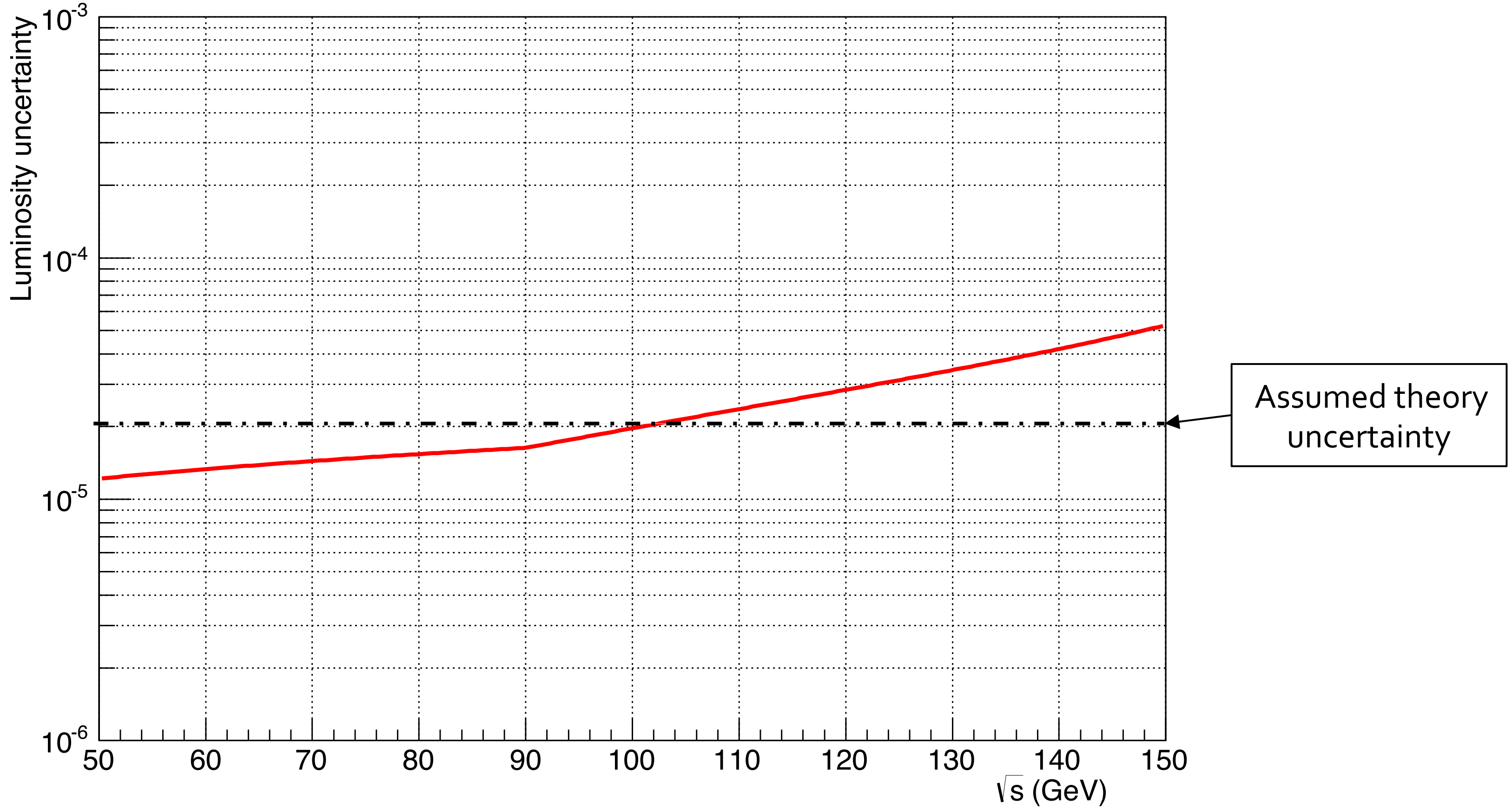
- **Photon-pair production cross-section larger than γ exchange in $\sigma_{\mu\mu}$**



- **Can also normalize $\sigma_{\gamma\gamma}$ to the Z peak cross section at $\sqrt{s} = m_Z$**
  - **In case theory uncertainties turn out to be too large**

Luminosity measurement (4)

- Statistical uncertainty within a year (w/ crabbed-waist config. , 4 IPs)



- In the following, assume a theory uncertainty of 2×10^{-5}
  - Matches well the possibility of normalization with the Z peak cross section

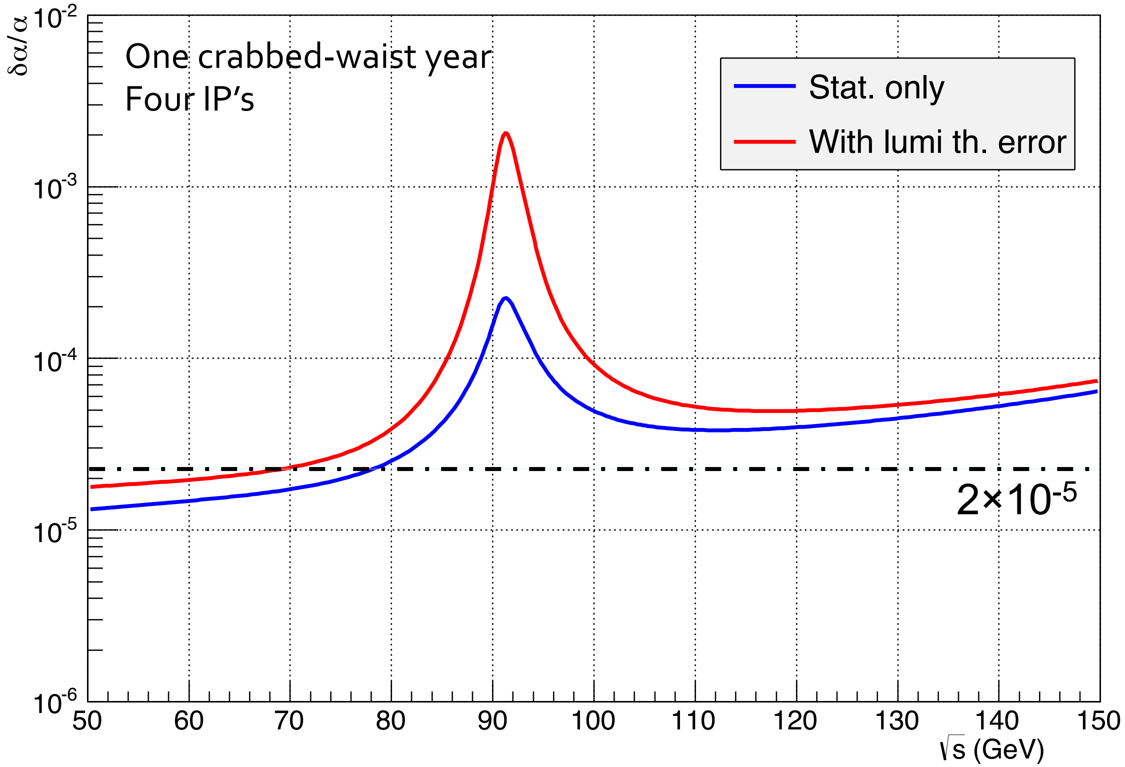
Uncertainty on α_{QED} from $\sigma_{\mu\mu}$ measurement (2)

- **Reminder: Statistical uncertainty (including theory error on lumi)**

$$\frac{\delta\alpha}{\alpha} = \frac{\delta\sigma_{\mu\mu}}{2\gamma(\alpha_0)}$$

with

$$\left(\frac{\delta\sigma_{\mu\mu}}{\sigma_{\mu\mu}}\right)^2 = \left(\frac{\delta N_{\mu\mu}}{N_{\mu\mu}}\right)^2 + \left(\frac{\delta L}{L}\right)^2$$



Uncertainty on α_{QED} from $\sigma_{\mu\mu}$ measurement (3)

- **Systematic uncertainties to be studied and controlled to ~10^{-5} !**
  - Detector acceptance modelling (for muons and photons)
  - Muon and photon reconstruction efficiency
  - Backgrounds from tau pairs and Bhabha
    - All the above can be controlled at the Z pole with the requested precision
  - Dependence on Z parameters ($\sin^2\theta_W$, Γ_Z, M_Z, σ_0, …)
  - Theory prediction for the dimuon and diphoton cross sections
    - "Radiative corrections" workshop (July 13-14) will pave the way — See Z. Was talk

- **Provided these systematic uncertainties are mastered (difficult)**
  - The requested precision for α_{QED} in within reach for $\sqrt{s} \leq 70$ GeV, with a year of running
    - In the crabbed-waist option, with 4 IP's

- **Additional channels can potentially further improve the precision**
  - $e^+e^- \rightarrow \tau^+\tau^-$
  - $e^+e^- \rightarrow$ hadrons
    - Experimentally more difficult (systematic uncertainties), to be studied

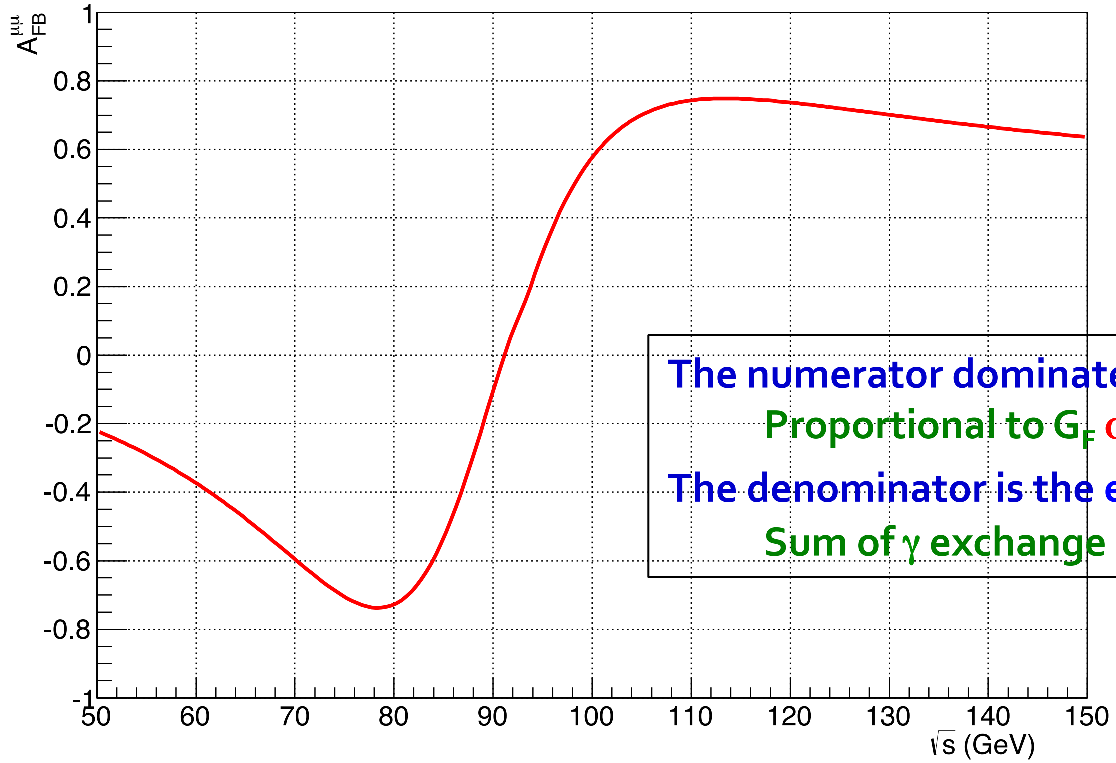
Summary for $\sigma_{\mu\mu}$

- **Systematic uncertainties are challenging**
  - Measuring a cross section to a precision of 10^{-5} is somewhat "crazy"
    - Need to predict absolute cross sections and efficiencies at this level
      - ➨ For the signal $e^+e^- \to \gamma \to \mu^+\mu^-$
      - ➨ For the irreducible background $e^+e^- \to Z \to \mu^+\mu^-$
      - ➨ For the luminosity measurement $e^+e^- \to \gamma\gamma$
      - ➨ For all the reducible backgrounds
    - Need to run the machine at yet another, very different, energy
      - ➨ $\sqrt{s}$ = 50 – 70 GeV might reveal difficult / different challenges
      - ➨ One year at this energy is one year less for the FCC-ee core programme
- **Is it possible to find another observable ?**
  - Less affected by systematic uncertainties
  - With a beam energy closer to those used for the core programme
- **The answer is … YES !**

The forward-backward asymmetry $A_{FB}^{\mu\mu}$

- **A measurement potentially free of theory error**
  - Self normalized
  - Lots of uncertainties cancel in the ratio
    - Including experimental uncertainties

$$A_{FB}^{\mu\mu} = \frac{N_F^{\mu\mu} - N_B^{\mu\mu}}{N_F^{\mu\mu} + N_B^{\mu\mu}}$$



The numerator dominated by interference term
Proportional to $G_F\ \alpha_{QED}(s)$ [Except at the Z peak]

The denominator is the $e^+e^- \to \mu^+\mu^-$ cross section
Sum of γ exchange ($\sim\alpha^2_{QED}(s)$) and Z exchange ($\sim G_F^2$)

Sensitivity of $A_{FB}^{\mu\mu}$ to α_{QED} (1)

- **Assume we can measure $A_{FB}^{\mu\mu}$ with a precision $\delta A_{FB}^{\mu\mu}$ at a given $\sqrt{s}$**
  - The asymmetry can be parameterized as a function of α_{QED} as follows

$$A_{FB}^{\mu\mu}(\alpha) \propto \frac{\frac{\alpha}{\alpha_0} I(\alpha_0)}{z + \frac{\alpha^2}{\alpha_0^2}\gamma(\alpha_0)}$$

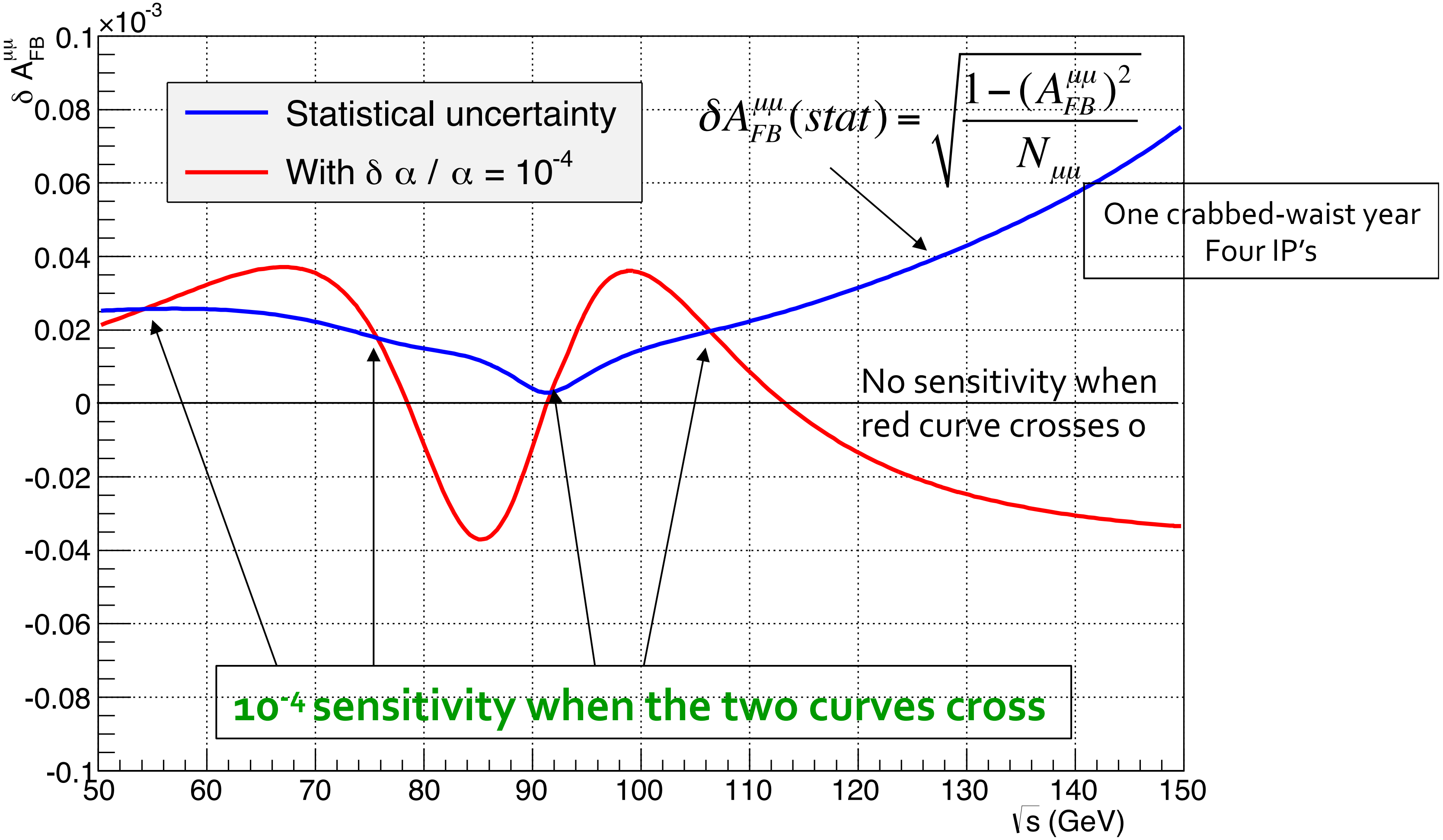
    - Where I, z, and γ are the interference and the Z/γ exchange terms ($\alpha = \alpha_0$)

  - Hence $\delta A_{FB}^{\mu\mu} \propto \frac{I(z-\gamma)}{\sigma_{\mu\mu}^2}\frac{\delta\alpha}{\alpha}$ or $\frac{\delta A_{FB}^{\mu\mu}}{A_{FB}^{\mu\mu}} = \frac{z-\gamma}{\sigma_{\mu\mu}}\frac{\delta\alpha}{\alpha}$

  - No dependence on α_{QED} when Z and γ exchange are equal
    - i.e., when the interference is maximal : $\sqrt{s}$ = 78 GeV and $\sqrt{s}$ = 114 GeV
  - No dependence on α_{QED} when the interference term / the asymmetry vanishes
    - i.e., at the Z pole : $\sqrt{s} = m_Z$
  - Relative sign of δA_{FB} and $\delta\alpha$ changes at these three values of $\sqrt{s}$
    - Important remark when it comes to evaluate systematic uncertainties (see later)

Sensitivity of $A_{FB}^{\mu\mu}$ to α_{QED} (2)

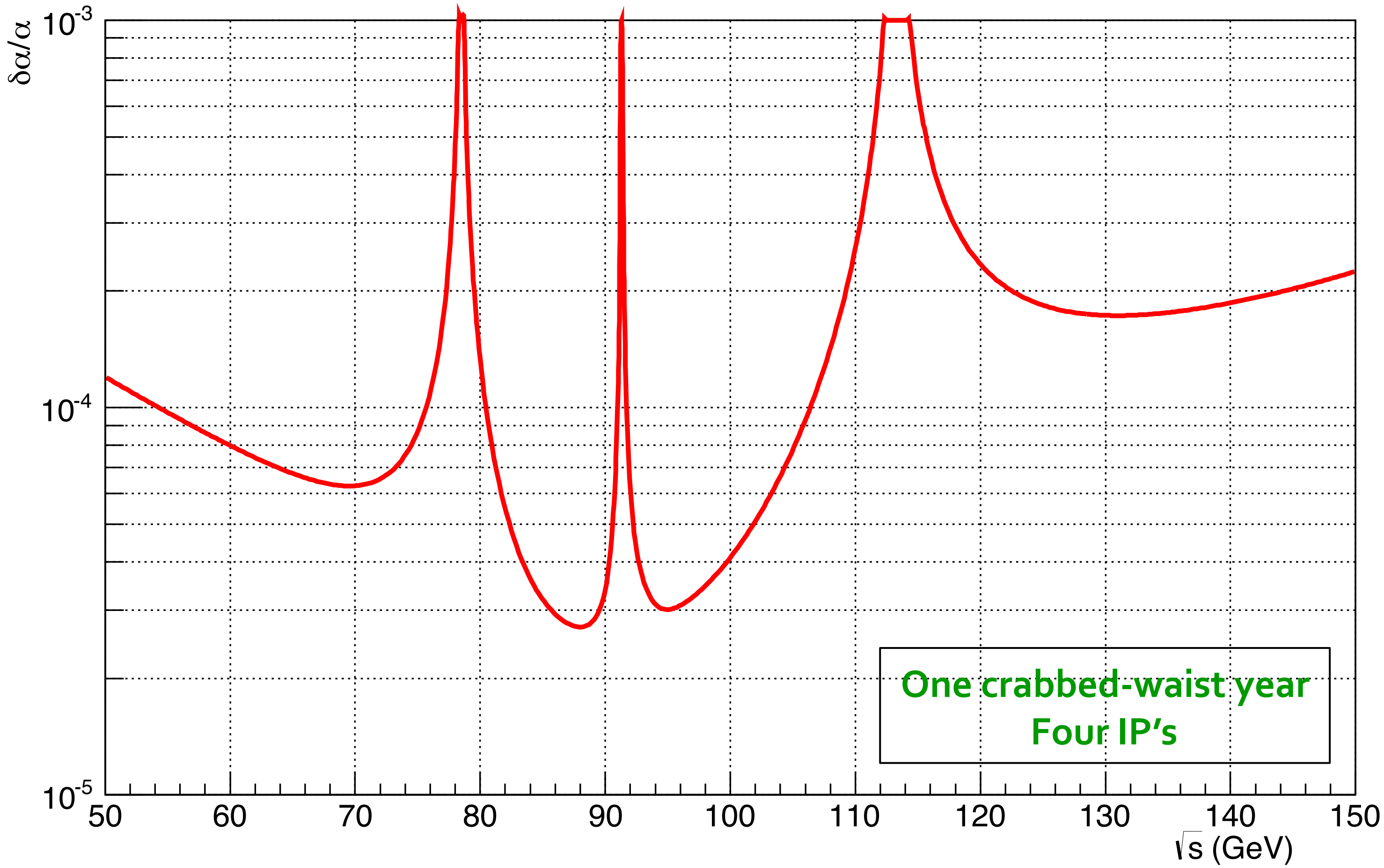
- **Sensitivity quite dependent on √s**



  - **Best just above and just below the Z pole**

Sensitivity of $A_{FB}^{\mu\mu}$ to α_{QED} (3)

- Turning the previous plot in a $|\delta\alpha/\alpha|$ plot, for a year of running at any $\sqrt{s}$



- Best measurement (2.5×10^{-5} within a year) at $\sqrt{s}$ = 88 GeV

Sensitivity of $A_{FB}^{\mu\mu}$ to α_{QED} (4)

- **Systematic uncertainties**
  - Charge inversion (from double charge mismeasurement): can be measured from data
    - Already at the level of 2×10^{-5} with the L3 detector – not a worry.
  - Acceptance asymmetries
    - Can be controlled at the Z pole
  - Cosmic ray background
    - Can be controlled with off-collision data
  - Effect of ISR on A_{FB}
    - Should be small as $\sqrt{s'}$ can be determined from muon angles (and momenta)
  - Dependence on Z parameters
    - See next slide
  - Theoretical prediction accuracy on $A_{FB}^{\mu\mu}$

See Z. Was talk at RadCor Workshop, 13-14 July 2015
Was taken to be 10^{-4} at LEP (ZFITTER/TOPAZ0 difference)
Need one order of magnitude better

- **Further improvement ?**
  - A factor of almost $\sqrt{2}$ can be gained right away with $e^+e^-\rightarrow\tau^+\tau^-$
    - Without additional systematic uncertainty
      - ➨ The 2×10^{-5} goal can be reached with one year at $\sqrt{s}$ = 88 ± 1 GeV

        Or one year at $\sqrt{s}$ = 95 GeV

        Or 6 months at each energy (better, see next slide)

Sensitivity of $A_{FB}^{\mu\mu}$ to α_{QED} (5)

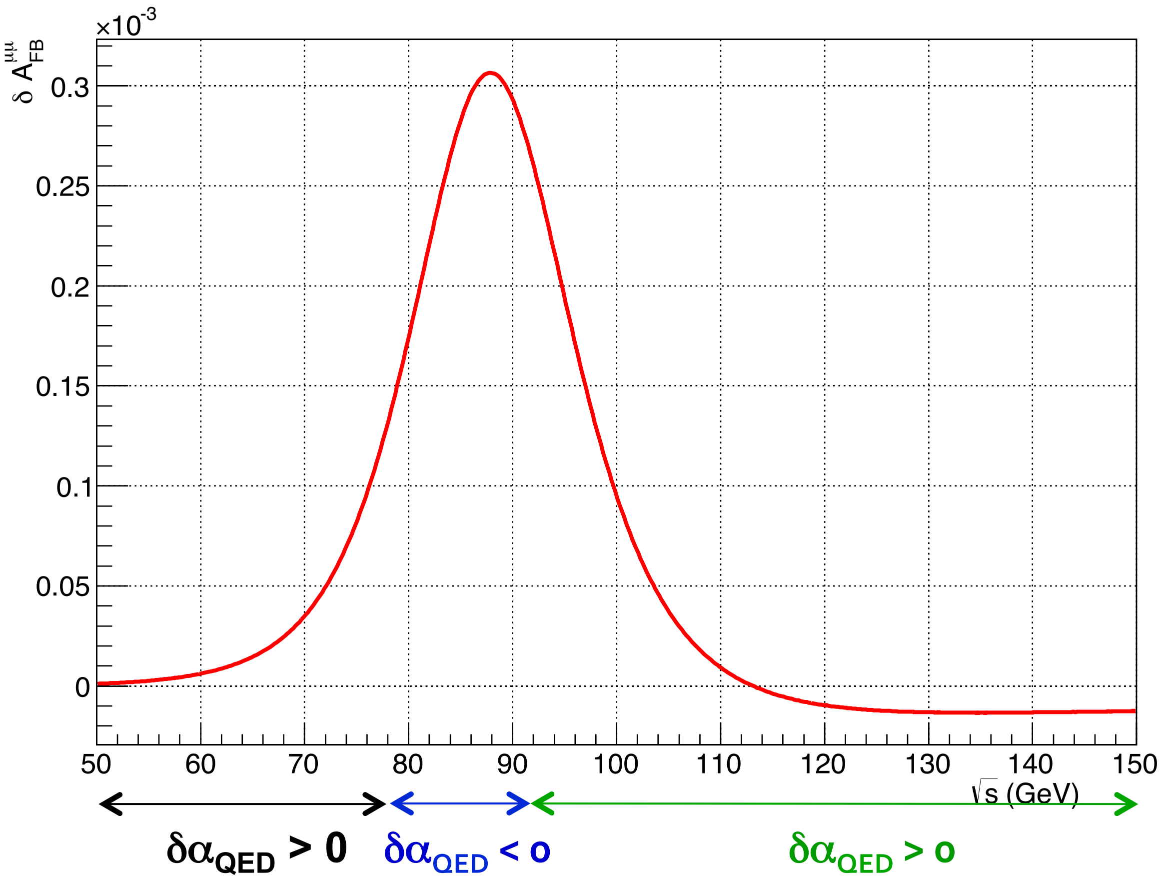
- **Dependence on Z parameters**
  - The derivative of A_{FB} with $\sqrt{s}$ is ~largest at 88 and 95 GeV
    - Hence, for a given $\sqrt{s}$, the absolute knowledge of m_Z is essential
      - ➨ An uncorrelated error of 50 keV on m_Z and $\sqrt{s}$ is equivalent to 2×10^{-5} on α_{QED} ☹
    - However, the only quantity of interest is $\sqrt{s} - m_Z$
      - ➨ A point-to-point energy calibration of 0.1 MeV leads to $\delta\alpha_{QED}/\alpha_{QED} = 2\times10^{-6}$ ☺
  - The dependence on Γ_Z is minute
    - $d\Gamma_Z = 0.1$ MeV leads to $\delta\alpha_{QED}/\alpha_{QED} = 10^{-6}$ ☺
  - The dependence on $\sin^2\theta_W$ is large (will be the dominant Z-parameter uncertainty)
    - Indeed $A_{FB}^{\mu\mu}$ is used at the Z pole (no dependence on α_{QED}) to measure $\sin^2\theta_W$
      - ➨ With the current uncertainty (1.6×10^{-4}), $\delta\alpha_{QED}(s) / \alpha_{QED}(s)$ ~ 10^{-3} at 88/95 GeV ! ☹
    - However, $\alpha_{QED}(88)$ and $\alpha_{QED}(95)$ vary in opposite directions with $\sin^2\theta_W$
      - ➨ Leads to large cancellations in the fit for $\alpha_{QED}(m_Z)$ [$1/\alpha(s) \approx a + b \, \mathrm{Log}(s/m_Z)$]

        With the current uncertainty, $\delta\alpha_{QED}(m_Z) / \alpha_{QED}(m_Z)$ ~ 2×10^{-4}
      - ➨ FCC-ee plans for $\delta\sin^2\theta_W$ ~ 6×10^{-6} with $A_{FB}^{\mu\mu}$ at the Z pole — A. Blondel, March '15

        Corresponds to $\delta\alpha_{QED}(m_Z) / \alpha_{QED}(m_Z)$ ~ 7×10^{-6} OK ! ☺☺

Sensitivity of $A_{FB}^{\mu\mu}$ to α_{QED} (6)

- Variation of $A_{FB}^{\mu\mu}$ for $\delta\sin^2\theta_W = +1.6\times10^{-4}$:



- (See slide 15 for the sign of the derivative of A_{FB} with respect to α_{QED})

Sensitivity of $A_{FB}^{\mu\mu}$ to α_{QED} (7)

Effect of Initial State Radiation

- The derivative of A_{FB} with √s is ~largest at 88 and 95 GeV
  - The effect of ISR on α_{QED} is therefore maximal at these energies
    - ➨ Without any anti-ISR cut, $\delta A_{FB}/A_{FB}$ = +3.7% (−7.1%) at 88 (95) GeV

- The hard collision centre-of-mass energy √s′ can be obtained from the muon angles
  - If exactly one photon is radiated (need the muon momenta for two photons)

$$\frac{s'}{s} = \frac{\sin\vartheta_1 + \sin\vartheta_2 + \sin(\vartheta_1+\vartheta_2)}{\sin\vartheta_1 + \sin\vartheta_2 - \sin(\vartheta_1+\vartheta_2)}$$

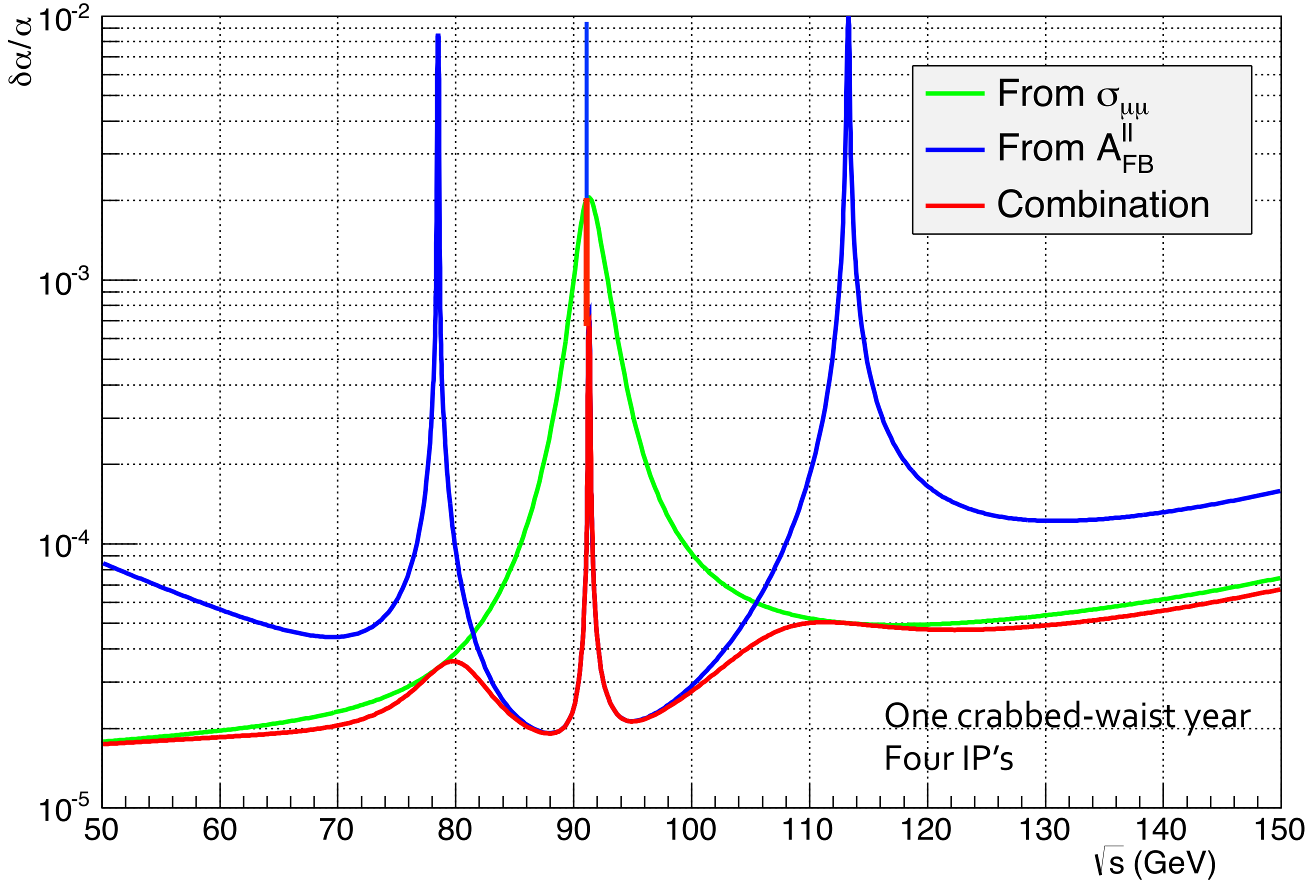
  - Precision on √s′ of the order of 10^{-3} for σ_θ = 1 mrad (multiple scattering)

- Preliminary Monte Carlo study (allowing up to two ISR photons)
  - Anti-ISR cuts : s′/s > 0.99, and $p_\mu > 0.99\ E_{beam}$
    - ➨ $\delta A_{FB}/A_{FB}$ = +0.75% (−0.70%) at 88 (95) GeV

      $\delta\alpha_{QED}/\alpha_{QED}$ = +0.84% (−0.88%) at 88 (95) GeV
  - Without any ISR spectrum knowledge, nice cancellation at the level of a few 10^{-4}
    - ➨ Reduced to a few 10^{-6} if ISR spectrum is known to 1% ☺☺☺

Summary (1)

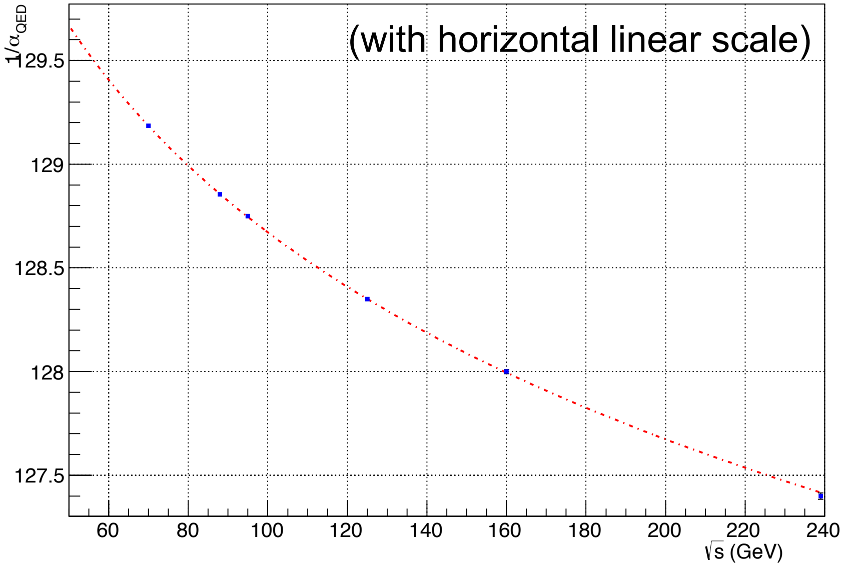
- Combination of cross section (μμ) and A_{FB} (μμ and ττ), in a year (CW, 4IPs)

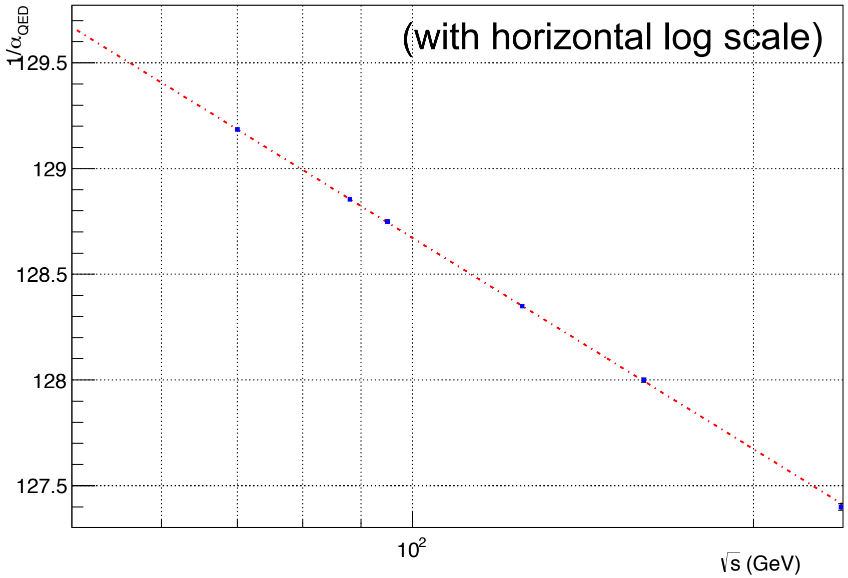


- Get to 2×10^{-5} at $\sqrt{s} \leq 70$ GeV (cross section) and 88 / 95 GeV (forward-backward asym.)
  - Also with cross section at 125 GeV (5×10^{-5}), 160 GeV (8×10^{-5}) or 240 GeV (1.2×10^{-4})

Summary (2)

- **Can even see the actual running of the EM coupling constant**
  - With incredible accuracy:





  - With a fit of $1/\alpha(s)$ to a + b Log(s/m_Z)
    - Get an uncertainty of 1.4×10^{-5} on $\alpha_{QED}(m_Z)$
      - ➨ With data at 88 (1y), 91 (<0.5y), 95 (1y), 125 (1y), 160 (1y) and 240 GeV (3y)

        Increased to 1.8×10^{-5} with only six months at 88 and 95 GeV each.
    - Reduced to 1.2×10^{-5} with 1 year at 70 GeV in addition

Conclusions and outlook (1)

- A good way to determine $\alpha_{QED}(m_Z)$ at FCC-ee is to measure $A^{FB}_{\mu\mu}$
  - At Peak−3.5 ($\sqrt{s} \sim 88$ GeV) and Peak+3.5 ($\sqrt{s} \sim 95$ GeV)
    - As well as at $\sqrt{s} \sim m_Z$ to determine $\sin^2\theta_W$ with the necessary accuracy

- With four IP's and crabbed-waist luminosity at full RF power
  - A total of six months running at each three points is enough
    - To reach a relative precision better than 2×10^{-5} on $\alpha_{QED}(m_Z)$ [Current: 1.1×10^{-4}]
  - It is at most a small extenstion of the FCC-ee core programme

- Experimental uncertainties need to be studied in details
  - Uncertainties on the Z parameters seem to be under control
  - The precision with which ISR needs to be simulated does not exceed 1%
    - Benefits from good μ direction and momentum measurement (how good?)
      - ➨ Detailed study might give some insight on the tracker characteristics
      - ➨ Di-tau final state requires more investigation

- Theory uncertainties are under study
  - See the presentation of Z. Was at the forthcoming RadCor workshop (13-14 July)

Conclusions and outlook (2)

- **A side note: the requested beam energies are also part of the Z scan**
  - They can be measured with resonant depolarization
    - Provided that the corresponding spin tune is half integer
      - For example

Spin Tune	√s (GeV)	Scan name	Run (cw, 4IP)	Physics
99.5	87.6890714	Peak -4	3 months	α_{QED}
100.5	88.5703686	Peak - 3	3 months	α_{QED}
101.5	89.4516658	Peak -2	1 month	m_Z, Γ_Z
103.5	91.2142602	Peak	2.5 years	m_Z, $\sin^2\theta_W$, α_s, N_ν
105.5	92.9768546	Peak+2	1 month	m_Z, Γ_Z
107.5	94.7394490	Peak+3	6 months	α_{QED}
108.5	95.6207462	Peak+4	?	?

Conclusions and outlook (3)

- **Bonus of the study: Absolute luminosity measurement with $e^+e^- \rightarrow \gamma\gamma$**
  - An experimental precision of $2{\times}10^{-5}$ is possible at $\sqrt{s} = m_Z$
    - In the crabbed-waist configuration to have enough events
    - Without the need of a small-angle luminometer
  - This precision needs to be backed-up with (N?)NNLO cross-section prediction
    - See talk from Zbigniew Was on 14 July
  - A small angle luminometer is still needed for the relative luminosity measurements
    - In order to measure the Z width with a scan of the resonance
      - ➨ Otherwise, the error will be dominated by the stat. uncertainty on L

        Because a moderate luminosity is needed at each of the points
  - The limited role of the small angle device may have consequences on its design
    - By relaxing a number of constraints otherwise imposed to it
  - Study to be taken over in the Detector Designs and Experimental Environment groups