KICK from straight wire element

Hamiltonian $H = -\frac{I\mu_0}{4\pi} * (asinh\frac{L/2-z}{\sqrt{x^2+y^2}} - asinh\frac{-L/2-z}{\sqrt{x^2+y^2}})$:

$$x_n = x \tag{1a}$$

$$p_{xn} = p_x - N_1\frac{x}{R}[\sqrt{((L_{emb}+L)^2+4R^2)} - \sqrt{((L_{emb}-L)^2+4R^2)}] \tag{1b}$$

$$y_n = y \tag{1c}$$

$$p_{yn} = p_y - N_1\frac{y}{R}[\sqrt{((L_{emb}+L)^2+4R^2)} - \sqrt{((L_{emb}-L)^2+4R^2)}] \tag{1d}$$

$$z_n = z; p_{zn} = p_z \tag{1e}$$

where: $N_1 = \frac{\mu_0 Ie}{4\pi P_0}$; $R = x^2 + y^2$; and L - the length of the element; L_{emb} - integration length (embedding drift).

KICK from straight wire element in SixTrack

Hamiltonian $H = -\frac{I\mu_0}{4\pi} * (asinh\frac{L/2-z}{\sqrt{x^2+y^2}} - asinh\frac{-L/2-z}{\sqrt{x^2+y^2}})$:

$$x_n = x \quad (2a)$$

$$p_{xn} = p_x - N_1[xi * wire(xi, yi) - dx * wire(dx, dy)^{optional}] \quad (2b)$$

$$y_n = y \quad (2c)$$

$$p_{yn} = p_y - N_1[yi * wire(xi, yi) - dy * wire(dx, dy)^{optional}] \quad (2d)$$

$$z_n = z; p_{zn} = p_z \quad (2e)$$

where:
$N_1 = \frac{\mu_0 Ie}{4\pi P_0}$;
$wire(x, y) = (\sqrt{((L_{emb} + L)^2 + 4R^2)} - \sqrt{((L_{emb} - L)^2 + 4R^2)})/R$;
$R = x^2 + y^2$;
dx,dy - wire displacements;
xi = x-dx; yi=y-dy;
L - the length of the element; L_{emb} - integration length (embedding drift);
optional (for SixTrack) - the subtraction is 'ON' if option 'ibeco'=1

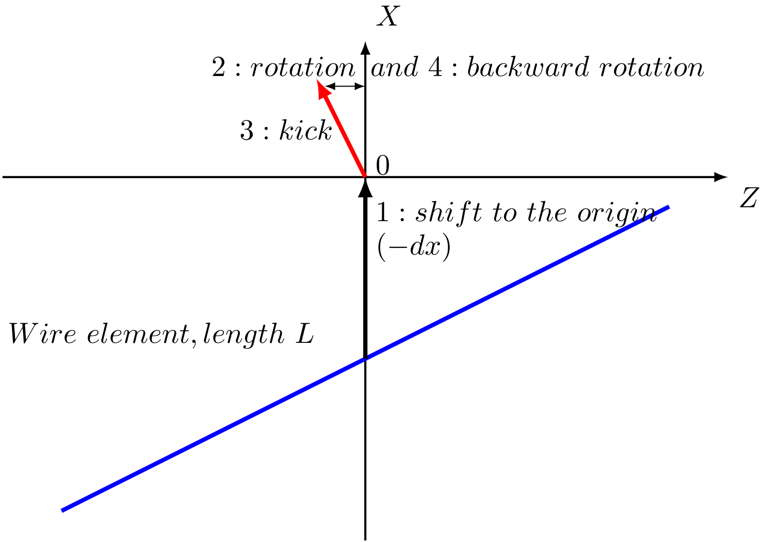
III. TRANSPORT MAP. The first order transport Map (General case)

Element length L ($L_{embedding}$ in our case):

1. Definition of Shifted variables: :
$(xi - > x - DX \ yi - > y - DY)$ DX,DY - wire shift
2. Rotation for 4 canonical variables (xi,px,yi,py) on angles TX, TY (as defined in Beam dynamics, 1998; E. Forest)
3. Wire element kick (slide 8)
4. Backward Rotation for px, py on -TY, -TX

III. TRANSPORT MAP. Tilted, thin element element

Tilted element:



NOTE: L - emb. drift; kick is a function of L and wire length l

The implementation was done on base of preexisting model (rotation part has been taken).
The wire element parameters are:
Single element:

Keyword	val. 1	val. 2	val. 3	val. 4	val. 5
SING	Name	15	current (Amp)	Embedding drift	Length (m)

Displacement:

Keyword	val. 1	val. 2	val. 3	val. 4	val. 5
DISP	Name	Dx(m)	Tx(deg)	Dy(m)	Ty(deg)

Subtraction of dipole kick: if 'ibeco'=1 (IN BEAM BLOCK)

V. VERIFICATION. Tracking in arbitrary magnetic field

From Euler method (and for the Hamiltonian which is used in SixTrack) the explicit map can be obtained:

$$p_{xn} = \frac{b_1 p_y + p_x a_1 - b_1 c_2 - a_2 c_1}{a_1 a_2 - b_1 b_2} \tag{3a}$$

$$p_{yn} = \frac{a_1 p_y - a_1 c_2 + b_2 p_x - b_2 c_1}{a_1 a_2 - b_1 b_2} \tag{3b}$$

$$p_{zn} = pz \tag{3c}$$

$$x_n = x + ds \frac{p_{xn} - A_x}{1 + \delta} \tag{3d}$$

$$y_n = y + ds \frac{p_{yn} - A_y}{1 + \delta} \tag{3e}$$

$$z_n = z + ds[1 - \frac{\beta_0}{\beta} - \frac{\beta_0 (p_{xn} - A_x)^2 + (p_{yn} - A_y)^2}{2\beta(1 + \delta)}] \tag{3f}$$

Ax; Ay and Az - any analytical functions (with the first derivative) - formula **??** for wire

Where:

$$a_1 = 1 - \frac{ds * dA_x/dx}{1+\delta} \tag{4a}$$

$$a_2 = 1 - \frac{ds * dA_y/dy}{1+\delta} \tag{4b}$$

$$b_1 = \frac{ds * dA_y/dx}{1+\delta} \tag{4c}$$

$$b_2 = \frac{ds * dA_x/dy}{1+\delta} \tag{4d}$$

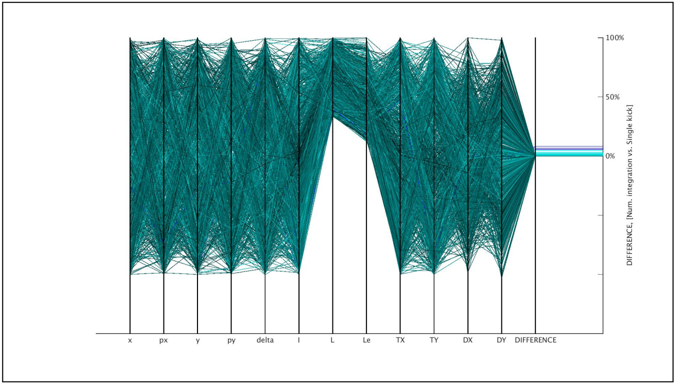
$$c_1 = -ds * dA_z/dx + \frac{ds * A_x * dA_x/dx + ds * A_y * dA_y/dx}{1+\delta} \tag{4e}$$

$$c_2 = -ds * dA_z/dy + \frac{ds * A_x * dA_x/dy + ds * A_y * dA_y/dy}{1+\delta} \tag{4f}$$

Ax; Ay and Az - any analytical functions (with the first derivative)

V. VERIFICATION. Firs order transport map vs. Numerical integration

Parallel Coordinates diagram:



The axis on the right shows the difference between models (in %) for arbitrary combinations of the variables: x,px,y,py,δ, tilts: tx,ty; and displacements: dx,dy. The limitations were: $x << L, px << 1, dx < L$ 20000 of combinations are shown; 99.8 % of cases provides the difference $<$ 0.1 % ; in a few cases the difference is about 1-5 % and can be explained by numerical rounding, when the angles is close to 90 degrees.