Non-locality in QFT due to Quantum Effects in Gravity

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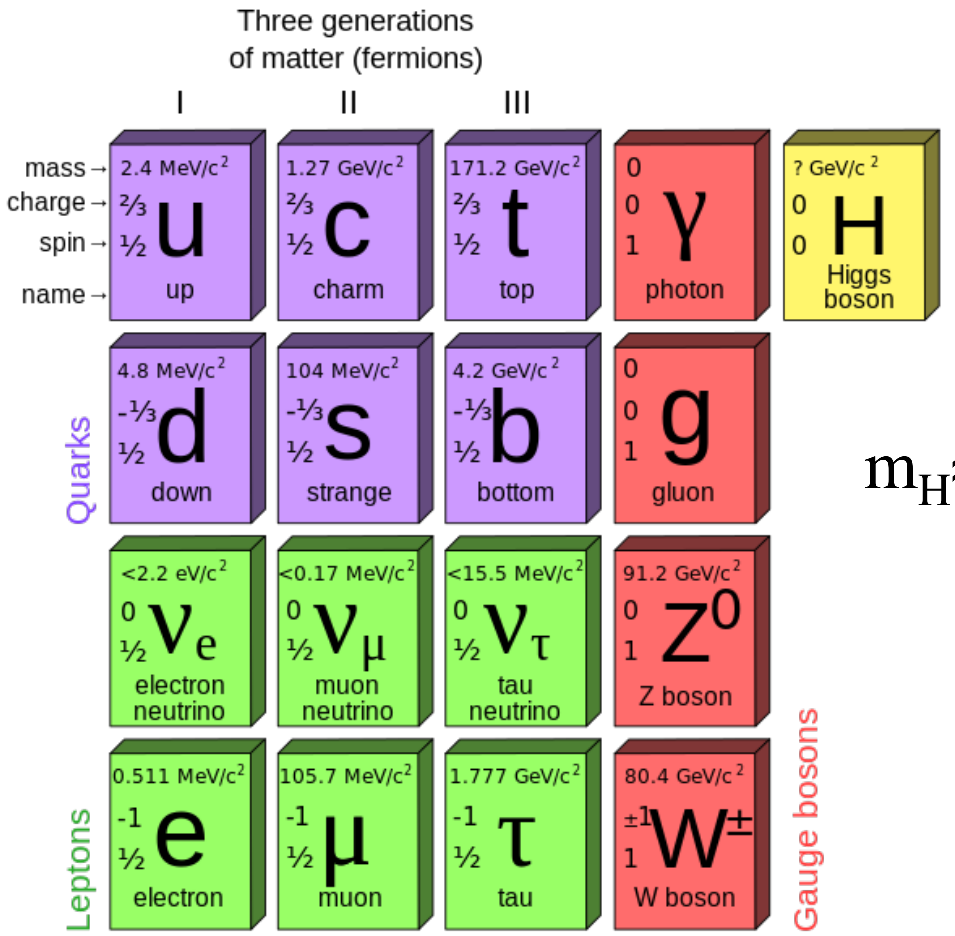
Effective action for GR

- How can we describe general relativity quantum mechanically?
- Well known issues with linearized GR: it is not renormalizable.
- This is the reason d'être of string theory, loop quantum gravity etc…
- How much can we understand using QFT techniques?
- We have good reasons to think that length scales smaller than the Planck scale are not observables due to the formation of small black holes.
- Effective field theories might be all we need to discuss physics at least up to the Planck scale.

Effective action for GR

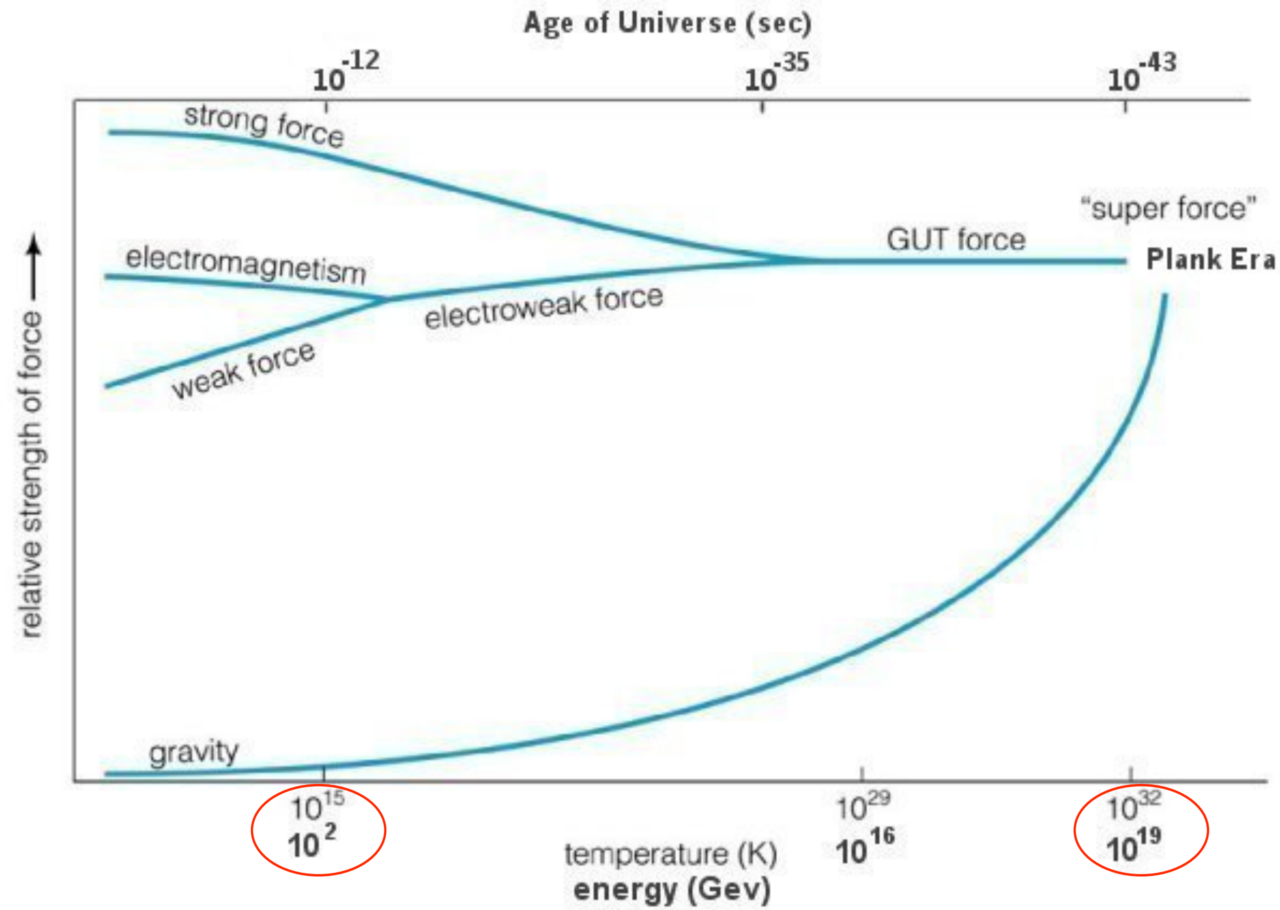
- The goal is to try to make the link with observables.
- Or at least with with thought experiments.
- It is very conservative.
- What can we learn using techniques we actually understand well, and which are compatible with nature as we know it: standard model and GR.

Why quantize gravity? Unification…

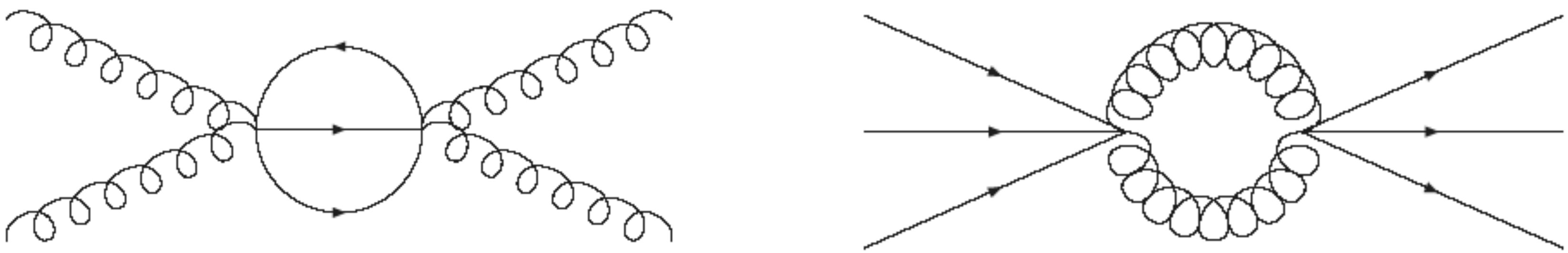


$m_H \approx 125$ GeV

Grand unification?



• Besides the beauty of grand unification, there are formal problems as well e.g.



• We thus have very good reasons to believe that gravity must be quantized.

• One feature we expect from such a unification is the existence of a minimal length.

A minimal length from QM and GR

Claim: GR and QM imply that no operational procedure exists which can measure a distance less than the Planck length.

Assumptions:

- Hoop Conjecture (GR): if an amount of energy E is confined to a ball of size R, where R < E, then that region will eventually evolve into a black hole.
- Quantum Mechanics: uncertainty relation.

Minimal Ball of uncertainty:

Consider a particle of Energy E which is not already a black hole.
Its size r must satisfy:

$$r \gtrsim \max\,[\,1/E\,,\;E\,]$$

where 1/E is the Compton wavelength and E comes from the Hoop Conjecture. We find:

$$r \sim l_P$$

Can we identify this minimal length in QFT? It is a form of non-locality.

Effective action for GR

- I am going to assume general covariance (diffeomorphism invariance)
- Quantum gravity has only 2 dofs namely the massless graviton (which has 2 helicity states).
- We know the particle content of the "matter theory" (SM, GUT, inflation etc).
- We can write down an effective action for quantum gravity.

Effective action for GR

- This program was started by Feynman in the 60’s using linearized GR.
- Try to find/calculate observables
- Try to find consistency conditions which could guide us on our path towards a quantization of GR.

Effective action for GR coupled to known matter

- We can describe any theory of quantum gravity below the Planck scale using effective field theory techniques:

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{1}{2} M^2 + \xi H^\dagger H \right) \mathcal{R} - \Lambda_C^4 + c_1 \mathcal{R}^2 + c_2 \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} + \mathcal{L}_{SM} + \mathcal{O}(M_\star^{-2}) \right]$$

- Planck scale $(M^2 + \xi v^2) = M_P^2$ $M_P = 2.4335 \times 10^{18}$ GeV
- $\Lambda_C \sim 10^{-12}$ GeV; cosmological constant.
- $M_\star >$ few TeVs from QBH searches at LHC and cosmic rays.
- Dimensionless coupling constants ξ, c_1, c_2
  - c_1 and $c_2 < 10^{61}$ [xc, Hsu and Reeb (2008)]

    R^2 inflation requires $c_1 = 5 \times 10^8$ (Faulkner et al. astro-ph/0612569]).
  - $\xi < 2.6 \times 10^{15}$ [xc & Atkins, 2013]

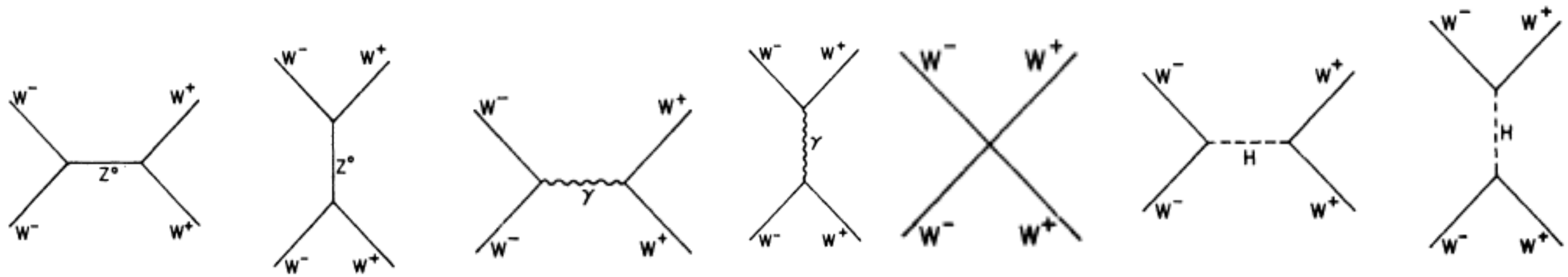
    Higgs inflation requires $\xi \sim 10^4$.

What do we know about $M_\star$?

(energy scale up to which one trusts the effective theory)

Unitarity in quantum field theory

- Follows from the conservation of probability in quantum mechanics.
- Implies that amplitudes do not grow too fast with energy.
- One of the few theoretical tools in quantum field theory to get information about the parameters of the model.
- Well known example is the bound on the Higgs boson's mass in the Standard Model (m<790 GeV).



Let us consider gravitational scattering of the particles included in that model (s-channel, we impose different in and out states) (Han & Willenbrock 2004, xc & Atkins 2011)

$$\mathcal{A} = 16\pi \sum_J (2J+1) a_J d^J_{\mu,\mu'}$$

$$|\mathrm{Re}\ a_J| \leq 1/2$$

Let us look at J=2 partial wave

$$a_2 = -\frac{1}{320\pi}\frac{s}{\bar{M}_P^2}N \qquad N = 1/3N_s + N_\psi + 4N_V$$

One gets the bound:

$$E^*_{\mathrm{CM}} \leq \bar{M}_P\sqrt{\frac{160\pi}{N}}$$

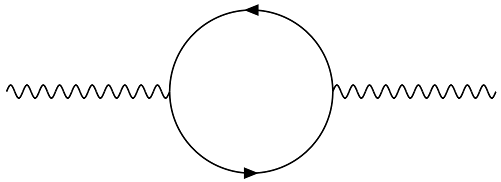
For large *N*, unitarity can be violated well below the Planck mass.

From the J=0 partial wave, one gets

$$\Lambda \simeq \bar{M}_P/\xi$$

Self-healing of unitarity

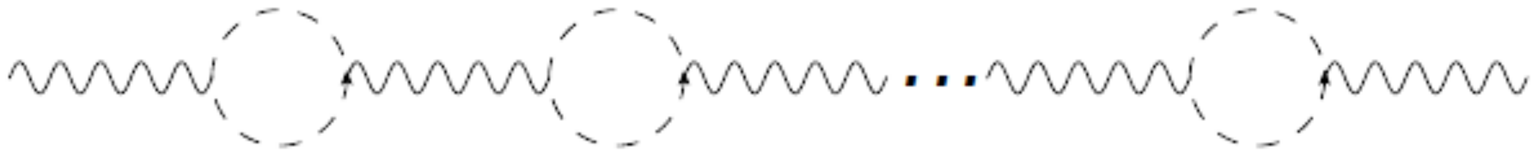
- Aydemir, Anber & Donoghue argued that the effective theory heals itself.
- First let's calculate the leading quantum corrections to the previous amplitude (still working in linearized GR in flat space-time)



- Insert any matter in your model in that loop.
- Typically there is more matter than gravitational degrees of freedom, we can thus ignore gravitons in that loops for energies below the Planck mass.
- Honest calculation: regularized using dim-reg and absorb divergencies in R^2 etc.
- Obviously the theory is still not renormalizable, but that's not an issue for an effective field theory.

Self-healing of unitarity

- In the case of linearized gravity coupled to the SM, resum:



- in the large N limit, keeping NG_N small. One obtains a resummed graviton propagator

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{i\left(L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}\right)}{2q^2\left(1 - \frac{NG_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right)}$$

$$N = N_s + 3N_f + 12N_V$$

$$L^{\mu\nu}(q) = \eta^{\mu\nu} - q^\mu q^\nu / q^2$$

- One can check explicitly

$$|A_{dressed}|^2 = \mathrm{Im}(A_{dressed})$$

Self-healing of unitarity non-minimal coupling

- One can also resum the infinite series of 1-loop polarization diagrams

- In the large ξ and N limits but keeping N ξG_N small, I get

$$iD^{\alpha\beta\mu\nu}_{dressed} = -\frac{i}{2s}\frac{L^{\alpha\beta}L^{\mu\nu}}{\left(1-\frac{sF_1(s)}{2}\right)}$$

$$F_1(q^2) = -N_s G_N \xi^2 \log\left(\frac{-q^2}{\mu^2}\right)$$

- The dressed amplitude fulfills exactly

$$|A_{dressed}|^2 = \mathrm{Im}(A_{dressed})$$

XC & Casadio 2014

Comments

- In linearized GR, the effective theory self-heals itself.
- In the large N limit one finds poles in the resummed graviton propagator: sign of strong interaction.
- The positions of these poles depend on the number of fields
- One finds

$$
\begin{aligned}
q_1^2 &= 0, \\
q_2^2 &= \frac{1}{G_N N} \frac{120\pi}{W\left(\frac{-120\pi M_P^2}{\mu^2 N}\right)}, \\
q_3^2 &= (q_2^2)^*,
\end{aligned}
$$

Comments

- It is tempting to interpret these poles as black hole precursors.
- In the SM

$$N_s = 4,\ N_f = 45,\ \text{and}\ N_V = 12$$

- We thus find

$$(7 - 3i) \times 10^{18}\ \text{GeV}\ \text{and}\ (7 + 3i) \times 10^{18}\ \text{GeV}$$

- The first one corresponds to a state with mass $p_0^2 = (m - i\Gamma/2)^2$

$$7 \times 10^{18}\ \text{GeV}$$

- And width

$$6 \times 10^{18}\ \text{GeV}$$

Comments

- Our interpretation is similar to the sigma-meson case which can be identified as the pole of a resummed scattering amplitude in the large N limit of chiral perturbation theory.

- This resummed amplitude is an example of self-healing in chiral perturbation theory.

- In low energy QCD, the position of the pole does correspond to the correct value of the mass and width of the sigma-meson.

Comments

- Note that the 2nd pole has the wrong sign for particle: it is a ghost

$$(7+3i)\times 10^{18}\ \mathrm{GeV}$$

- Acausal effects: connection to black hole information paradox? Could be canceled by e.g. Lee and Wick's mechanism.

- Non local effects

$$S=\int d^4x\sqrt{g}\left[R\log\left(\frac{\Box}{\mu^2}\right)R\right]$$

- Can these effects soften singularities?

Comments

- With our interpretation in mind, an interesting picture emerges.
- Self-healing in the case of gravitational interactions implies unitarization of quantum amplitudes via quantum black holes.
- As the center of mass energy increases so does the mass of the black hole and it becomes more and more classical.
- This is nothing but classicalization.
- What we call Planck scale (first QBH mass/cut off for the EFT) is now a dynamical quantity which depends on the number of fields.
- The effective theory certainly breaks down at the Planck scale.
- Self-healing makes the link between several concepts that had been proposed previously.

Comments

- Let's think about perturbative unitarity again.
- We are taught that a breakdown of perturbative unitarity is a sign of new physics or strong dynamics.
- In the case of quantum gravity in the large N, we have identified the strong dynamics as quantum black holes: this is not a surprise.
- More surprising is the case of a large nonminimal coupling of scalars to R, here we found a resummed propagator that does not have poles beyond the one at $q^2=0$.
- Unitarity is restored by the self-healing mechanism without new physics or strong dynamics.

Gravity leads to non-local effects in Matter

XC, Croon & Fritz (2015)

- Let's reconsider the resummed graviton propagator

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{i\left(L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}\right)}{2q^2\left(1 - \frac{NG_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right)}$$

- Using this propagator we can now calculate the dressed amplitude for the gravitational scattering of 2 scalar fields.
- The tree-level amplitude has been known for a long time:

$$\begin{aligned} A_{tree} \;\; &= \;\; 16\pi G\left(m^4\left(\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\right) + \frac{1}{2s}(2m^2+t)(2m^2+u)\right. \\ &\left. +\frac{1}{2t}(2m^2+s)(2m^2+u) + \frac{1}{2u}(2m^2+s)(2m^2+t)\right) \end{aligned}$$

Gravity leads to non-local effects in Matter

- Let me rewrite the dressed propagator as

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{P^{\alpha\beta,\mu\nu}(q^2)}{1+f(q^2)}, \qquad f(q^2) = -\frac{NG_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right).$$

- We find the Taylor expanded dressed amplitude:

$$\begin{aligned} A_{dressed} &= A_{tree} + A^{(1)} + \dots \\ A^{(1)} &= \frac{2}{15}G_N^2 N \left(m^4\left(\log\left(-\frac{stu}{\mu^6}\right)\right)\right. \\ &\quad + \log\left(-\frac{s}{\mu^2}\right)(2m^2+t)(2m^2+u) + \log\left(-\frac{t}{\mu^2}\right)(2m^2+s)(2m^2+u) \\ &\quad \left. + \log\left(-\frac{u}{\mu^2}\right)(2m^2+s)(2m^2+t)\right). \end{aligned}$$

Gravity leads to non-local effects in Matter

- It is easy to see that $A^{(1)}$ can be obtained from this effective operator:

$$O_8 \;=\; \frac{2}{15} G_N^2 N \left(\partial_\mu \phi(x) \partial^\mu \phi(x) - m^2 \phi(x)^2\right) \log\left(-\frac{\Box}{\mu^2}\right) \left(\partial_\nu \phi(x) \partial^\nu \phi(x) - m^2 \phi(x)^2\right)$$

- This is a non-local operator, we need to make sense of the log term to obtain a causal theory (Espiru et al. (2005), Donoghue &El-Menoufi (2014) and Barvinsky et al in the 80's.)

$$\begin{aligned} S \;=\; & \int d^4x d^4y \sqrt{-g} \left(\frac{1}{16\pi G_N} R - \frac{1}{2} \partial_\mu \phi(x) \partial^\nu \phi(x) + \frac{m^2}{2} \phi^2 \right. \\ & + \left(\frac{2}{15} G_N^2 N \right) \times \end{aligned}$$

$$\left(\left(\partial_\mu \phi(x) \partial^\mu \phi(x) + m^2 \phi(x)^2\right) \int d^4y \sqrt{-g(y)} \langle x | \log\left(-\frac{\Box}{\mu^2}\right) | y \rangle \left(\partial_\nu \phi(y) \partial^\nu \phi(y) - m^2 \phi(y)^2\right) \right) \Bigg)$$

Gravity leads to non-local effects in Matter

- One can define the interpolating function:

$$\mathcal{L}(x,y) = \langle x| \log\left(-\frac{\Box}{\mu^2}\right)|y\rangle$$

- which can be evaluated $\log(x) \approx -1/\epsilon + \; x^\epsilon/\epsilon$

$$\begin{aligned} -\langle x|\frac{1}{\epsilon}|y\rangle + \langle x|\frac{(\Box/\mu^2)^\epsilon}{\epsilon}|y\rangle &= -\frac{1}{\epsilon}\delta(x-y) + \frac{1}{\epsilon}\frac{2\pi^2}{\mu^{2\epsilon}}\int d^4k k^{2+2\epsilon}\frac{1}{|x-y|}J_1(k|x-y|) \\ &\sim -\frac{1}{\epsilon}\delta(x-y) - \frac{8\pi^2}{\mu^{2\epsilon}}\frac{1}{|x-y|^{4+2\epsilon}}, \end{aligned}$$

- For a purely time-dependent problem one has

$$\mathcal{L}(t,t') = -2\lim_{\epsilon\to 0}\left(\frac{\Theta(t-t'-\epsilon)}{t-t'} + \delta(t-t')(\log(\mu\epsilon)+\gamma\right)$$

Gravity leads to non-local effects in Matter

- We have seen that the non-local effects observed in gravity feed back into matter.

- This is compatible with our interpretation of the poles of the resummed propagators as quantum black holes (black hole precursors) which are extended objects.

- The new higher dimensional operators have an approximate shift symmetry

$$\phi \to \phi + c, \text{ where } c \text{ is a constant}$$

- which is broken explicitly by the mass of the scalar field.

- This is interesting for models of inflation.

Gravity leads to non-local effects in Matter

- Are there any observational consequences of this short distance non-locality?
- The effect is suppressed by powers of the Planck scale, one can see that it leads to a small non-Gaussianities even for a single scalar inflation model.
- However the effect is too small to be observable.
- Let's considering the following Lagrangian

$$
\begin{aligned}
L(x) &= X + \frac{m^2}{2}\phi^2(x) + \\
&+ \frac{8}{15}G_N^2 N\left(X(x) + \frac{m^2}{2}\phi^2(x)\right)\int d^4y\sqrt{-g(y)}\mathcal{L}(x,y)\left(X(y) + \frac{m^2}{2}\phi^2(y)\right)
\end{aligned}
$$

$$X(x) = -1/2\partial_\mu\phi(x)\partial^\mu\phi(x) \quad X(y) = -1/2\partial_\mu\phi(y)\partial^\mu\phi(y)$$

Gravity leads to non-local effects in Matter

- We can calculate the speed of sound:

$$c_s^2 = \frac{L(x)_{,X(x)}}{L(x)_{,X(x)} + 2X(x)L(x)_{,X(x)X(x)}} \approx 1 - \frac{32}{15} X(x) G_N^2 N$$

- which remarkably to leading order does not depend on the specific form of the nonlocal function.

- GR coupled to a single scalar field thus predict a small amount from non-Gaussianity, but with a speed of sound very close to one.

- Non-locality is a generic feature of quantum field theory coupled to GR.

QBHs in Effective Field Theory

- We have seen that the QBHs like objects naturally appear in the context of EFT.

- This is precisely what one would expect from the thought experiment we started from.

- The mass of the lightest black holes is close to the Planck mass and depends on the number of fields in nature.

- They lead to non-local effects at a scale which is model dependent.

- This is a new mechanism to lower the Planck mass in 4-dimensions which could lead to interesting model building ideas.

Conclusions

- We have discussed a conservative effective action for quantum gravity within usual QFTs such as the standard model.
- Thought experiments point towards a minimal length in nature when combining GR and QM.
- We can identify these effects in QFT coupled to GR.
- We have found the sign of strong dynamics in the resummed graviton propagator.
- These poles can be interpreted as black hole precursors and their masses and width can be calculated.
- These quantum black holes lead to non-local effects in QFT.

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Thanks for your attention!