Top Quark Pole Mass Measurements at the LHC

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LHC TOPWG Meeting @ CERN
17-18 November 2015

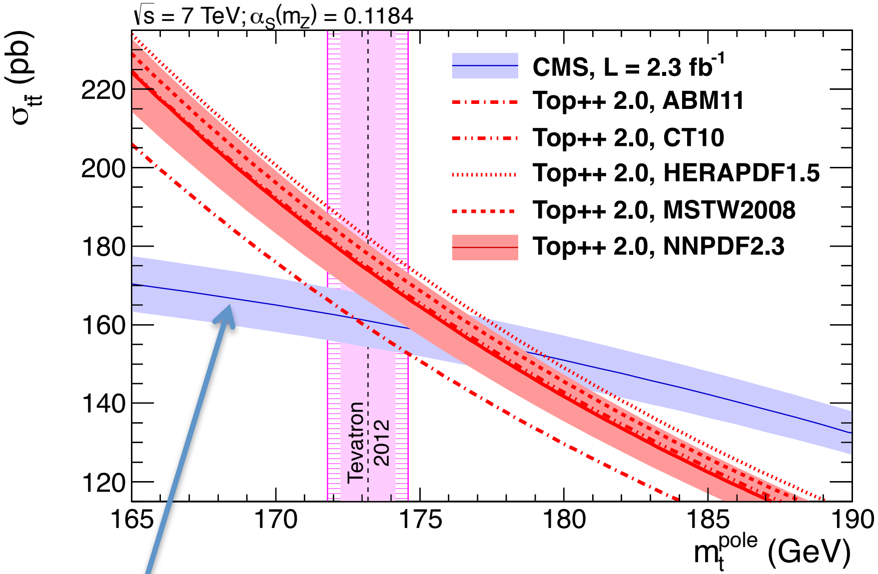
Measuring a Well-Defined Top Quark Pole Mass

- Recipe
  - Pick an observable that can be calculated in perturbative QCD with a well-defined mass definition.
  - Measure distribution (or number) at detector level
  - Unfold to a (parton?) level that can be calculated
    - Or forward fold from parton level.
  - Fit calculation to the (un)folded quantity
  - Evaluate all uncertainties, including dependence of (un)folding on the top mass assumption.

Outline

- m_t^{pole} from inclusive cross section
  - from CMS at 7 TeV.
  - from ATLAS at 7 and 8 TeV.
  - from CMS at 7 and 8 TeV.
- Projections
- m_t^{pole} from $t\bar{t}$+1 jet events from ATLAS at 7 TeV.
- Top quark mass from lepton-bjet invariant mass from CMS at 8 TeV.
- Summary

m_t^{pole} from $t\bar{t}$ Cross Section



Compare measured inclusive tt production cross-section to fully inclusive calculations at NNLO QCD that involve an unambiguous m_t definition.

Joint likelihood:

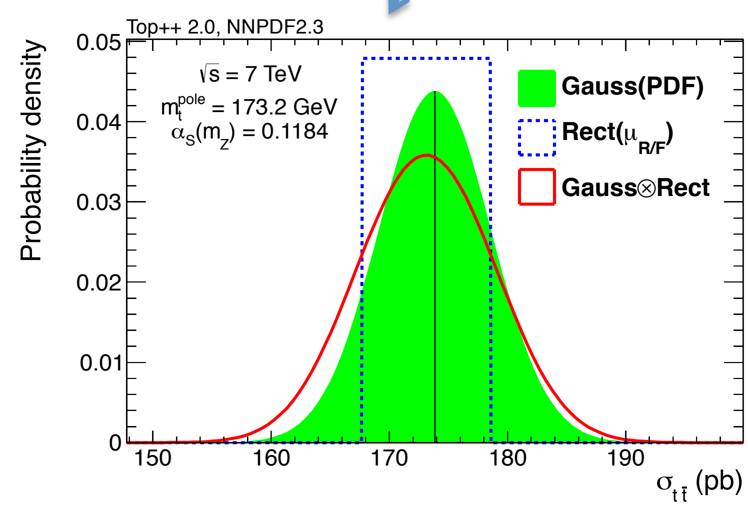
$$P\left(m_t^{pole}\right) = \int f_{\exp}\left(\sigma_{t\bar{t}} \mid m_t^{pole}\right) f_{th}\left(\sigma_{t\bar{t}} \mid m_t^{pole}\right) d\sigma_{t\bar{t}}$$

Gaussian probability distribution

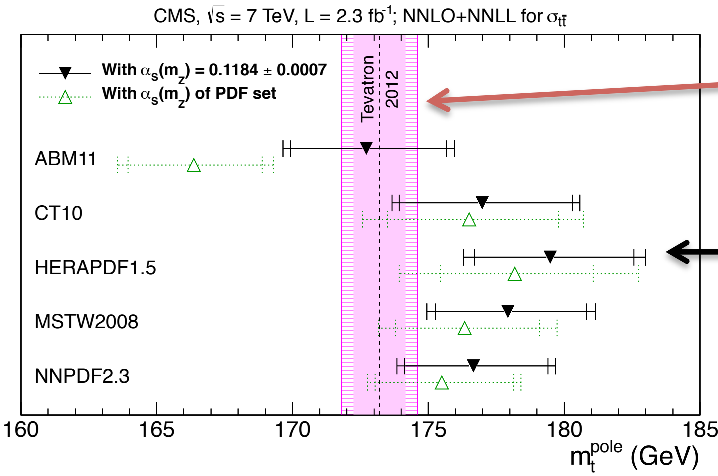
Dependence due to acceptance and efficiency depending on top mass.

$$m_t^{pole} = 176.7^{+3.0}_{-2.8}\, GeV$$

→*First NNLO top mass.*



PLB 728, 496 (2014) (arXiv:1307.1907v3)



outer band:
MC-pole mass.

Inner error bar: cross-section, LHC beam energy, PDF, scale.

Outer error bar: + $\delta(\alpha_s)$.

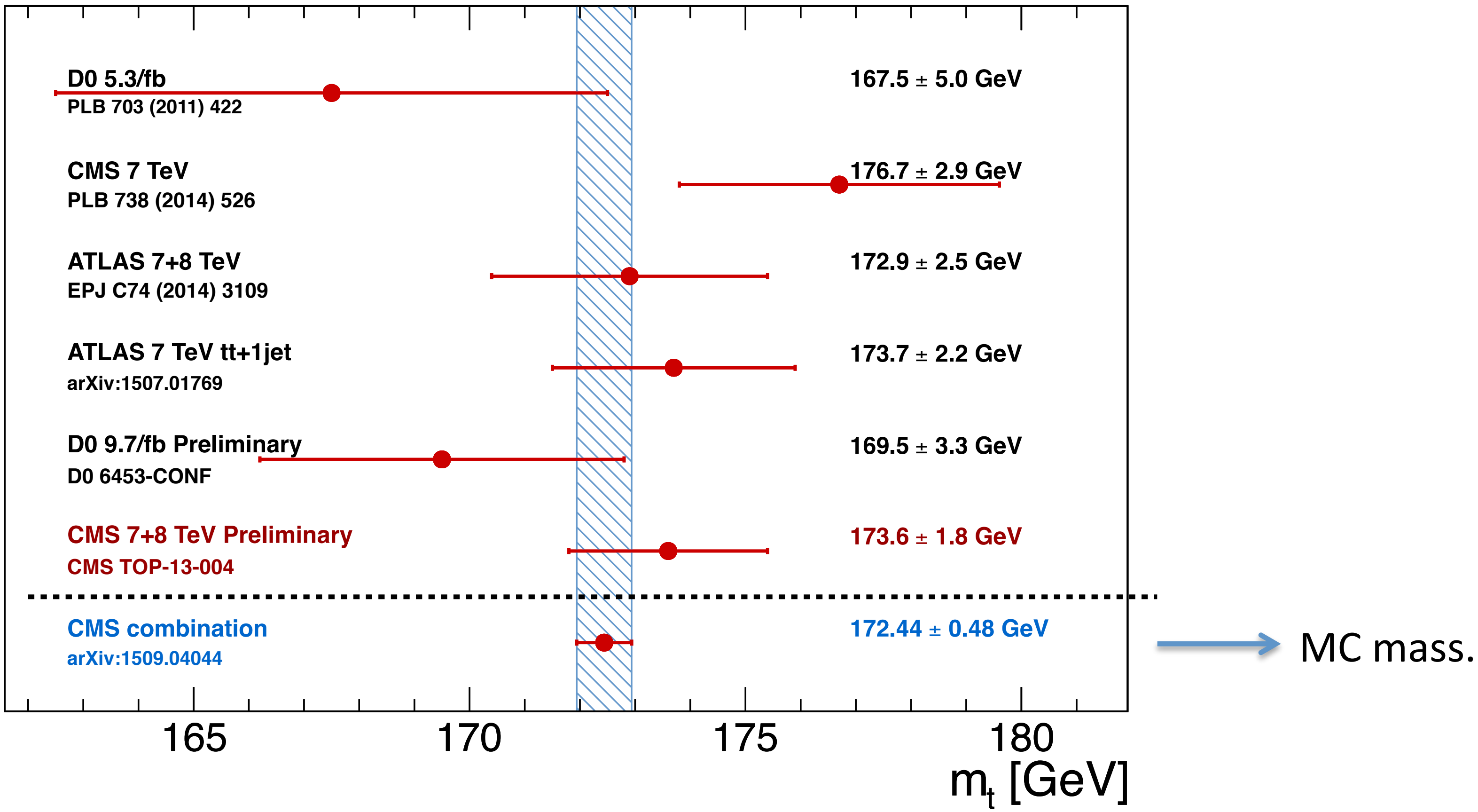
PLB 728, 496 (2014)
(arXiv:1307.1907v3)

	m_t^{pole} (GeV)	Uncertainty on m_t^{pole} (GeV)						
		Total	$\sigma_{t\bar{t}}^{meas}$	PDF	$\mu_{R,F}$	α_S	E_{LHC}	m_t^{MC}
ABM11	172.7	$^{+3.2}_{-3.1}$	$^{+1.8}_{-1.8}$	$^{+2.2}_{-2.0}$	$^{+0.7}_{-0.7}$	$^{+1.0}_{-1.0}$	$^{+0.8}_{-0.8}$	$^{+0.4}_{-0.3}$
CT10	177.0	$^{+3.6}_{-3.3}$	$^{+2.2}_{-2.1}$	$^{+2.4}_{-2.0}$	$^{+0.9}_{-0.9}$	$^{+0.8}_{-0.8}$	$^{+0.9}_{-0.9}$	$^{+0.5}_{-0.4}$
HERAPDF1.5	179.5	$^{+3.5}_{-3.2}$	$^{+2.4}_{-2.2}$	$^{+1.7}_{-1.5}$	$^{+0.9}_{-0.8}$	$^{+1.2}_{-1.1}$	$^{+1.0}_{-1.0}$	$^{+0.6}_{-0.5}$
MSTW2008	177.9	$^{+3.2}_{-3.0}$	$^{+2.2}_{-2.1}$	$^{+1.6}_{-1.4}$	$^{+0.9}_{-0.9}$	$^{+0.9}_{-0.9}$	$^{+0.9}_{-0.9}$	$^{+0.5}_{-0.5}$
NNPDF2.3	176.7	$^{+3.0}_{-2.8}$	$^{+2.1}_{-2.0}$	$^{+1.5}_{-1.3}$	$^{+0.9}_{-0.9}$	$^{+0.7}_{-0.7}$	$^{+0.9}_{-0.9}$	$^{+0.5}_{-0.4}$

- ABM11: significantly smaller gluon density → larger α_s or smaller m_t^{pole} to match the inclusive $t\bar{t}$ cross section measurement by CMS.
- m_t^{pole} from other PDF sets compatible with
  - each other
  - direct measurements within < 2 sigma

m_t^{pole} from $t\bar{t}$ Cross Section

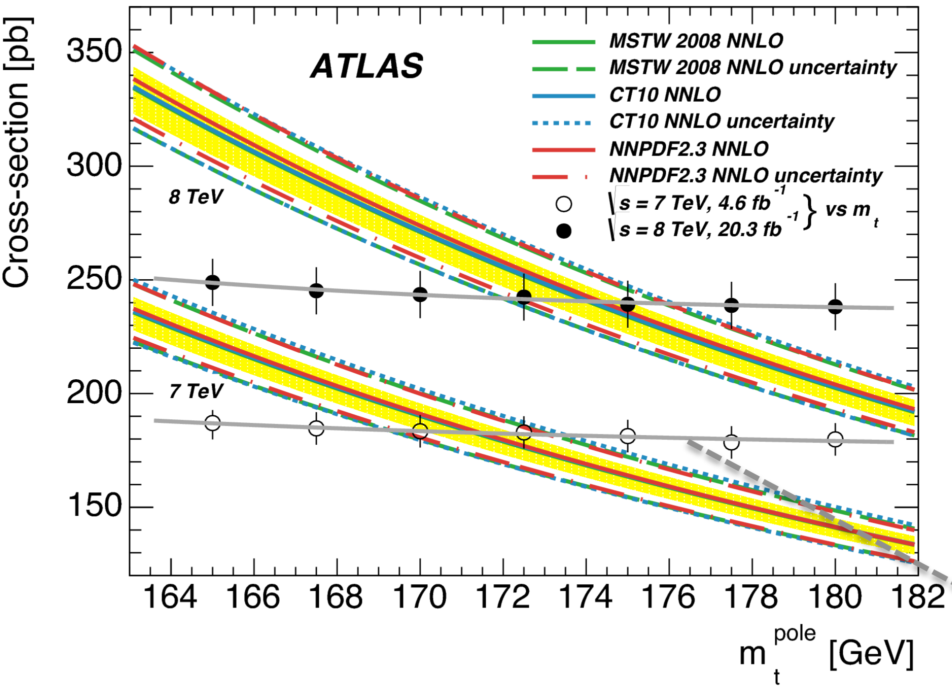
- The 7 TeV measurement was only the beginning.
- It became obsolete after the following measurements from ATLAS and CMS described in the next slides.

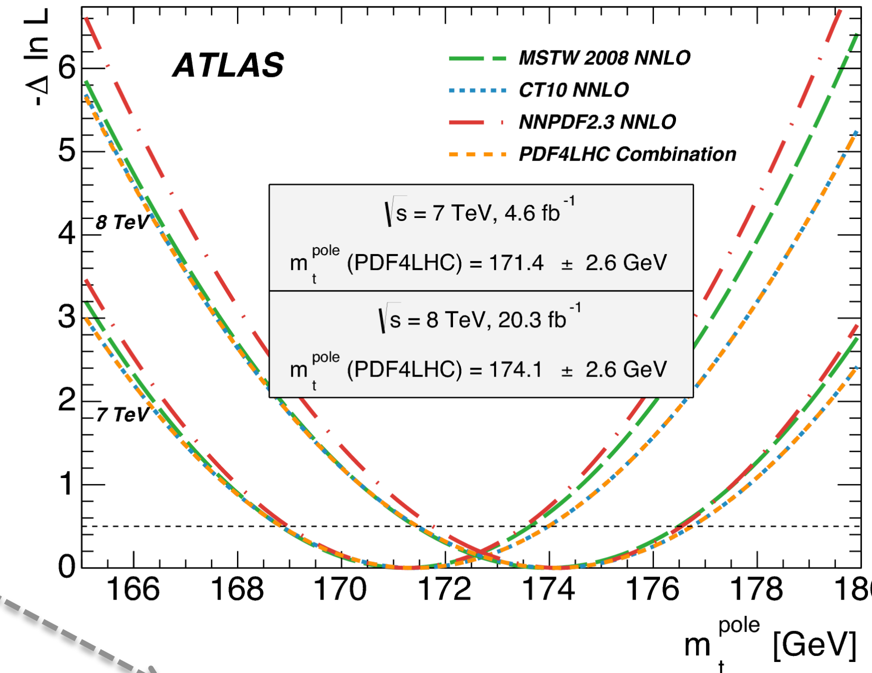


m_t^{pole} from $t\bar{t}$ Cross Section

eμ channel

EPJ C74 (2014) 3109





$$\sigma_{t\bar{t}}^{theo,\exp}\left(m_t^{pole}\right) = \sigma\left(m_t^{ref}\right)\left(\frac{m_t^{ref}}{m_t^{pole}}\right)\left(1 + a_1 x + a_2 x^2\right)$$

Acceptance and background effects partially cancel → Final dependence:

$$\frac{d\sigma_{t\bar{t}}}{dm_t} = -0.28\% / GeV$$

Remaining small slope from acceptance and Wt single top background.

m_t^{pole} from $t\bar{t}$ Cross Section

PDF	m_t^{pole} (GeV) from $\sigma_{t\bar{t}}$	
	$\sqrt{s} = 7$ TeV	$\sqrt{s} = 8$ TeV
CT10 NNLO	171.4 ± 2.6	174.1 ± 2.6
MSTW 68 % NNLO	171.2 ± 2.4	174.0 ± 2.5
NNPDF2.3 5f FFN	$171.3^{+2.2}_{-2.3}$	174.2 ± 2.4

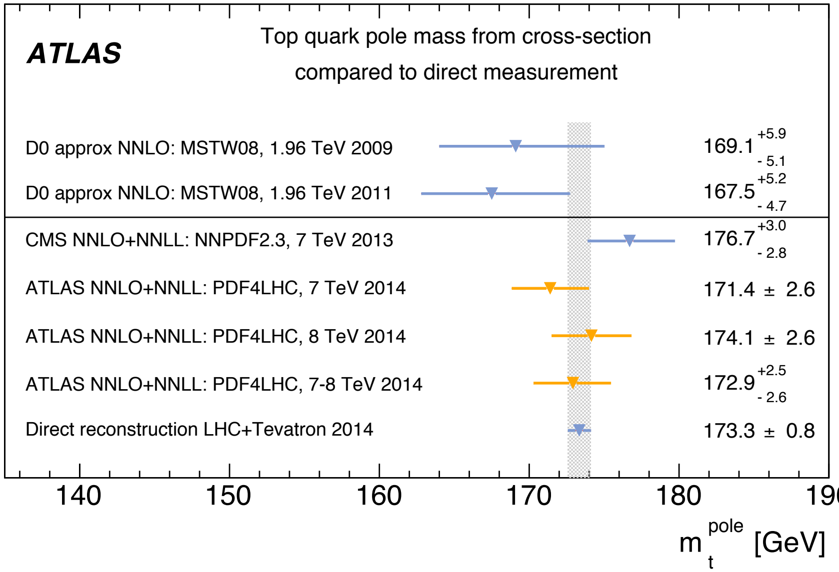
$$m_t^{pole} = 172.9^{+2.5}_{-2.6} \quad GeV$$

→ assuming 100% correlated uncertainties for the theory predictions at 7 and 8 TeV.

CT10

Δm_t^{pole} (GeV)	$\sqrt{s} = 7$ TeV	$\sqrt{s} = 8$ TeV
Data statistics	0.6	0.3
Analysis systematics	0.8	0.9
Integrated luminosity	0.7	1.2
LHC beam energy	0.7	0.6
PDF+α_s	1.8	1.7
QCD scale choice	$^{+0.9}_{-1.2}$	$^{+0.9}_{-1.3}$

ATLAS: Using env. of 3 PDF sets
CMS: Using individual PDF sets



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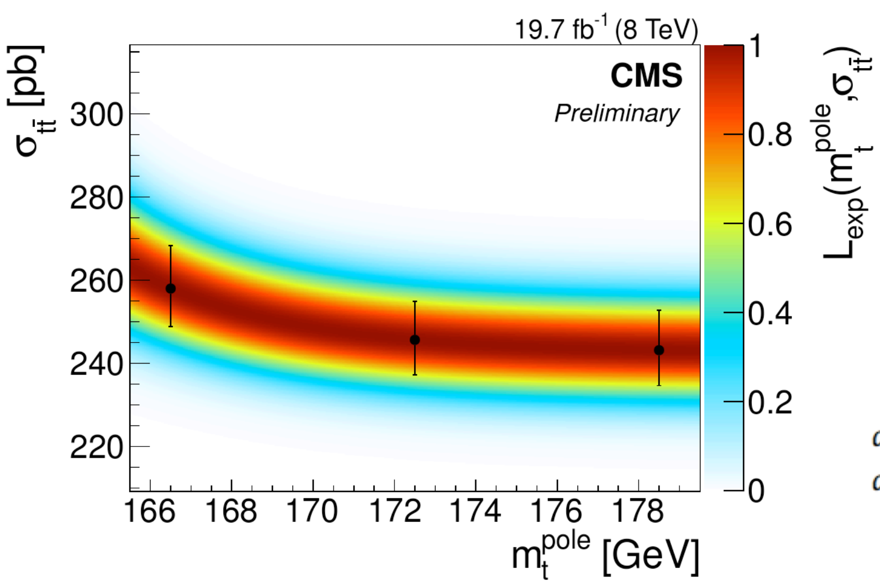
m_t^{pole} from $t\bar{t}$ Cross Section

CMS-PAS-TOP-13-004

- Full phase space cross-sections at parton level with full Run-I data at 7 and 8 TeV in the most precise channel (eμ).
  - The cross section fit repeated for m_t=169.5, 172.5 and 175.5 GeV
    - Uncertainties from detector effects evaluated separately for each mass point.
    - Modelling uncertainties vary little in 169.5-175.5 GeV range → use uncertainties for 172.5 GeV.
    - Measurements represented by a Gaussian:



$$L(m_t,\sigma_{t\bar{t}}) = \exp\left[\frac{\left(\sigma_{t\bar{t}}(m_t)-\sigma_{t\bar{t}}\right)^2}{-2\left(\Delta^2+\Delta^2_{m_t^\pm}\right)}\right]$$

Δ: total uncertainty on each measurement

$\Delta_{mt}^{+/-}$: +/-1 GeV from m_t^{MC}-m_t^{pole}.

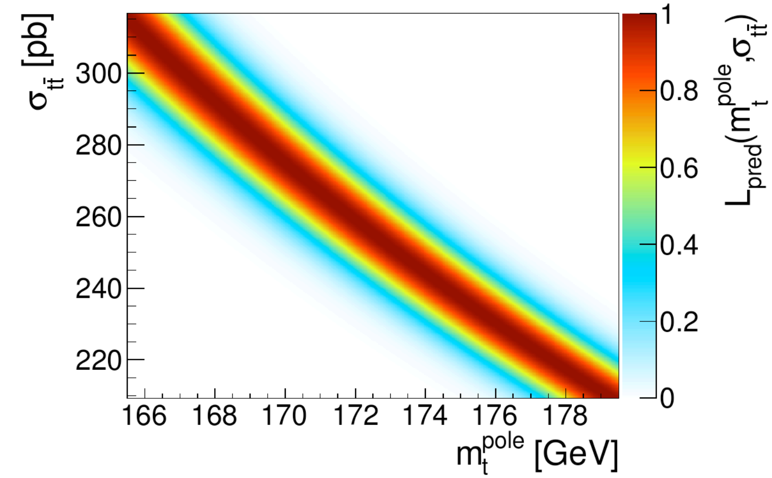
$$\sigma_{t\bar{t}}(8\,\text{TeV}, m_t) = \exp\left(-0.267617\cdot(m_t/\,\text{GeV}-176.729)\right)\,\text{pb}+242.549\,\text{pb}$$

$$\sigma_{t\bar{t}}(7\,\text{TeV}, m_t) = \exp\left(-0.130183\cdot(m_t/\,\text{GeV}-184.100)\right)\,\text{pb}+169.924\,\text{pb}$$

m_t^{pole} from $t\bar{t}$ Cross Section

CMS-PAS-TOP-13-004

- σ_{tt} vs m_t^{pole} from NNLO+NNLL prediction using TOP++ with different PDF sets with α_s=0.118+/-0.001.
- + beam energy uncertainty
  - 1.79% @ 7 TeV
  - 1.72% @ 8 TeV



From variations of renorm and fact. scales

$$L_{\text{pred}}(m_t,\sigma_{t\bar{t}}) = \frac{1}{C(m_t)}\left(\text{erf}\left[\frac{\sigma_{t\bar{t}}^{(h)}(m_t)-\sigma_{t\bar{t}}}{\sqrt{2}\Delta_{p,+}}\right] - \text{erf}\left[\frac{\sigma_{t\bar{t}}^{(l)}(m_t)-\sigma_{t\bar{t}}}{\sqrt{2}\Delta_{p,-}}\right]\right)$$

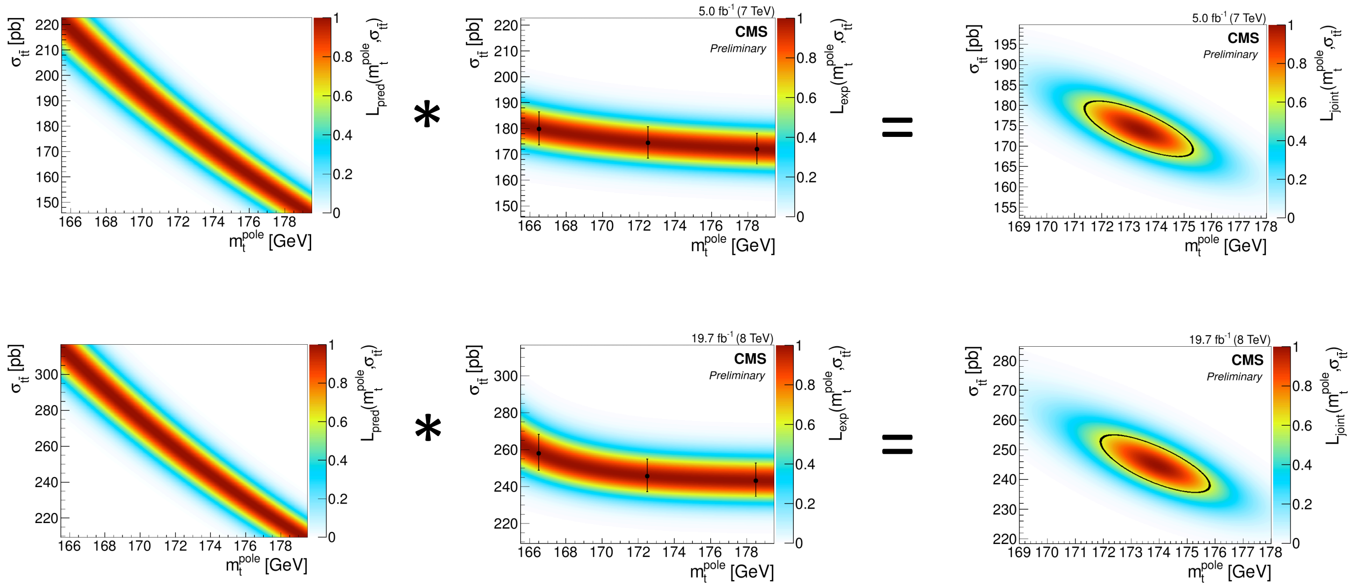
Normalization to assure max L = 1 for each m_t.

Includes PDF, α_s and beam energy uncertainties summed in quadrature.

m_t^{pole} from $t\bar{t}$ Cross Section

CMS-PAS-TOP-13-004

Minimize theory x experimental likelihoods



	m_t(7 TeV)	m_t(8 TeV)
NNPDF3.0	$173.4 \pm^{2.0}_{2.0}$ GeV	$173.9 \pm^{1.9}_{2.0}$ GeV
MMHT2014	$173.7 \pm^{2.0}_{2.1}$ GeV	$174.2 \pm^{1.9}_{2.2}$ GeV
CT14	$173.9 \pm^{2.3}_{2.4}$ GeV	$174.3 \pm^{2.2}_{2.4}$ GeV

m_t^{pole} from $t\bar{t}$ Cross Section

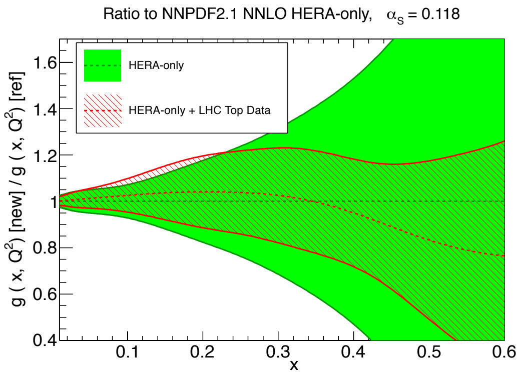
- Weighted average sys. unc. correlations between measured cross sections at 7 and 8 TeV and assuming 100% correlated uncertainties for the theory predictions at 7 and 8 TeV.

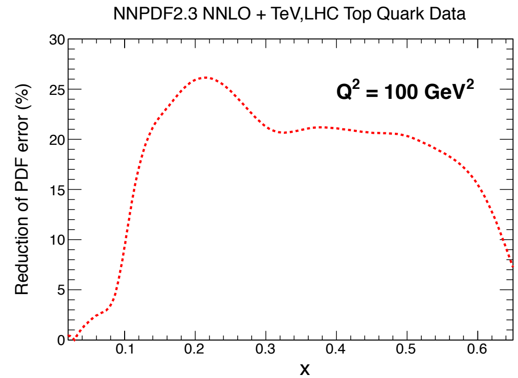
	m_t
NNPDF30	$173.6 \pm^{1.7}_{1.8}$ GeV
MMHT2014	$173.9 \pm^{1.8}_{1.9}$ GeV
CT14	$174.1 \pm^{2.1}_{2.2}$ GeV

CMS-PAS-TOP-13-004

Theory Uncertainties

e.g.



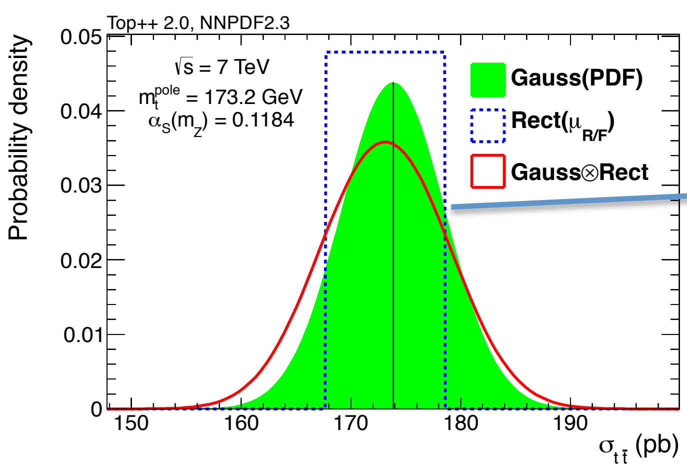


Czakon et al.
arXiv:1303.7215

Relative reduction of error due to the inclusion of top data in the PDF fit.

Used LHC $t\bar{t}$ inclusive cross section measurements as one of the ingredients.
→ Potential bias or underestimated errors in top pole mass?

	m_t
NNPDF30	$173.6 \pm^{1.7}_{1.8}$ GeV
MMHT2014	$173.9 \pm^{1.8}_{1.9}$ GeV
CT14	$174.1 \pm^{2.1}_{2.2}$ GeV



Using a Gaussian for the $\mu_{R/F}$ uncertainty might make it easier in comparisons or combinations with other measurements and would slightly reduce quoted uncertainty.

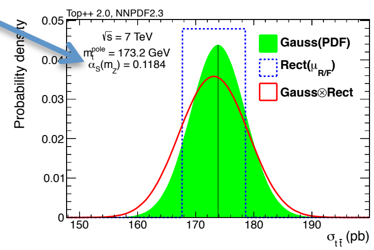
Measurement Uncertainties

CMS-PAS-TOP-13-004

Source	Uncertainty [%]	
	7 TeV	8 TeV
Total (vis)	$\pm^{3.5}_{3.4}$	$\pm^{3.7}_{3.4}$
Q^2 scale (extrapol.)	$\pm^{0.4}_{0.0}$	$\pm^{0.2}_{0.1}$
ME/PS matching (extrapol.)	$\mp^{0.1}_{0.1}$	$\pm^{0.3}_{0.3}$
Top p_T (extrapol.)	$\pm^{0.4}_{0.2}$	$\pm^{0.8}_{0.4}$
PDF (extrapol.)	$\mp^{0.2}_{0.1}$	$\mp^{0.1}_{0.2}$
Total	$\pm^{3.6}_{3.4}$	$\pm^{3.8}_{3.5}$

$$P\left(m_t^{pole}\right) = \int f_{\exp}\left(\sigma_{t\bar{t}} \mid m_t^{pole}\right) f_{th}\left(\sigma_{t\bar{t}} \mid m_t^{pole}\right) d\sigma_{t\bar{t}}$$

Gaussian



Crude estimates from the convolution of two Gaussians:

$$\left.\begin{array}{l}\sigma_{theo} \to \sigma_{theo}/1 \\ \sigma_{\exp} \to \sigma_{\exp}/2.0\end{array}\right\} \delta\left(m_t^{pole}\right) \approx 1.6\ GeV$$

$$\left.\begin{array}{l}\sigma_{theo} \to \sigma_{theo}/1.5 \\ \sigma_{\exp} \to \sigma_{\exp}/2.0\end{array}\right\} \delta\left(m_t^{pole}\right) \approx 1.1\ GeV$$

Source	Uncertainty [%]	
	7 TeV	8 TeV
Trigger	1.2	1.2
Lepton ID/isolation	1.4	1.5
Lepton energy scale	0.1	0.1
Jet energy scale	0.7	0.9
Jet energy resolution	0.1	0.1
Single top	0.9	0.6
DY	1.2	1.2
$t\bar{t}$ other	0.1	0.1
$t\bar{t}+V$	0.0	0.1
Diboson	0.2	0.6
W+jets	0.0	0.0
QCD	0.0	0.0
B-tag	0.5	0.5
Mistag	0.2	0.1
Pileup	0.3	0.3
Q^2 scale	0.3	0.3
ME/PS matching	0.2	0.1
MG+PY → PH+PY	0.2	0.4
Hadronization (JES)	0.6	0.8
Top p_T	0.3	0.3
Color reconnection	0.1	0.0
Underlying event	0.0	0.1
PDF	0.2	0.7
Luminosity	2.2	2.6
Statistical	1.2	0.6

Projections

CMS-PAS-FTR-13-017

	Current	Future A		Future B		Future C	
PDF uncertainty on predicted $\sigma_{t\bar{t}}$ (%)	2.5	2.5	(—)	1.3	(1.4 GeV)	1.3	(—)
Scale uncertainty on predicted $\sigma_{t\bar{t}}$ (%)	3.5	3.5	(—)	1.8	(1.2 GeV)	1.8	(—)
Uncertainty on measured $\sigma_{t\bar{t}}$ (%)	4.2	3.0	(2.2 GeV)	2.0	(1.6 GeV)	2.0	(—)
Slope of measured $\sigma_{t\bar{t}}$ (%/GeV)	−0.8	0.0	(2.6 GeV)	0.0	(—)	1.6	(1.0 GeV)
Uncertainty on LHC beam energy (%)	0.7	0.7	(—)	0.4	(1.0 GeV)	0.4	(—)
Uncertainty on $\alpha_S(m_Z)$ (%)	0.6	0.6	(—)	0.3	(1.2 GeV)	0.3	(—)
Uncertainty on $m_t^{MC} = m_t^{pole}$ (GeV)	1.0	0.5	(1.9 GeV)	0.25	(1.0 GeV)	0.25	(—)
Uncertainty on extracted mass	3.1 GeV	1.9 GeV		1.0 GeV		0.7 GeV	

Hypothetical analysis in which experimental acceptance decreases with top quark mass.

- PDF uncertainties will decrease at 13 TeV
  - With improved fits (adding more LHC data + technical improvements in PDF fits, …).
  - And we are probing lower x values w.r.t. 8 TeV.
  - Need to have PDF sets/versions without top quark data included
- Scale uncertainties
  - Expect mild improvements.
- α_s uncertainties
  - Expect small improvements?
  - World average uncertainty → determined by the lattice determination.

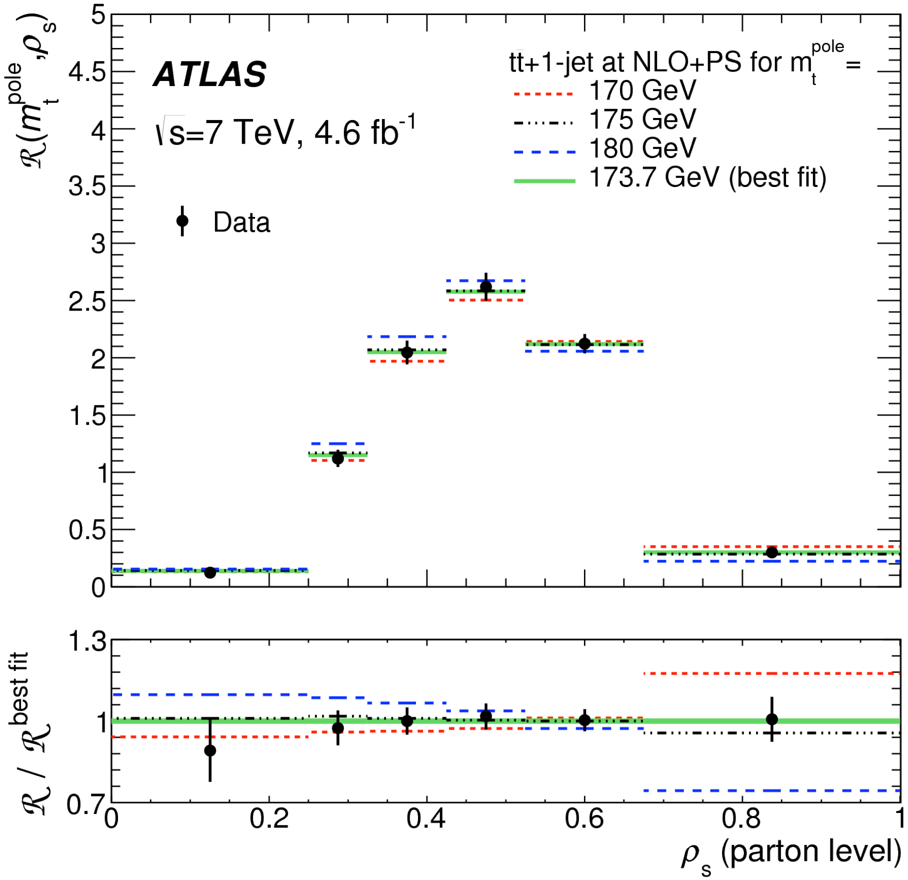
Other Mass Measurement Methods

- Pole mass (or $\overline{\text{MS}}$ mass) from Differential distribution from $t\bar{t}$ + 1 jet events (@NLO)
- Lepton-bjet Invariant Mass
- …

Top Quark Pole Mass Determination from $t\bar{t}$+1 Jet Events

- $t\bar{t}$+1 jet system → gluon radiation depends on the quark mass [S. Alioli et al. EPJ C 73 (2013) 2438].
- Pole mass at NLO from normalized cross section vs the inverse of the invariant mass of the $t\bar{t}$ + 1-jet system.
- Using lepton+jets channel.

$$\mathcal{R}(m_t^{\text{pole}}, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\text{-jet}}} \frac{d\sigma_{t\bar{t}+1\text{-jet}}}{d\rho_s}(m_t^{\text{pole}}, \rho_s),$$



$$\rho_s = \frac{2m_0}{\sqrt{s_{t\bar{t}j}}},$$

m_0 :arbitrary constant of $O(m_t)$
m_0 = 170 GeV

Extra jet: p_T > 50 GeV, $|\eta|$<2.5.

→ Fit with NLO+PS theory (POWHEG + PYTHIA8)

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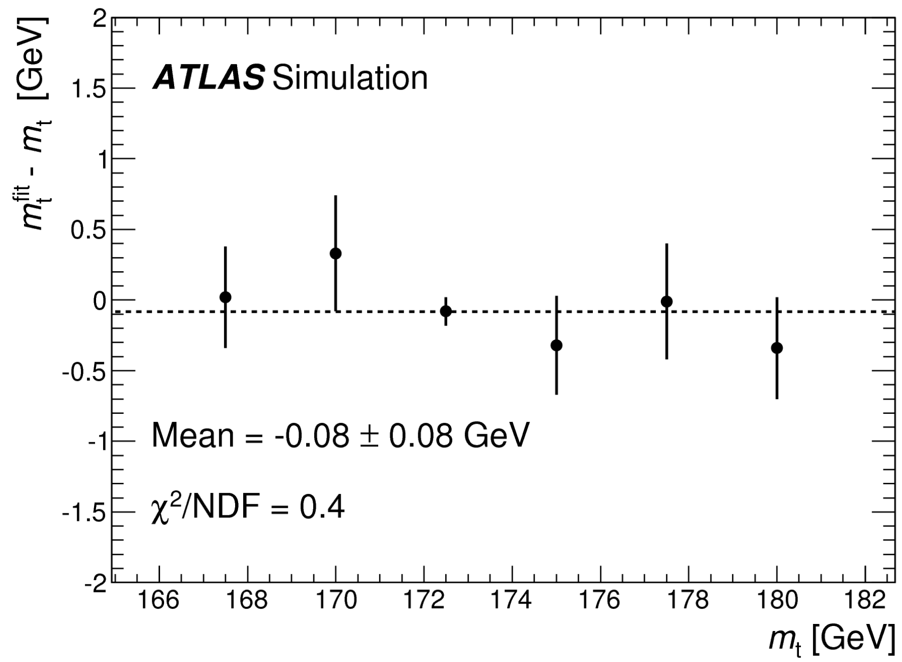
Top Quark Pole Mass Determination from $t\bar{t}$+1 Jet Events

- Fit is done at parton level: Top pole mass with acceptance corrections, color reconnection, …
  - but unbiased unfolding to parton level: good closure of the final fit.



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Top Quark Pole Mass Determination from $t\bar{t}$+1 Jet Events

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$$m_t = 173.7 \pm 1.5(stat.) \pm 1.4(syst.)^{+1.0}_{-0.5}(theo.)\ GeV$$

Description	Value [GeV]
m_t^{pole}	**173.71**
Statistical uncertainty	**1.50**
Monte Carlo statistics	0.13
Signal MC Generator	0.28
Hadronization	0.33
Proton PDF	0.54
ISR/FSR	0.72
Color reconnection	0.14
Underlying Event	0.25
b-tagging efficiency and mistag rate	0.17
Jet reconstruction efficiency	0.05
Jet energy resolution	0.02
Jet energy scale (including b–jet energy scale)	0.94
Missing Transverse Momentum	0.02
Lepton uncertainties	0.07
Background	0.16
Total experimental syst. uncertainty	**1.43**
Scale uncertainty	(+0.93, −0.44)
Theory PDF uncertainty	0.21
Total theory syst. uncertainty	**(+0.95, −0.49)**
Total uncertainty	**(+2.27, −2.12)**

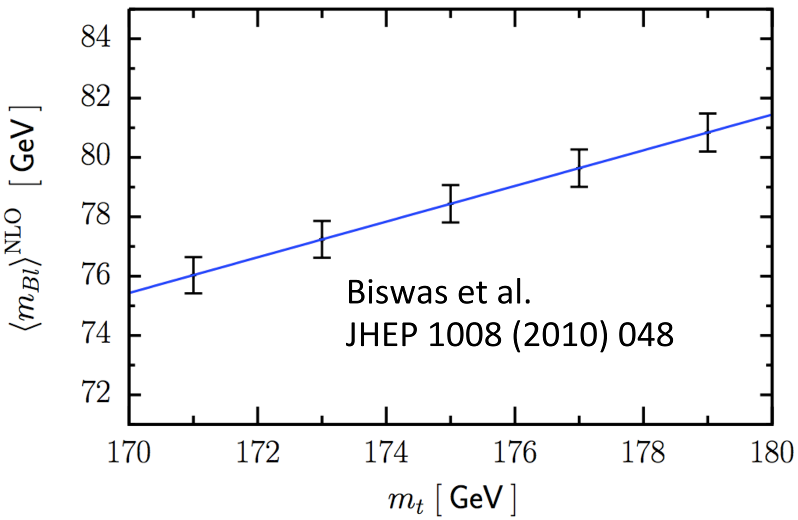
← will improve with 8 and 13 TeV data.

Dominant systematic uncertainties

- Statistically uncorrelated to the pole mass extracted from $t\bar{t}$ inclusive cross section in the dilepton channel.

Top quark mass from lepton-bjet Invariant Mass

CMS-PAS-TOP-14-014



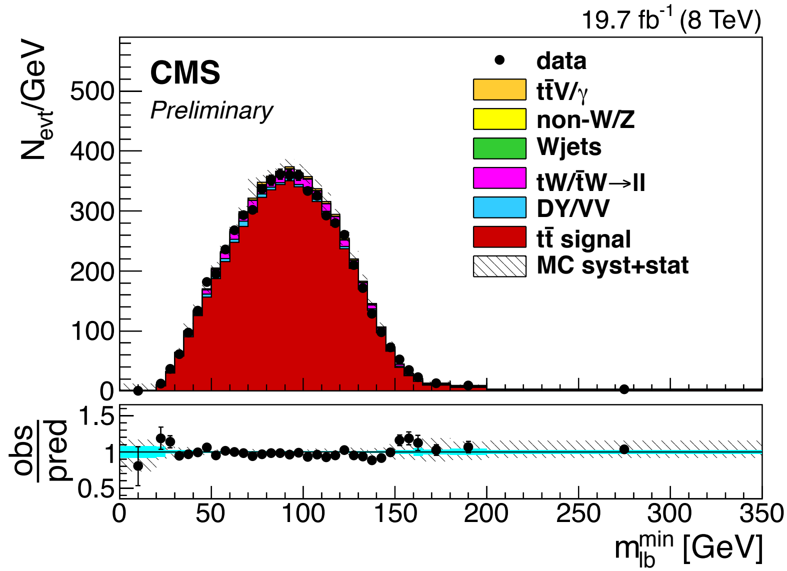
- m_{lb}
  - sensitive to top mt
  - Not so sensitive to the details of the production or the choice of PDF set.

At LO: $m_{lb}^2 = \frac{m_t^2 - m_W^2}{2}(1-\cos\theta_{lb})$

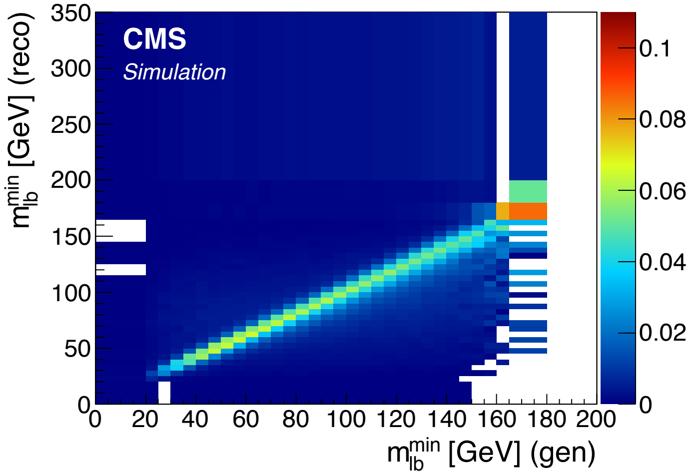
Endpoint at $\max(m_{lb}) \approx \sqrt{m_t^2 - m_W^2}$

modified by higher order corrections and experimental effects.

- m_{lb} generated by MCFM
  - at LO QCD
  - at NLO QCD for production but LO for decay
- Predictions at parton level folded to the detector level.
  - Response matrix to correct: acceptance, bin-to-bin migrations, detector efficiencies, ..
- eμ channel for high precision.
- Select the permutation that minimizes the m_{lb} in each event.



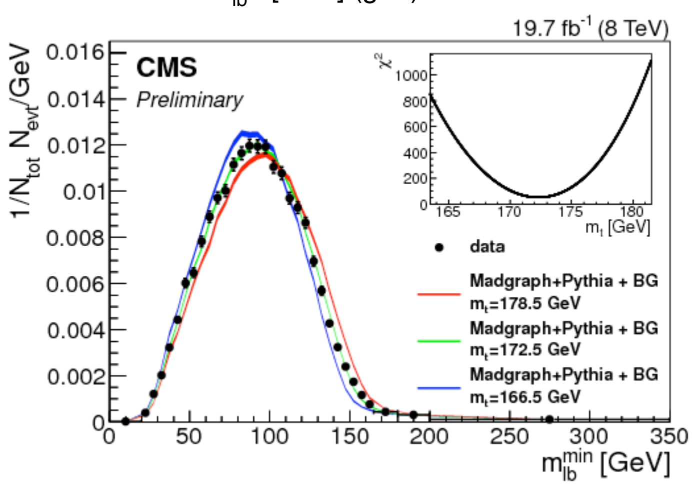
Top quark mass from lepton-bjet Invariant Mass

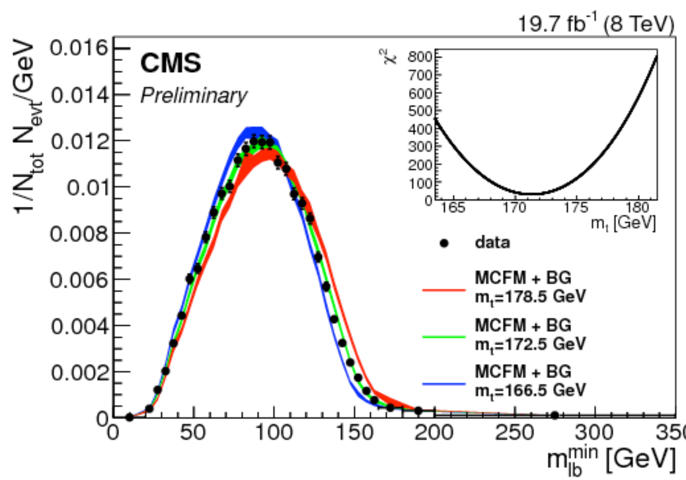


Prediction	Fit method	Fitted m_t [GeV] from m_{lb}^{min}
MADGRAPH+PYTHIA	shape+rate	$173.1\ ^{+1.9}_{-1.8}$
MADGRAPH+PYTHIA	rate	$173.7\ ^{+3.5}_{-3.4}$
MADGRAPH+PYTHIA	shape	$172.3\ ^{+1.3}_{-1.3}$
MCFM (LO)	shape	$171.5\ ^{+1.1}_{-1.1}$
MCFM (NLO)	shape	$171.4\ ^{+1.0}_{-1.1}$

Main uncertainties: renorm. x factor. scales and b-fragmentation.

CMS-PAS-TOP-14-014



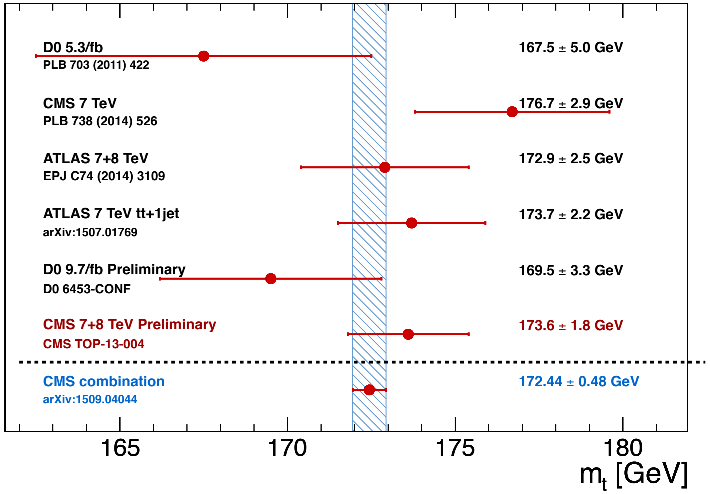


- Using MCFM including real and virtual corrections to NLO *also* in the decay, a shift of +0.9 GeV is observed → Next step: use NLO b-***jet*** (instead of LO b-quark) in the decay in MCFM to have a proper folding matrix.

Summary

- Top pole mass measurement using inclusive cross section and NNLO+NNLL calculation reached a precision of 1.8 GeV.
  - Will be hard to have a significantly improved precision - ultimately limited by uncertainty in luminosity.
  - Focus on improving uncertainties on the experimental side: JES, background, lepton-id…
  - And on improving Scale+PDF uncertainty on the theory calculations.
- Top pole mass from differential distributions in $t\bar{t}$+jet, precision ~2.2 GeV.
  - Statistics → Including 8 TeV measurement the precision will be ~1.8 GeV (if no improvement on JES, ISR/FSR, scales, PDF uncertainties)
- Measurements with m_{lb} very promising, precision ~1.2 GeV
  - Ability to bring calculations/simulations to detector level
  - Work to be done to measure top pole mass from m_{lb}
    - NLO corrections in the decay being studied in more detail
    - Main challenge: Definition of the b-quark / jet in the interface with a QCD prediction.

Summary



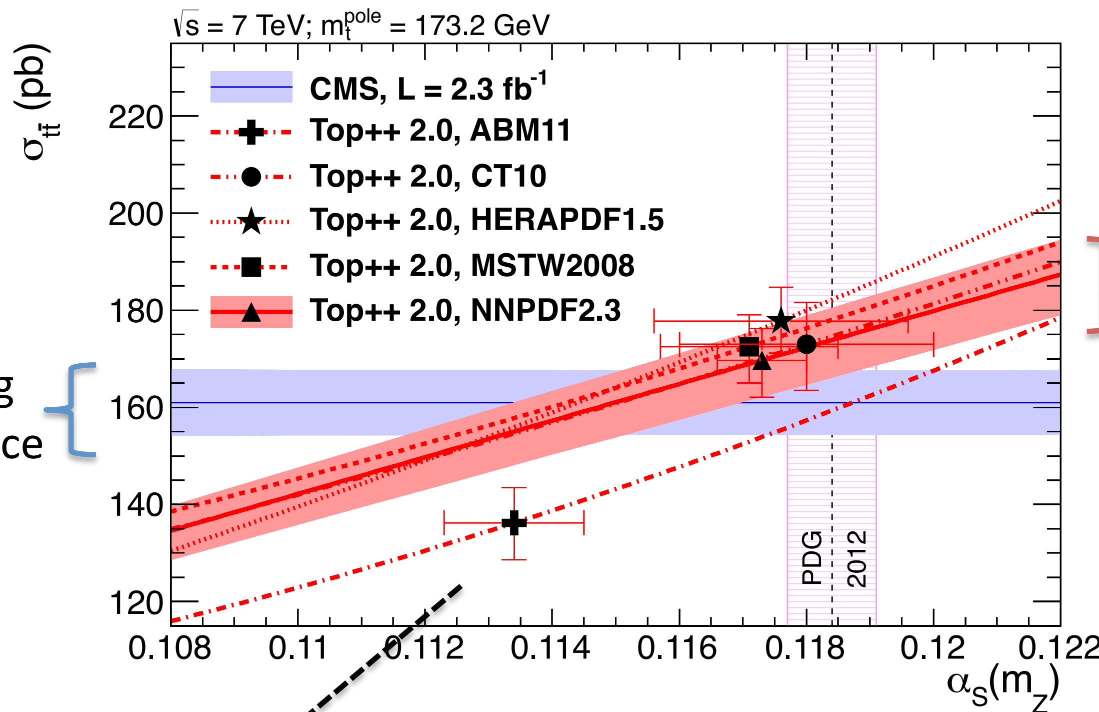
- We should be able to reach ~<1.4 GeV uncertainty by the end of LHC run II
  - Using top quark pole mass from inclusive cross section + NNLO predictions.
    - Using combined inclusive cross section measurement.
  - Using the measurements using $t\bar{t}$+1jet events.
  - Combining inclusive cross section (eμ) and $t\bar{t}$+1 jet (lepton+jet) top pole mass results.
    - Statistically independent samples.
    - Detailed studies needed to understand correlations of the uncertainties.

Additional Slides

Determination of α_s

- $\sigma_{t\bar{t}}$ (CMS; 7 TeV; dilepton) [JHEP, 11 (2012) 067]
- TOP++ 2.0 calculation (@ NNLO+NNLL) with $\alpha_s(M_Z)$ scan.

PLB 728, 496 (2014) (arXiv:1307.1907v3)



4.1% (including acceptance effects)

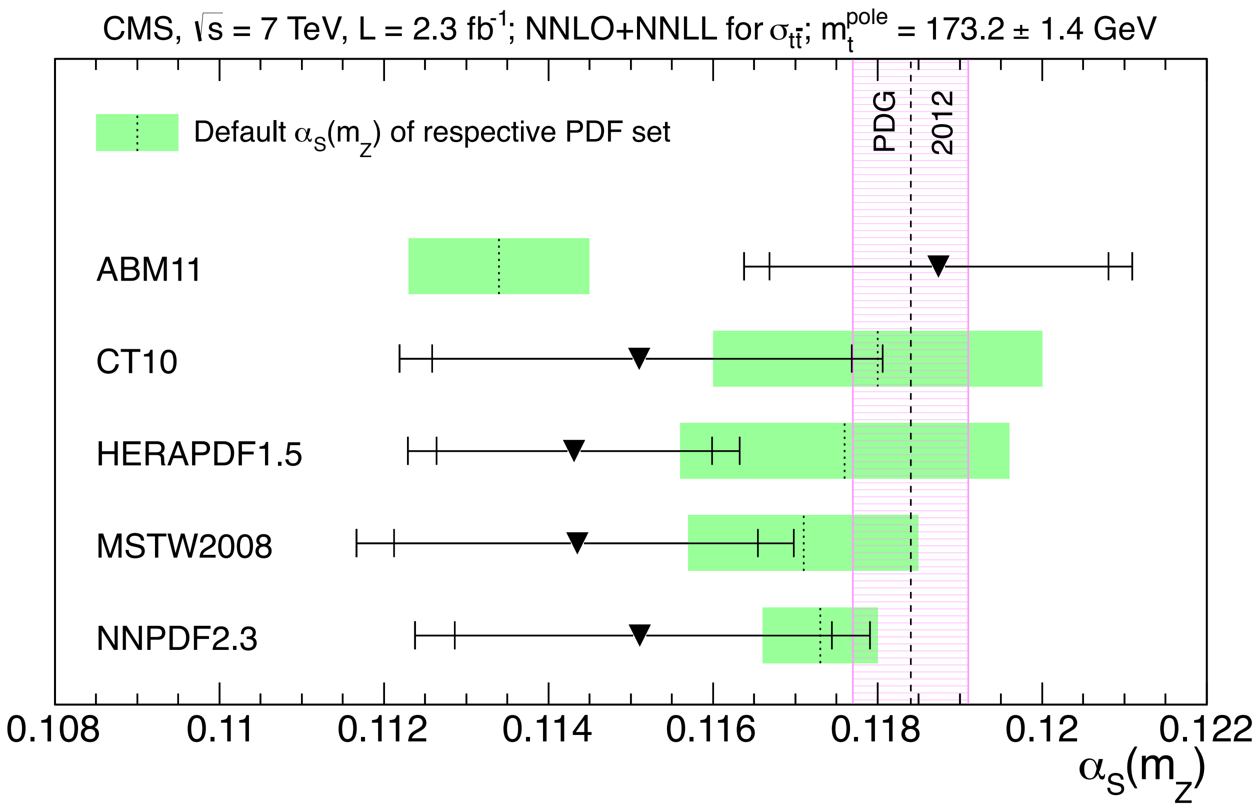
scale & PDF uncertainties ($\mu_r=\mu_f=m_t^{pole}$ → 0.5 – 2 x μ)

ABM: smaller gluon density.

$$m_t^{pole} = m_t^{Tevatron} = 173.18 \pm 0.94\ GeV \oplus 1\ GeV$$

assumed difference between m_t^{pole} & m_t^{MC}

Determination of α_s



PLB 728, 496 (2014) (arXiv:1307.1907v3)

$\delta(\alpha_s)$=2.4%
→ δ(scale): 0.8%
→ $\delta(m_t)$: 1.1%
→ LHC beam energy: $\delta(\sqrt{s})$=46 GeV → 0.7%

using NNPDF2.3:

$$\alpha_s(m_Z) = 0.1151^{+0.0028}_{-0.0027}$$

- The first determination of $\alpha_s(m_Z)$ from top quark production.
- The first $\alpha_s(m_Z)$ result at a hadron collider at full NNLO QCD.