A few comments on the top quark measurements

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The outline

I want to make a few comments concerning the top quark mass measurement at a hadron collider. These comments do not address specific measurements, or suggest particular improvements, or sharply point problems with existing analyses.

What I am going to tell you are really just a few things that are related to top quark physics in general and the top quark mass measurement in particular, and that were bothering me for a while. I think those things are not discussed enough in the context of the top quark measurements.

These things include:

1) Non-perturbative corrections to production cross sections and kinematic distributions as precision limiting factor for the determination of the top quark mass.

2) How does a finite life-time of the top quark impacts the non-perturbative corrections to top quark pair production ?

3) Top quark lifetimes are distributed exponentially; this means that there should be a few that live longer than the hadronization time. Are there mesons composed of the top quark and the light quark? How would one see them? Can one use those mesons to measure the top quark mass without an ambiguity?

Non-perturbative effects and the top quark mass

Similar to the measurement of any observable at a hadron collider, extraction of the top quark mass is affected by non-perturbative effects. This is an issue that exists even if a short-distance mass definition for the top quark mass is chosen.

Let us imagine an idealized situation where the parton shower is not needed for the extraction of the top quark mass but an observable, from which the top quark mass is determined, is predicted with the standard QCD accuracy, i.e. up to power corrections.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}M} \approx T(M, m_t, \alpha_s) \left[1 + c \left(\frac{\Lambda_{\rm QCD}}{M} \right)^n \right]$$

$$\delta m_t \sim \frac{c\, T}{\partial T / \partial m_t} \left(\frac{\Lambda_{\rm QCD}}{M^*} \right)^n$$

$$\frac{\partial T}{\partial m_t} \sim k \frac{T}{m}$$

$$\delta m_t \sim \frac{c\, m_t}{k} \left(\frac{\Lambda_{\rm QCD}}{m_t} \right)^n$$

For a typical observable, k=1, n=1 (cross section is, perhaps, the only known exception -- n = 2 in that case); this implies that the top quark mass can not be extracted with precision that is better than the non-perturbative QCD scale. However, if we find an observable where the non-perturbative effects are quadratic in the ratio of QCD scale to the top quark mass, we can determine the top quark mass from it with a much higher precision. So understanding non-perturbative corrections -- and not the mass definition -- seems to be a limiting factor to me.

The lifetime of a top quark

There are three reason to talk about the top quark lifetime in that context.

1) Can a finite lifetime of a top quark impact the non-perturbative corrections that we assign to the production and decay processes of a heavy stable quark?

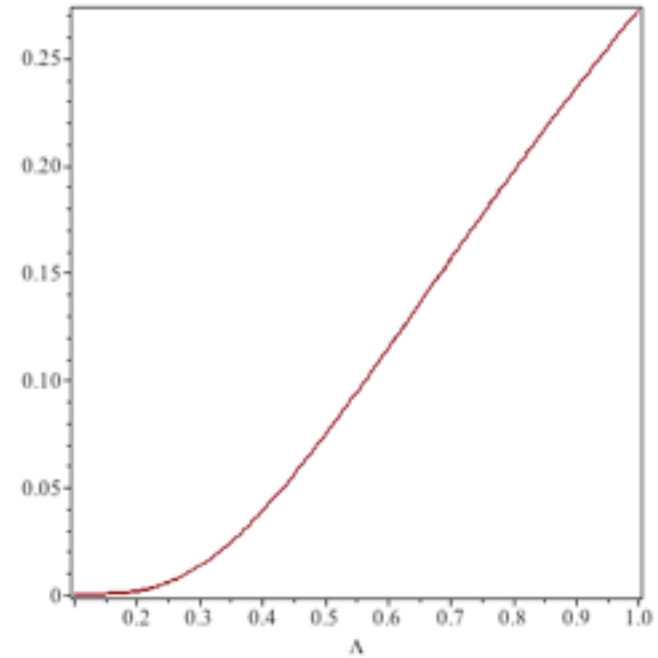
2) Is it beneficial to study top quarks that hadronize before decays? Is it possible to understand non-perturbative corrections to top quark production by thinking about how non-perturbative physics affects decays of colorless heavy meson?

3) Can we imagine a limiting case such that the measurement of the top quark mass at a hadron collider can be understood perfectly?

Non-perturbative effects and the top quark lifetime

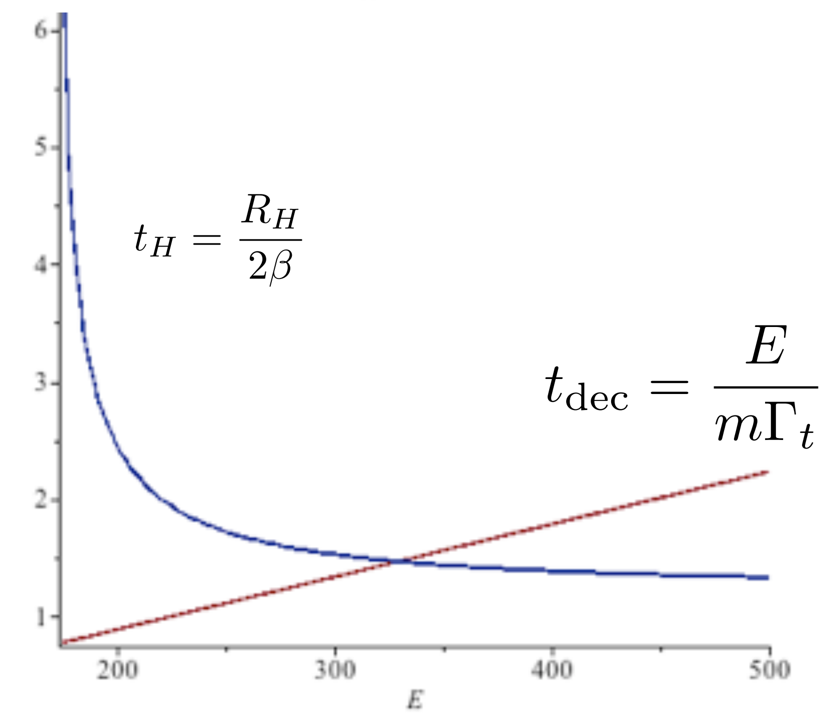
We often say that non-perturbative effects in top quark physics do not play a big role because top quarks decay before hadronization. How accurate is this statement?

A naive estimate: number of hadronized top quarks is given by $N_{\rm hadr} = N_{\rm tot} e^{-\Gamma_t/\Lambda}$. We do not know what the QCD scale is; for values higher than 400 MeV, more than five percent of all top quarks should hadronize before the decay.



However, the story might be even more convoluted since the ``lifetime" and the "hadronization time" have different dependences on the top quark energy; for $R_H = (0.4~{\rm GeV})^{-1}$ the cross-over occurs for top quarks with the energy of about 300 GeV.

For larger energies, top quarks will hadronize before they decay (again, this is statement that strongly depends on the assumed value of $\Lambda_{\rm QCD}$).



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Non-perturbative effects and the top quark lifetime

Are there observable effects of the energy-dependent hierarchy between top lifetime and the hadronization?

Consider a model were top quarks are produced from a source with a fixed energy. We will measure the energy of top decay products and deduce the mass of the top quark from there. For this, we need to understand the top quark energy.

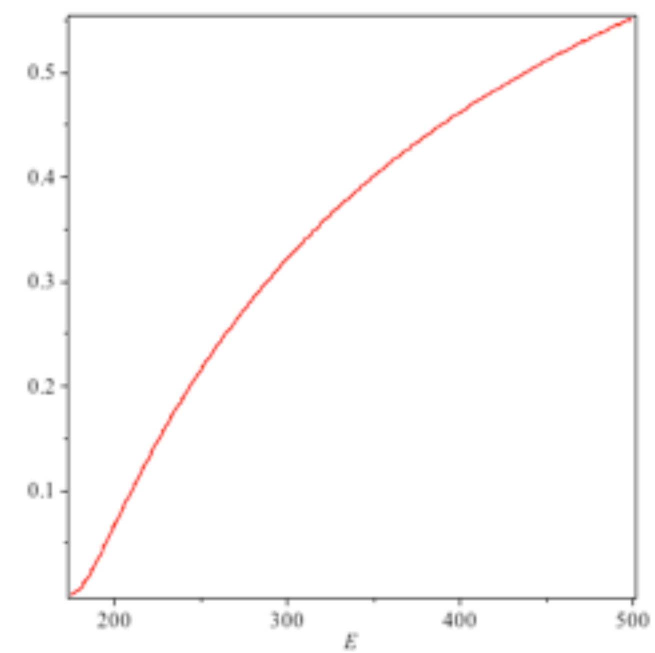
Top and anti-top fly away from each other and have to reach a certain spatial separation from a source to experience non-perturbative effects. Suppose that these non-perturbative effects change the energy of the tops by a fixed amount ($\Delta E \sim \Lambda_{\rm QCD}$). What is the average energy that will be measured?

$$\langle E \rangle = N_{\rm t}^{-1} \left(N_t^{\rm free}\, E + N_t^{\rm hadr}\, \left(E - \Lambda_{\rm QCD}\right)\right).$$

$$N_t^{\rm free} = N_t \left(1 - e^{-t^*/\tau_E}\right), \qquad N_t^{\rm hard} = N_t e^{-t^*/\tau_E}.$$

$$t^* = \frac{R}{2\beta} \qquad\qquad \tau_E = \frac{E}{m\Gamma_t}.$$

$$\langle E \rangle = E \left(1 - \frac{\Lambda_{\rm QCD}}{E} e^{-\frac{Rm\Gamma_t}{2\beta E}}\right)$$



Fraction of the non-perturbative correction Λ/E that contributes at a given energy E.

The non-perturbative effects seem to be reduced in case of an unstable top quark case compared to similar effects in case of stable quarks.

The small weak coupling limit

The width of the top quark is determined by the top quark mass, the W-boson mass and the weak coupling:

$$\Gamma_t \sim \frac{g_W^2 m_t^3}{m_W^2}$$

Of course, there are many reasons why the weak coupling should not be smaller than its value but -- focusing only on the top quark physics -- what would change if the weak coupling is 3-4 times smaller?

Of course, hadronization will occur before the decay most of the time but it seems that ``hadronization before decay" scenario can be reconciled with the existence of top quark spin correlations. Indeed, to have spin correlations we require that non-perturbative fields flip the top quark spin by a negligible amount. The non-perturbative QCD fields are strong but the top quark is so heavy that its chromomagnetic moment is absolutely tiny.

$$\frac{\Delta S}{S} \approx \frac{\Lambda_{\rm QCD}^2}{m_t \Gamma_t} \ll 1 \qquad \Lambda_{\rm QCD} \times \frac{\Lambda_{\rm QCD}}{m_t} \ll \Gamma_t \quad \leftrightarrow \quad \Gamma_t \ll \Lambda_{\rm QCD}$$

In this case, the hadronization occurs before the decay but top quark decay products are fully spin-correlated. There seems then not so much difference in observables related to top quark production and decay.

The small weak coupling limit

In the limit when the weak coupling is small, we have production of T-hadrons. The decay of the top quark occurs inside a T-hadron; such semi-leptonic decays are well-understood (B-physics).

The average energy of a lepton in the rest frame of a T-hadron is determined by a quark decay up to corrections that scale like $\mathcal{O}(\Lambda_{\rm QCD}^2/m_t^2)$ (provided that the short-distance mass is used in the calculation).

$$\langle E_l \rangle = {\rm N}^{-1} \int {\rm d}E_l \; E_l \frac{{\rm d}\Gamma_T}{{\rm d}E_l} = f(\bar{m}_t, m_W) \left(1 + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{m_t^2} \right) \right).$$

To use this equation one needs to understand how to reconstruct a reference frame of each of the hypothetical T-hadron and what is the mistake that one introduces by failing to do that.

The mistake is related with the mismatch between the top quark mass and the T-meson mass that differ by $\mathcal{O}(\Lambda_{\rm QCD})$. It seems that this mistake should then propagate into the reconstructed average energy of a lepton and, eventually, give us the non-perturbative effect of a canonical strength. However, the fact that this limit suggest that major non-perturbative effects are in a reference-frame mismatch is an interesting perspective in my opinion.

Conclusions

1) The main limiting factor in the determination of the top quark mass at hadrons colliders is our current inability to control the non-perturbative effects (and not the MC mass issue).

We will not be able to move past a few hundred MeV precision in the top quark mass determination unless the non-perturbative effects are understood.

2) Finite top quark lifetime may have an impact on the non-perturbative corrections to top quark pair production, perhaps making them smaller.

3) An interesting theoretical limit is that of the small weak coupling. This limit interchanges the hierarchy of hadronization and decay times -- without doing much else -- and allows us to think about production and decay of top quarks inside T-mesons. Relevant non-perturbative effects in this case seem to be related to a problem of reconstructing the rest frame of the T-mesons; all other non-perturbative effects are strongly suppressed.