Precision determinations of the top-quark mass

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Based on work done in collaboration with:

- *Calibration of the Top-Quark Monte-Carlo Mass*
  J. Kieseler, K. Lipka and S. M. `arXiv:1511.00841`
- *High precision fundamental constants at the TeV scale (Procs. MITP workshop)*
  S. M., S. Weinzierl et. al. `MITP workshop procs. arXiv:1405.4781`
- *A new observable to measure the top-quark mass at hadron colliders*
  S. Alioli, P. Fernandez, J. Fuster, A. Irles, S. M., P. Uwer and M. Vos
  `arXiv:1303.6415`
- *The top quark and Higgs boson masses and the stability of the electroweak vacuum*
  S. Alekhin, A. Djouadi and S. M. `arXiv:1207.0980`
- *Measuring the running top-quark mass*
  U. Langenfeld, S. M. and P. Uwer `arXiv:0906.5273`
- Many more papers with friends . . .

Top-quark mass

What is the value of the top-quark mass ?

$$m_t = ?$$

World combination

Experiment: ATLAS, CDF, CMS & D0 coll. 1403.4427

$$m_t = 173.34 \pm 0.76\,\text{GeV}$$

In all measurements considered in the present combination, the analyses are calibrated to the Monte Carlo (MC) top-quark mass definition.

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Folklore:

That is, we can state as the final result for the likely relation between the top-quark mass measured using a given Monte Carlo event generator ("MC") and the pole mass as

$$m^{\mathrm{pole}} = m^{\mathrm{MC}} + Q_0 \left[\alpha_s(Q_0) c_1 + \dots\right]$$

where $Q_0 \sim 1$ GeV and c_1 is unknown, but presumed to be of order 1 and, according to the argument above, presumed to be positive.

A. Buckley et al. arXiv:1101.2599

Conversion Monte Carlo mass to pole mass (I)

Theory

- Appropriate description of the content of Ref. Hoang, Stewart '08 is as follows:

  The uncertainty on the translation from the MC mass definition to a theoretically well defined short-distance mass definition at a low scale is currently estimated to be of the order of 1 GeV.
  [MITP workshop procs. arXiv:1405.4781]

Assumption

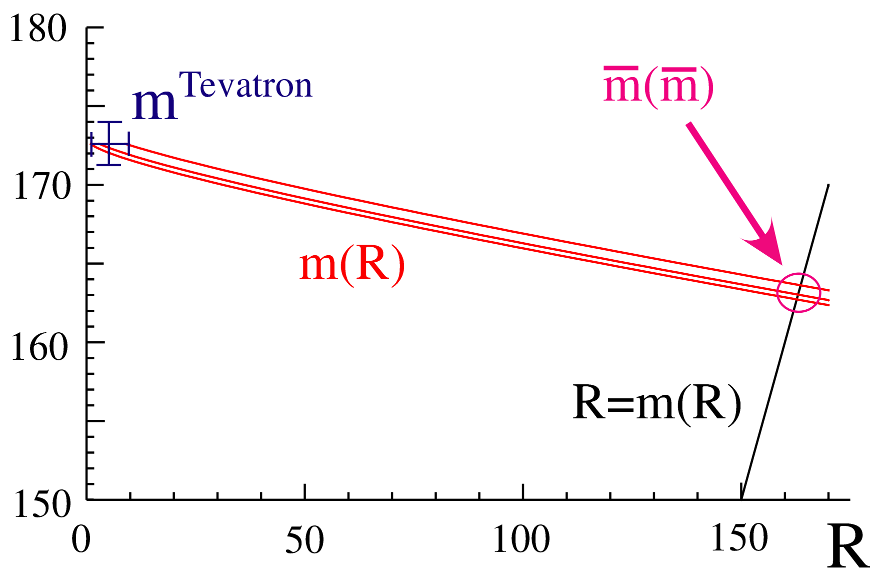
- Identify Monte Carlo mass $m^{\rm MC}$ with short distance mass $m^{\rm MSR}(R)$ at low scale $\mathcal{O}(1)$ GeV
  - choice for range of scale $R \simeq 1 \dots 9\mathrm{GeV}$

$$m^{\rm MC} \quad = \quad m^{\rm MSR}(R = 3^{+6}_{-2}\text{GeV})$$

Conversion Monte Carlo mass to pole mass (II)

Strategy

- Use perturbation theory to convert $m^{\rm MSR}(R)$ to $m^{\rm pole}$
- Running of $m^{\rm MSR}(R)$ mass
  Hoang, Stewart '08



- Run $m^{\rm MSR}(R)$ from low scale to $R = m_t$: $m^{\rm MSR}(R) \to m(m)$ and convert from $m(m)$ to pole mass [MITP workshop procs. arXiv:1405.4781]

$m^{\rm MSR}(1)$	$m^{\rm MSR}(3)$	$m^{\rm MSR}(9)$	$\overline{m}(\overline{m})$	$m^{\rm pl}_{\rm 1lp}$	$m^{\rm pl}_{\rm 2lp}$	$m^{\rm pl}_{\rm 3lp}$
173.72	173.40	172.78	163.76	171.33	172.95	173.45

- Upshot:
  $\Delta m(\text{th}) =^{+0.32}_{-0.62}$ GeV $(m^{\rm MC} \to m^{\rm MSR}(3\text{GeV})) + 0.50$ GeV $(m(m) \to m_{\rm pole})$

Conversion Monte Carlo mass to pole mass (III)

Summary [MITP workshop procs. arXiv:1405.4781]

$$m_{\rm pole} \; = \; 173.34 \pm 0.76 \, \text{GeV (exp)} \; + \; \Delta m\text{(th)} \; + \; \Delta m\text{(unknown)}$$

with

$$\Delta m\text{(th)} \quad = \quad {}^{+0.82}_{-0.62}\text{GeV}$$

and in addition

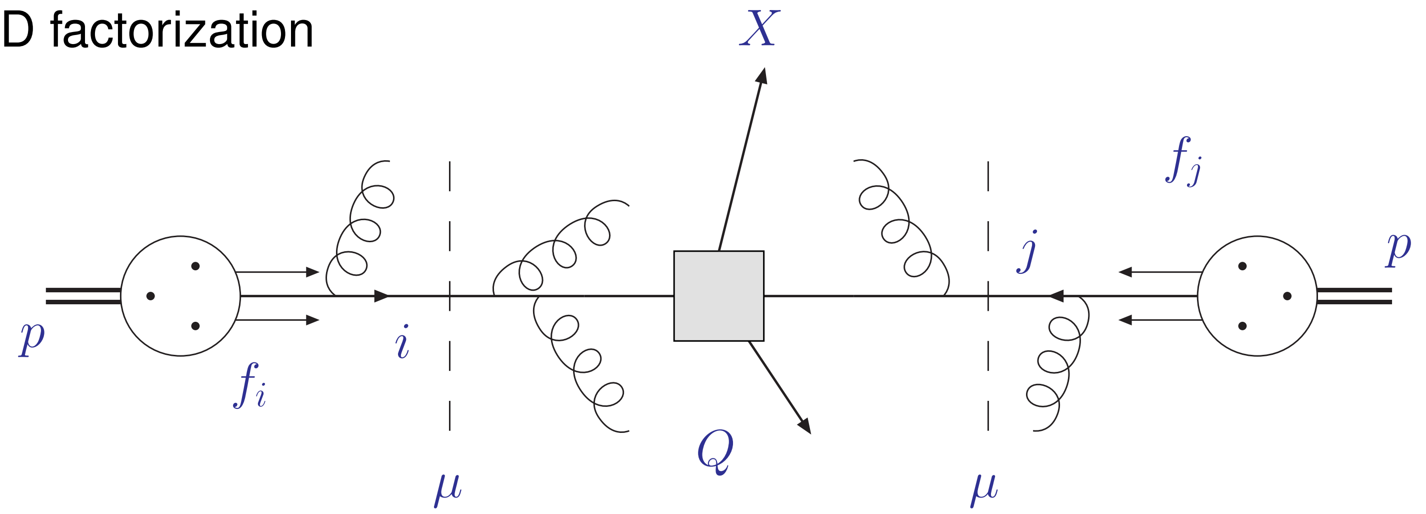
$$\Delta m\text{(unknown)} \quad = \quad ???$$

Δm(unknown) is systematic mass shift of $\mathcal{O}(1)$ GeV due to non-perturbative effects on peak position of invariant jet-mass distribution $M^{\rm peak}$ with decaying top-quark for short distance mass m_t

$$M^{\rm peak} = m_t + \Gamma_t(\alpha_s + \alpha_s^2 + \dots) + \frac{Q\Lambda_{\rm QCD}}{m_t}$$

Top mass from cross sections

- QCD factorization



$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X} \left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Joint dependence on non-perturbative parameters:
  parton distribution functions f_i, strong coupling α_s, masses m_X

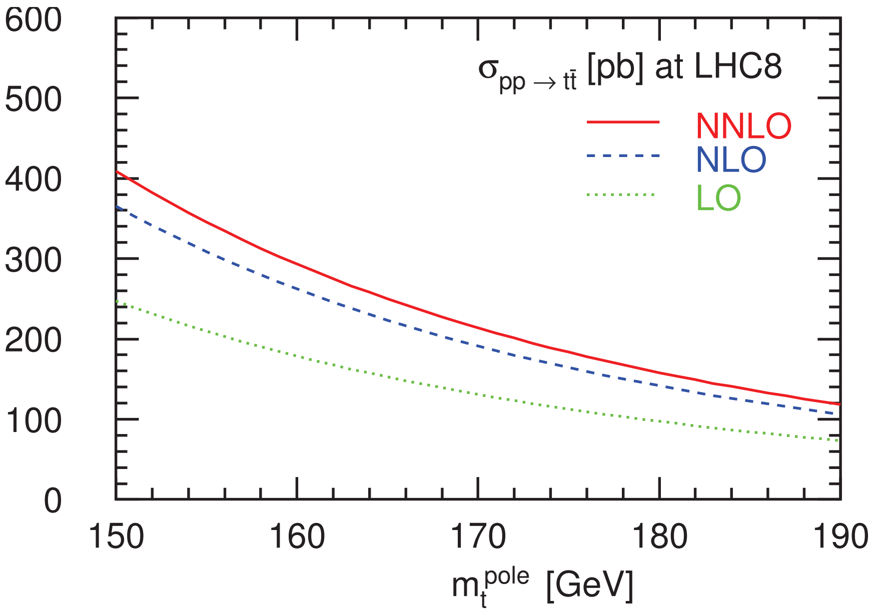
- Total cross section: intrinsic limitation in through sensitivity $\mathcal{S} \simeq 5$

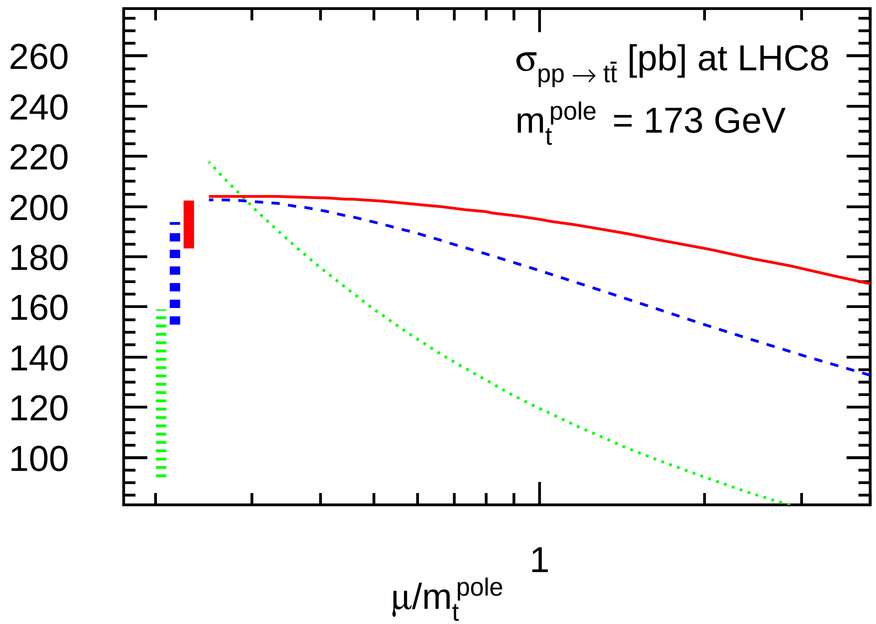
$$\left|\frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}\right| \simeq 5 \times \left|\frac{\Delta m_t}{m_t}\right|$$

Total cross section

Exact result at NNLO in QCD

Czakon, Fiedler, Mitov '13



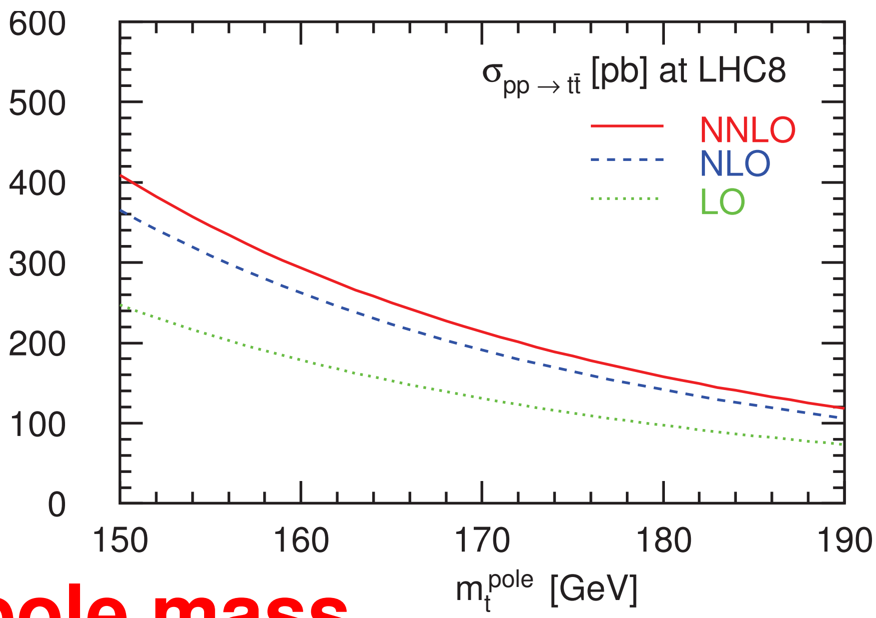


- NNLO perturbative corrections (e.g. at LHC8)
  - K-factor (NLO $\to$ NNLO) of $\mathcal{O}(10\%)$
  - scale stability at NNLO of $\mathcal{O}(\pm 5\%)$

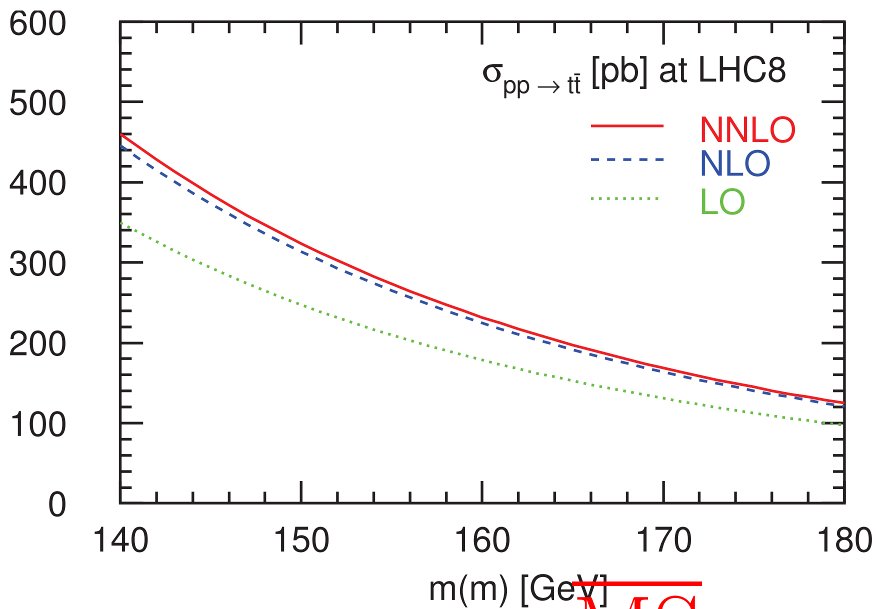
Total cross section with running mass

Comparison pole mass vs. $\overline{\text{MS}}$ mass (I)

Dowling, S.M. '13



pole mass



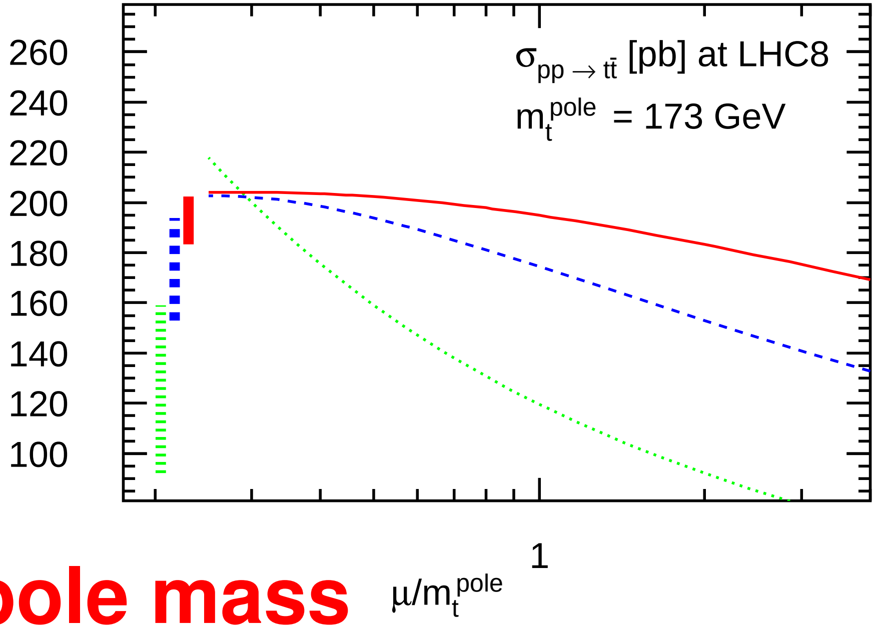
$\overline{\text{MS}}$ mass

- NNLO cross section with running mass significantly improved
  - good apparent convergence of perturbative expansion
  - small theoretical uncertainity from scale variation

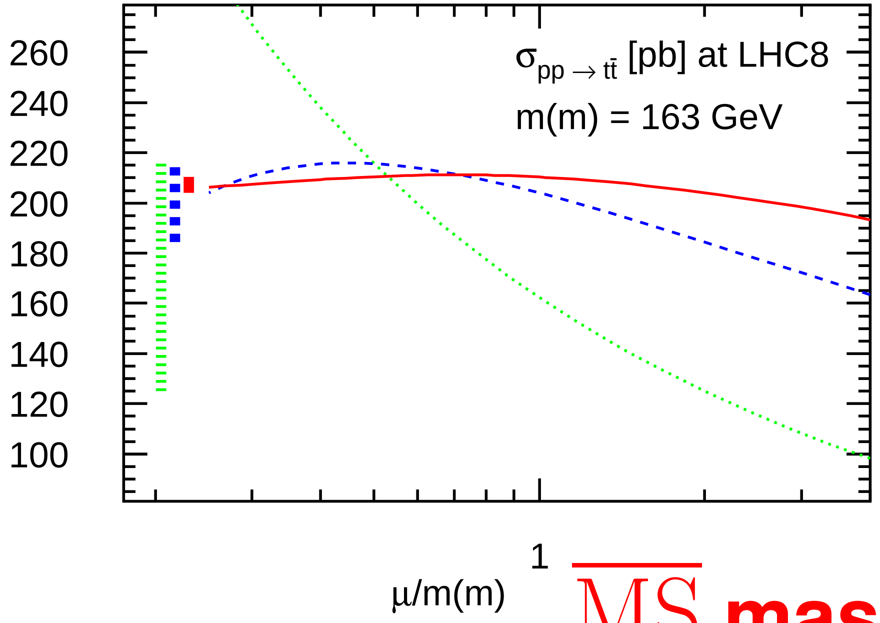
Total cross section with running mass

Comparison pole mass vs. $\overline{\text{MS}}$ mass (II)

Dowling, S.M. '13



pole mass



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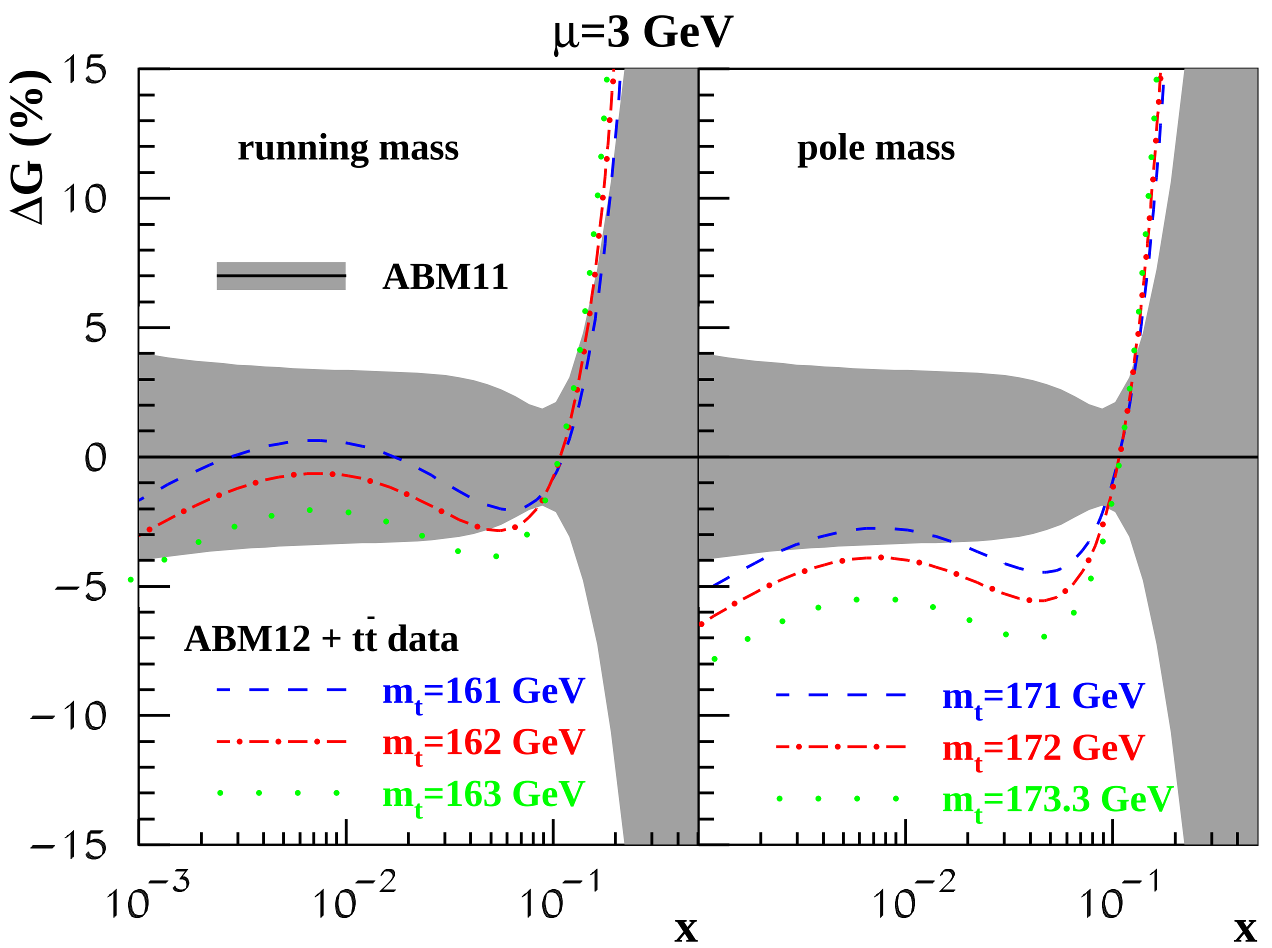
Top-quark mass from total cross section

- Total cross section has correlations $\sigma_{t\bar{t}} \sim \alpha_s^2 \, m_t^2 \, g(x) \otimes g(x)$
  - correlation of gluon PDF $g(x)$ with value of m_t
  - PDFs and $\alpha_s(M_Z)$ already well constrained by global fit (typically no changes)



Top-quark mass determination

- Cross section measurement ATLAS arXiv:1406.5375

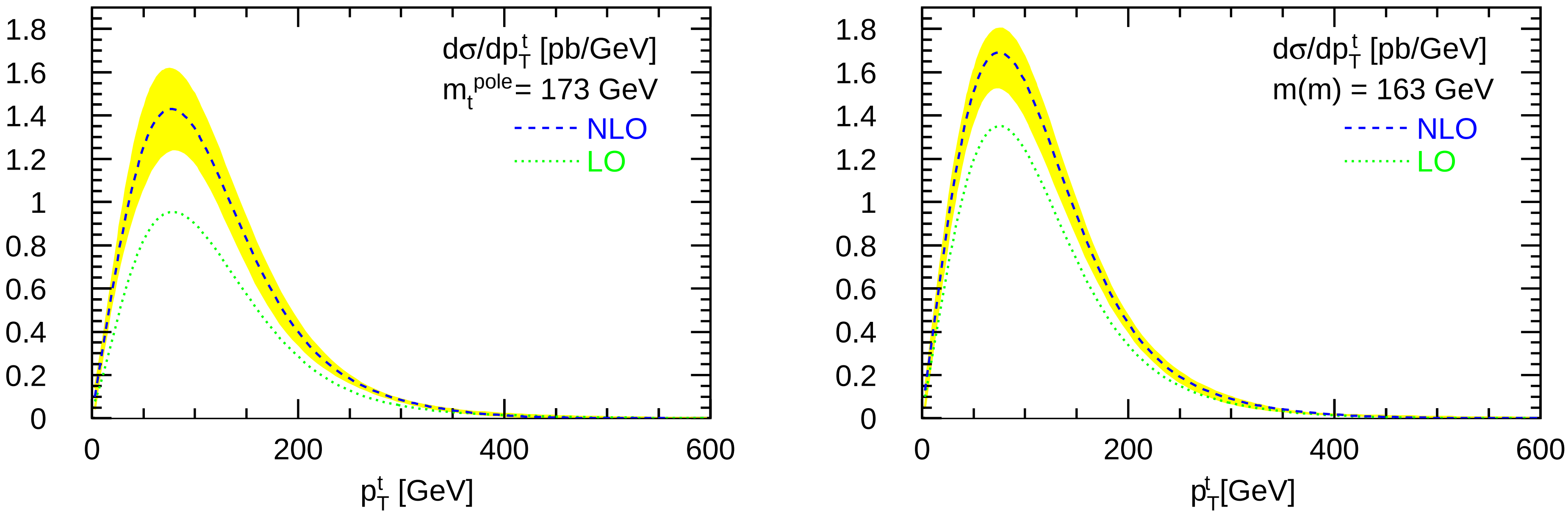
$$\sigma_{t\bar{t}} = 242.4 \pm 9.5 \text{ pb}$$

	$m^{\rm pole} + \Delta^{\rm exp} + \Delta^{\rm th+PDF}$	$m(m) + \Delta^{\rm exp} + \Delta^{\rm th+PDF}$	$m^{\rm pl}_{\rm 1lp}$	$m^{\rm pl}_{\rm 2lp}$	$m^{\rm pl}_{\rm 3lp}$
ABM12	$166.4 \pm 1.3 \pm 2.1$	$159.1 \pm 1.2 \pm 1.2$	166.2	167.8	168.4
CT14	$173.8 \pm 1.3 \pm 2.2$	$165.9 \pm 1.3 \pm 1.3$	173.5	175.4	176.0
MMHT	$173.7 \pm 1.3 \pm 2.0$	$165.8 \pm 1.3 \pm 1.0$	173.4	175.2	175.9
NNPDF3.0	$173.5 \pm 1.3 \pm 2.0$	$165.6 \pm 1.3 \pm 1.0$	173.2	175.0	175.7

- m_t from total cross section sensitive to PDFs
  - pole mass from $\overline{MS}$ mass $m_t(m_t)$ gives spread $m^{\rm pole} = 168.4 \dots 176.0$ GeV
- Scale uncertainty from range $m_t/2 \leq \mu_r, \mu_f \leq 2m_t$ GeV

Differential cross sections

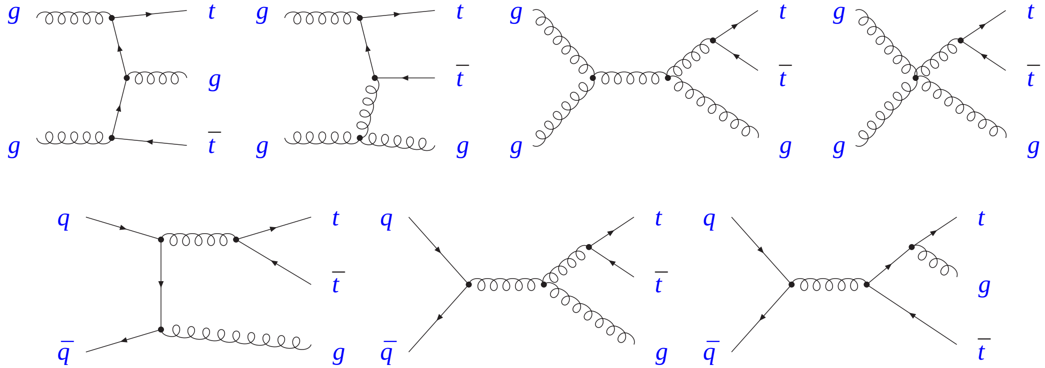
NLO in QCD

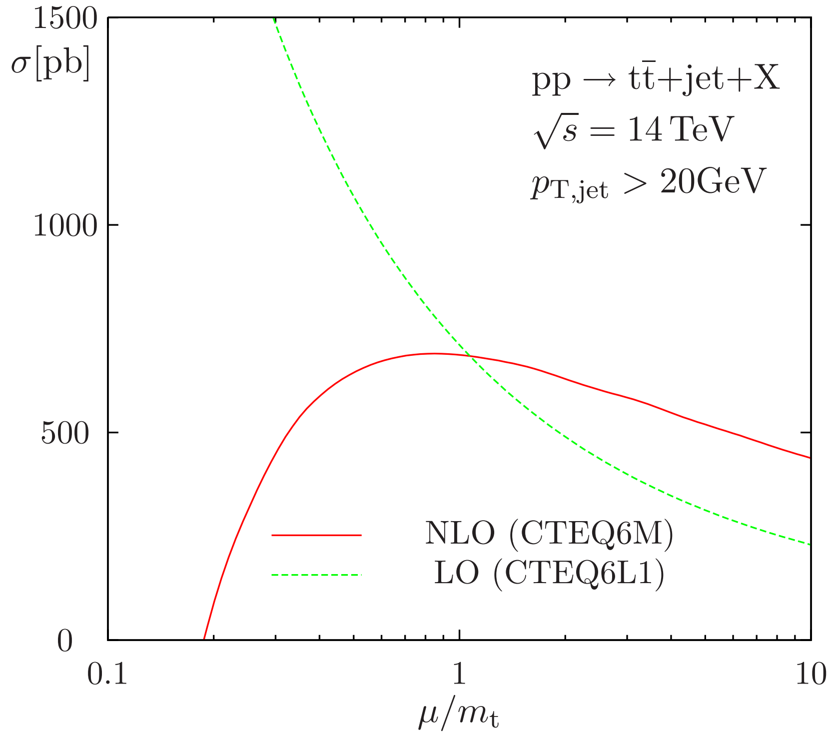


- Differential distributions for running mass show same features, e.g., p_T^t-distribution Dowling, S.M. '13
  - better convergence of perturbative expansion
  - smaller theoretical uncertainity from scale variation
- Possible measurement of $m(\mu = p_T)$ with high statistics at $\sqrt{s} = 13$ TeV

Top-quark pairs with one jet

- LHC: large rates for production of $t\bar{t}$-pairs with additional jets
- NLO QCD corrections for $t\bar{t} + 1\mathrm{jet}$ Dittmaier, Uwer, Weinzierl '07-'08
  - scale dependence greatly reduced at NLO
  - corrections for total rate at scale $\mu_r = \mu_f = m_t$ are almost zero





- Additional jet raises kinematical threshold
  - invariant mass $\sqrt{s_{t\bar{t}+1\mathrm{jet}}}$

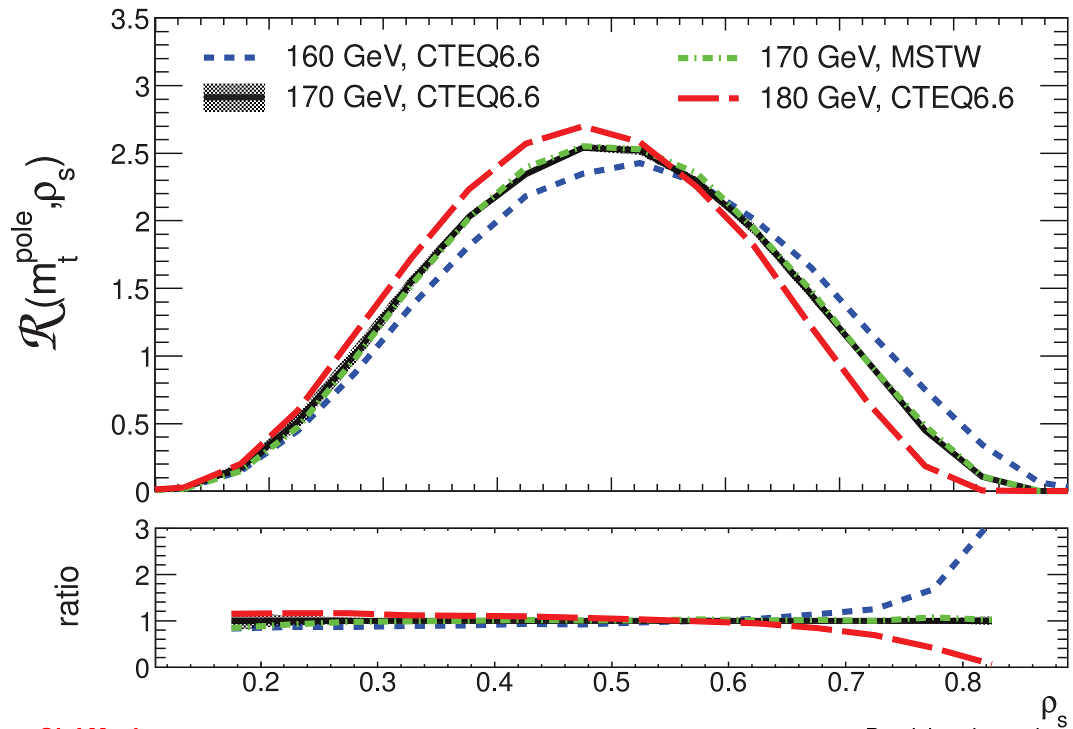
Top mass with $t\bar{t}$ + jet*-samples*

- Normalized-differential $t\bar{t} + \text{jet}$ cross section

Alioli, Fernandez, Fuster, Irles, S.M., Uwer, Vos '13

$$\mathcal{R}(m_t, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\text{jet}}} \frac{d\sigma_{t\bar{t}+1\text{jet}}}{d\rho_s}(m_t, \rho_s)$$

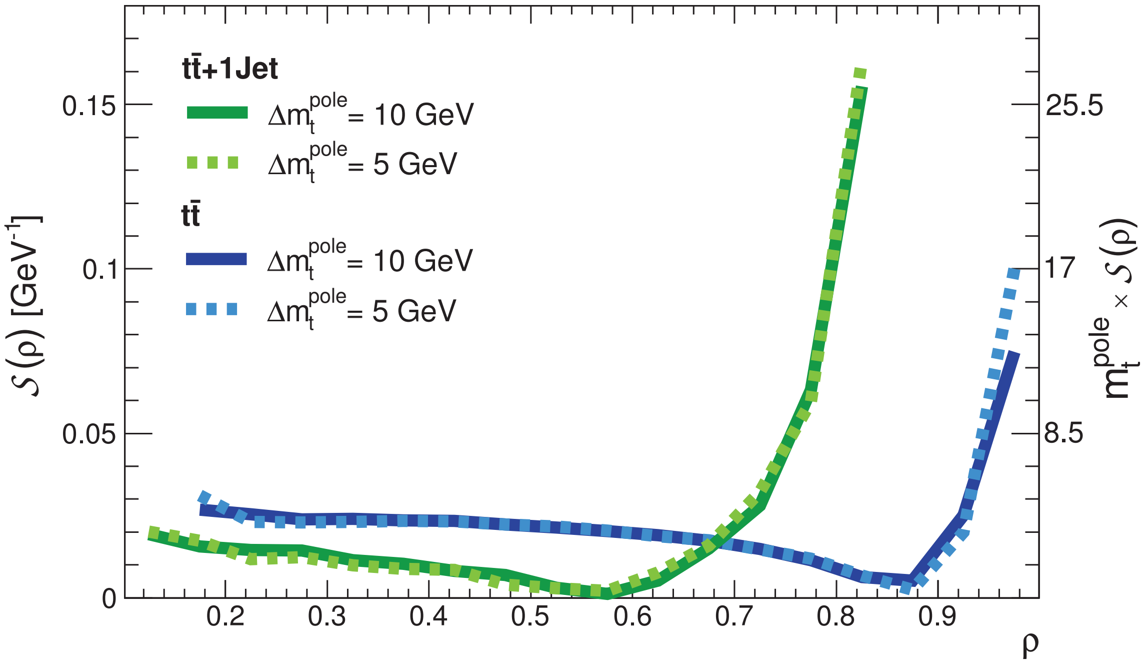
  - variable $\rho_s = \frac{2 \cdot m_0}{\sqrt{s_{t\bar{t}+1\text{jet}}}}$ with invariant mass of $t\bar{t} + 1\text{jet}$ system and fixed scale $m_0 = 170\,\text{GeV}$
- Significant mass dependence for $0.4 \le \rho_s \le 0.5$ and $0.7 \le \rho_s$



Mass sensitivity of $t\bar{t} + \text{jet}$*-samples*

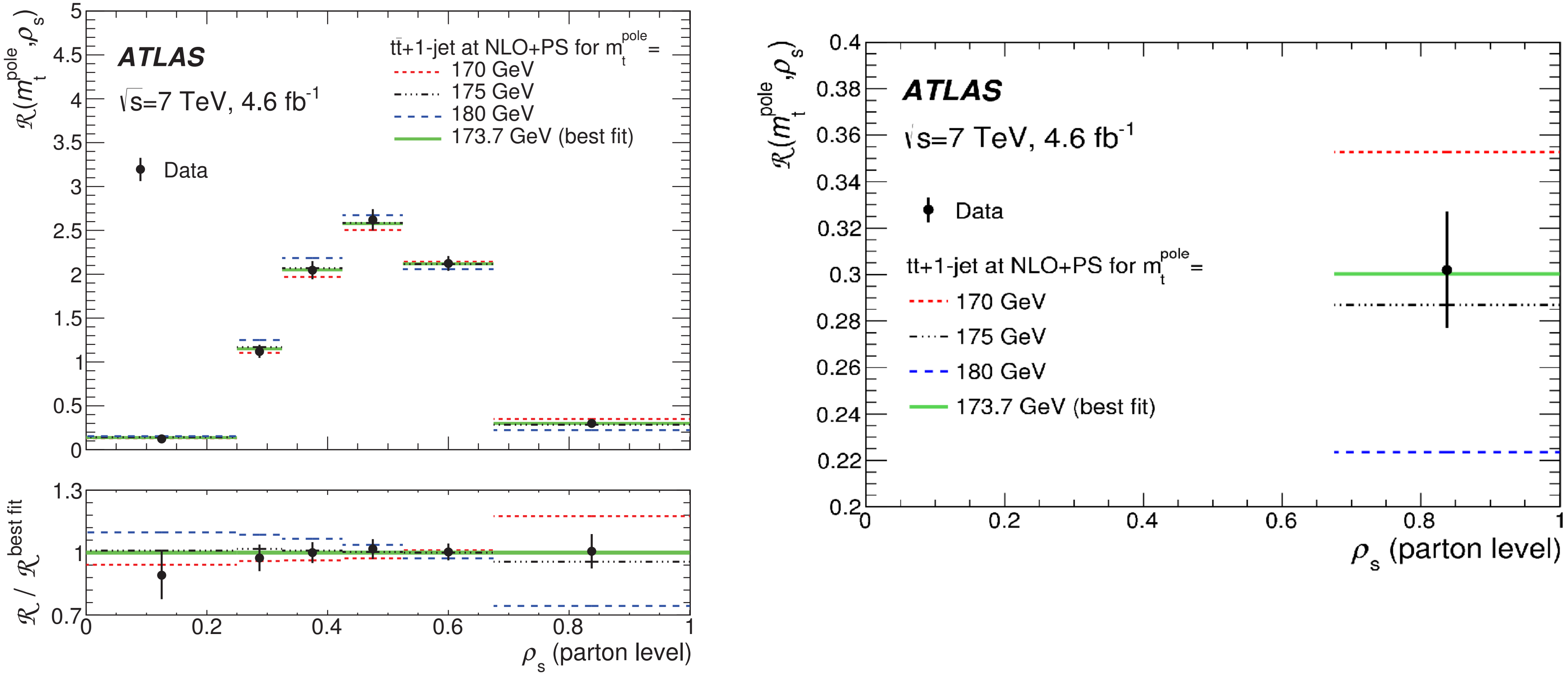
- Differential cross section $\mathcal{R}(m_t, \rho_s)$
  - good pertubative stability, small theory uncertainties, small dependence on experimental uncertainties, ...
- Increased sensitivity for system $t\bar{t} + \text{jet}$ compared

$$\left|\frac{\Delta\mathcal{R}}{\mathcal{R}}\right| \simeq (m_t \mathcal{S}) \times \left|\frac{\Delta m_t}{m_t}\right|$$



ATLAS analysis

- Top-quark mass measurement ATLAS arxiv:1507.01769
  With this method, the value of top-quark pole mass $m_{\rm pole}$ is:
  $m_{\rm pole} = 173.7 \pm 1.5(\text{stat.}) \pm 1.4(\text{syst.})^{+1.0}_{-0.5}(\text{theo.})$



- High precision data in threshold region $0.7 \leq \rho_s$
  - cancellation of systematics in the normalized distribution
  - promising method for LHC at $\sqrt{s} = 13$ TeV

Calibration of Monte-Carlo Mass (I)

Idea Kieseler, Lipka, S.M. '15

- Simultaneous fit of $m^{\rm MC}$ and observable $\sigma(m_t)$ sensitive to m_t, e.g., total cross section, differential distributions, ...
- Observable σ does not rely on any prior assumptions about relation between m_t and $m^{\rm MC}$
- Extraction of m_t from $\sigma(m_t)$ calibration of $m^{\rm MC}$, e.g. pole mass

$$\Delta_m = m_t^{\rm pole} - m^{\rm MC}$$

Implementation [Ph.D. thesis Jan Kieseler]

- Confront N^d reconstructed events to N^p simulated ones
  - model parameters $\vec{\lambda}$

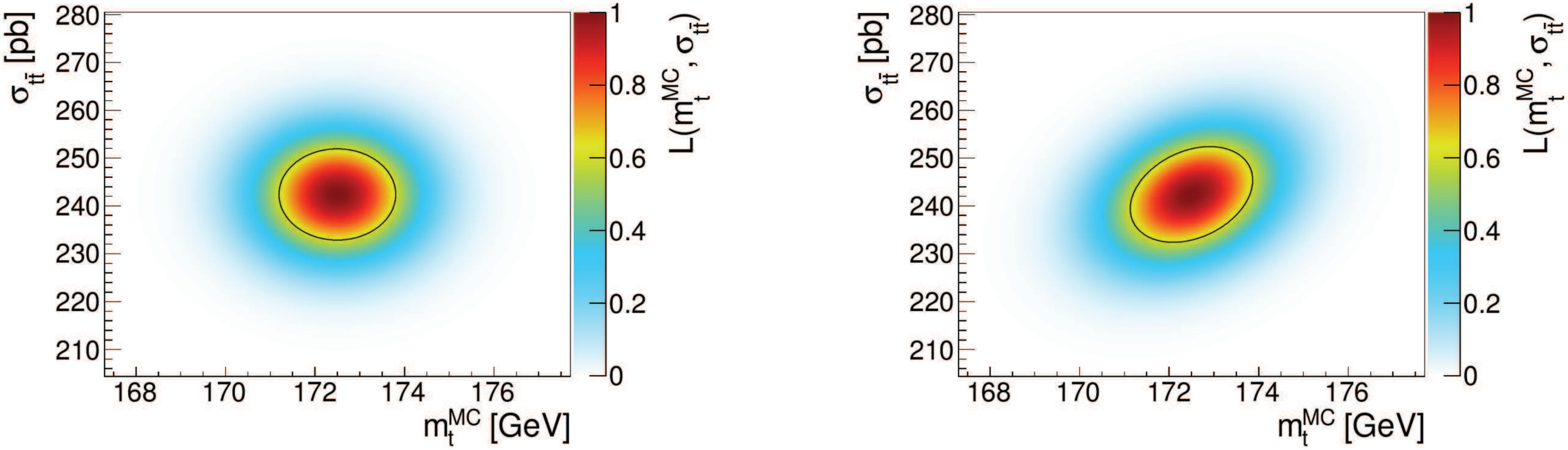
$$N^p = \mathcal{L} \cdot \underbrace{\epsilon(m^{\rm MC}, \vec{\lambda})}_{\text{efficiency}} \cdot \underbrace{\sigma}_{\text{observable}} \cdot \underbrace{n^p(m^{\rm MC}, \vec{\lambda})}_{\text{predicted shape contribution}} + \underbrace{N^{bg}(\vec{\lambda})}_{\text{background}}$$

  - shape of distribution constrains $m^{\rm MC}$, normalization determines σ

Top-Quark Monte-Carlo Mass (II)

Likelihood fit [Ph.D. thesis Jan Kieseler]

- Correlations between $m^{\rm MC}$ and σ present in $\epsilon(m^{\rm MC}, \vec{\lambda})$
  - minimize in $m^{\rm MC}$ dependence in efficiency
- Reduce contribution of $m^{\rm MC}$ to total uncertainty of σ
  - constrain $m^{\rm MC}$ in predicted events $n^p(m^{\rm MC}, \vec{\lambda})$



- Illustration for cross section measurement $\sigma_{t\bar{t}} = 242.4 \pm 9.5$ pb ATLAS arXiv:1406.5375 assuming $m^{\rm MC} = 172.0 \pm 1.3$ GeV
- Calibration of $m^{\rm MC}$ with uncertainty on $\Delta_m = m_t^{\rm pole} - m^{\rm MC}$ of approximately 2 GeV possible

Summary

Top-quark Monte-Carlo mass

- Monte-Carlo mass $m^{\rm MC}$ is (as the name says) a Monte-Carlo parameter
- Monte-Carlo mass $m^{\rm MC}$ needs calibration with data

Well-defined top-quark masses

- Pole mass $m^{\rm pole}$ or short distance masses $m^{\rm MS}(\mu)$
- Quality of perturbative expansion depends on scheme for top-quark mass
  - cross sections for $m^{\rm MS}(\mu)$ shows better convergence and smaller scale uncertainity

Measurements of top-quark mass

- Measurements of m_t require careful definition of observable
- Absolute relates (total cross sections, etc.) have m_t correlated with gluon PDF $g(x)$ and $\alpha_s(M_Z)$
- Normalized shapes help to cancel systematics