A viscous blast-wave model for heavy-ion collisions

Amaresh Jaiswal

GSI Darmstadt

Based on A. Jaiswal and V. Koch, arXiv:1508.05878 [nucl-th]

Strangeness in Quark Matter 2016

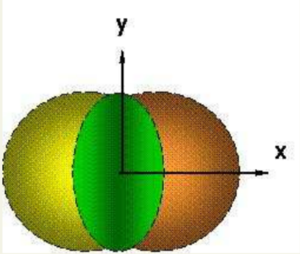
June 28, 2016

High-energy heavy-ion collisions and QGP

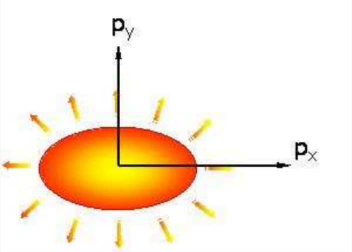
- The Quark-Gluon Plasma (QGP) is a phase of QCD which is expected to be created at sufficiently high temperatures and/or densities.
- It is now well established that QGP is formed in high-energy heavy-ion collision experiments at RHIC and LHC.
- Our aim: to study and understand the thermodynamical and transport properties of QGP.
- QGP exhibit strong collective behaviour and therefore can be studied within the framework of relativistic hydrodynamics.
- Relativistic hydrodynamics has been applied quite successfully to describe the space-time evolution of the QGP.
- Hydrodynamical analyses suggests that QGP has an extremely small shear viscosity.
- An alternative model to estimate viscosity of QGP will be presented.

Conversion of initial anisotropy to final flow

initial spatial anisotropy converts to final momentum space anisotropy



$\longrightarrow$



$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$

$$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \equiv \langle \cos 2\phi_p \rangle$$

hydrodynamic models can generate the large v_2 observed at RHIC

How to measure η/s of QGP

- Role of Hydrodynamics:

Initial state spatial deformation $\xrightarrow{Hydro}$ Final state momentum anisotropy

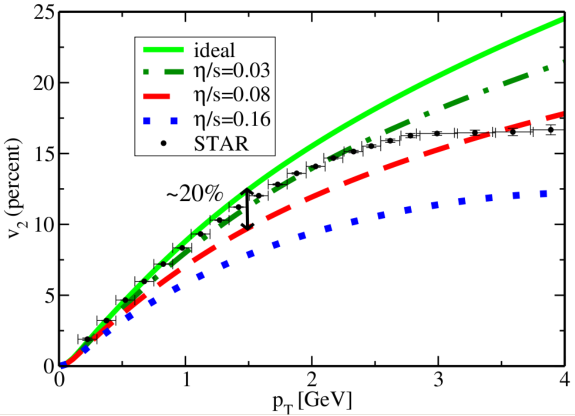


Figure: Viscosity degrades conversion efficiency.

Blast wave model: A simple freezeout model

- Extensively used to fit transverse momentum spectra of particles.
- The hydrodynamic fields at freeze-out are parametrized as:

$$T = T_f, \qquad u^r = u_0 \frac{r}{R}, \qquad u^\varphi = u^{\eta_s} = 0, \qquad u^\tau = \sqrt{1 + (u^r)^2}.$$

- The hadron spectra can be obtained using the Cooper-Frye freeze-out prescription:

$$\frac{dN}{d^2p_T dy} = \frac{1}{(2\pi)^3} \int p_\mu d\Sigma^\mu f(x,p).$$

- $d\Sigma_\mu$ is the oriented freeze-out hyper-surface and $f(x,p)$ is the phase-space distribution function of the particles at freeze-out.
- The distribution function: $f = f_0 + \delta f$, where

$$f_0 = \frac{1}{\exp(u_\mu p^\mu / T) + a}, \quad a = \begin{cases} +1 \text{ for baryons} \\ -1 \text{ for mesons} \end{cases}; \quad \delta f = \frac{f_0 \tilde{f}_0}{T^3} \left(\frac{\eta}{s}\right) p^\alpha p^\beta \nabla_{\langle\alpha} u_{\beta\rangle}$$

Hydrodynamic conversion efficiency

- Participant anisotropies, ε_n, via the Fourier expansion for a single-particle distribution is:

$$f(\varphi) = \frac{1}{2\pi}\left[1 + 2\sum_{n=1}^{\infty} \varepsilon_n \cos[n(\varphi - \psi_n)]\right].$$

- ε_n eventually converts to anisotropies in the radial fluid velocity

$$u^r = u_0 \frac{r}{R}\left[1 + 2\sum_{n=1}^{\infty} u_n \cos[n(\varphi - \psi_n)]\right].$$

- Important question: what is the hydrodynamic conversion efficiency?

$$\frac{u_n}{\varepsilon_n} = ?$$

- Answer comes from "acoustic damping" formula.

Acoustic damping [P. Staig and E. Shuryak, PRC **84**, 034908 (2011)]

- Dispersion relation for sound in a viscous medium:

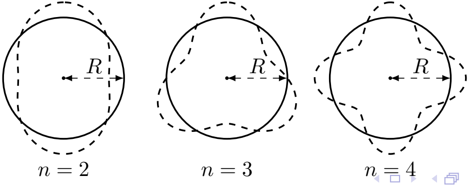
$$\omega = c_s k + ik^2 \frac{1}{T}\left(\frac{2}{3}\frac{\eta}{s}\right).$$

- Using a plane-wave Fourier ansatz, $\exp(i\omega t - ikx)$,

$$\delta T^{\mu\nu}(t,k) = \exp\left[-\left(\frac{2}{3}\frac{\eta}{s}\right)\frac{k^2 t}{T}\right]\delta T^{\mu\nu}(0,k).$$

- Each harmonics is a damped oscillator with wave-vector k which form standing waves on the fireball circumference:

$$2\pi R = n\frac{2\pi}{k}.$$



Viscous blast-wave model

- At the freeze-out time t_f, the wave amplitude reaction is given by

$$\frac{\delta T^{\mu\nu}|_{t=t_f}}{\delta T^{\mu\nu}|_{t=0}} = \exp\left[-n^2\left(\frac{2}{3}\frac{\eta}{s}\right)\frac{t_f}{R^2 T_f}\right].$$

- The conversion efficiency is proportional to the wave amplitude reaction [AJ and V Koch, arXiv:1508.05878 [nucl-th]]:

$$\frac{u_n}{\varepsilon_n} = \alpha_0 \exp\left[-n^2\left(\frac{2}{3}\frac{\eta}{s}\right)\frac{t_f}{R^2 T_f}\right].$$

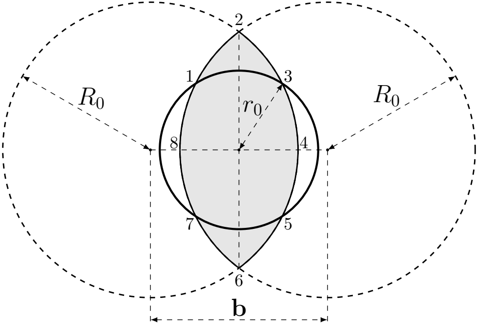
- Remember that the radial velocity is:

$$u^r = u_0 \frac{r}{R}\left[1 + 2\sum_{n=1}^{\infty} u_n \cos[n(\varphi - \psi_n)]\right].$$

- And other blast wave fields are:

$$T = T_f, \qquad u^{\varphi} = u^{\eta_s} = 0, \qquad u^{\tau} = \sqrt{1 + (u^r)^2}.$$

Initial geometry and fixing R



$$\widehat{123} = \widehat{345} = \widehat{567} = \widehat{781}$$

- The initial radius r_0 of the expanding fireball is given by

$$r_0 = \frac{1}{2}\left(b^2 - 2\,bR_0\sqrt{2 + \frac{b}{R_0}} + 4R_0^2\right)^{1/2}$$

- The final radius using perturbation-free expression velocity

$$u^r \equiv \frac{dr}{d\tau} = u_0\frac{r}{R} \quad \Rightarrow \quad \int_{r_0}^{R}\frac{dr}{r} = \int_0^{\tau_f}\frac{u_0}{R}\,d\tau \quad \Rightarrow \quad R = r_0\exp\left(\frac{u_0\,\tau_f}{R}\right)$$

Parameters: viscous hydrodynamics vs. viscous blast-wave

Viscous hydrodynamics	Viscous blast-wave
T_f: Freeze-out temperature	T_f: Freeze-out temperature
τ_i: Initialization time	τ_f: Freeze-out time
η/s: Shear viscosity	η/s: Shear viscosity
ϵ_0: Initial energy density	u_0: Radial freeze-out velocity
σ: Smearing parameter	α_0: Conversion efficiency strength

- To remove centrality dependence of τ_f, we use Bjorken expansion results:

$$\epsilon \propto \tau^{-4/3} \quad \Rightarrow \quad \tau_f = \tau_{f0} \left(\frac{\epsilon_i}{\epsilon_{i0}} \right)^{3/4} .$$

- τ_{f0}: freeze-out time for central collision.
- ϵ_{i0}: initial energy density for central collisions.
- ϵ_i/ϵ_{i0} and ε_n obtained from Monte-Carlo Glauber model.

Freezze-out and flow

- The hadron spectra can be obtained using the Cooper-Frye formula

$$\frac{dN}{d^2p_T dy} = \frac{1}{(2\pi)^3} \int p_\mu d\Sigma^\mu f(x,p).$$

- The oriented freeze-out hyper-surface $d\Sigma_\mu$

$$p^\mu d\Sigma_\mu = m_T \cosh(y - \eta_s)\, \tau d\eta_s\, r dr\, d\varphi.$$

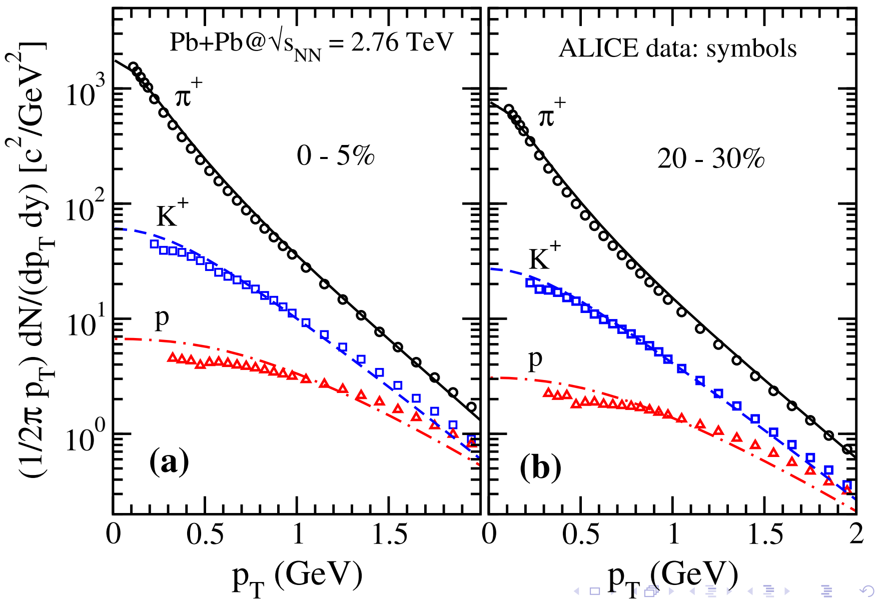
- We determine the form of the viscous correction $\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle}$ and analytically confirm that $g_{\mu\nu}\nabla^{\langle\mu}u^{\nu\rangle} = 0$.

- The anisotropic flow is defined as

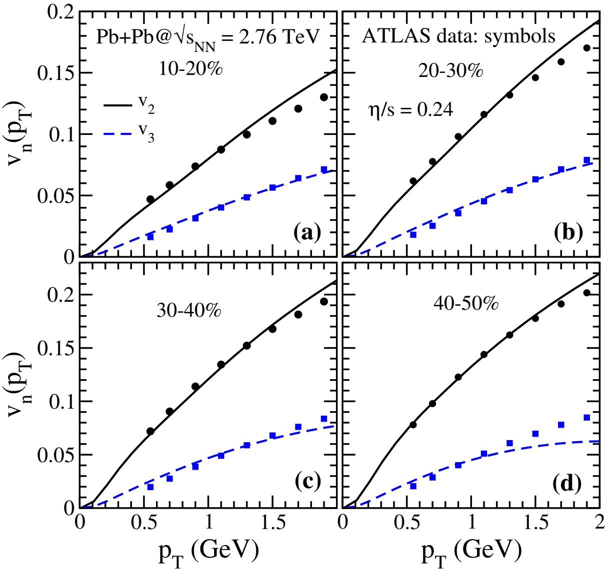
$$v_n(p_T) \equiv \frac{\displaystyle\int_{-\pi}^{\pi} d\phi\, \cos[n(\phi - \Psi_n)]\, \frac{dN}{dy\, p_T\, dp_T\, d\phi}}{\displaystyle\int_{-\pi}^{\pi} d\phi\, \frac{dN}{dy\, p_T\, dp_T\, d\phi}},$$

- We consider only first-order corrections in η.

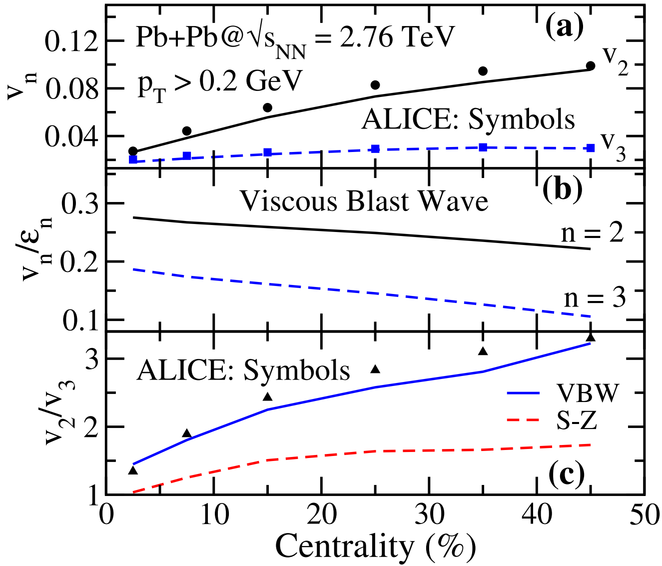
Results: Transverse momentum spectra



Results: p_T distribution of anisotropic flow



Results: Integrated anisotropic flow



Conclusions

- Generalized the blast-wave model to include viscous effects.
- Employed a viscosity-based survival scale for geometrical anisotropies formed in the early stages.
- This viscous damping is introduced in the parametrization of the radial flow velocity.
- This model incorporates important features of viscous hydrodynamic evolution but does not require to do the actual evolution.
- The blast-wave model parameters were fixed by fitting the transverse momentum spectra of identified particles.
- Demonstrated that a fairly good agreement is achieved for transverse momentum distribution of elliptic and triangular flow for various centralities.
- Within the present model, we estimated the shear viscosity to entropy density ratio $\eta/s \simeq 0.24$ at the LHC ($\eta/s \simeq 0.2$ obtained from hydro).