JOHANN WOLFGANG GOETHE UNIVERSITÄT FRANKFURT AM MAIN

HIC for FAIR
Helmholtz International Center

Fast Dynamical Evolution of Hadron Resonance Gas via Hagedorn States

C. Greiner,

SQM 2016, Berkeley, 27th June – 1st July

in collaboration with:

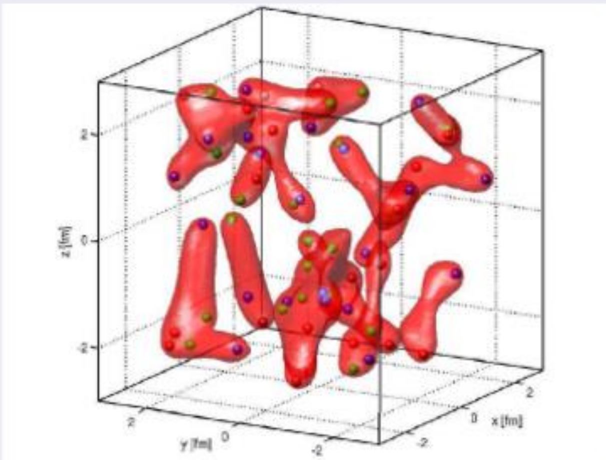
M. Beitel, K. Gallmeister and H. Stöcker

- (personal) history of Hagedorn States
- chemical equilibration at the phase boundary
- why so thermal ? … via phase space 2body decay of HS
- implementation into URQMD and chemicalization

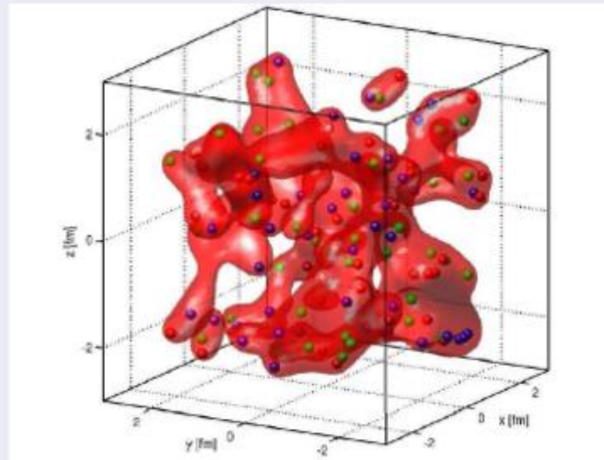
Deconfinement: transition to quark phase

G. Martens et al. Phys. Rev. D 70 / 73 (2010)

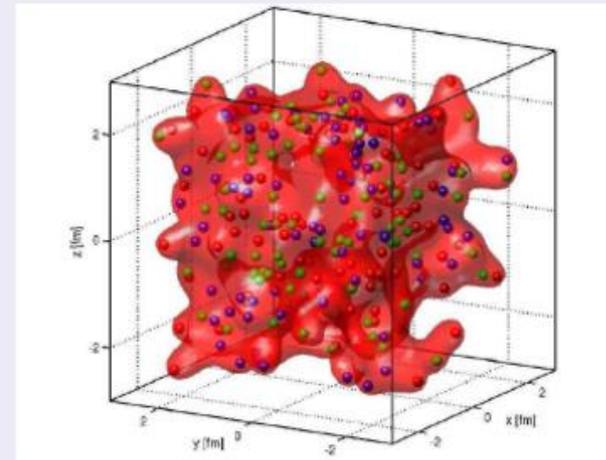
$n = 0.5\text{fm}^{-3}$



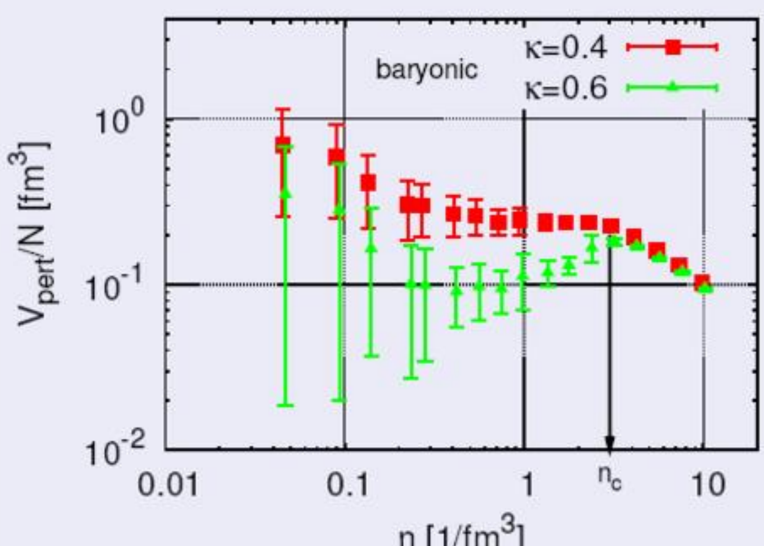
$n = 1.0\text{fm}^{-3}$



$n = 2.0\text{fm}^{-3}$



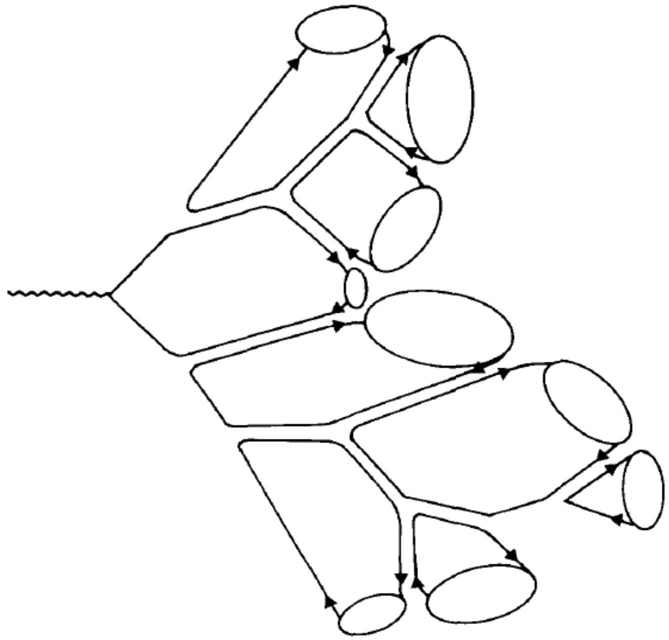
bag volume/particle

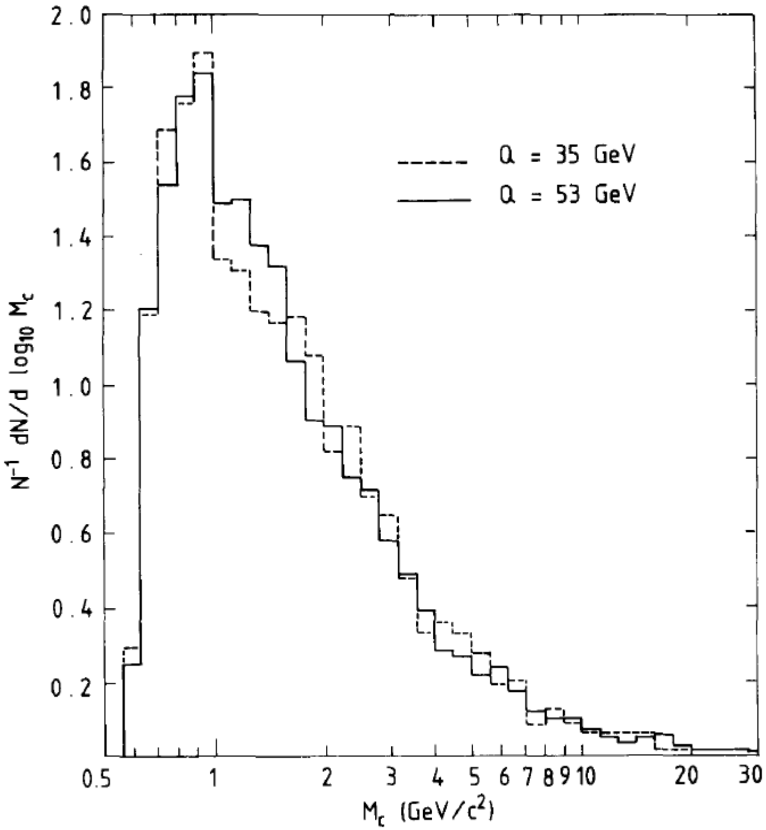


- formation of color neutral clusters at small densities
- particle number/cluster rises
- critical density at maximal overlap ($n \approx 2\text{fm}^{-3}$ or $\varepsilon \approx 1.1\text{GeV}/\text{fm}^3$)
- percolation transition

Color Singlet cluster and their distribution

B.R. Webber, Nucl. Phys. B 238 (1984)





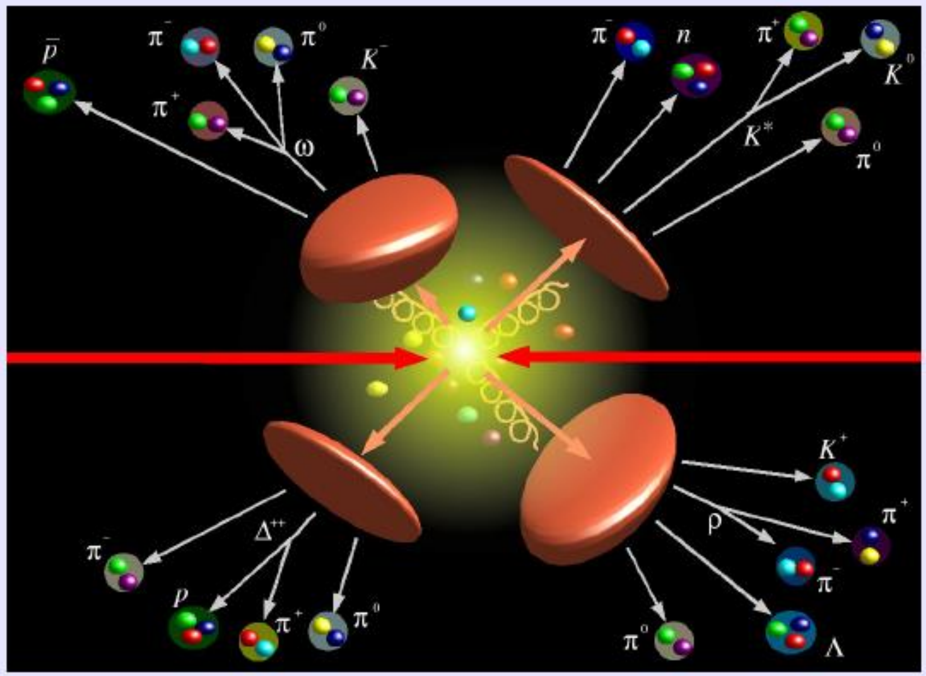
- The blobs (right) represent colour singlet clusters as basis for hadronization

- Distribution of colour singlet cluster mass (left) in e+-e- annihilation at c.m. energies of Q=35 GeV and Q=53 GeV

- this colour singlet clusters might be identified as Hagedorn States

Statistical Model

In modern view, the statistical model is a model of hadronization, describing the process of hadron formation at the scale where QCD is no longer perturbative



History

- 1965 R. Hagedorn postulated the “Statistical Bootstrap Model” before QCD
- fireballs and their constituents are the same
- nesting fireballs into each other leads to self-consistency condition (bootstrap equation)
- solution is exponentially rising common known as Hagedorn spectrum
- slope of Hagedorn Spectrum determined by Hagedorn temperature

Maciej Sobczak – analysis of states listed in PDG2008 compilation

$$f_{FIT}(m) = log_{10}\left(\int_0^m \frac{c}{(x^2+m_0^2)^{5/4}} exp(x/T_H)\right)$$

$$\rho(m) = \frac{c}{(m^2+m_0^2)^{5/4}} exp(m/T_H)$$

$$N_{exp}(m) = \sum_i g_i \Theta(m - m_i)$$

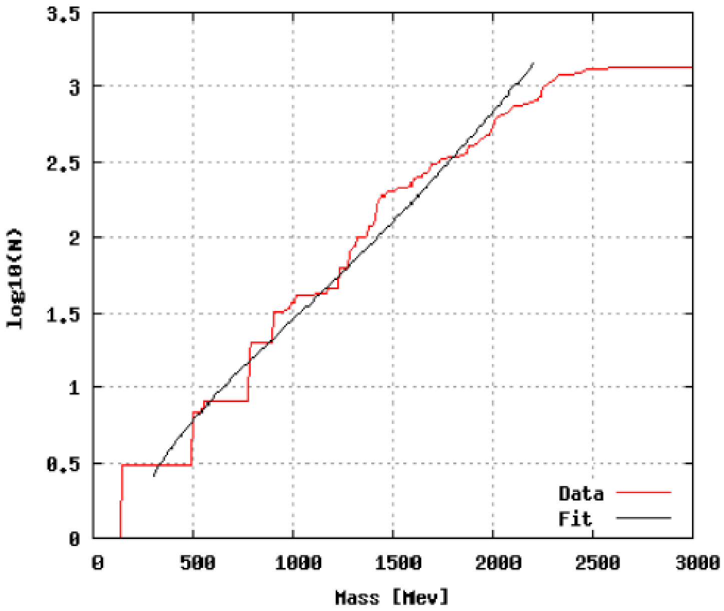


Figure 2: All mesons $T_H = 203.315$, $c = 25132.674$, range: $300 - 2200\ MeV$

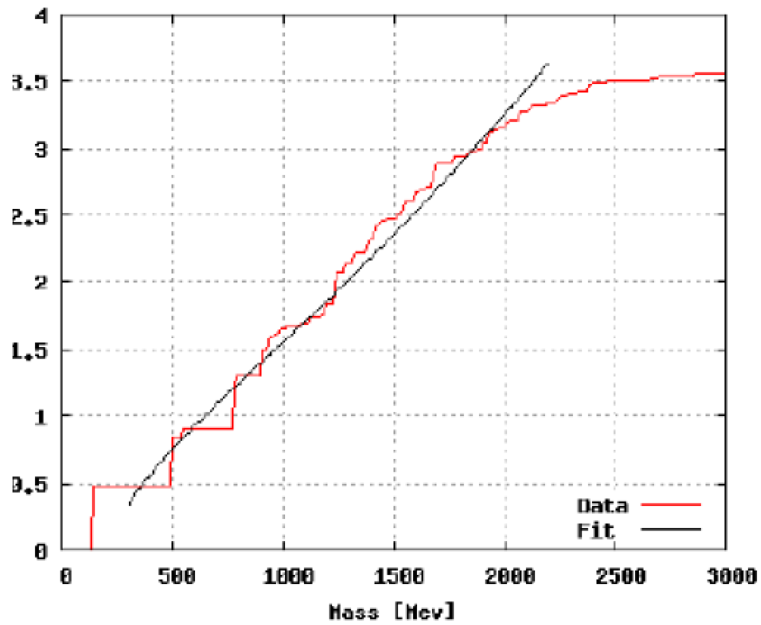


Figure 3: All hadrons $T_H = 177.086$, $c = 18726.494$, range: $300 - 2200\ MeV$

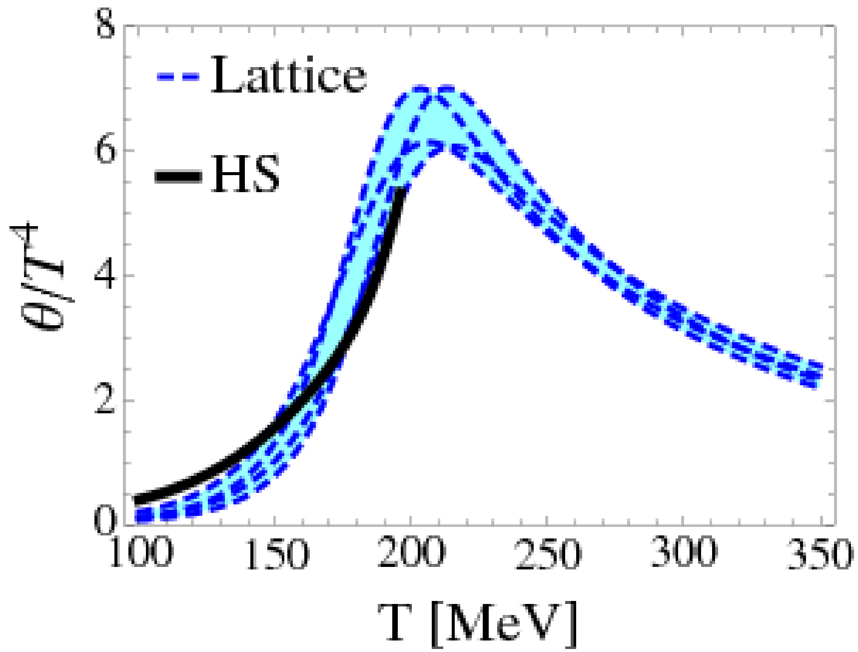
Hadron Resonance Gas with Hagedorn States and comparison to lattice QCD close to T_{critical}

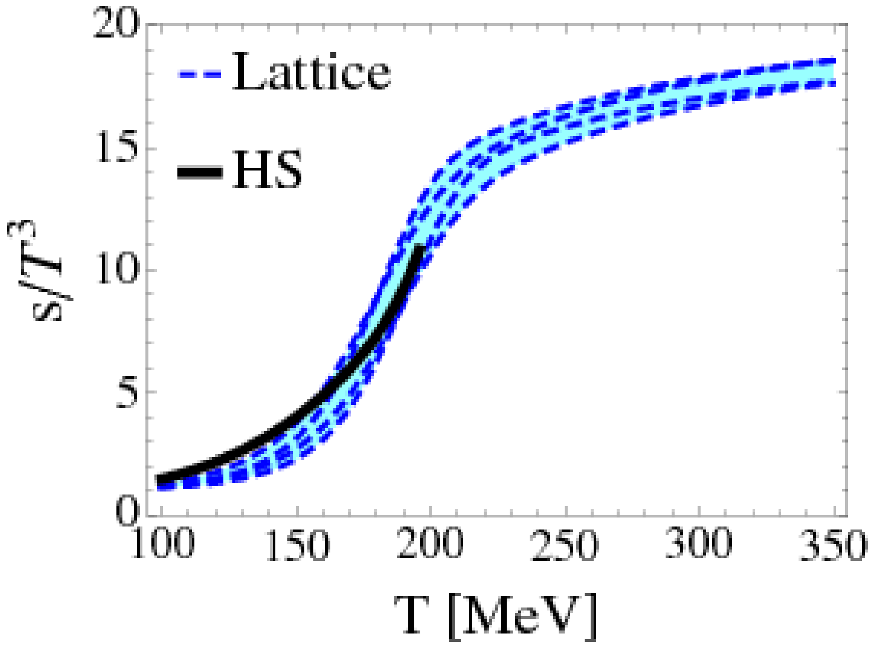
J. Noronha-Hostler, J. Noronha, CG, PRL 103 (2009), PRC 86 (2012)

- **Hagedorn spectrum**: $\rho_{HS} \sim m^{-a} \exp[m/T_H]$

$$\longrightarrow \quad \rho = \int_{M_0}^{M} \frac{A}{[m^2+m_r^2]^{\frac{5}{4}}} e^{\frac{m}{T_H}} dm$$

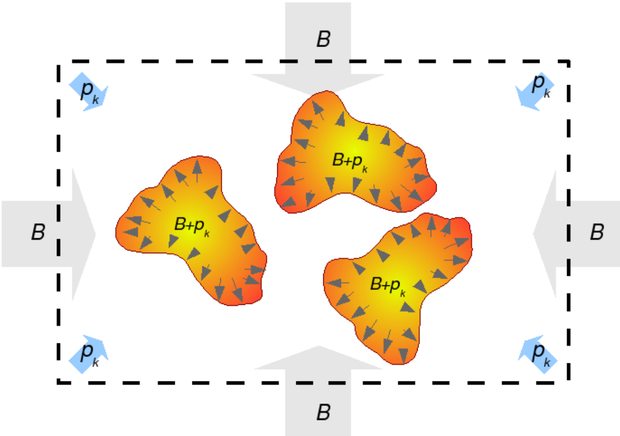
- **RBC collaboration**:



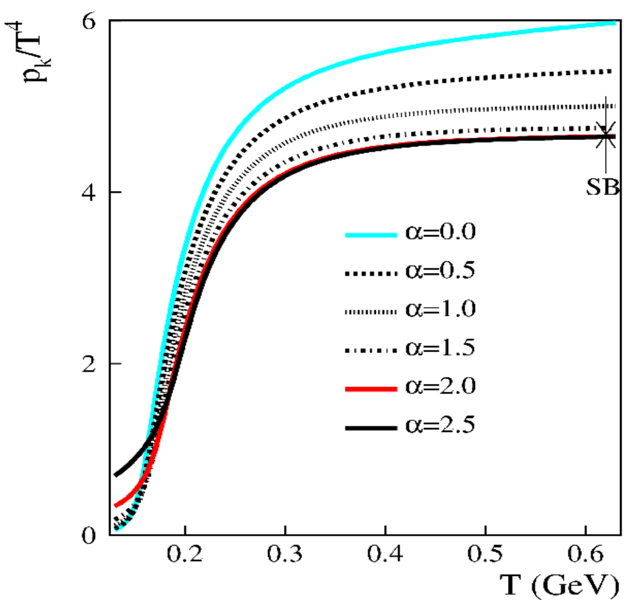


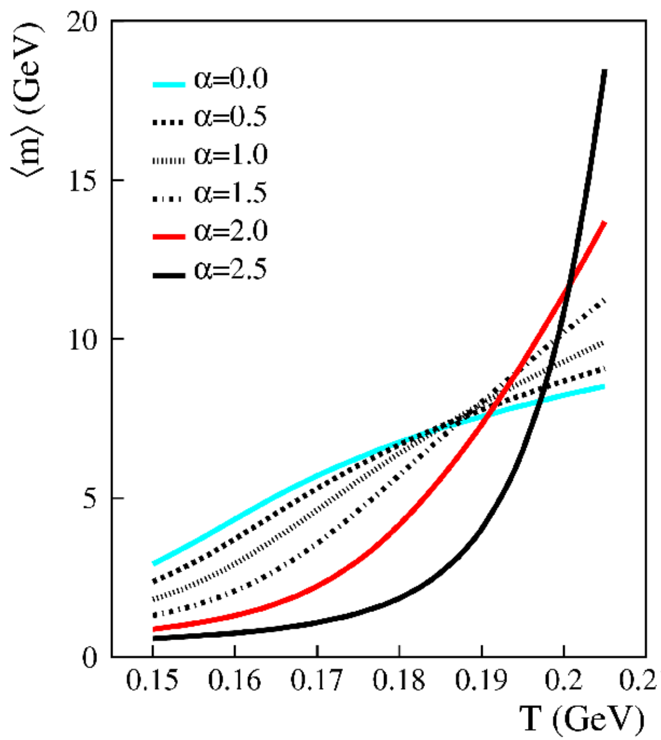
Crossover transition in bag-like models

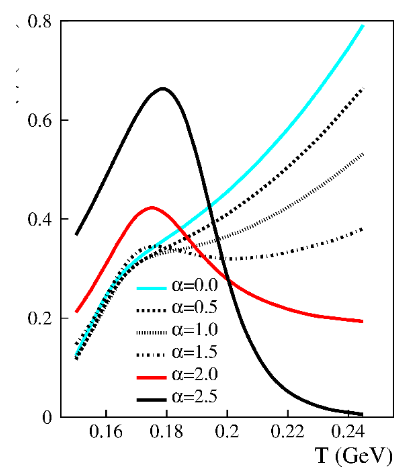
L. Ferroni and V. Koch, PRC79 (2009) 034905

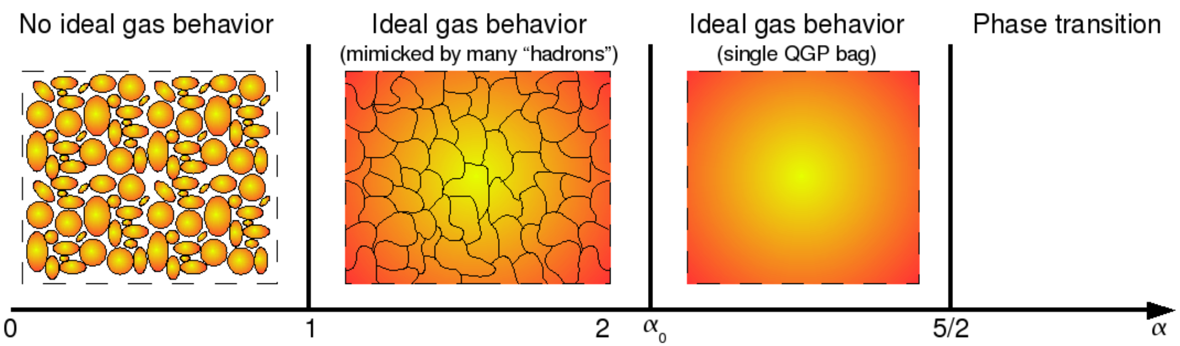


density of states: $\rho(m) \sim c\, m^{-(\alpha)}\, e^{\frac{m}{T_H[B]}}$









Application of Hagedorn states

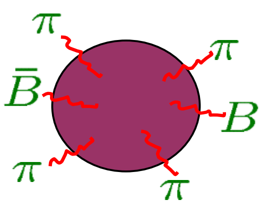
- at SPS energies chem. equil. time is 1-3 fm/c

$$\mathrm{n}_1\pi + n_2 K \leftrightarrow \bar{Y} + p$$ (CG, Leupold, 2000)

- at RHIC energies chem. equil. time is 10 fm/c

  with same approach

- fast chem. equil. mechanism through Hagedorn states

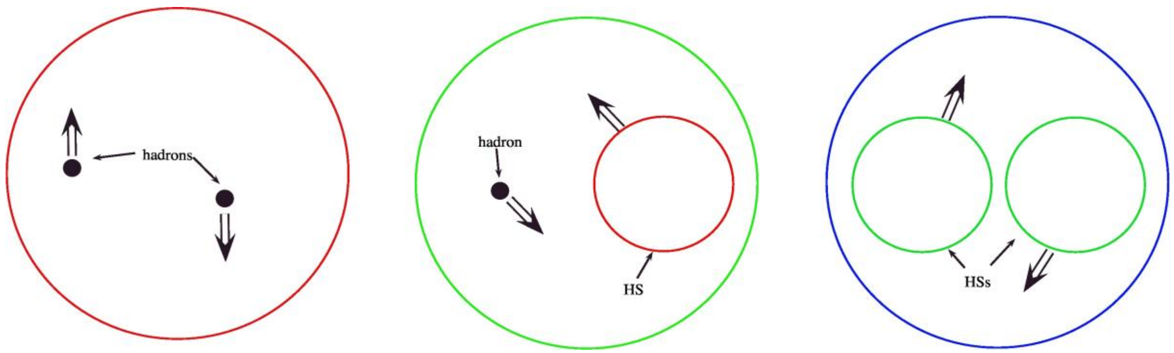


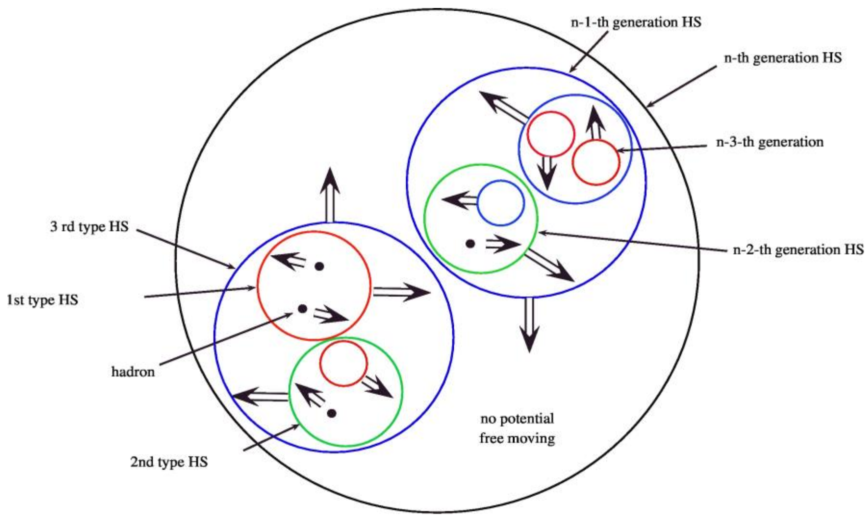
$$\left(n_1\pi + n_2 K + n_3 K \leftrightarrow\right) \; HS \leftrightarrow \bar{B} + B + X$$

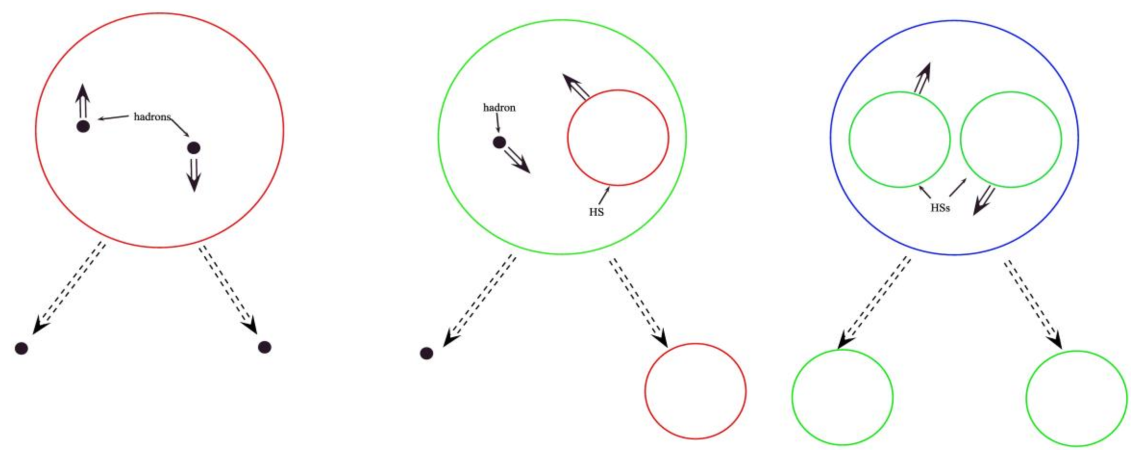
- dyn. evolution through set of coupled rate equations
  leads to 5 fm/c for BB pairs

J. Noronha-Hostler et al. PRL 100 (2008)
J. Noronha-Hostler et al. J. Phys. G 37 (2010)
J. Noronha-Hostler et al. Phys. Rev. C 81 (2010)

Basics: Build up and decay of Hagedorn states







M.Beitel, K: Gallmeister, CG, PRC 90 (2014) 045203

Bootstrap equation and Hagedorn state total decay width

(S. Frautschi PRD 3, C. Hamer et al. PRD 4)

$$\tau_{\vec{C}}(m) = \frac{R^3}{3\pi m} \sum_{\vec{C}_1,\vec{C}_2} \iint dm_1 dm_2\, \tau_{\vec{C}_1}(m_1)\, m_1$$

$$\times\, \tau_{\vec{C}_2}(m_2)\, m_2\;\, p_{cm}(m, m_1, m_2)\, \delta^{(3)}\left(\vec{C} - \vec{C}_1 - \vec{C}_2\right)$$

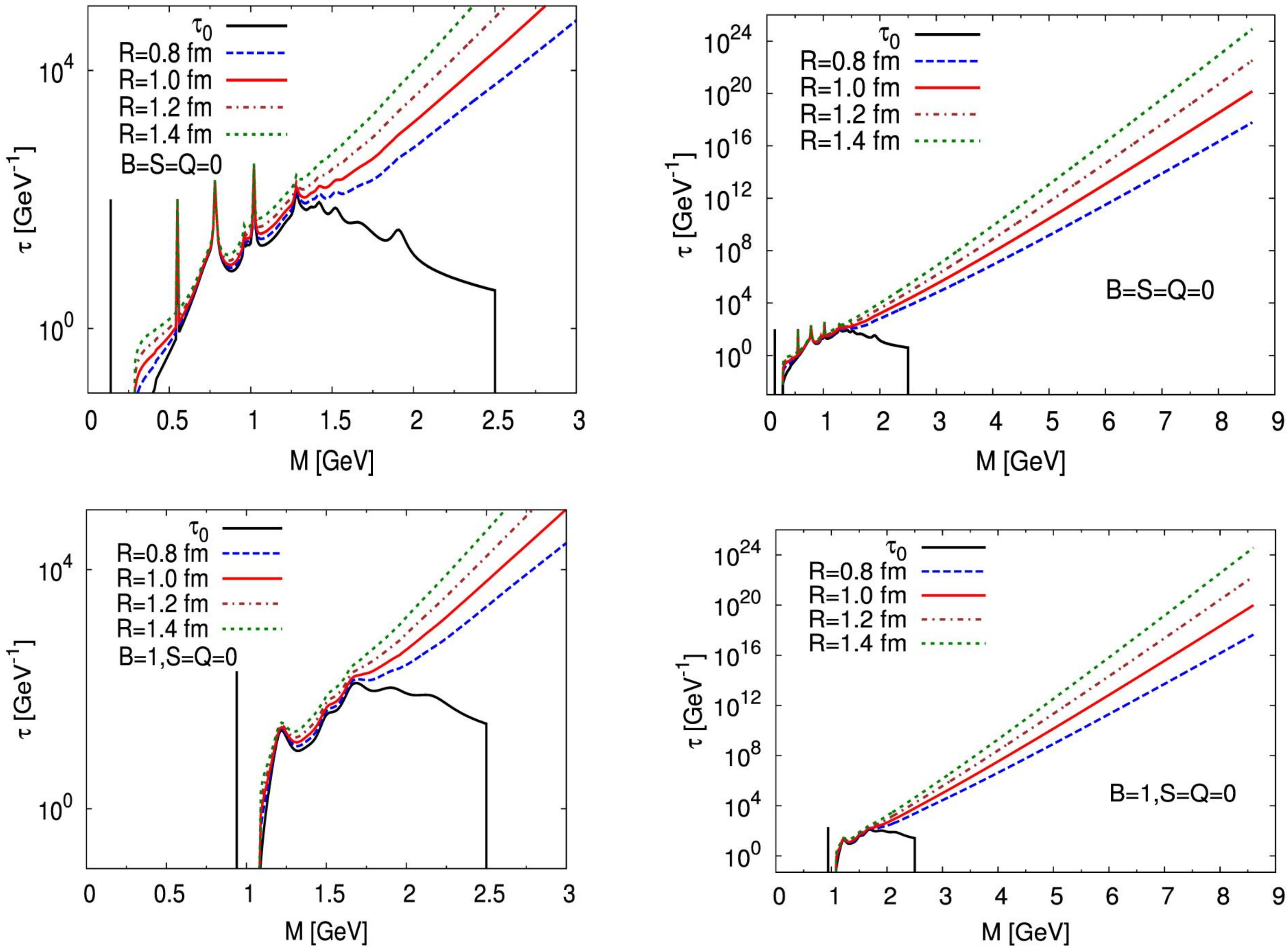
- Bootstrap equation with four-momentum and strict charge conservation **(B,S,Q)**

$$\Gamma_{\vec{C}}(m) = \frac{\sigma}{2\pi^2 \tau_{\vec{C}}(m)} \sum_{\vec{C}_1,\vec{C}_2} \iint dm_1 dm_2 \tau_{\vec{C}_1}(m_1)\, \tau_{\vec{C}_2}(m_2)$$

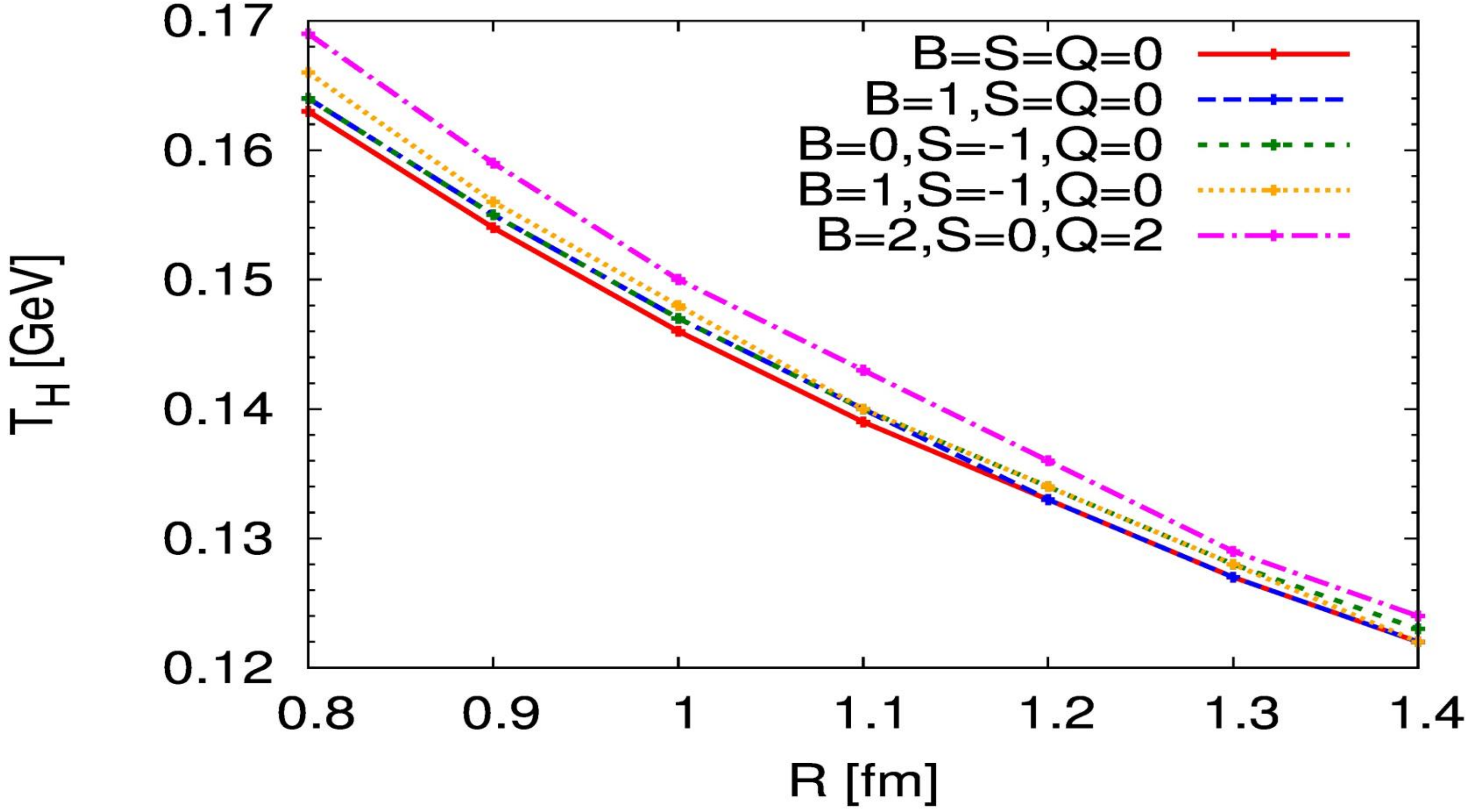
$$\times\, p_{cm}(m, m_1, m_2)^2\, \delta^{(3)}\left(\vec{C} - \vec{C}_1 - \vec{C}_2\right)$$

- Total decay width of Hagedorn state by application of the principle of detailed balance

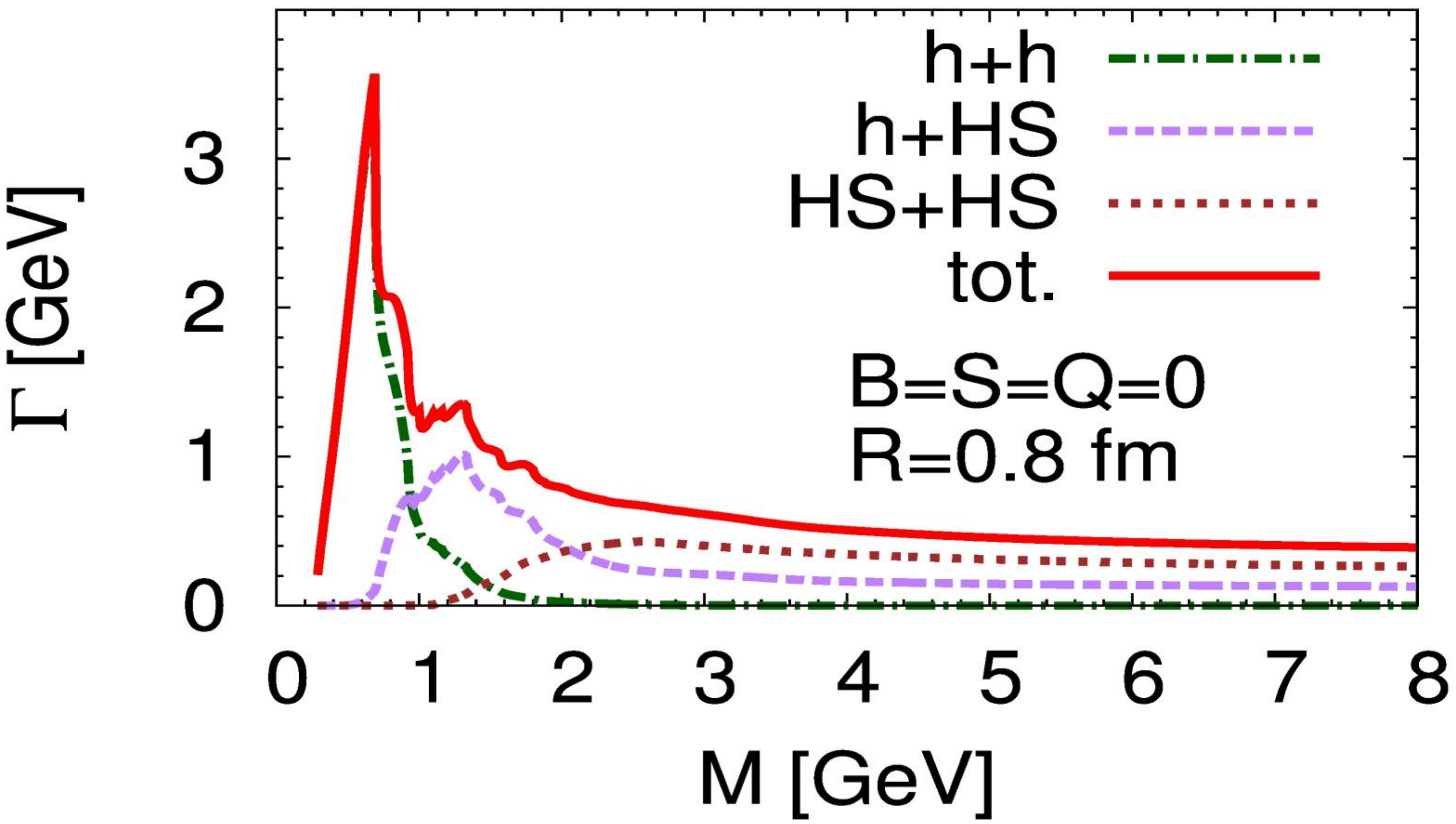
Hagedorn spectra

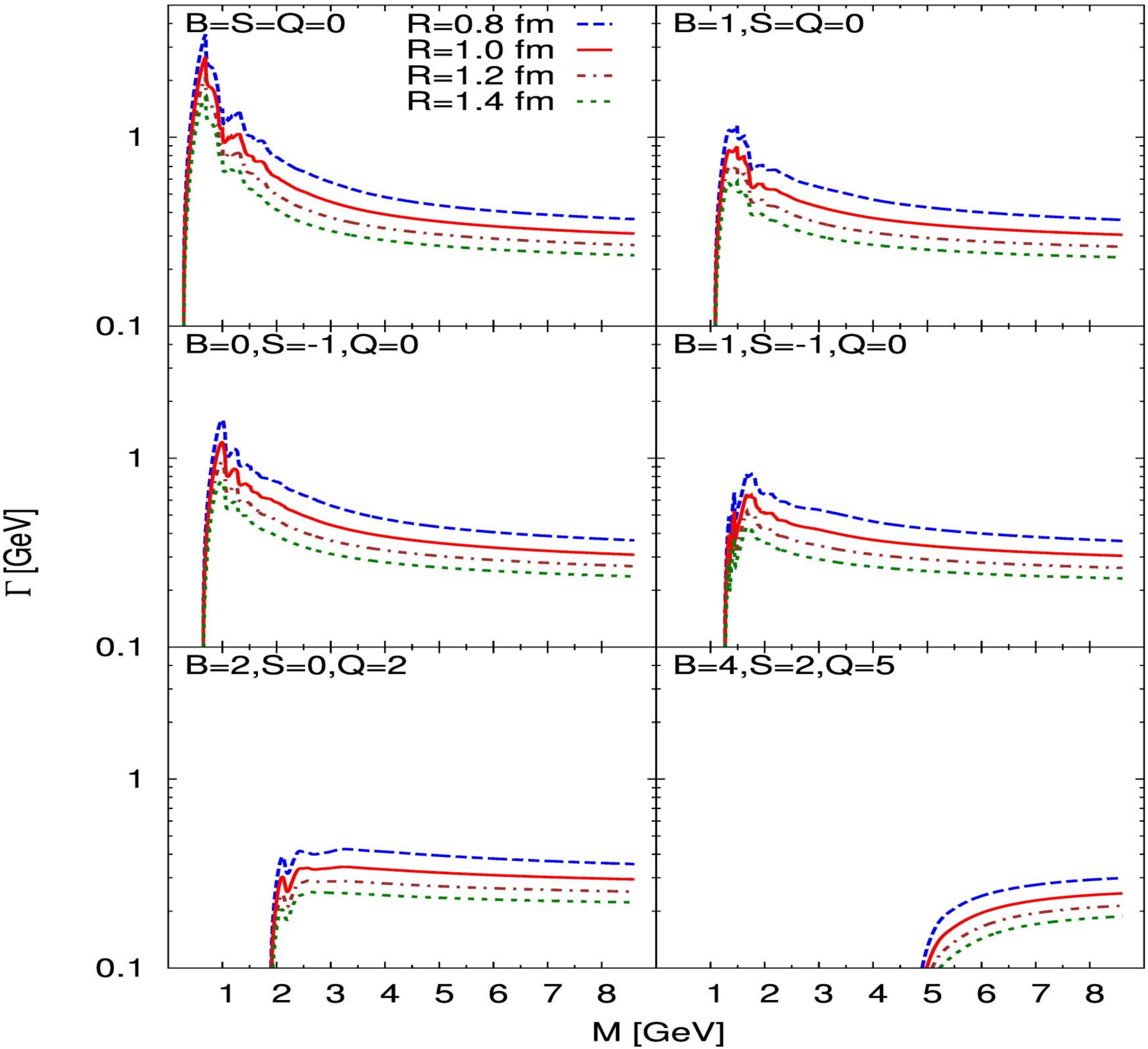


Hagedorn Temperature Radius Dependence



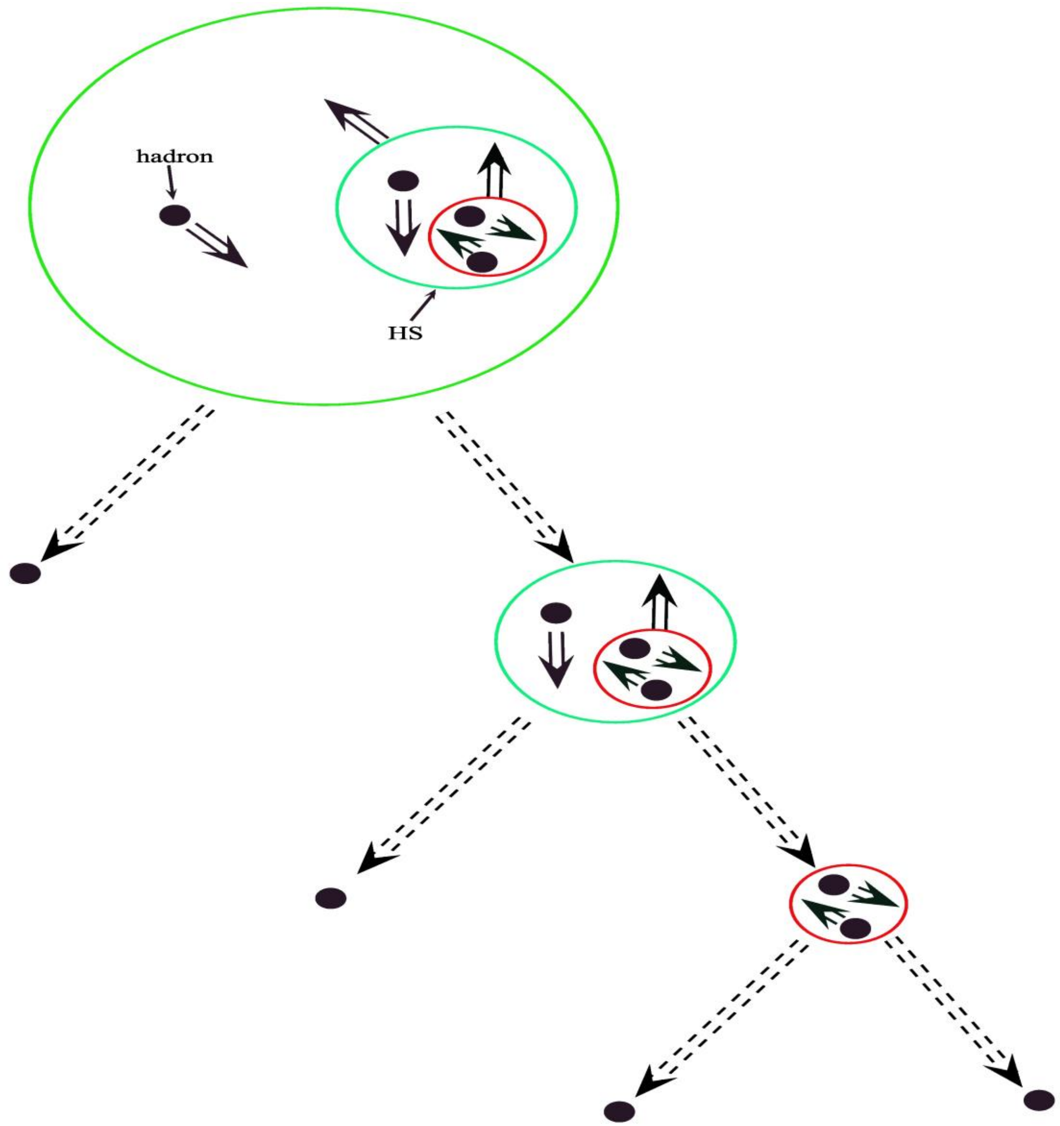
Hagedorn state decay widths



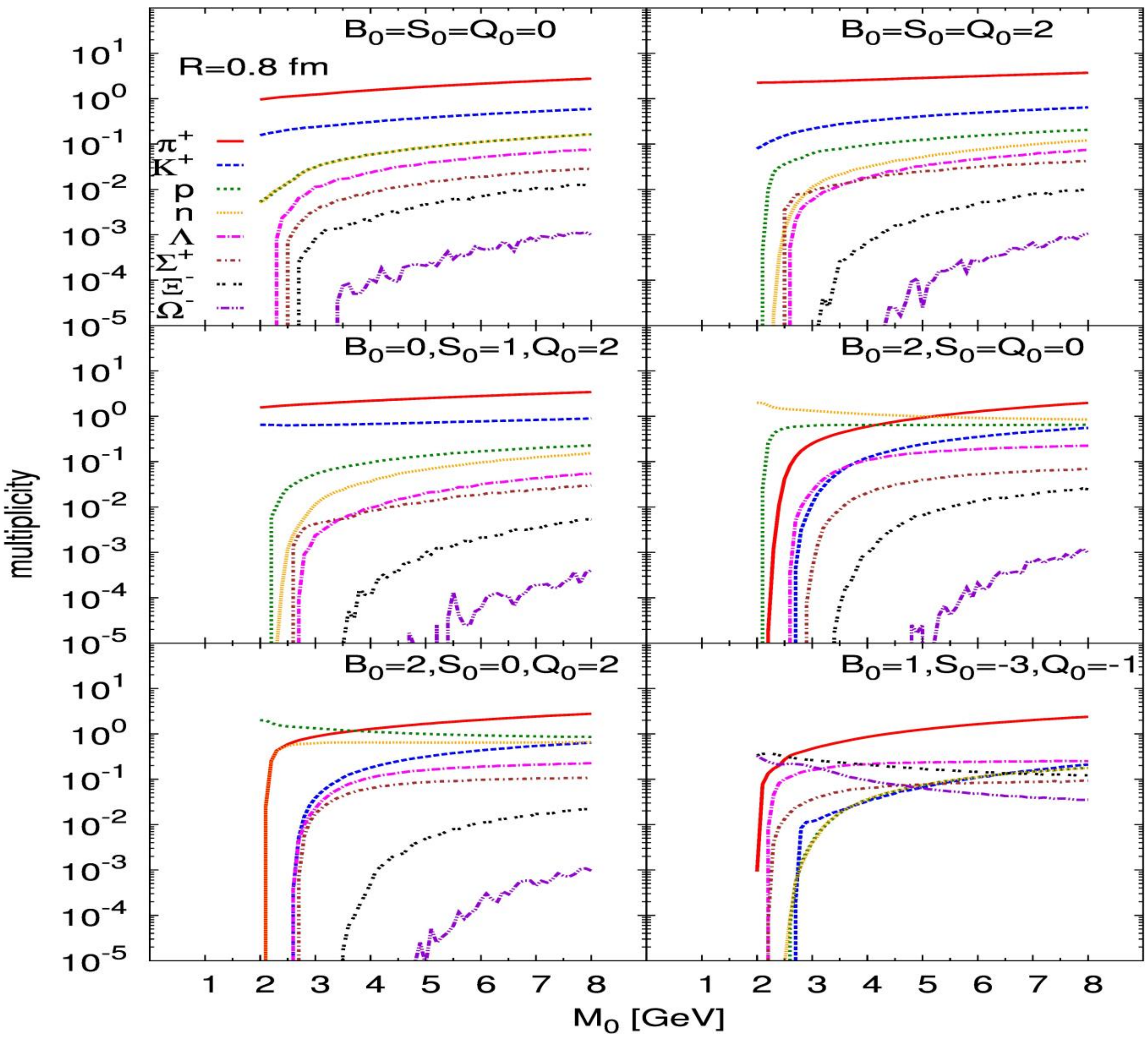


B=S=Q=0
R=0.8 fm
R=1.0 fm
R=1.2 fm
R=1.4 fm
B=1,S=Q=0
B=0,S=-1,Q=0
B=1,S=-1,Q=0
B=2,S=0,Q=2
B=4,S=2,Q=5
Γ [GeV]
M [GeV]
0.1
1
1 2 3 4 5 6 7 8

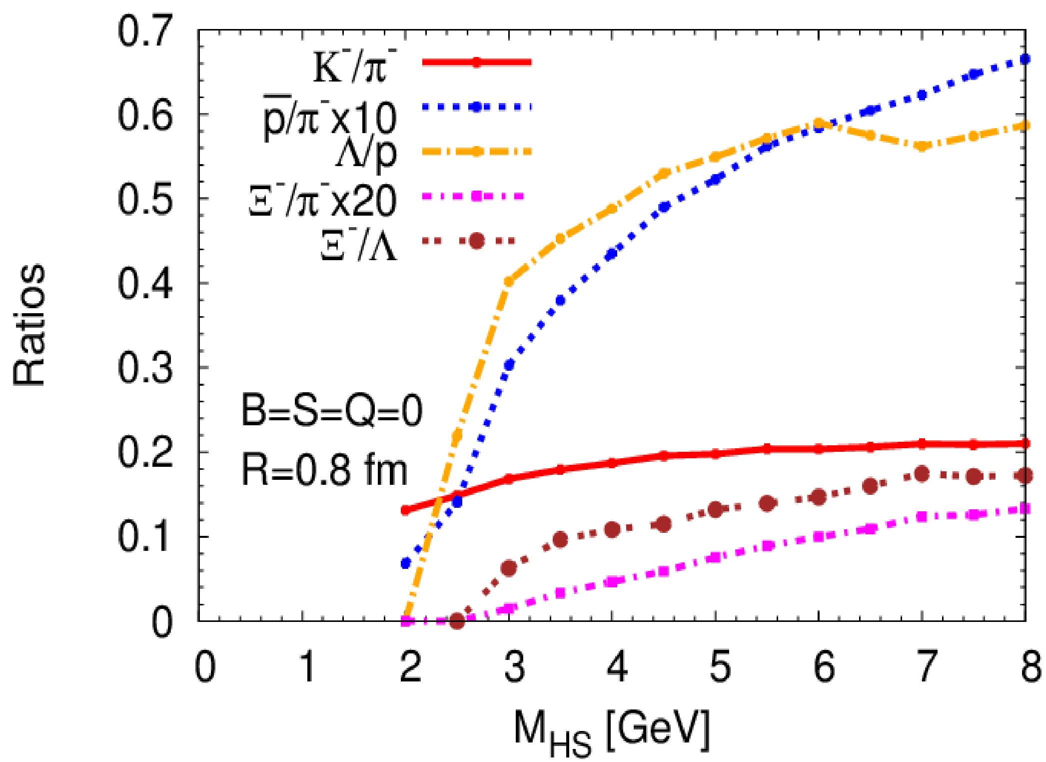
multiple Hagedorn state decay chain



Hadronic multiplicities after Hagedorn state cascading (incl. feeddown)



Hadronic ratios from Hagedorn state cascading decay



	p-p	Pb-Pb	4 GeV	8 GeV
K^-/π^-	0.123(14)	0.149(16)	0.187	0.210
$\overline{p}/\pi^-$	0.053(6)	0.045(5)	0.043	0.066
Λ/π^-	0.032(4)	0.036(5)	0.021	0.038
$\Lambda/\overline{p}$	0.608(88)	0.78(12)	0.494	0.579
Ξ^-/π^-	0.003(1)	0.0050(6)	0.0023	0.0066
$\Omega^-/\pi^- \cdot 10^{-3}$	-	0.87(17)	0.086	0.560

ALICE at LHC Ratios:

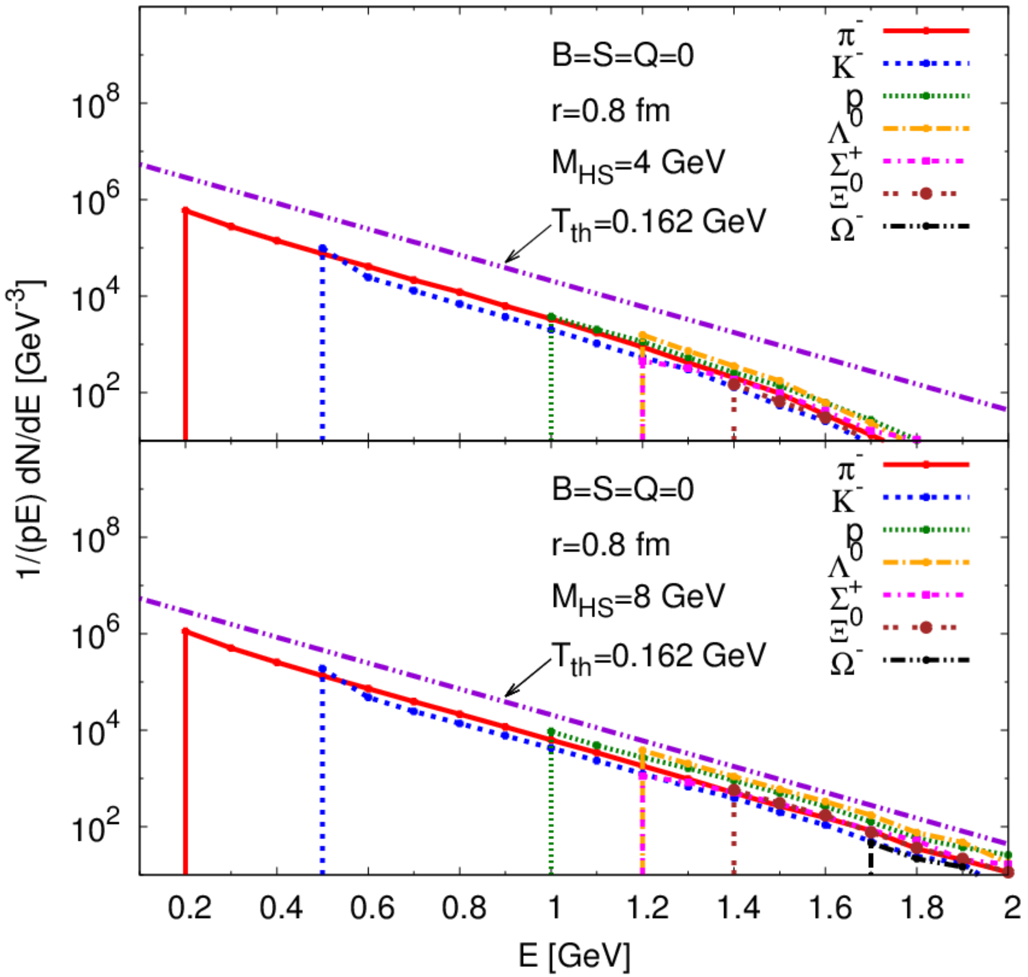
p-p @ 0.9 TeV
Pb-Pb @ 2.76 TeV

K. Aamodt et al. Eur. Phys J. C 71
B. Abelev et al. Phys Rev. C. 88
B. Abelev et al. Phys. Lett. B 728

Single HS cascading decay: Spectra - look thermal

Thermal temperature equals Hagedorn temperature, independent of:

Initial Hagedorn state mass
Hagedorn state radius
Hagedorn state charges

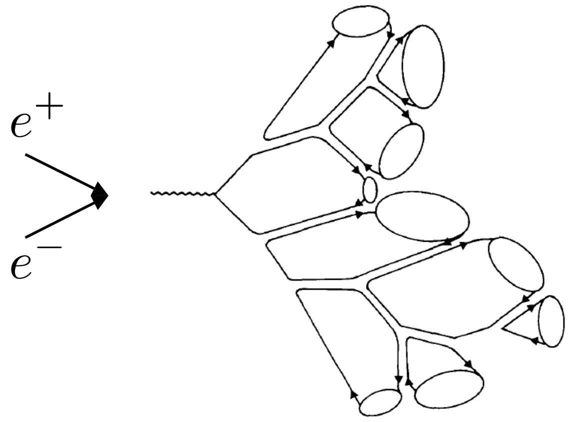


M.Beitel, K: Gallmeister, CG, PRC 90 (2014) 045203

Colorless Heavy Objects

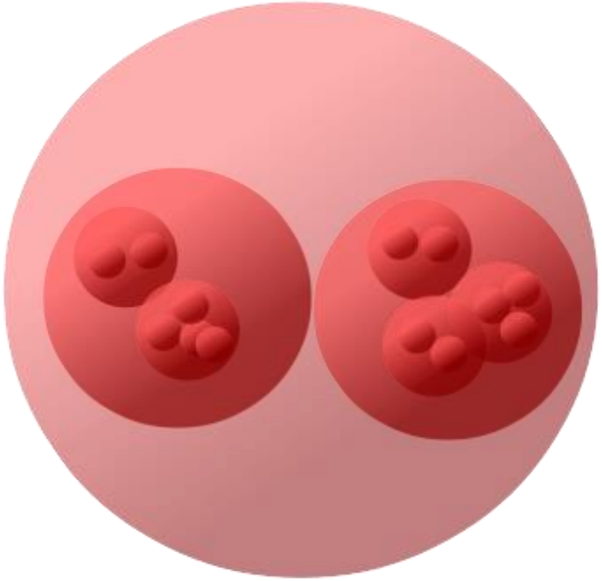
- Cluster (HERWIG)

B. Webber, Nucl.Phys.B 238 (1984) 492



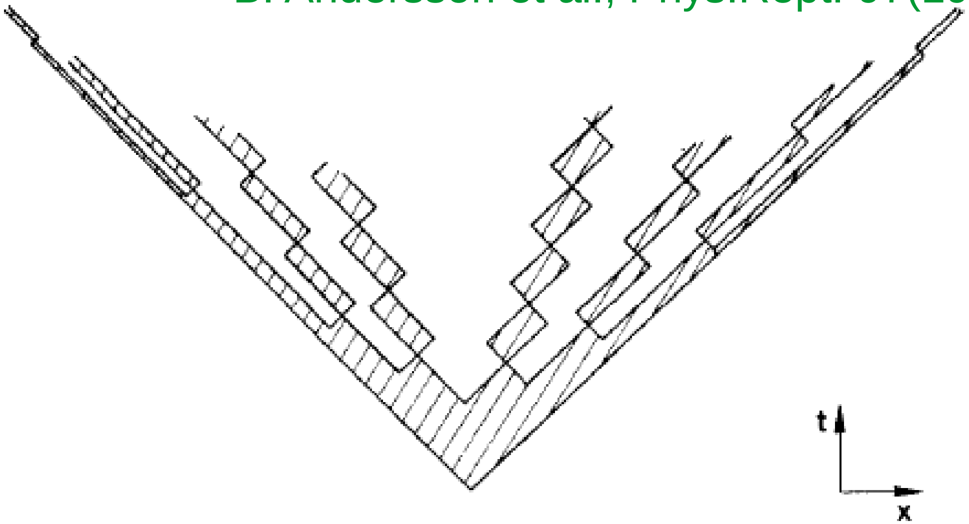
- Hagedorn states

R. Hagedorn, Nuovo Cim. Suppl. 3 (1965) 147



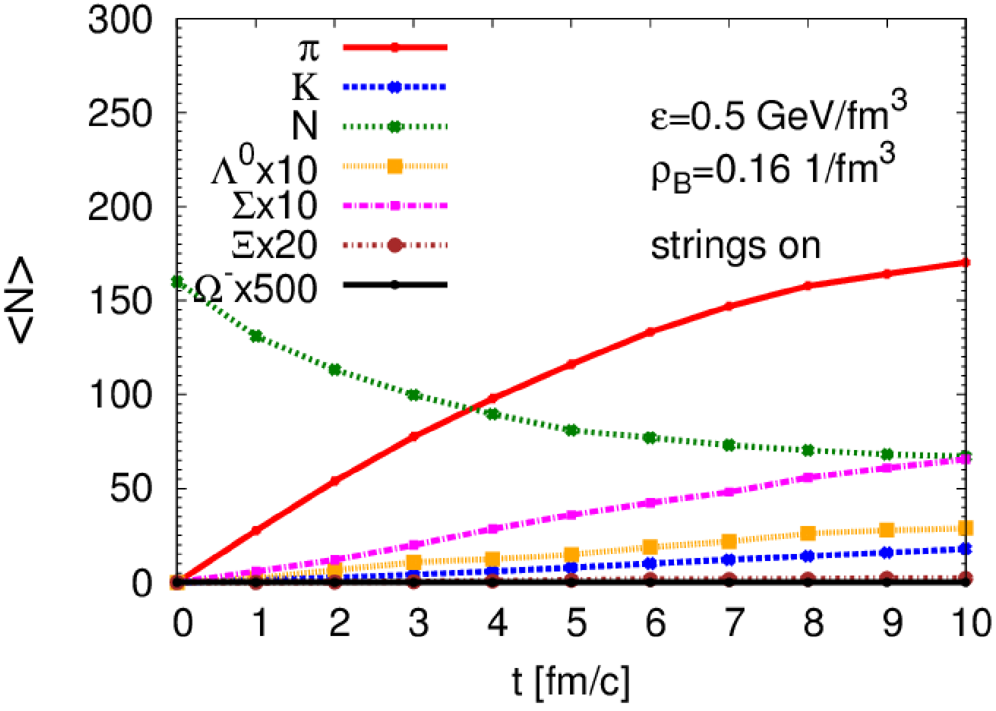
- Strings (Lund)

B. Andersson et al., Phys.Rept. 97(1983) 31

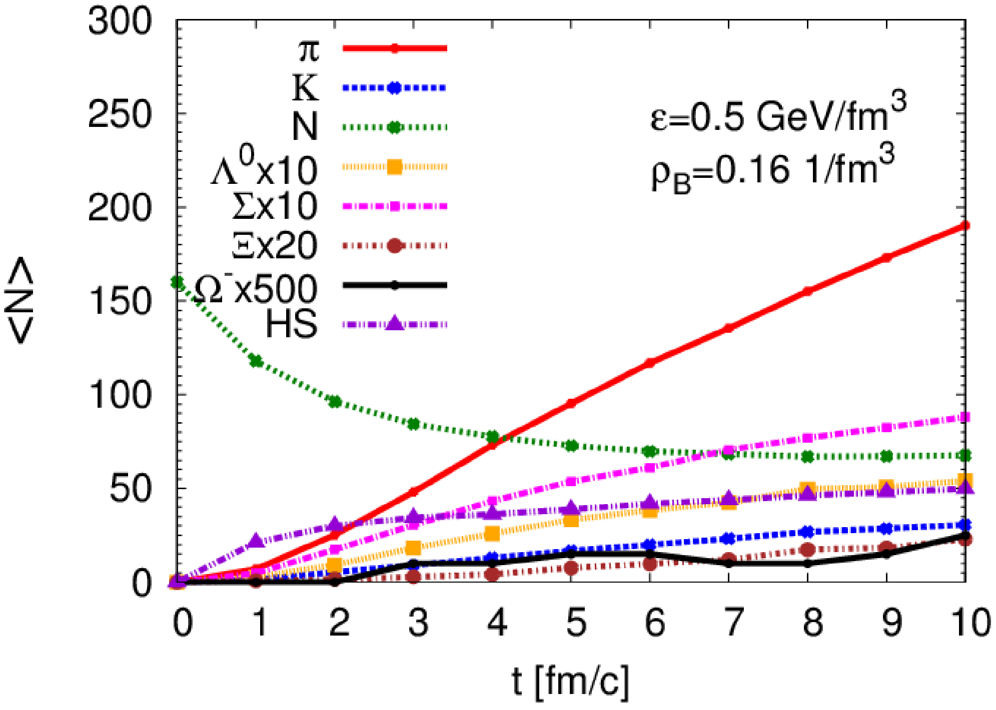


allow for
decay & recombination !

Box simulation: nucleon-nucleon initialization

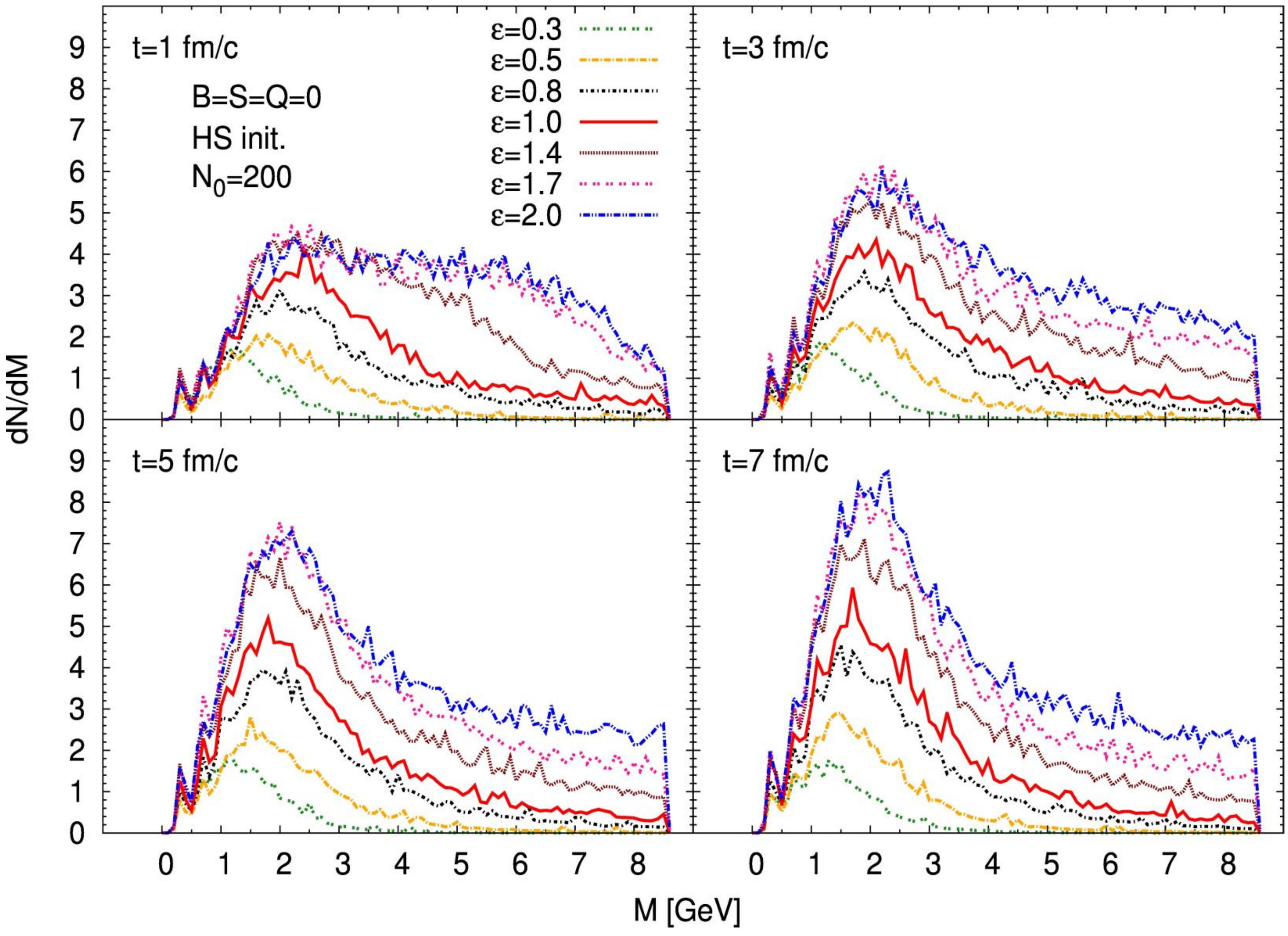


•Original UrQMD

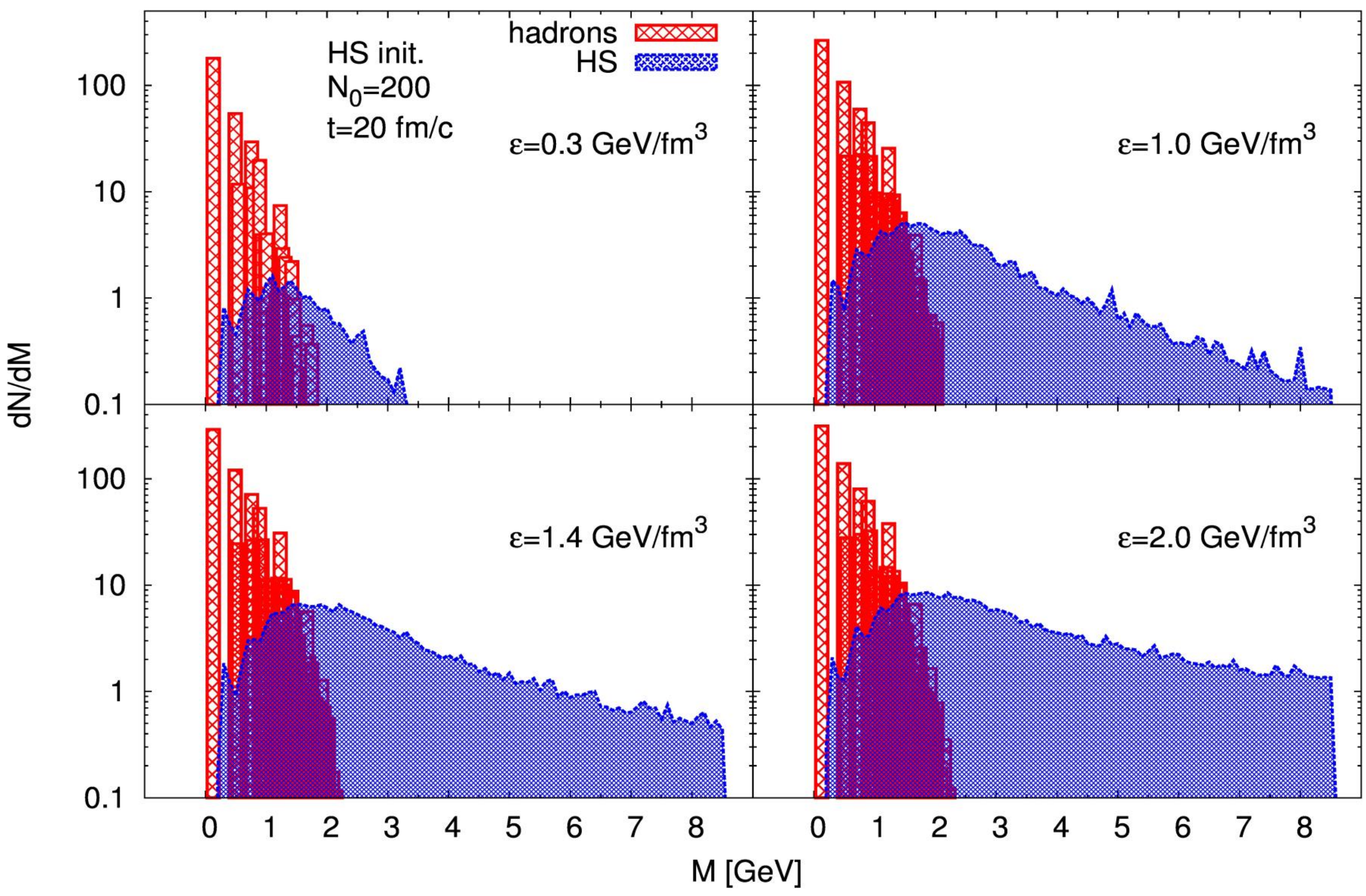


•.HS

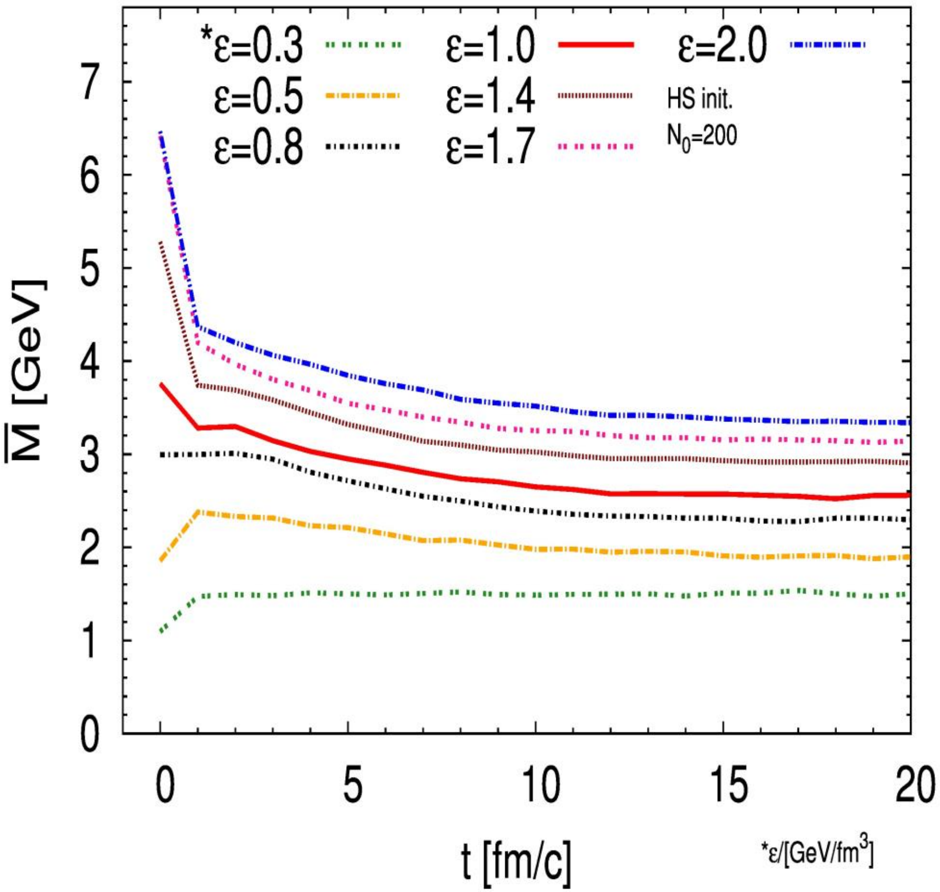
Box simulation: HS initialization … and its mass distribution in time

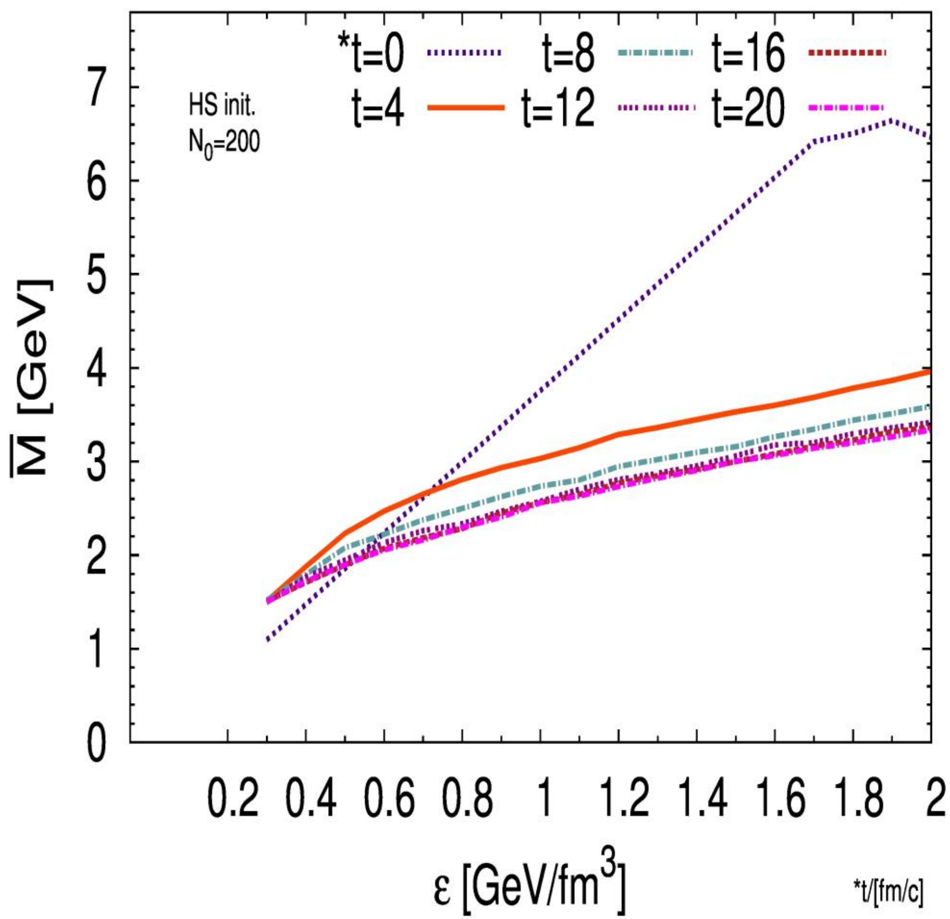


mass distribution of HS and Hadrons

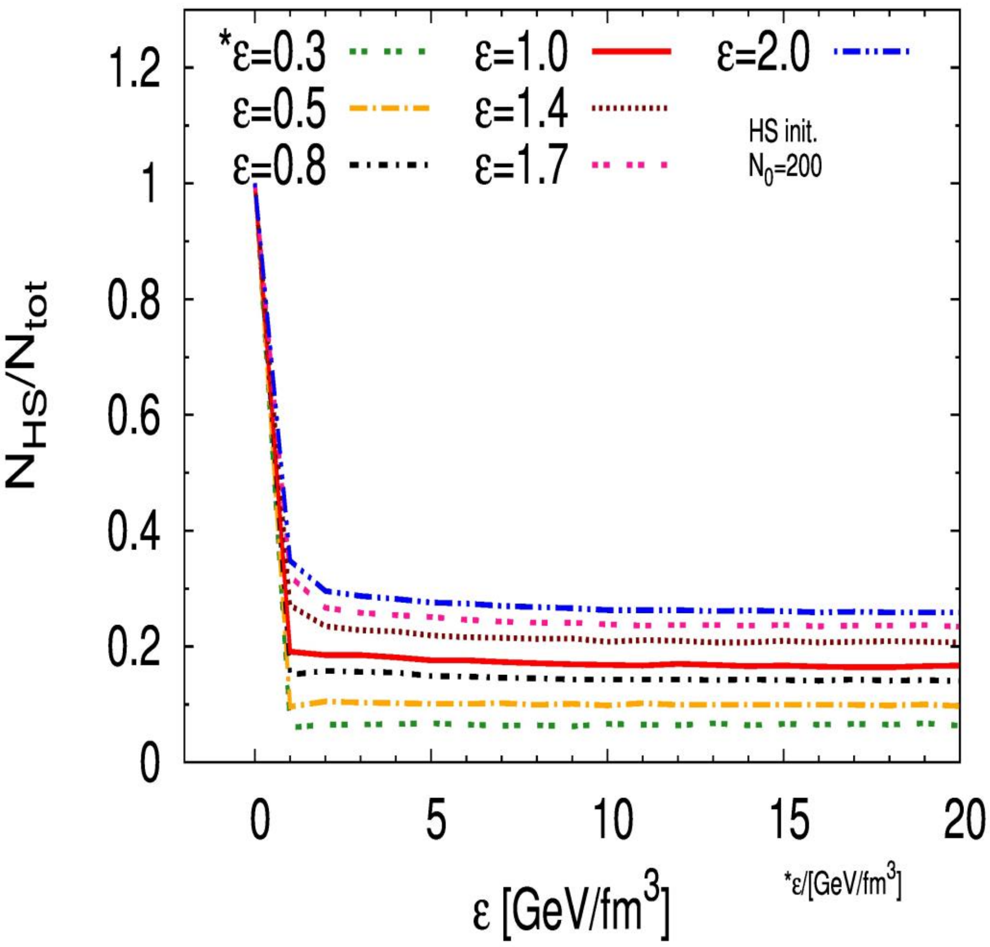


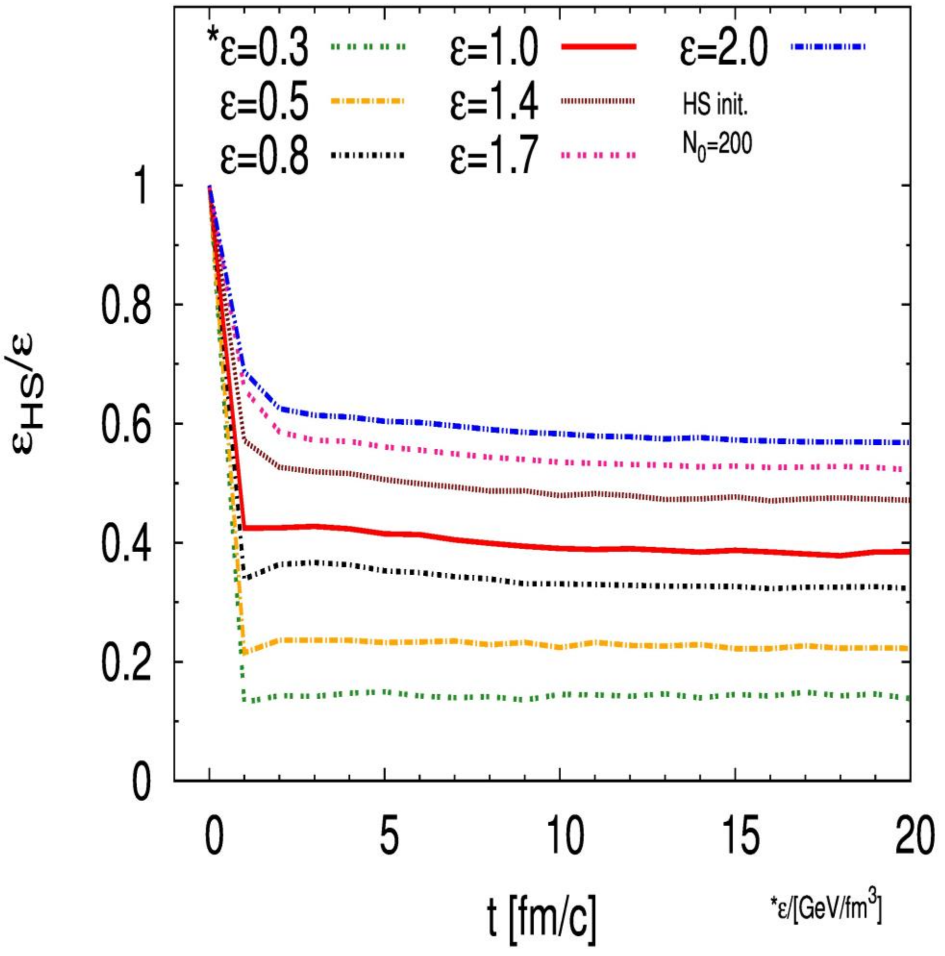
mean mass of HS



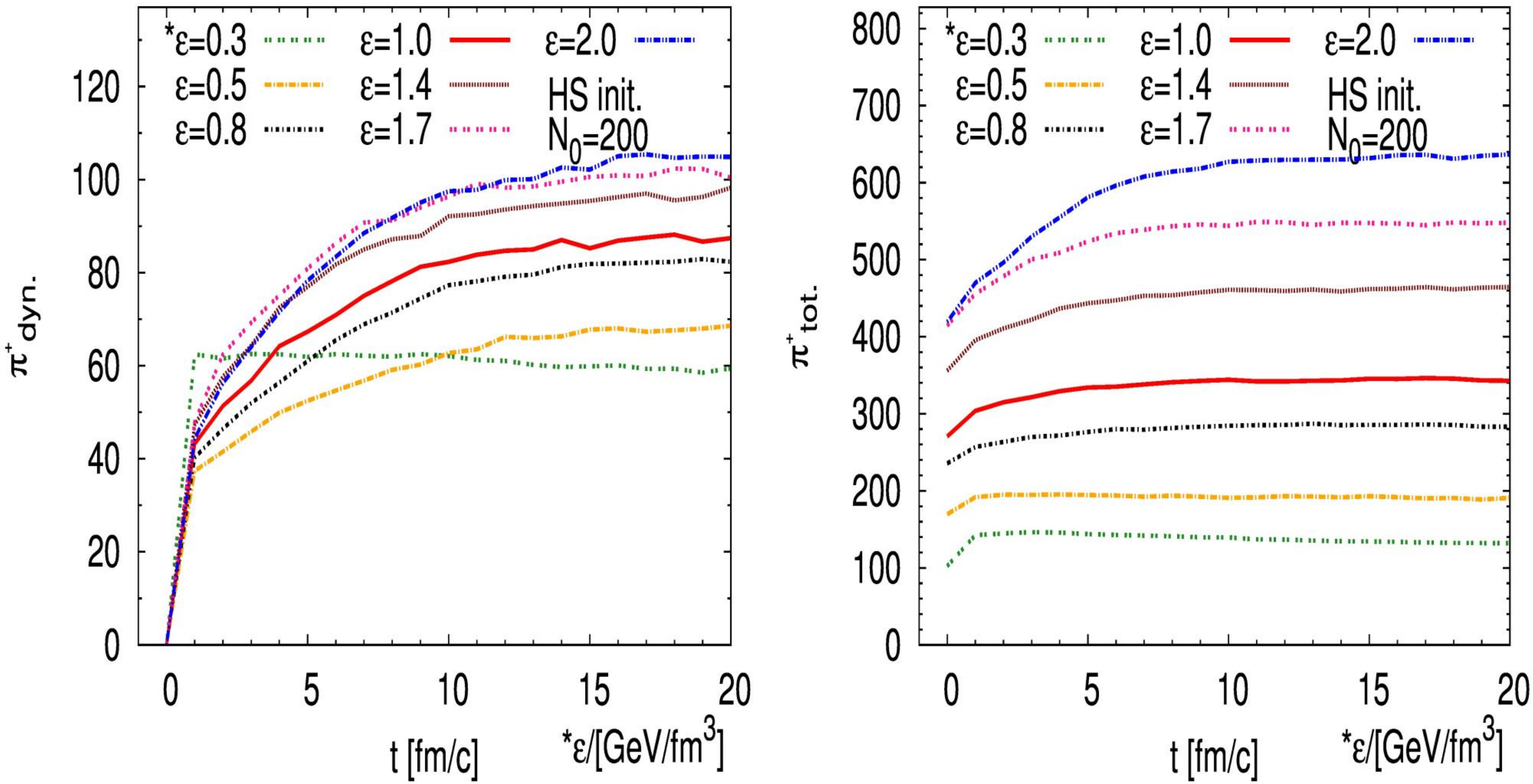


Fractions of HS

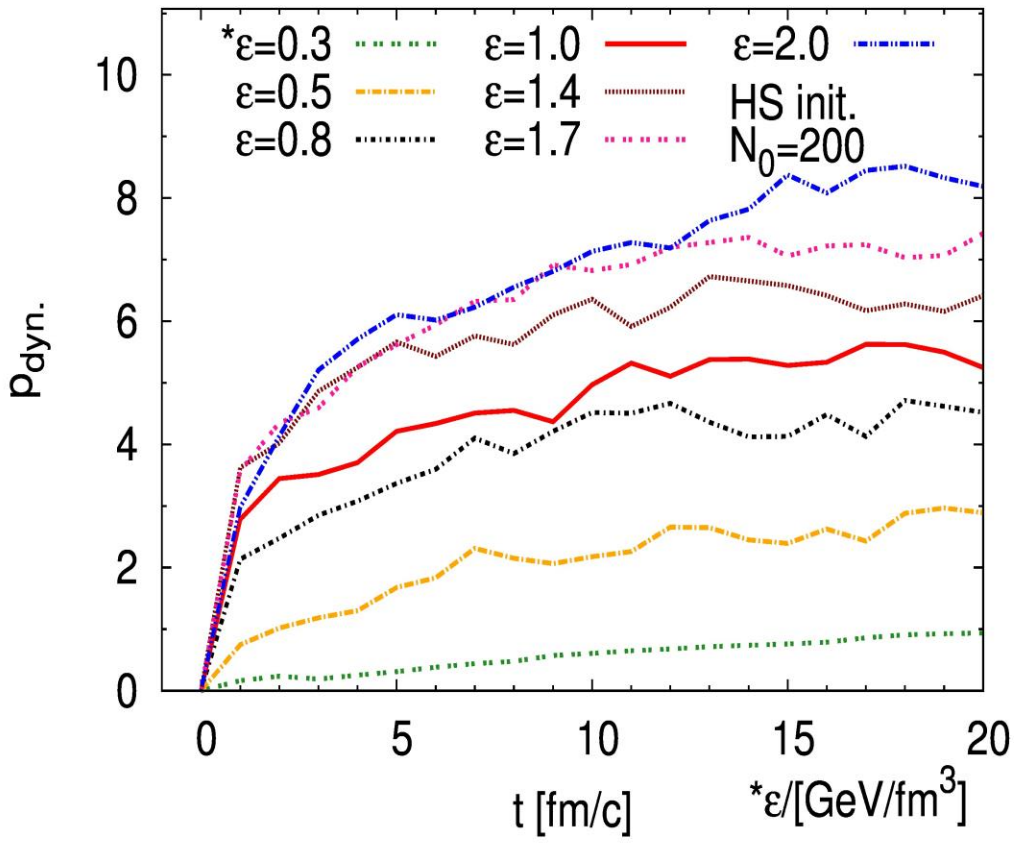


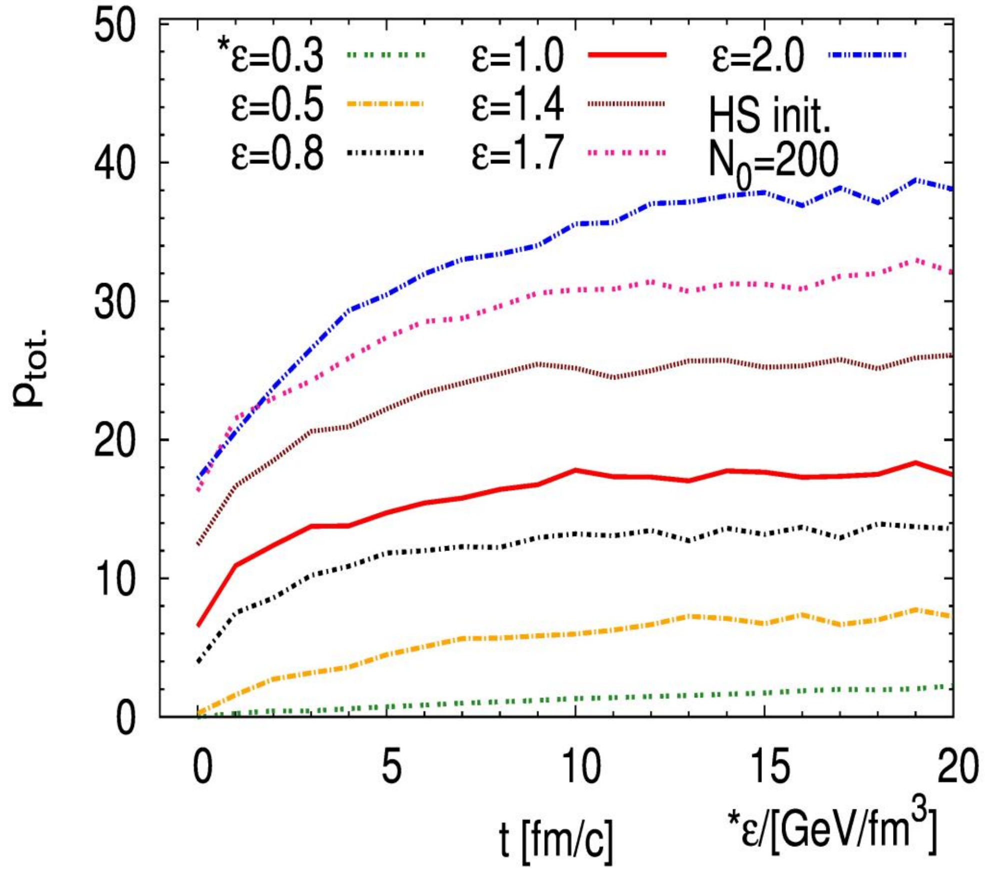


time evolution of pions

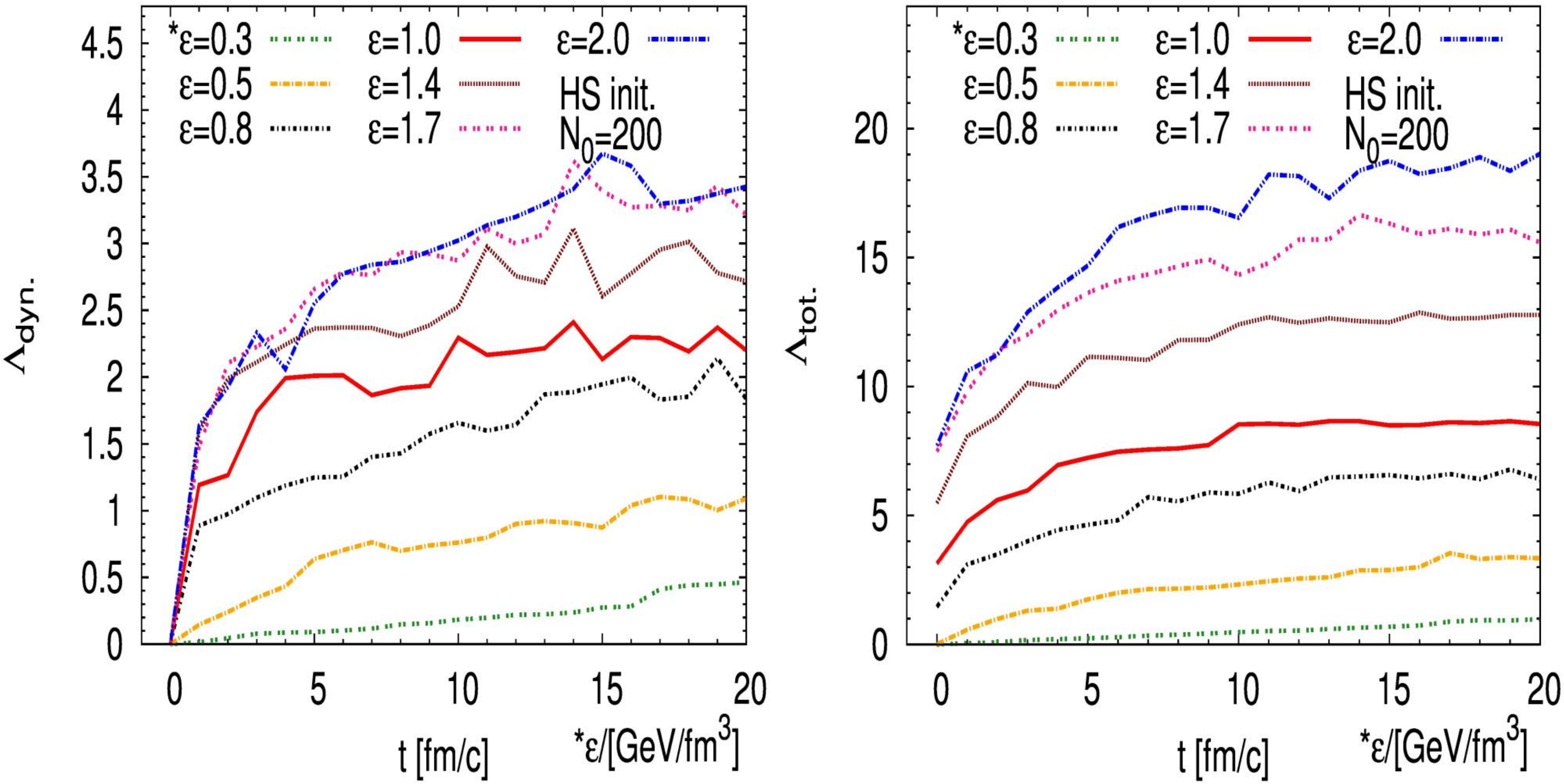


time evolution of protons

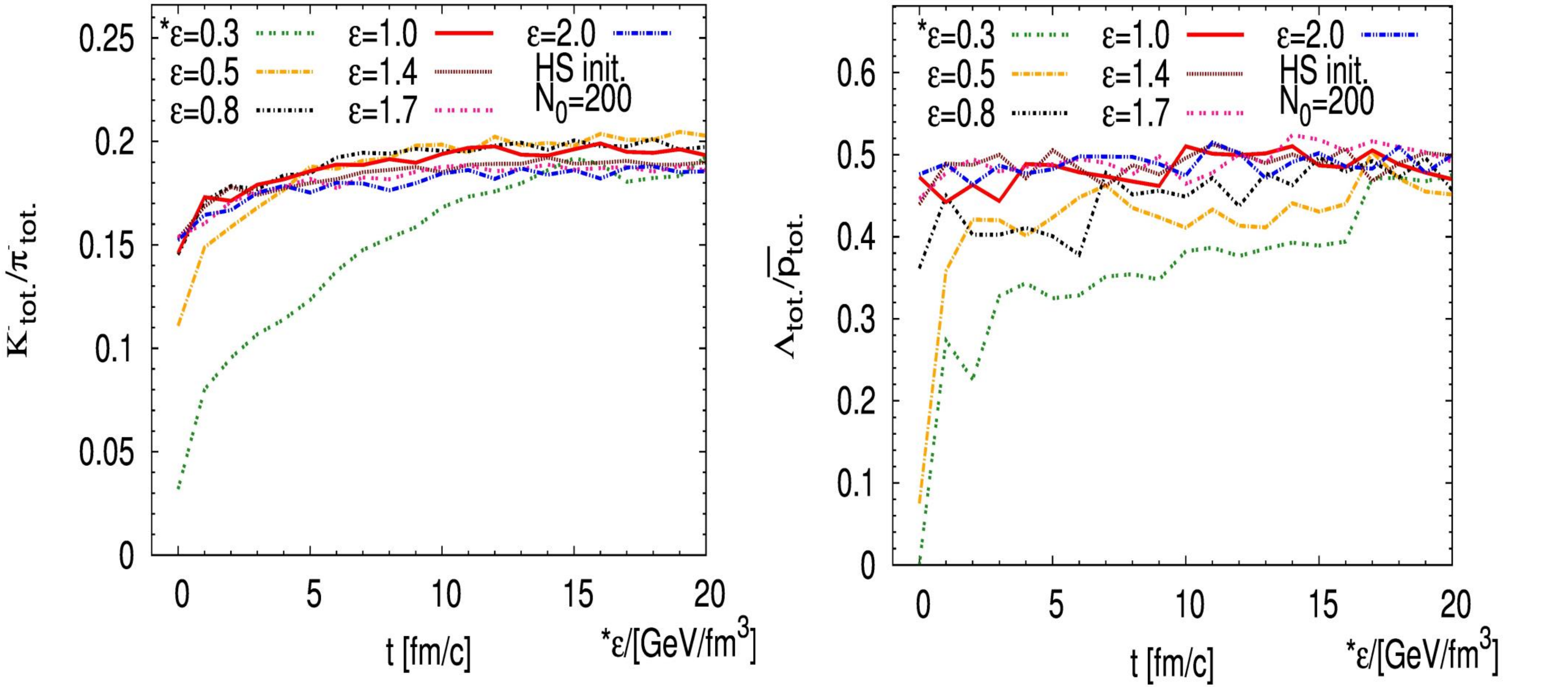




time evolution of Lambdas



time evolution of multiplicity Ratios



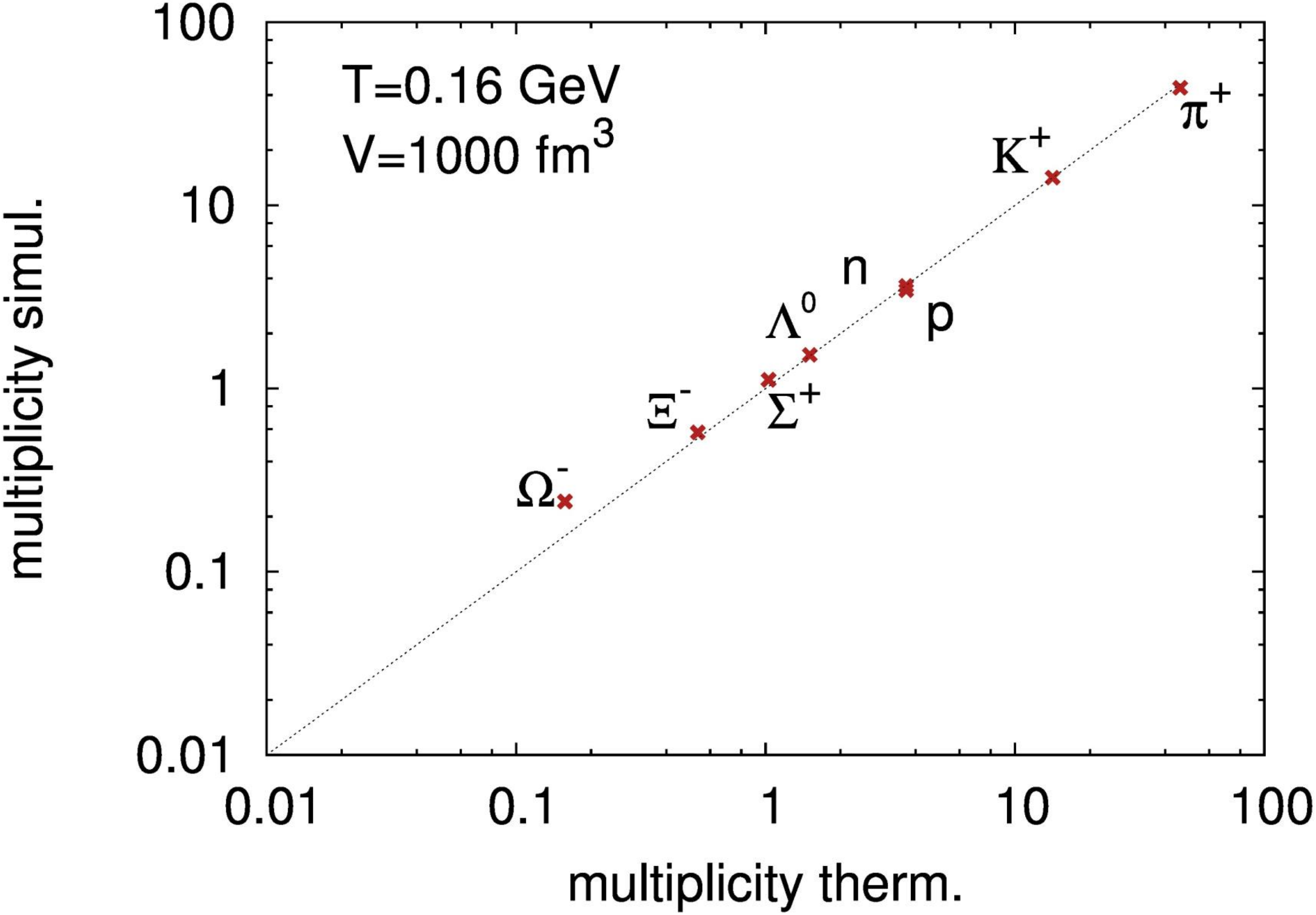
comparison to experiment

	p-p	Pb-Pb	0.3	0.5	0.8	1.0	1.5	2.0[a]
K^-/π^-	0.123(14)	0.149(16)	0.192	0.202	0.197	0.193	0.188	0.185
$\bar{p}/\pi^-$	0.053(6)	0.045(5)	0.015	0.038	0.049	0.052	0.056	0.060
Λ/π^-	0.032(4)	0.036(5)	0.007	0.017	0.022	0.024	0.028	0.029
$\Lambda/\bar{p}$	0.608(88)	0.78(12)	0.475	0.451	0.456	0.469	0.503	0.499
$\Xi^-/\pi^- * 10^3$	3.000(1)	5.000(6)	1.565	3.735	6.492	5.769	7.177	7.106
$\Omega^-/\pi^- * 10^3$	-	0.87(17)	0.137	0.612	0.815	0.823	1.191	0.994

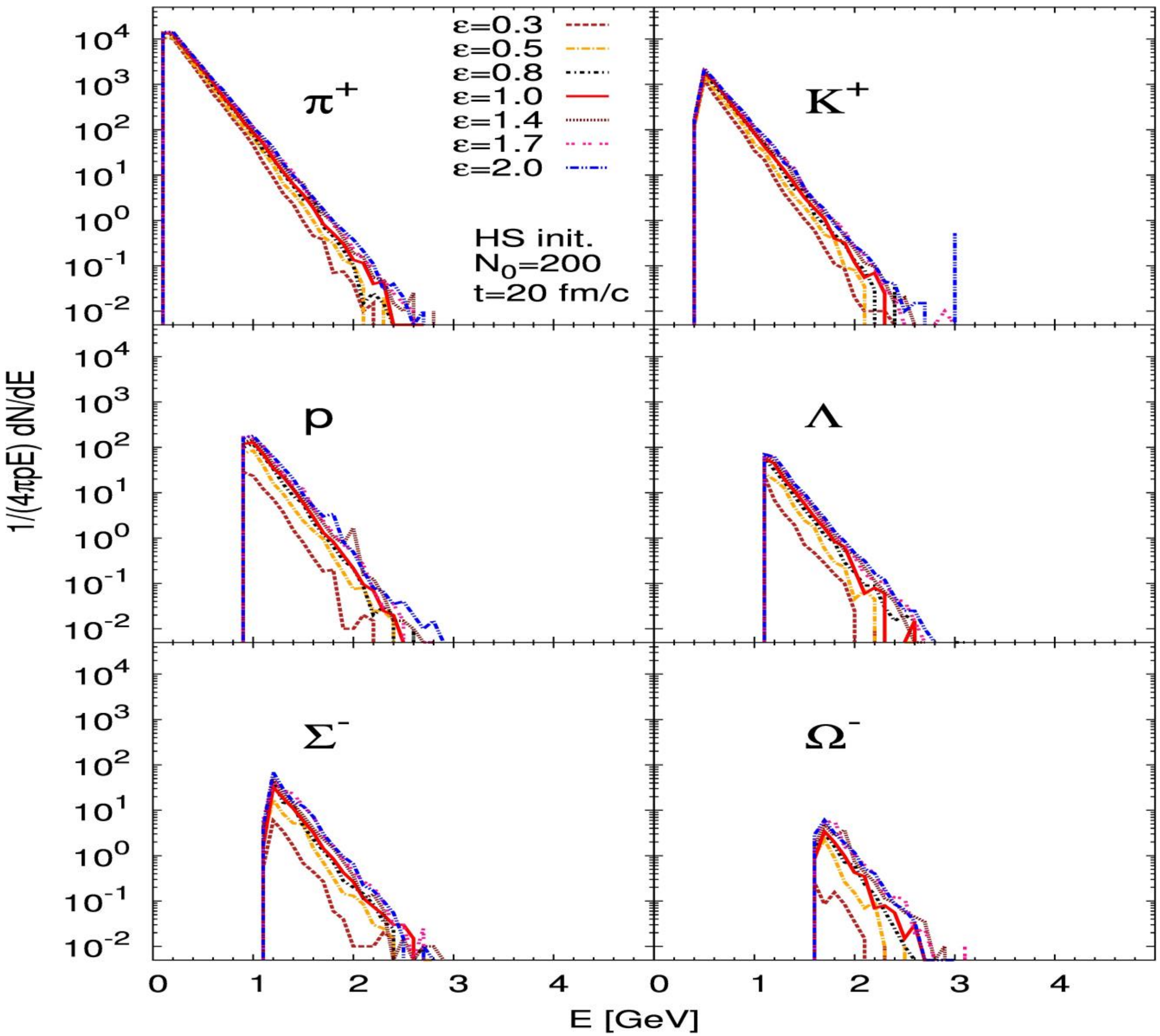
[a] [$\mathrm{GeV/fm^3}$]

Table 4.3: Experimental multiplicity ratios for p-p at $\sqrt{s_{NN}} = 0.9\,\mathrm{TeV}$ [8] and Pb-Pb at $\sqrt{s_{NN}} = 2.76\,\mathrm{TeV}$ [9, 10, 11] from ALICE and LHC. Values in the brackets denote the error in the last digits. Same theoretical ratios in a with *HS* initialized calculation after $t = 20\,\mathrm{fm}/c$ are listed for some energy densities in range $\epsilon = 0.3 - 2.0\,\mathrm{GeV/fm^3}$.

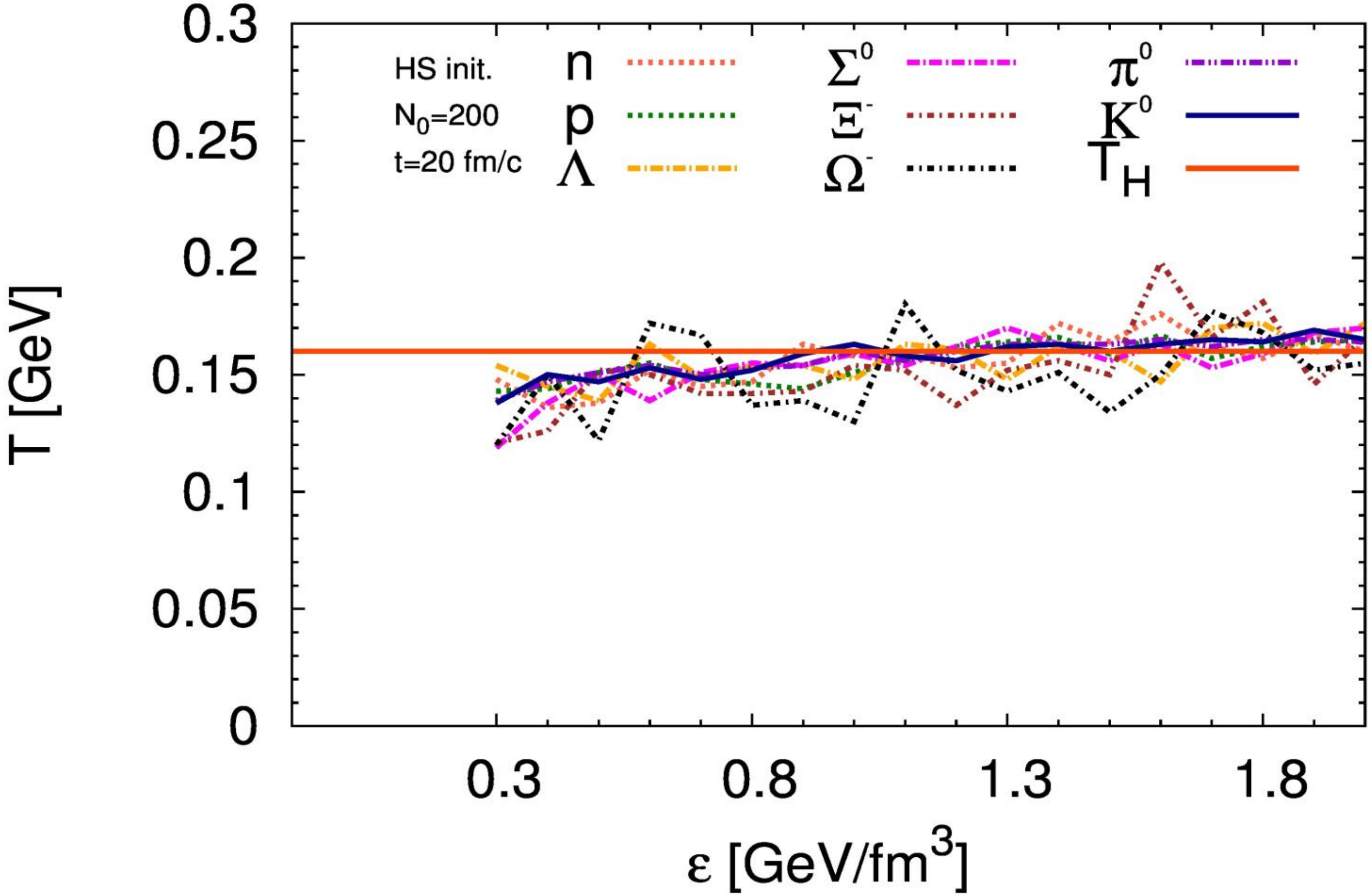
Comparison with Thermal Model



energy distributions of hadrons



Final Temperatures (slopes)



$T_{\rm H}$

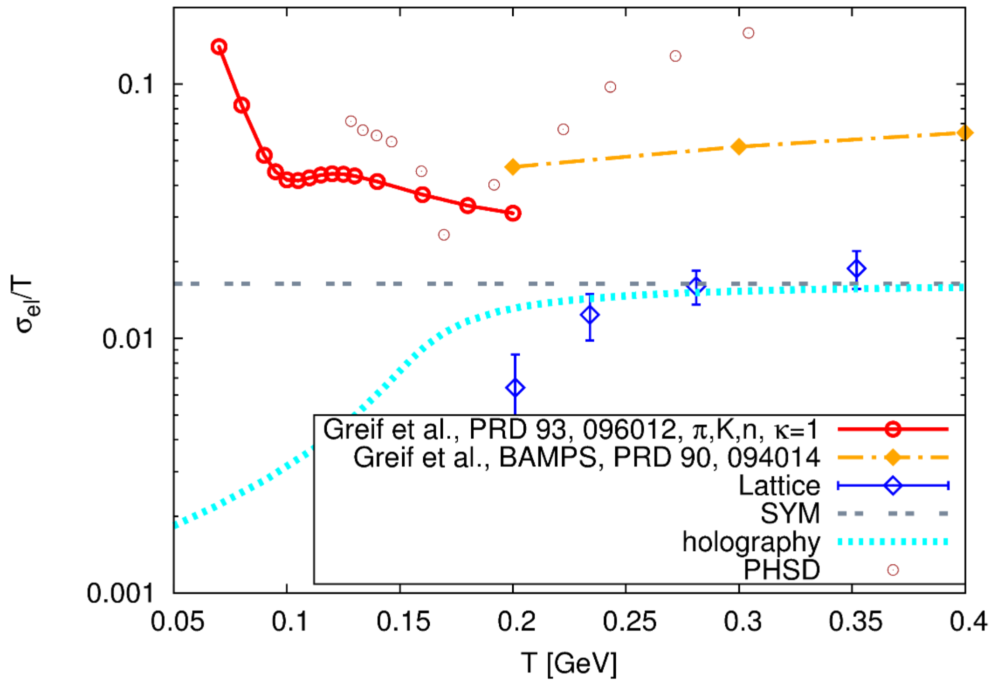
$= 162\,\mathrm{MeV}$

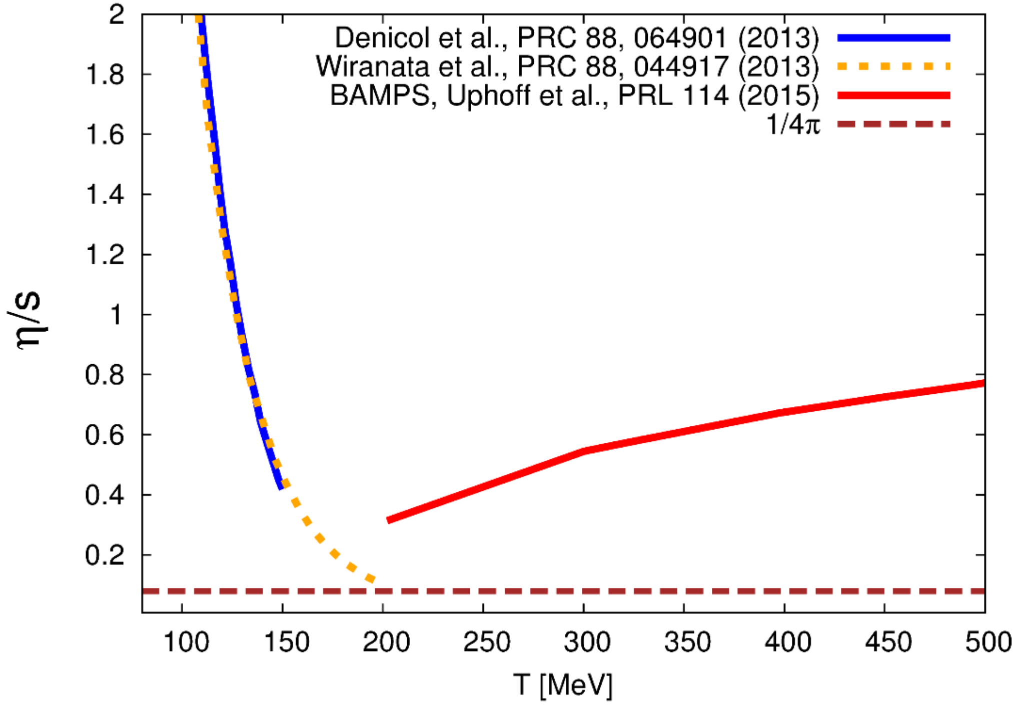
Summary & Outlook

- Hagedorn spectra derived from known hadronic spectral functions
- Energy spectra of decay products in Hagedorn state cascading simulations are thermal
- implementation into hadronic transport (URQMD, GiBUU)
- Regeneration of particles explains fast chem. equilibration
- explains the success of Statistical Hadronization Model
- future : … full 3+1 – dim. HIC simulation

  for RHIC BES, NICA, CBM
- fluctuations and cumulants

… transport coefficients



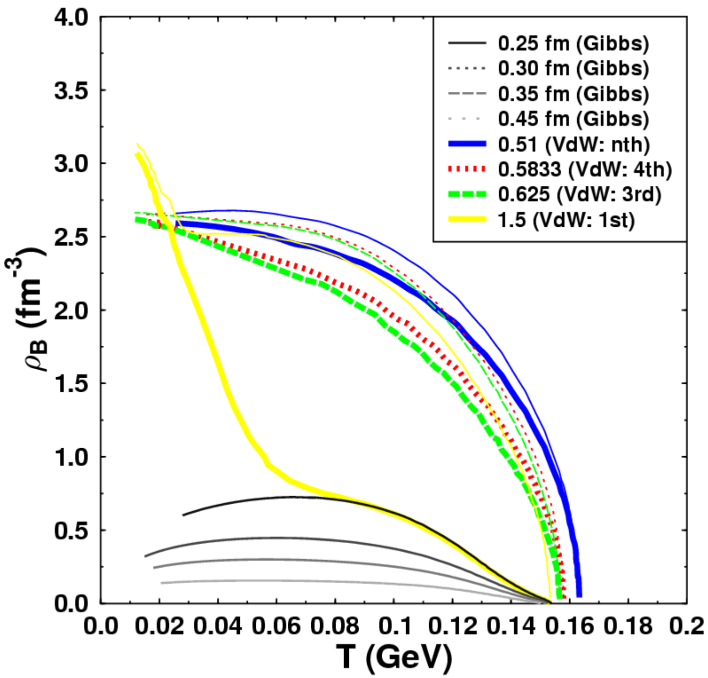


- electric conductivity
- shear viscosity

… J. Noronha-Hostler, J. Noronha, CG, PRL 103 (2009)

The order and shape of QGP phase transition

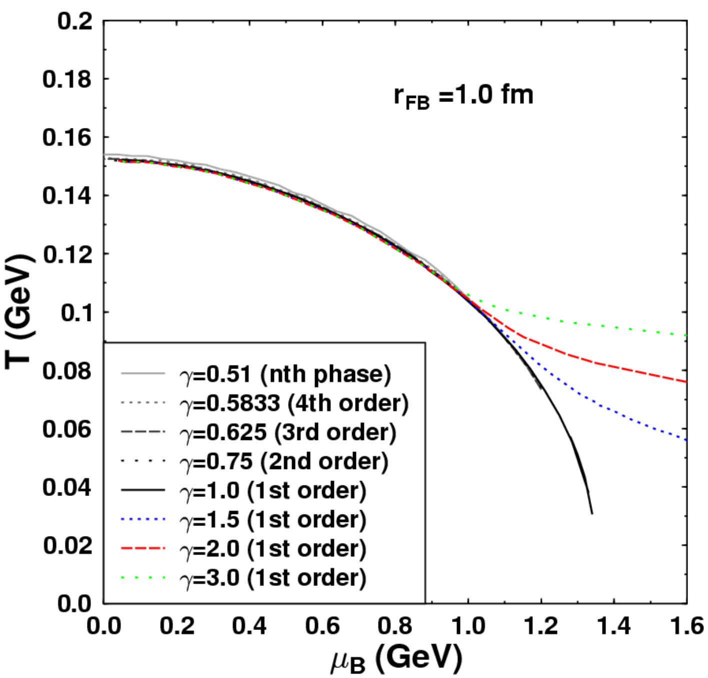
I.Zakout, CG and J. Schaffner-Bielich, NPA 781 (2007) 150, PRC78 (2008)

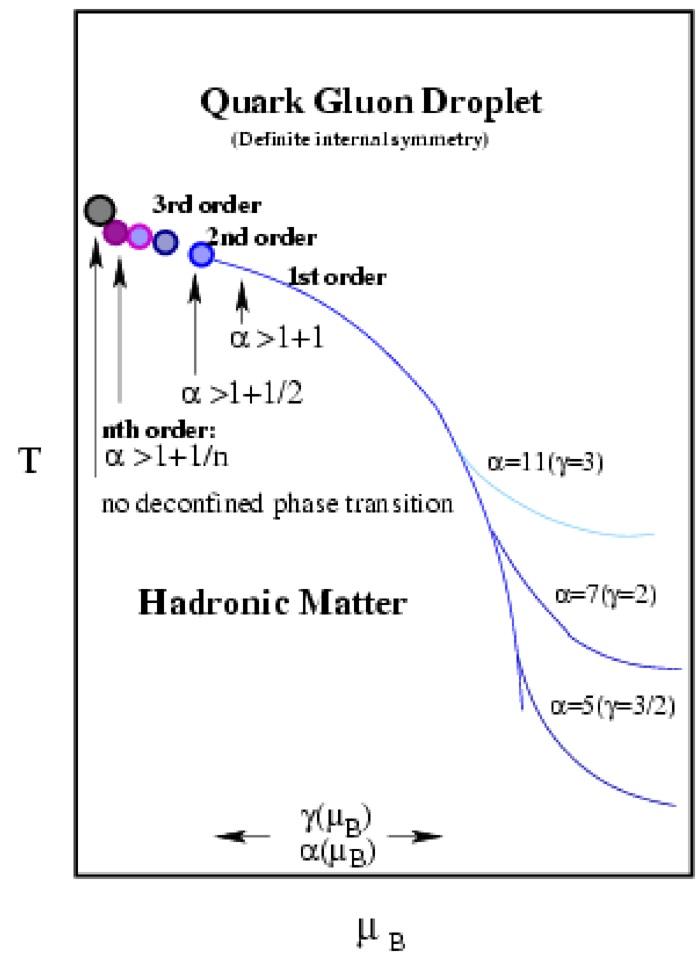


$$\}\ \gamma = \frac{\alpha+1}{4}$$

density of states:

$$\rho(m,v) \sim c\, m^{-(\alpha+2)}\, e^{\frac{m}{T_{H}[B]}}\, \delta(m-4Bv)$$





$$\alpha(\mu_B)$$

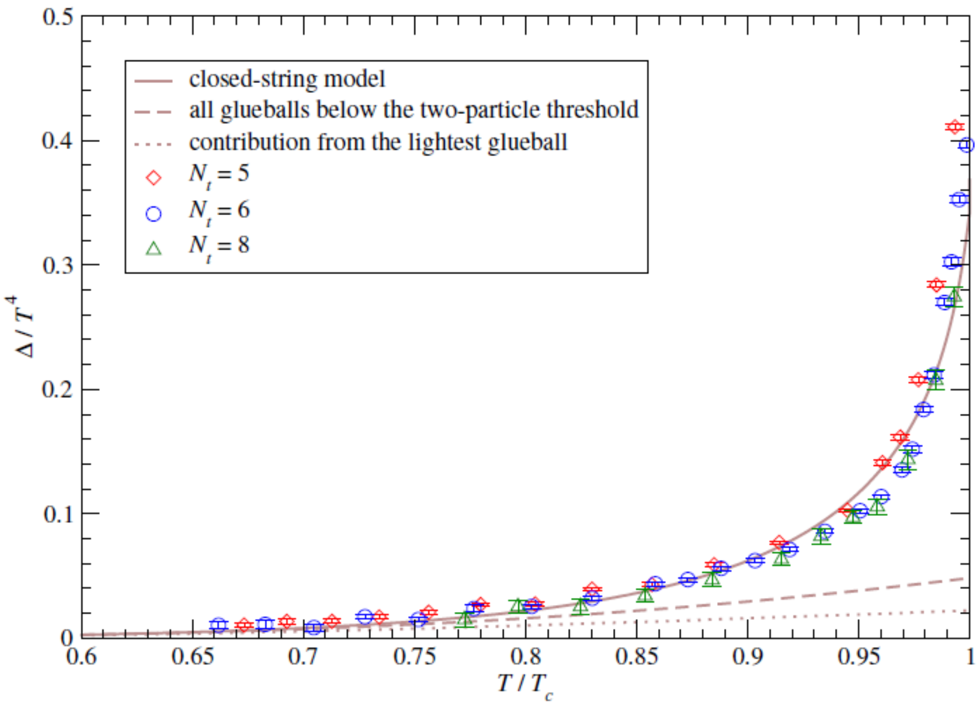
Hagedorn spectrum and thermodynamics of SU(2) and SU(3) Yang-Mills theories

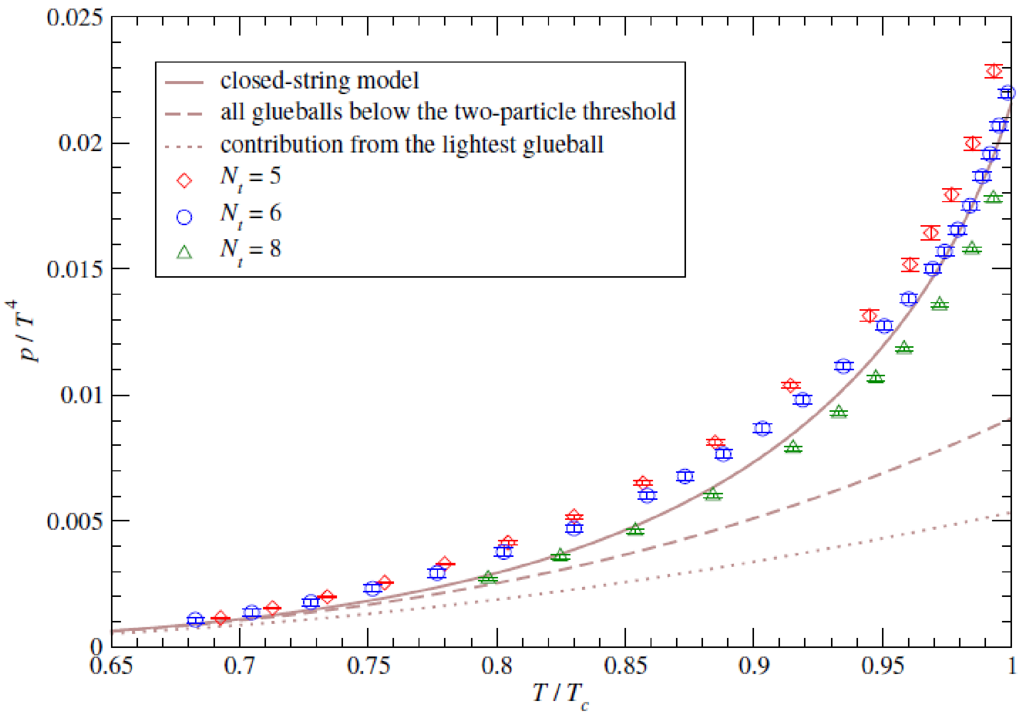
Michele Caselle, Alessandro Nada and Marco Panero arxive: 1505.01106

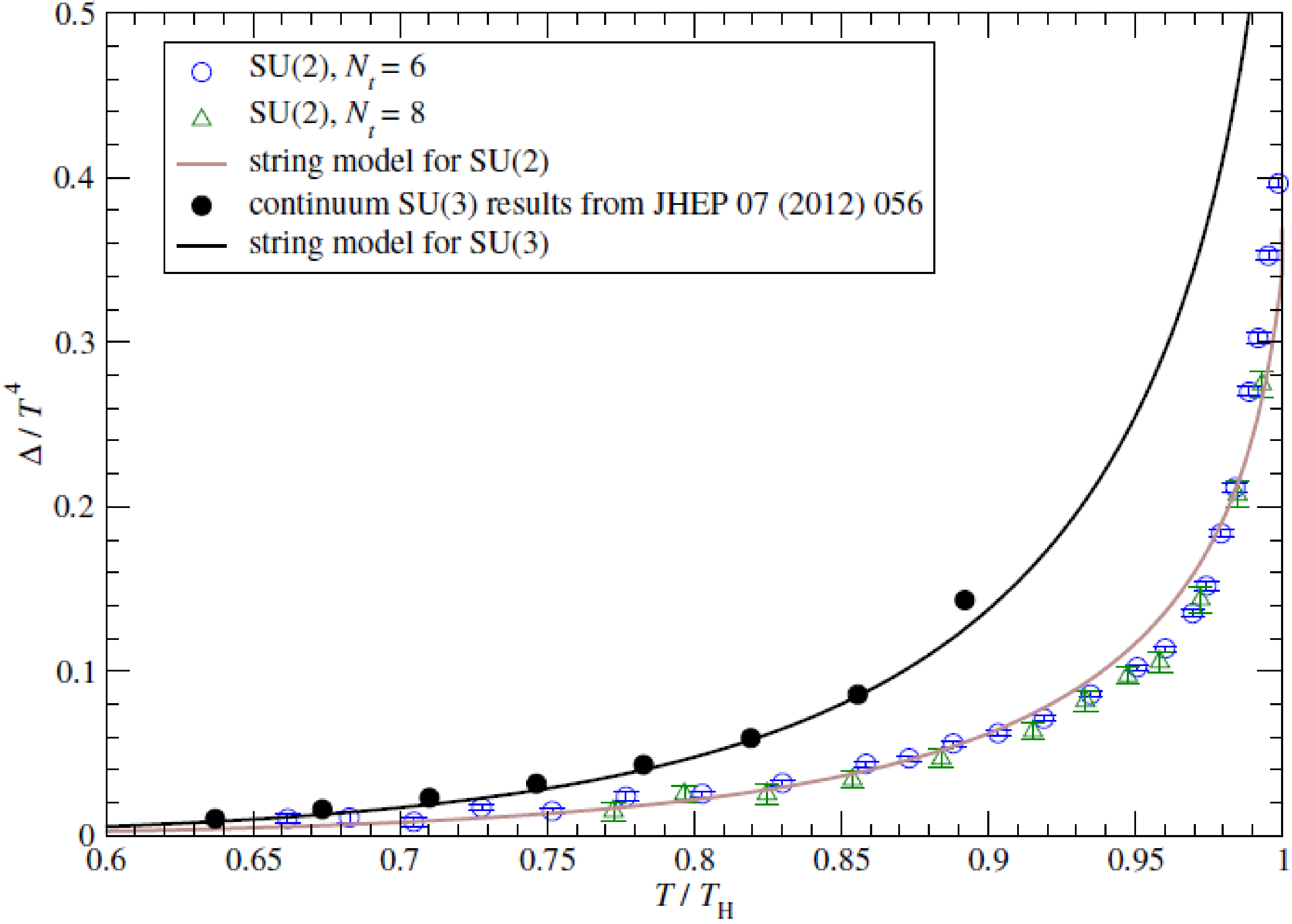
A high-precision lattice calculation of the equation of state in the confining phase of SU(2) and SU(3) Yang-Mills theory.

The results are described very well by a gas of massive, non-interacting glueballs, provided one assumes an exponentially growing Hagedorn spectrum.

SU(2) Yang-Mills



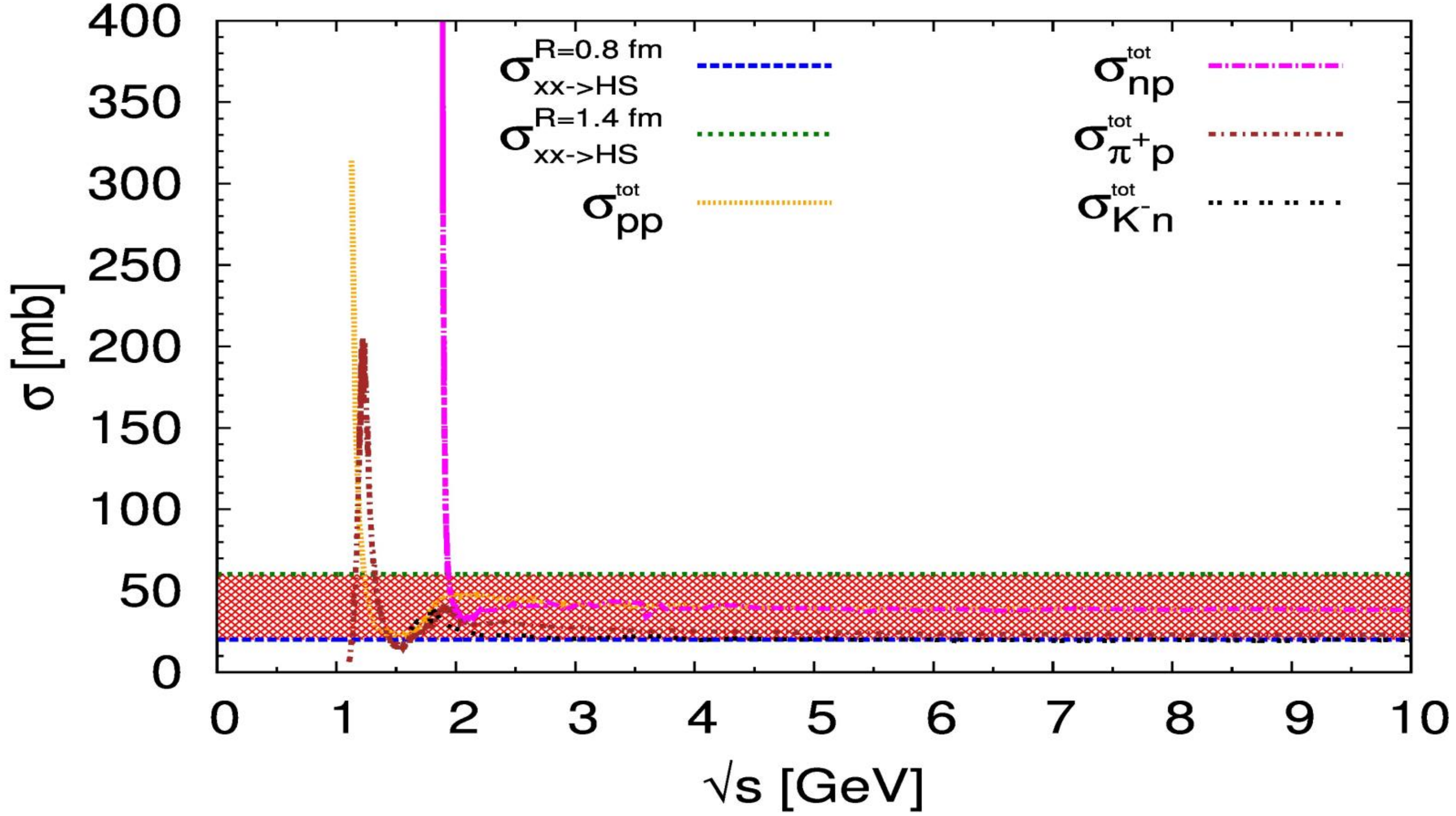




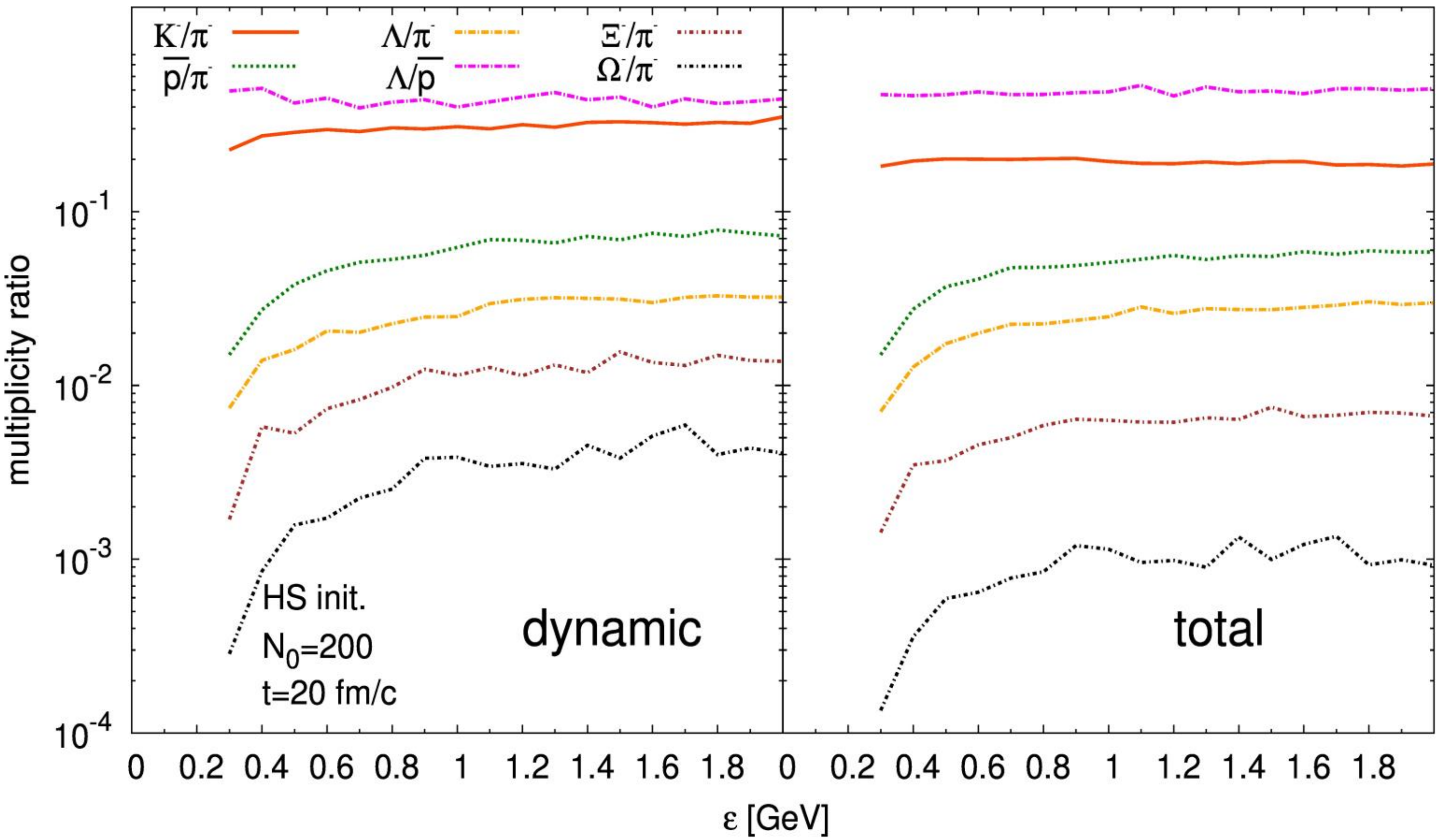
$$aV = a\sigma r + aV_0 - \frac{\pi a}{12r}.$$

a=lattice spacing

Creation Cross Sections of HS



Hadronic Multiplicity Ratios



Rate Equations

$$n\pi \leftrightarrow HS \leftrightarrow n'\pi + B\bar{B}$$

J. Noronha-Hostler, CG, I. Shovkovy, PRL 100:252301, 2008

$$\begin{aligned}
\dot{N}_i &= \Gamma_{i,\pi}\left[N_i^{eq}\sum_n B_{i,n}\left(\frac{N_\pi}{N_\pi^{eq}}\right)^n - N_i\right] \\
&+ \Gamma_{i,X\bar{X}}\left[N_i^{eq}\left(\frac{N_\pi}{N_\pi^{eq}}\right)^{\langle n_{i,x}\rangle}\left(\frac{N_{X\bar{X}}}{N_{X\bar{X}}^{eq}}\right)^2 - N_i\right] \\
\dot{N}_\pi &= \sum_i \Gamma_{i,\pi}\left[N_i\langle n_i\rangle - N_i^{eq}\sum_n B_{i,n} n\left(\frac{N_\pi}{N_\pi^{eq}}\right)^n\right] \\
&+ \sum_i \Gamma_{i,X\bar{X}}\langle n_{i,x}\rangle\left[N_i - N_i^{eq}\left(\frac{N_\pi}{N_\pi^{eq}}\right)^{\langle n_{i,x}\rangle}\left(\frac{N_{X\bar{X}}}{N_{X\bar{X}}^{eq}}\right)^2\right] \\
\dot{N}_{X\bar{X}} &= \sum_i \Gamma_{i,X\bar{X}}\left[N_i - N_i^{eq}\left(\frac{N_\pi}{N_\pi^{eq}}\right)^{\langle n_{i,x}\rangle}\left(\frac{N_{X\bar{X}}}{N_{X\bar{X}}^{eq}}\right)^2\right]
\end{aligned}$$