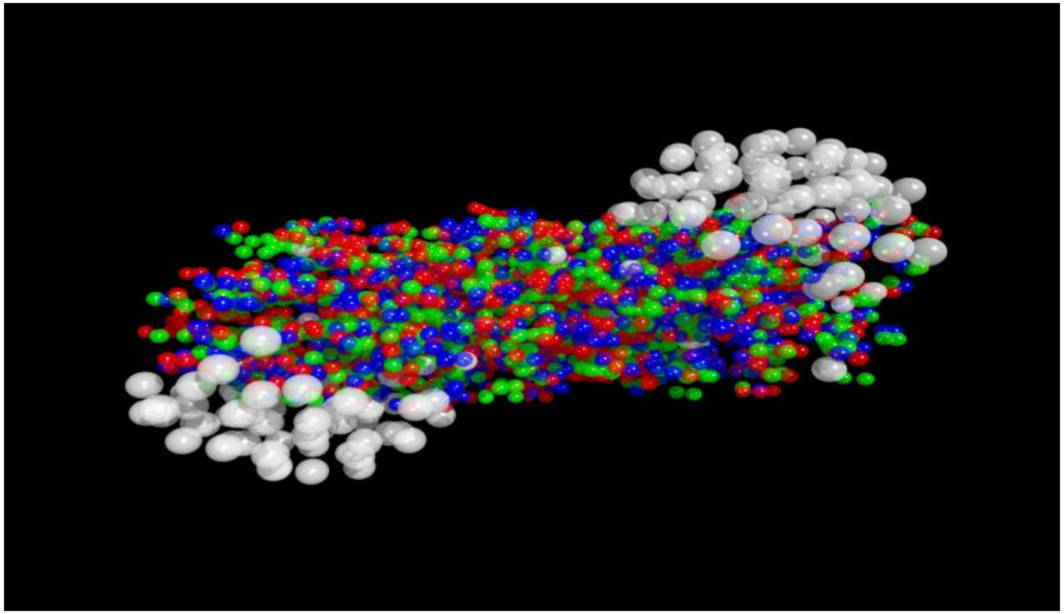
University of Catania
INFN-LNS, Catania, Italy

Heavy Quark Dynamics in QCD matter



Santosh Kumar Das

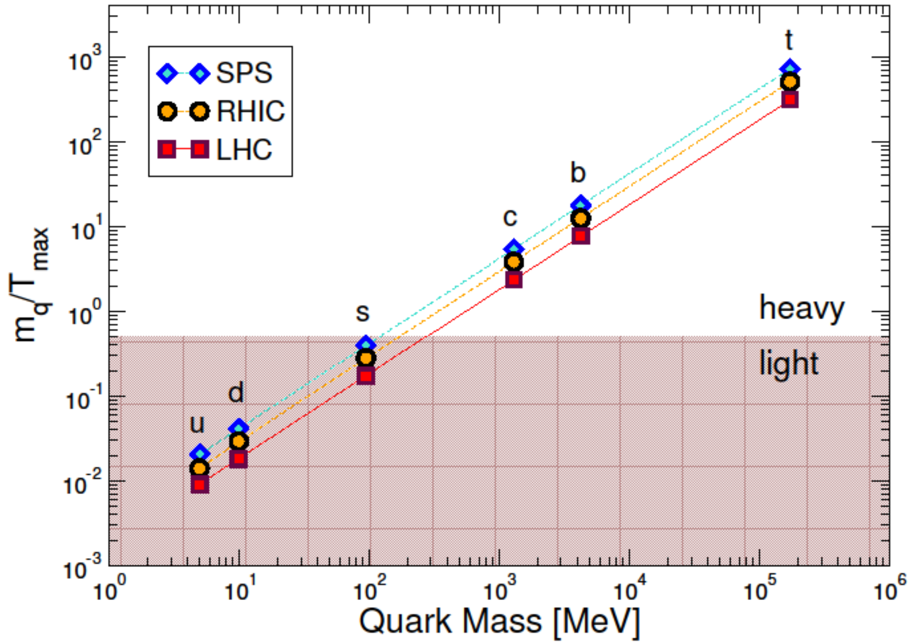
In collaboration with: Vincenzo Greco
Francesco Scadina
Salvatore Plumari

OUTLINE OF MY TALK…………

- ☐ Introduction
- ☐ Impact of T dependence drag on heavy quarks observables
    - I) Nuclear suppression factor
    - II) Elliptic flow
- ☐ Heavy quark momentum evolution: Langevin vs Boltzmann
- ☐ Impact of pre-equilibrium phase on heavy quark observables.
- ☐ Impact of the electromagnetic filed on heavy quark dynamics (sizable heavy quark v1)
- ☐ Summary and outlook

Heavy Quark & QGP

At very high density and temperature hadrons melt to a new phase of matter called Quark Gluon Plasma (QGP).



SPS to LHC

$\sqrt{s} = 17.3 GeV\ to\ 2.76 TeV$ **~100 times**

$T_i = 200\ MeV\ to\ 600\ MeV$ **~3 times**

$$M_{c,b} >> \Lambda_{QCD}$$

Produced by pQCD process (out of Equil.)

$$\tau_{c,b} >> \tau_{QGP}$$

They go through all the QGP life time

$$M_{c,b} >> T_0$$

No thermal production

Boltzmann Kinetic equation

$$\left(\frac{\partial}{\partial t}+\frac{P}{E}\frac{\partial}{\partial x}+\mathrm{F}.\frac{\partial}{\partial p}\right)f\left(x,\,p,\,t\right)=\left(\frac{\partial f}{\partial t}\right)_{col}$$

- The plasma is uniform ,i.e., the distribution function is independent of **x.**
- In the absence of any external force, F=0

$$\boxed{R\left(p,t\right)=\left(\frac{\partial f}{\partial t}\right)_{col}=\int d^{3}k\left[\omega\left(p+k,k\right)f\left(p+k\right)-\omega\left(p,k\right)f\left(p\right)\right]}$$

$$\omega(p,k)=g\int\frac{d^{3}q}{(2\pi)^{3}}f'(q)v_{q,p}\sigma_{p,q\rightarrow p-k,q+k}$$

→ is rate of collisions which change the momentum of the charmed quark from p to p-k

$$\omega\left(p+k,k\right)f\left(p+k\right)\approx\omega\left(p,k\right)f\left(p\right)+k.\frac{\partial}{\partial p}\left(\omega f\right)+\frac{1}{2}k_{i}k_{j}\frac{\partial^{2}}{\partial p_{i}\partial p_{j}}\left(\omega f\right)$$

$$\boxed{\frac{\partial \mathbf{f}}{\partial \mathbf{t}}=\frac{\partial}{\partial \mathbf{p_i}}\left[\mathbf{A_i}(\mathbf{p})\mathbf{f}+\frac{\partial}{\partial \mathbf{p_j}}\left[\mathbf{B_{ij}}(\mathbf{p})\mathbf{f}\right]\right]}$$

B. Svetitsky PRD 37(1987)2484

where we have defined the kernels

’ $\mathbf{A_i}=\int \mathbf{d^3k}\,\omega(\mathbf{p},\mathbf{k})\mathbf{k_i}$ → **Drag Coefficient**

$\mathbf{B_{ij}}=\int \mathbf{d^3k}\,\omega(\mathbf{p},\mathbf{k})\mathbf{k_i}\mathbf{k_j}$ → **Diffusion Coefficient**

Langevin Equation

$$dx_j = \frac{p_j}{E} dt$$

$$dp_j = -\Gamma p_j dt + \sqrt{dt} C_{jk}(t, p + \xi dp) \rho_k$$

where Γ **is the deterministic friction (drag) force**

C_{ij} **is stochastic force in terms of independent Gaussian-normal distributed random variable**

$$\rho = (\rho_x, \rho_y, \rho_z) \quad , \quad P(\rho) = \left(\frac{1}{2\pi}\right)^3 \exp\left(-\frac{\rho^2}{2}\right)$$

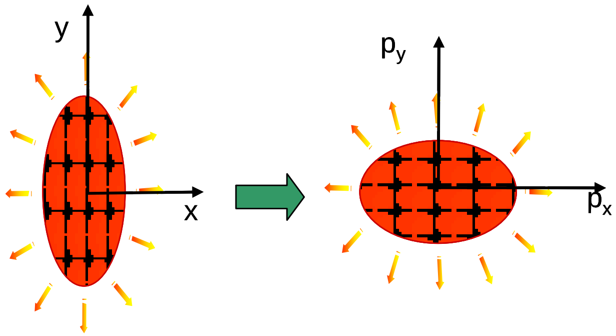
With $< \rho_i(t) \rho_k(t') > = \delta(t - t') \delta_{jk}$

H. v. Hees and R. Rapp
arXiv:0903.1096

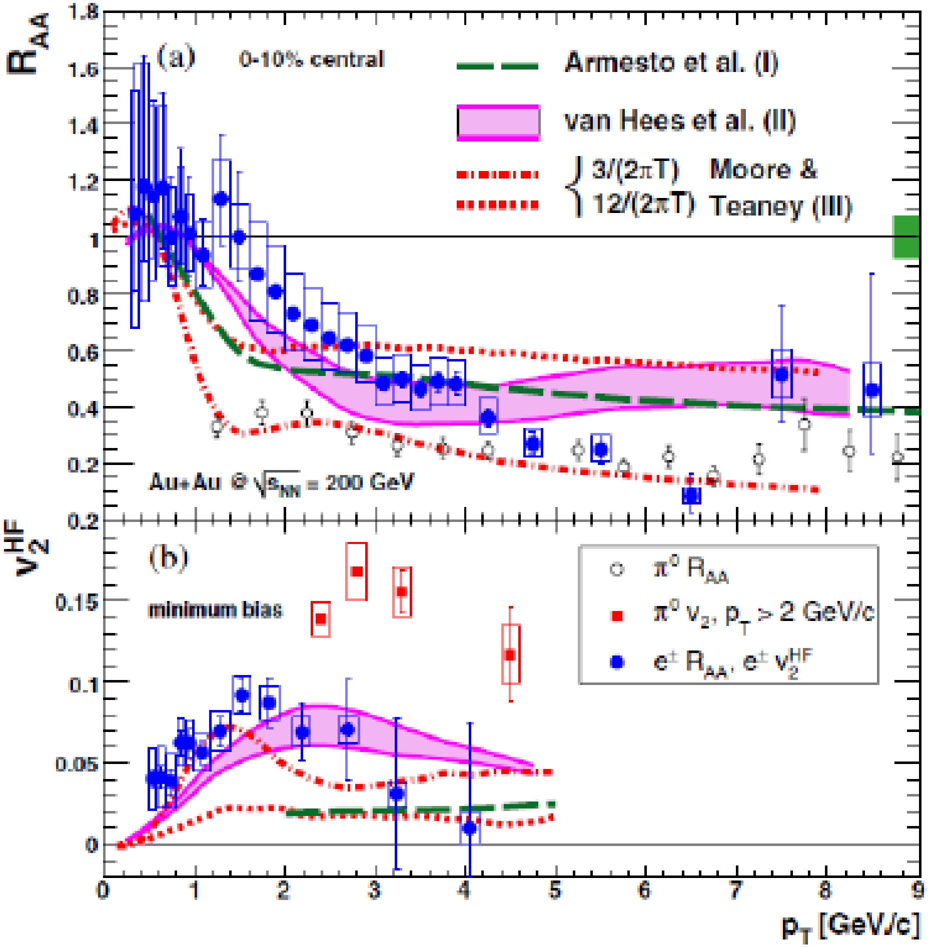
$\xi = 0$ **the pre-point Ito interpretation of the momentum argument of the covariance matrix.**

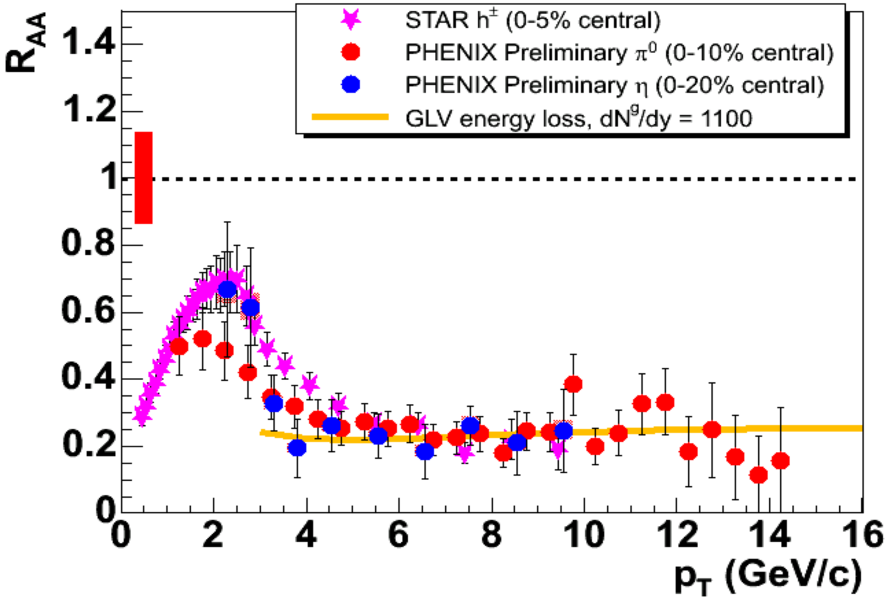
$$\boxed{R_{AA} = \frac{\left(\frac{dN}{d^2 p_T dy}\right)^{Au+Au}}{N_{coll} \left(\frac{dN}{d^2 p_T dy}\right)^{p+p}}}$$

$$\boxed{v_2(p_T) = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_x^2} \right\rangle}$$



Heavy flavor at RHIC

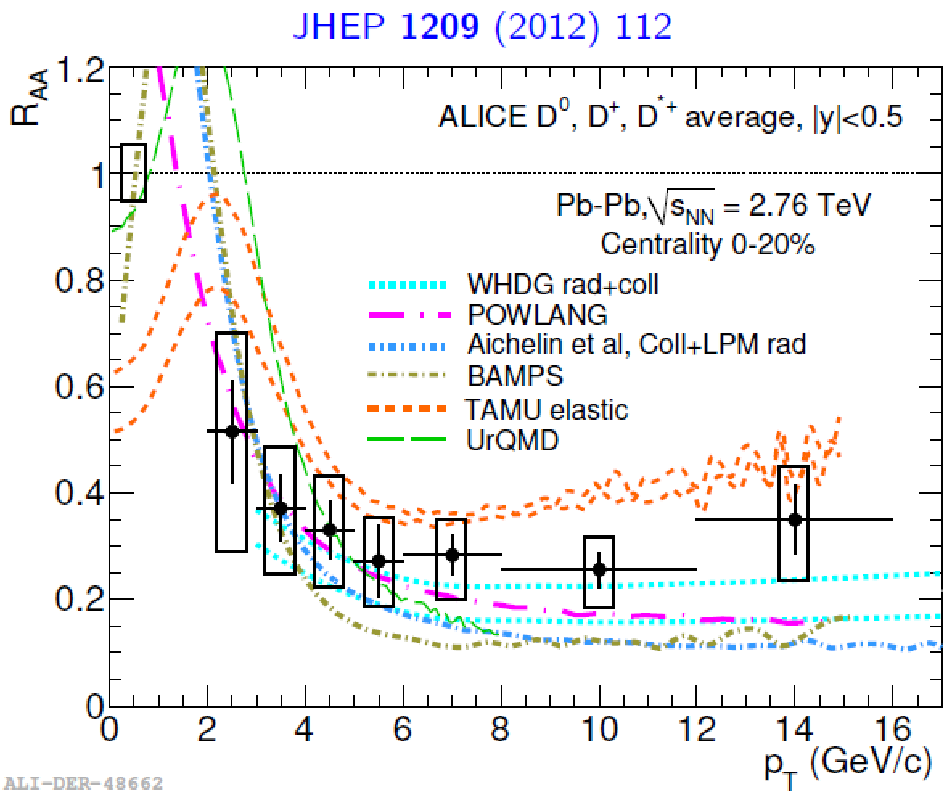


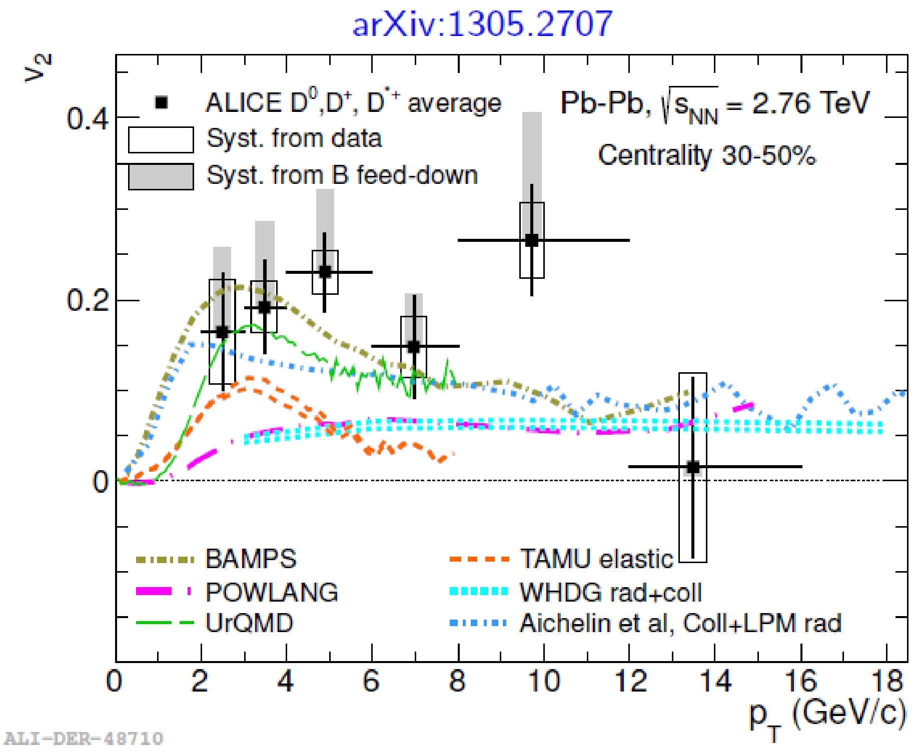


At RHIC energy heavy flavor suppression is similar to light flavor

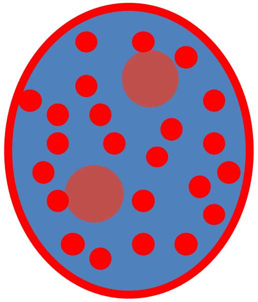
Simultaneous description of RAA and v2 is a tough challenge for all the models.

Heavy Flavors at LHC



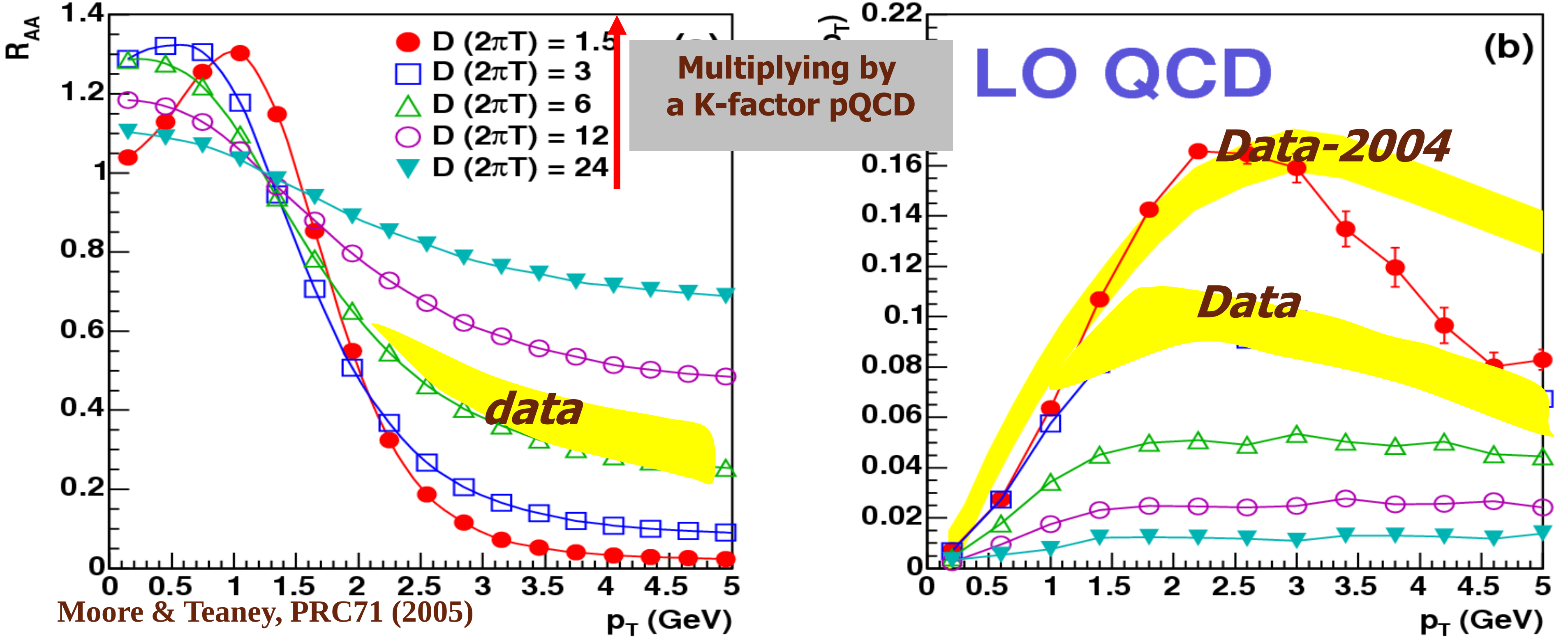


Charm dynamics with upscaled pQCD cross section



Fokker-Plank for charm interaction in a hydro bulk

$$D \propto \int d^3k \left|M_{g(q)c\to g(q)c}(k,p)\right|^2 k^2$$

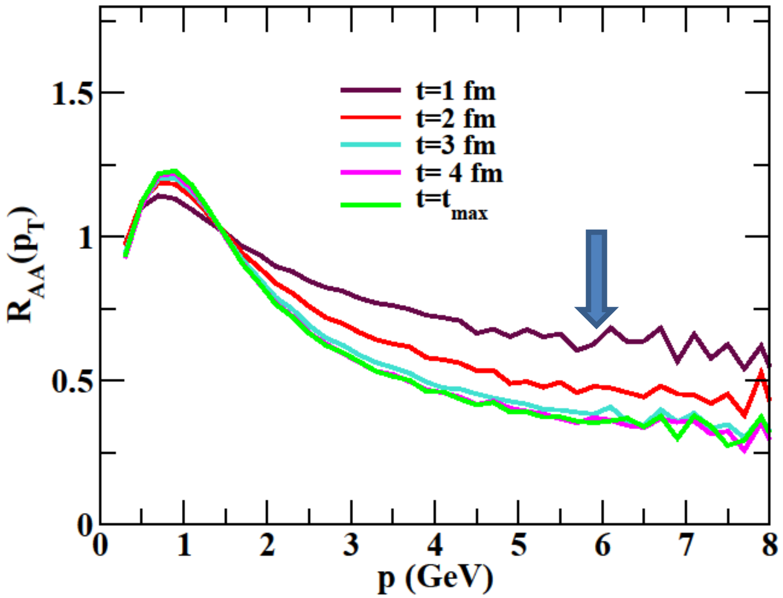


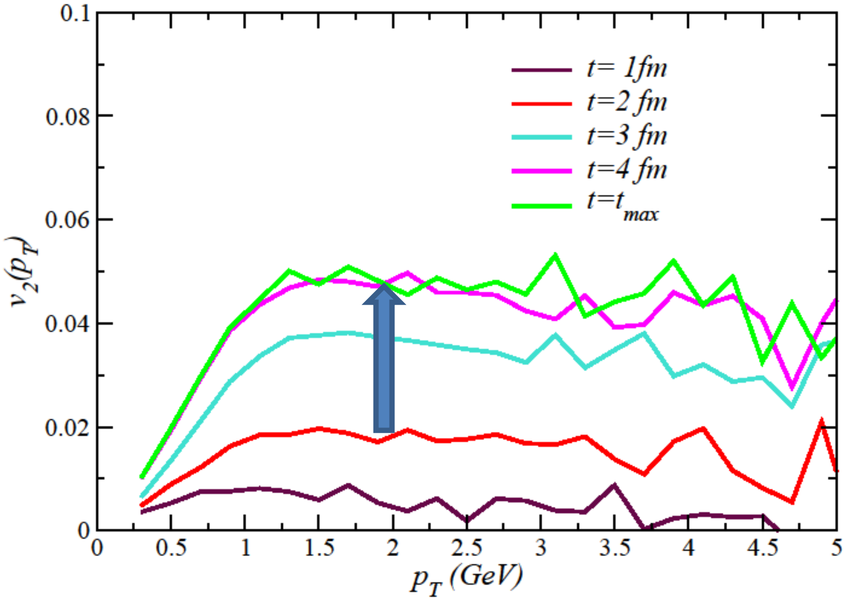
Multiplying by a K-factor pQCD

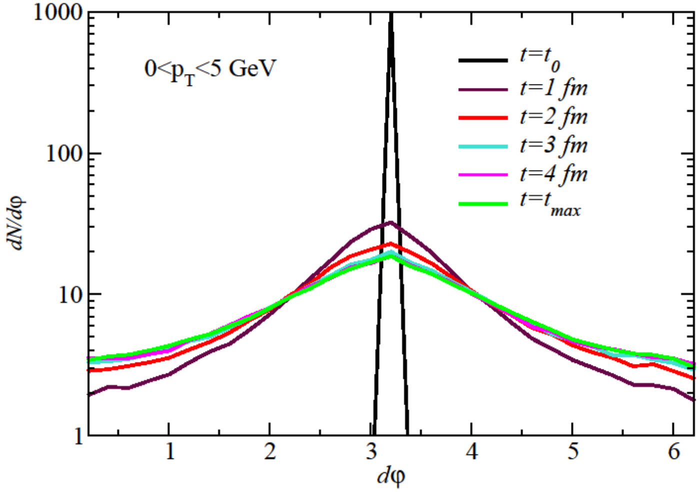
Moore & Teaney, PRC71 (2005)

It’s not just a matter of pumping up pQCD elastic cross section: too low R_{AA} or too low v_2

Time evolution of Heavy quarks observables







Das, Scardina, Plumari, Greco
arXiv:1509:06307

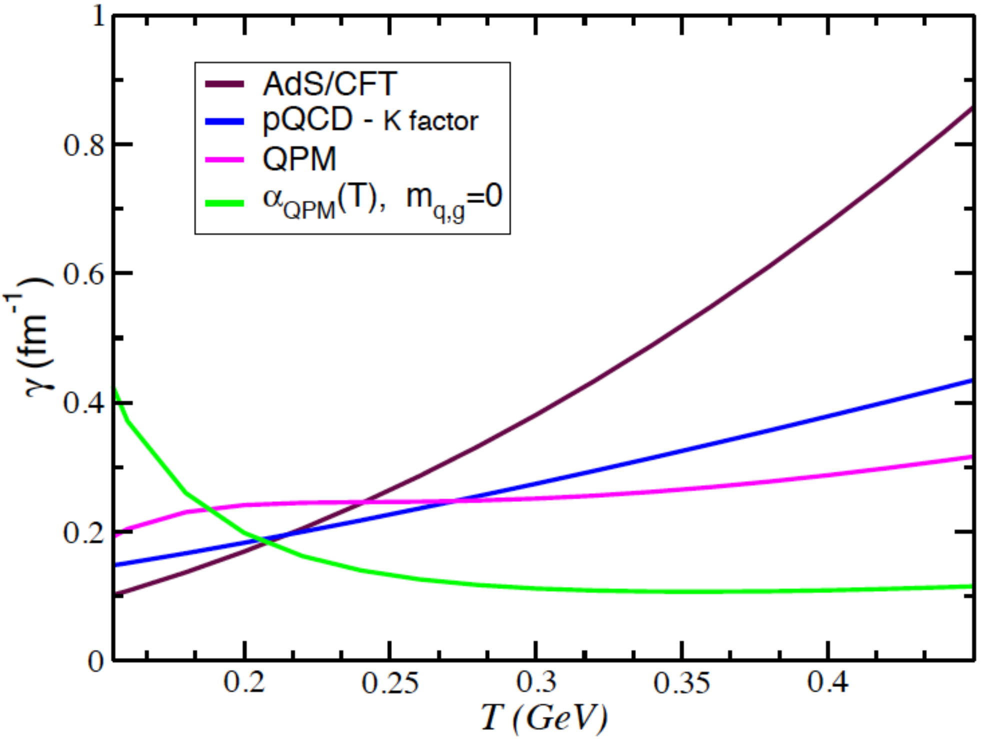
RAA and dN/dphi_ccbar developed during the early stage of the evolution → T_i

V2 developed duringing the later stage of the evolution → T_c

T dependence of the interaction i.e the transport Coefficients are the essential ingradiant for the symultanious description of HQ observabbles

T- dependence of the Drag Coefficient

Drag Coefficient



Das, Scardina, Plumari, Greco
PLB 747 (2015)260-264

Plumari at. al. PRD, 84, 094004 (2011)

pQCD *(Combridge)*

$$\alpha_{pQCD} = \frac{4\pi}{11\ln\left(2\pi T\Lambda^{-1}\right)}, \quad m_D^2 = 4\pi\alpha_{pQCD}(\mathrm{T})T^2$$

AdS/CFT

$$\gamma_{AdS/CFT} = k\frac{T^2}{M}$$

Gubser
PRD,74,126005 (2006)
Akamatsu, Hatsuda, Hirano
PRC, 79, 054907 (2009)
Das and Davody
PRC, 89,054912 (2014)

Quasi-Particle-Model *(fit to lQCD ε,P)*

$$g^2_{QP}(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[\lambda\left(\frac{T}{T_c} - \frac{T_s}{T_c}\right)\right]^2}$$

λ=2.6
T_s=0.57

$$m_g^2 = \frac{1}{6}\left(N_c + \frac{1}{2}N_f\right)g^2T^2$$

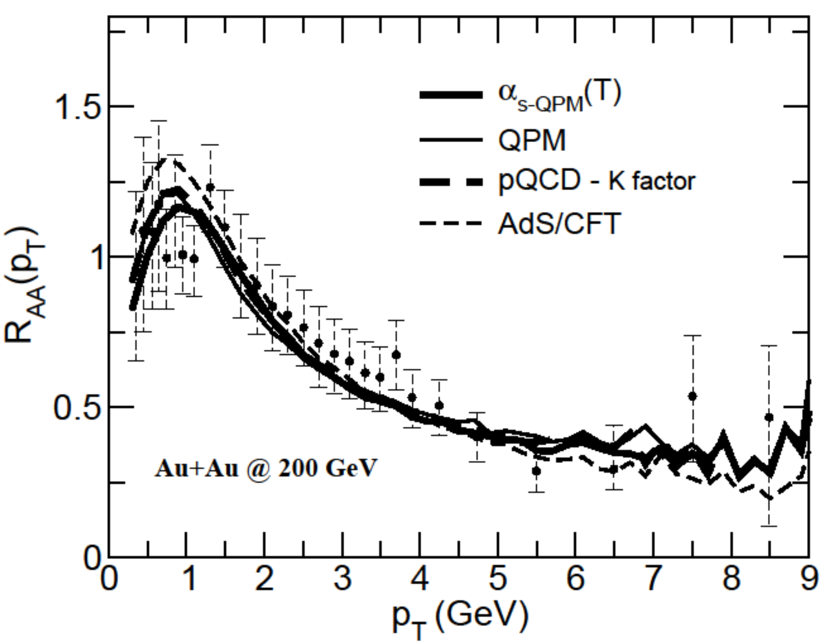
$$m_q^2 = \frac{N_c^2-1}{8N_c}g^2T^2($$

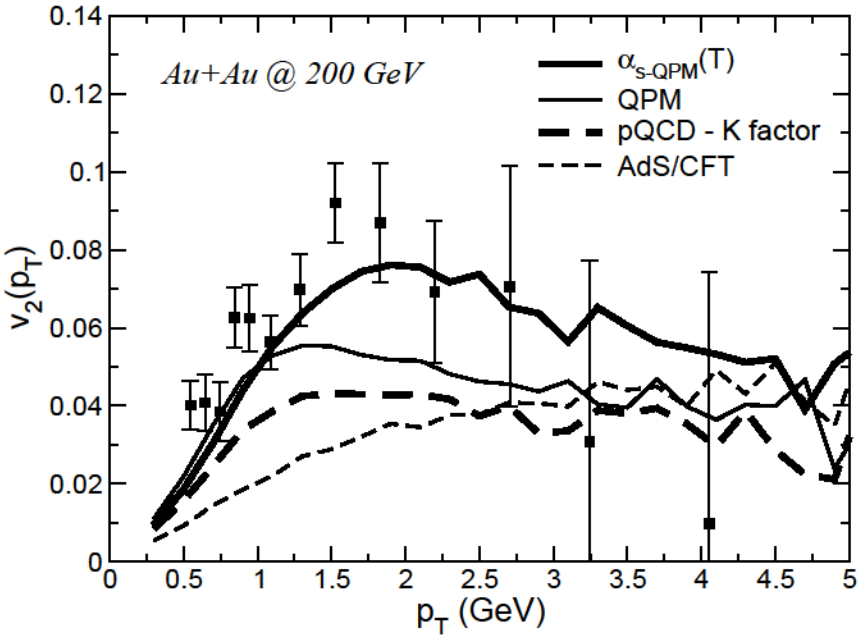
α_{QPM}(T) , $m_{q,g}$=0

we mean simply the coupling of the QPM, but with a bulk of massless q and g .

RAA and v2 @ RHIC

(Au+Au@200AGeV, b=8 fm)





For same RAA, large interaction at Tc develop larger v2

Light flavor sector:

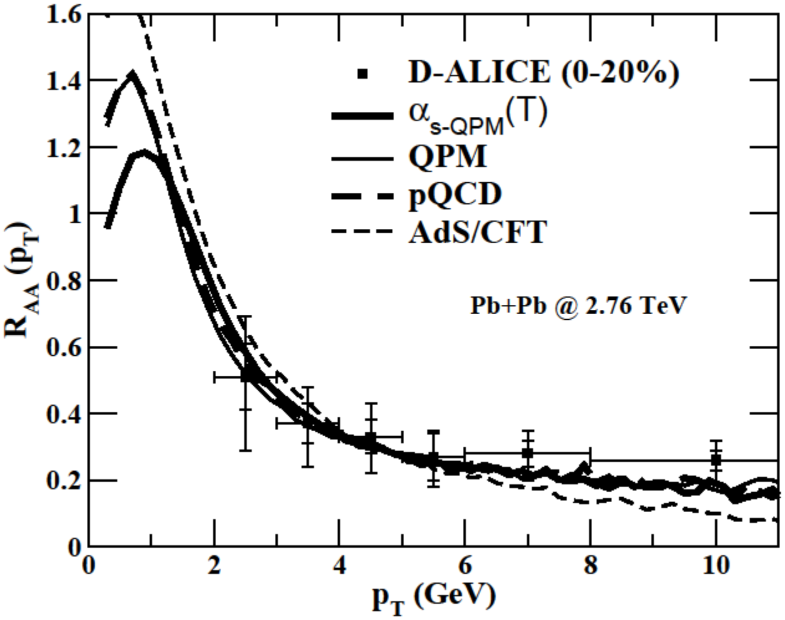
Liao, Shuryak, PRL102 (2009) 202302

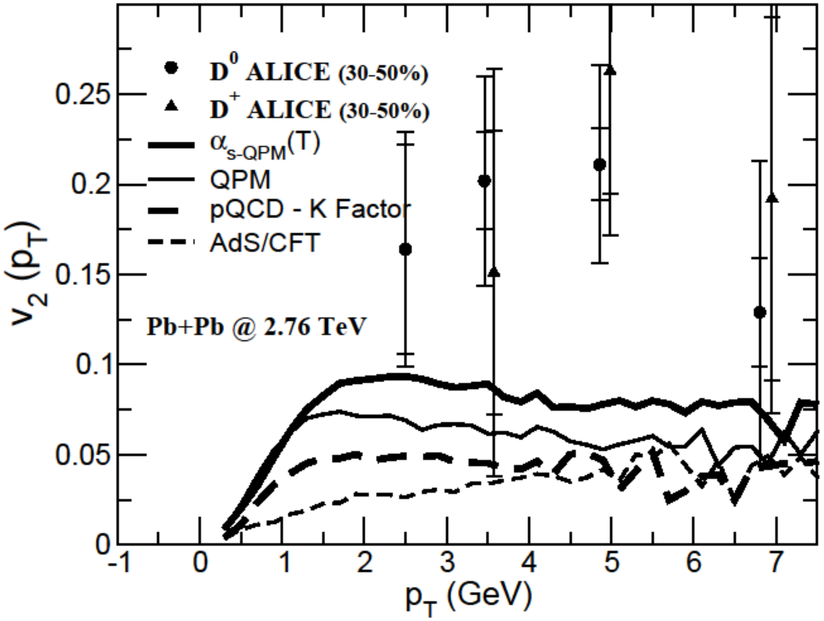
Scardina, Toro, Greco, PRC,82 (2010) 054901

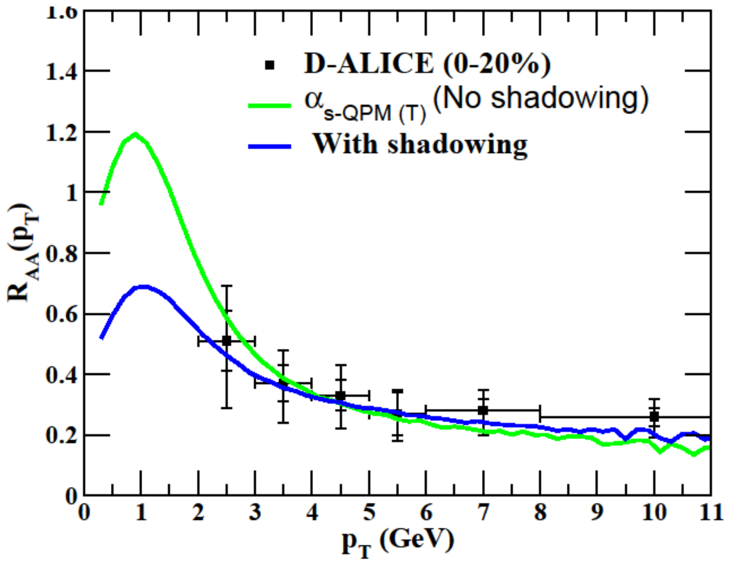
Xu, Liao, Gyulassy, arXiv:1411.3637

Das, Scardina, Plumari, Greco
PLB 747 (2016)260-264

RAA and v2 @ ALICE





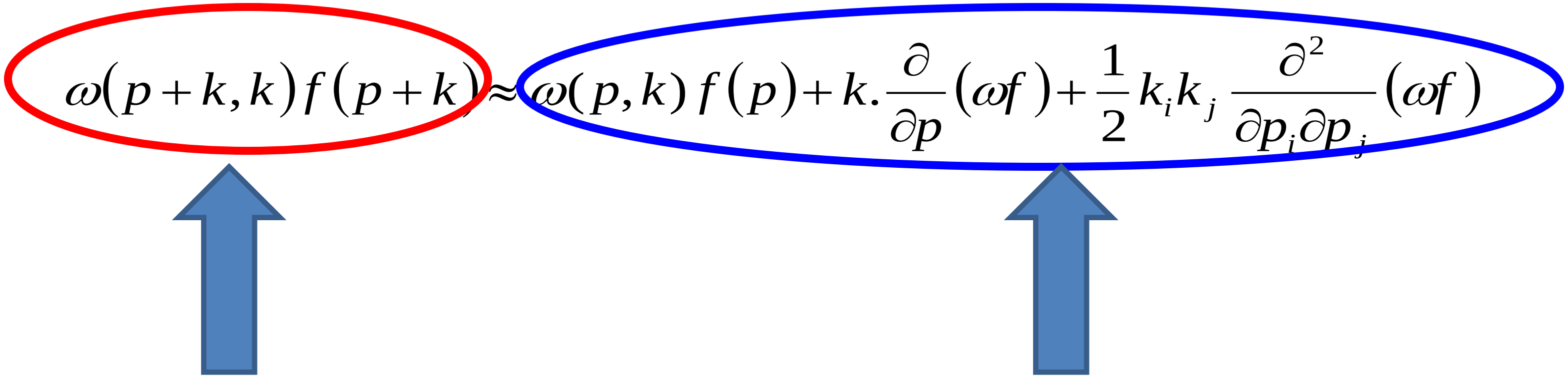


Das, Scardina, Plumari, Greco
PLB 747 (2015) 260-264

Eskola, Paukkunen, Salgado
JHEP,0807, 102 (2008)

S. Cao, G.-Y. Qin, and S. A. Bass
PRC 88(2013)044907

Heavy quark momentum evolution: Langevin vs Boltzmann



$$\omega(p+k,k)f(p+k)\approx\omega(p,k)f(p)+k.\frac{\partial}{\partial p}(\omega f)+\frac{1}{2}k_ik_j\frac{\partial^2}{\partial p_i\partial p_j}(\omega f)$$

Boltzmann Equation

Fokker Planck

It will be interesting to study both the equation in a identical environment to ensure the validity of this assumption at different momentum transfer and their subsequent effects on RAA and v2.

Langevin dynamics:

$$dx_j=\frac{p_j}{E}dt$$

$$dp_j=-\Gamma p_j dt+\sqrt{dt}C_{jk}(t,p+\xi dp)\rho_k$$

H. v. Hees and R. Rapp
arXiv:0903.1096

Γ **is the deterministic friction (drag) force**

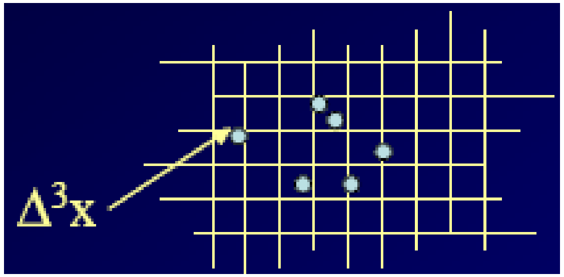
C_{ij} **is stochastic force in terms of independent Gaussian-normal distributed random variable.**

Transport theory

$$p^{\mu}\partial_{\mu}f(x,p)=C_{22}$$

We consider two body collisions

$$\begin{aligned}C_{22} = & \frac{1}{2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int\frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2}f'_1 f'_2|\mathcal{M}_{1'2'\to 12}|^2(2\pi)^4\delta^{(4)}(p'_1+p'_2-p_1-p_2)\\ & -\frac{1}{2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int\frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2}f_1 f_2|\mathcal{M}_{12\to 1'2'}|^2(2\pi)^4\delta^{(4)}(p_1+p_2-p'_1-p'_2)\end{aligned}$$



$$\Delta t \to 0$$

$$\Delta^3 x \to 0$$

Exact solution

Collision integral is solved with a local stochastic sampling

[Z. Xhu, et al. PRC71(04)]
Greco et al PLB670, 325 (08)]

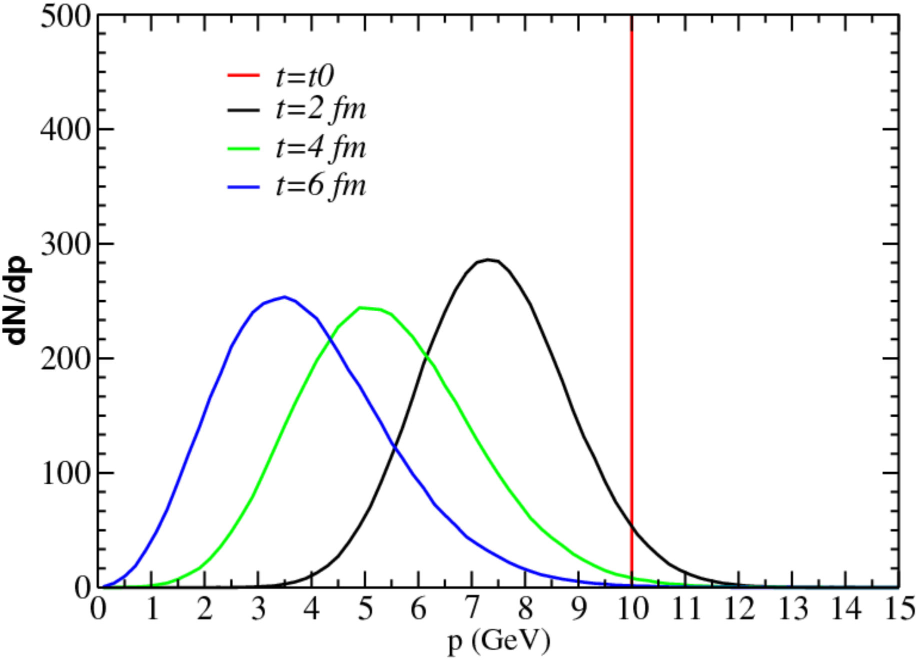
$$P_{22}=\frac{\Delta N^{2\to 2}_{\text{coll}}}{\Delta N_1\,\Delta N_2}=v_{\text{rel}}\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$

Evolution: Boltzmann vs Langevin (Charm)

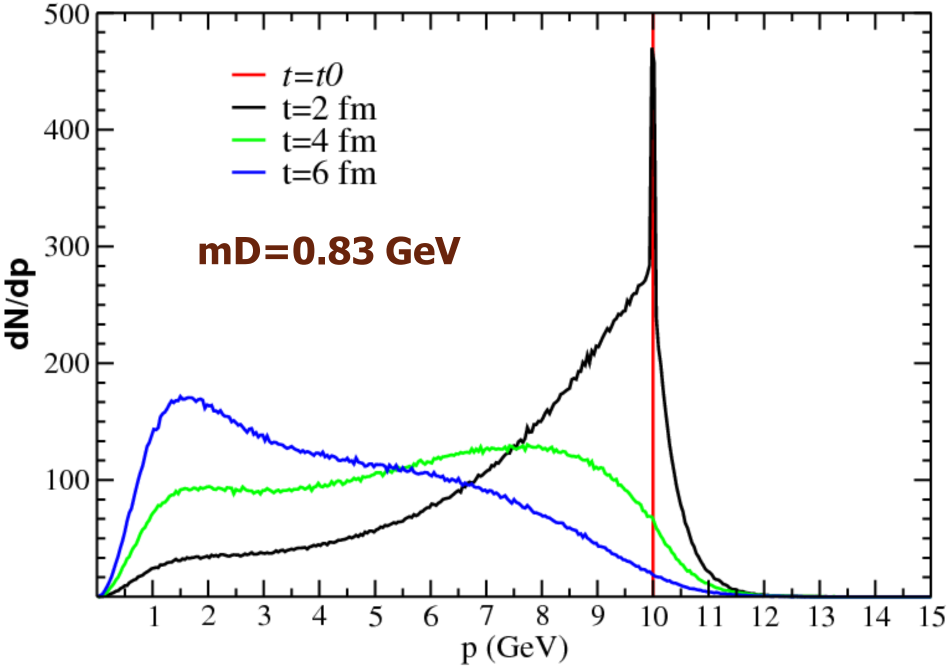
Momentum evolution starting from a δ (Charm) in a Box

$$\frac{dN}{d^3p}_{initial} = \delta\left(p - 10GeV\right)$$

Langevin



Boltzmann



In case of Langevin the distributions are Gaussian as expected by construction

In case of Boltzmann the charm quarks does not follow the Brownian motion

Das, Scardina, Plumari and Greco PRC,90,044901(2014)

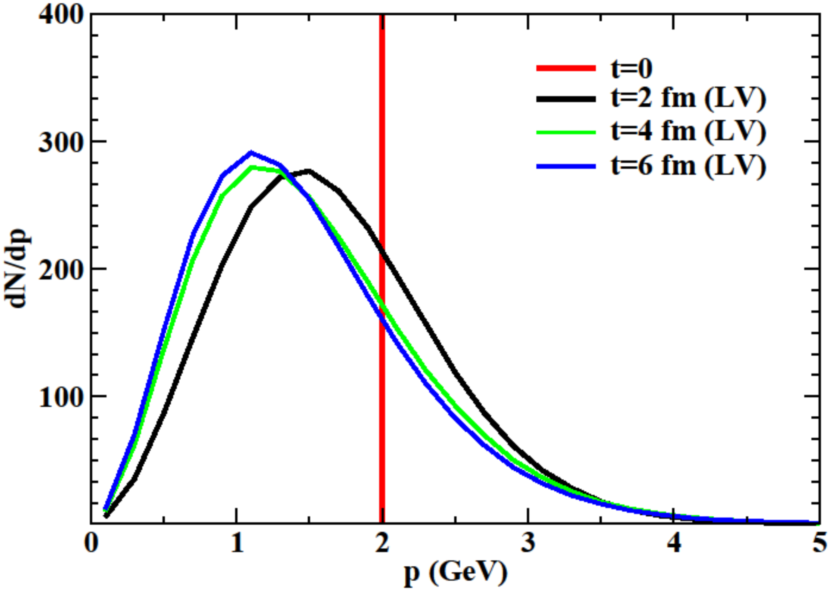
Evolution: Boltzmann vs Langevin (Charm)

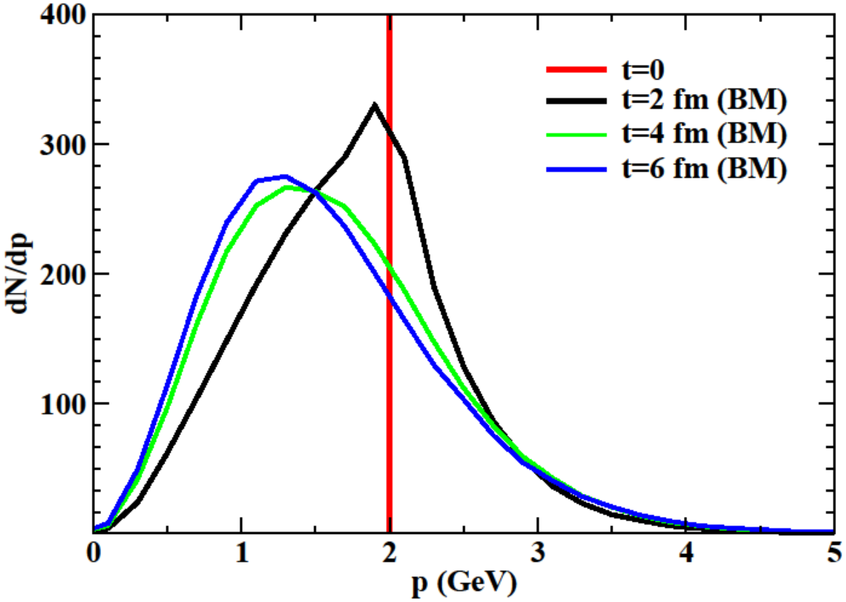
Momentum evolution starting from a δ (Charm) in a Box

$$\frac{dN}{d^3p}_{initial} = \delta(p - 2GeV)$$

Langevin

Boltzmann





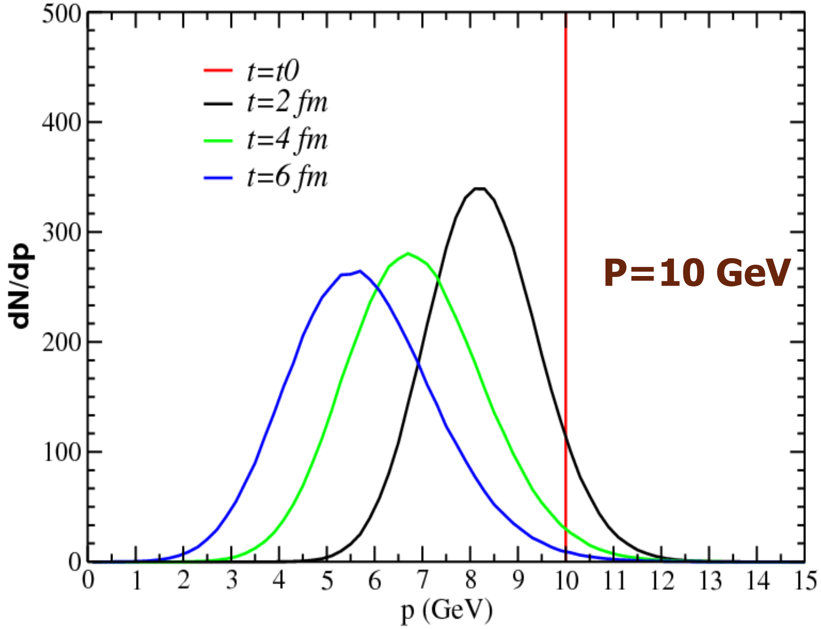
In case of Langevin the distributions are Gaussian as expected by construction

In case of Boltzmann the charm quarks follow the Brownian motion: At Low Momentum.

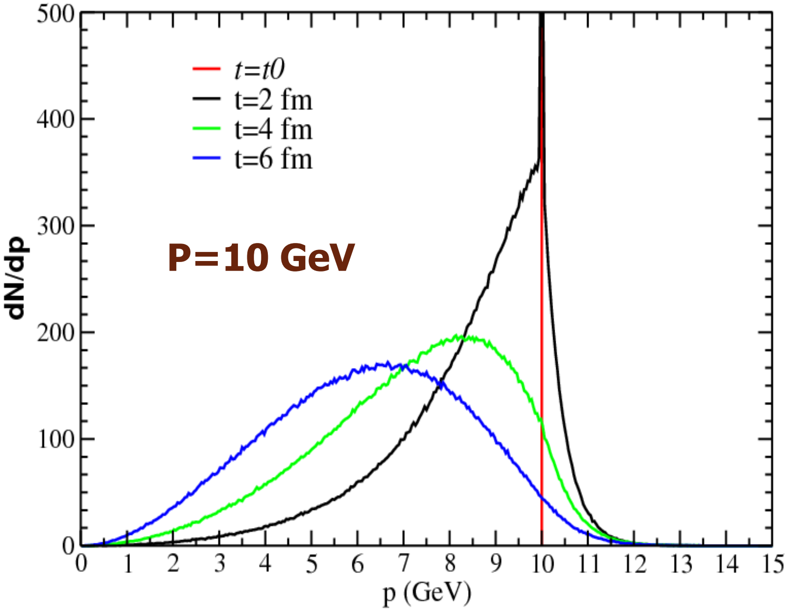
Momentum evolution starting from a δ (Bottom)

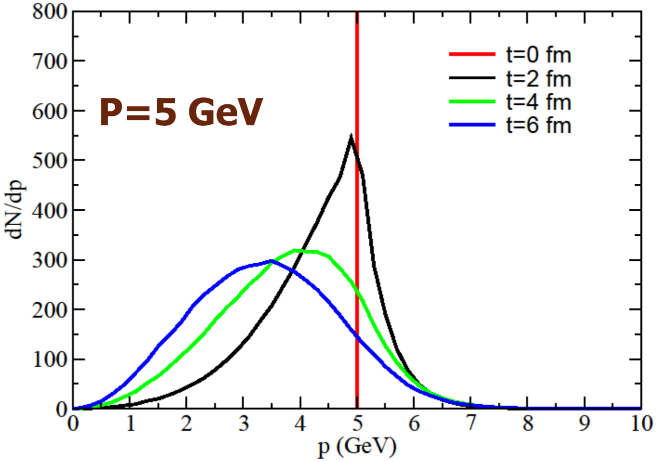
In a Box

Langevin



Boltzmann

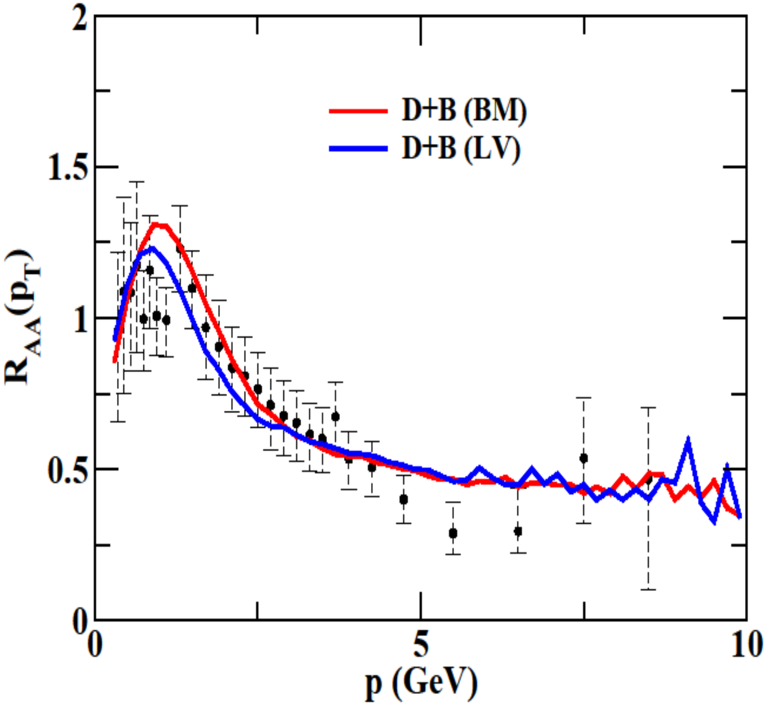


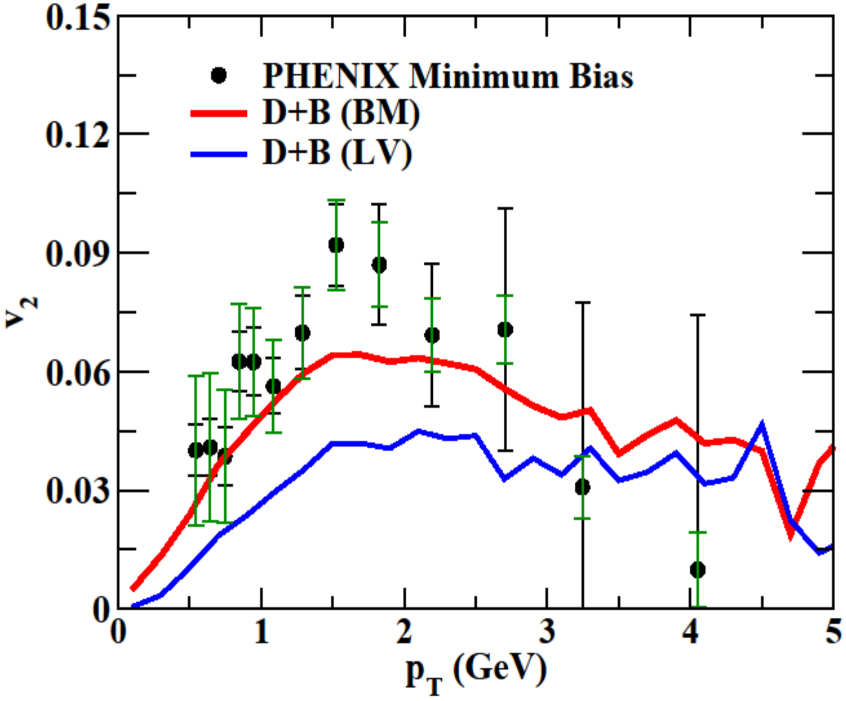


For bottom quarks it works better.

R_{AA} and v2 at RHIC

(With near isotropic cross-section)



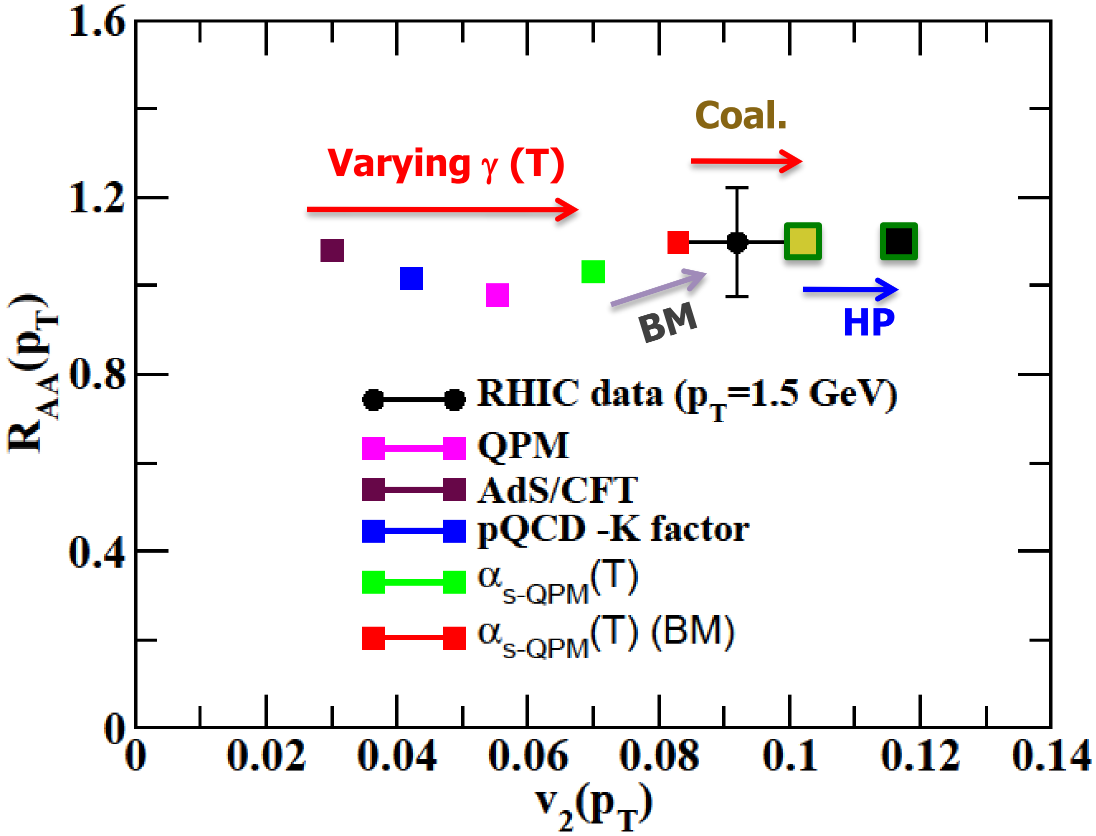


Das, Scardina, Plumari and Greco
PRC,90,044901(2014)

At fixed RAA Boltzmann approach generate larger v2 .
(depending on mD and M/T)

With isotropic cross section one can describe both RAA and V2 simultaneously within the Boltzmann approach !

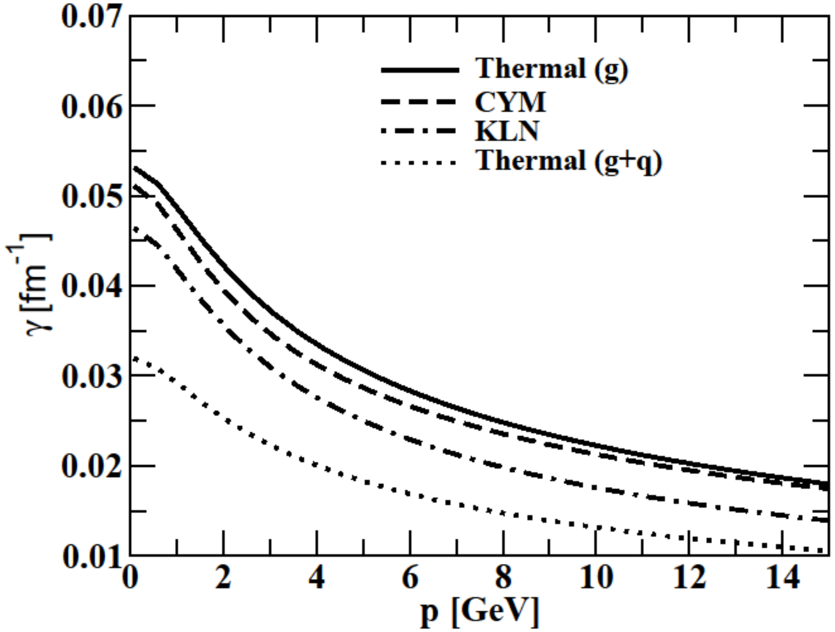
Summary on the build-up of v_2 at fixed R_{AA}



Das, Scardina, Plumari, Greco
arXiv:1509:06307

R_{AA} and V_2 are correlated but still one can have
R_{AA} about the same while V_2 can change up to a factor 2-3
γ(T) + Boltzmann dynamics+ hadronization+ hadronic phase

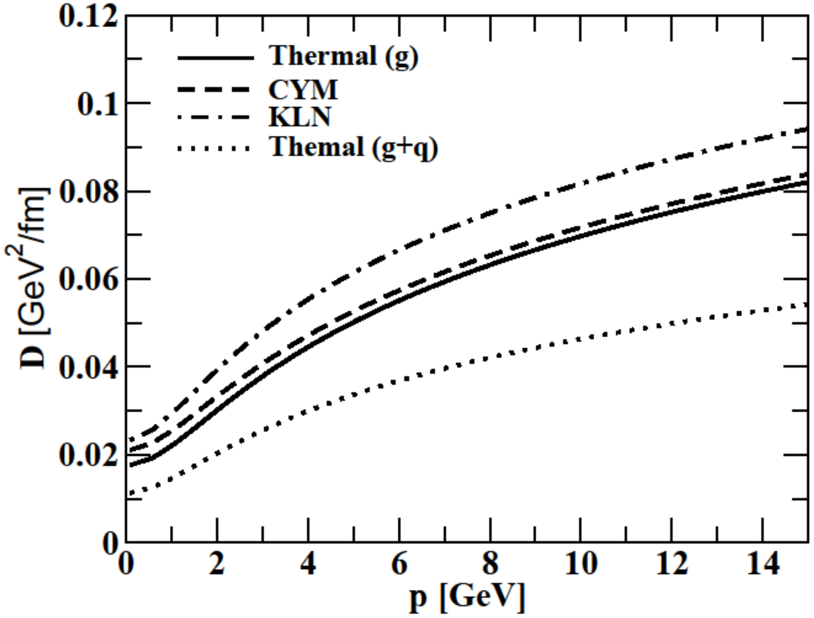
Impact of Pre-equilibrium Phase



Drescher, Nara PRC 75, 034905 (2007)
Hirano, Nara Y PRC 79, 064904 (2009)

Schenke , Tribedy, Venugopalan
PRC 89, 024901 (2013).

Ruggieri, Scardina, Plumari, Greco
PRC 89, 054914 (2014)

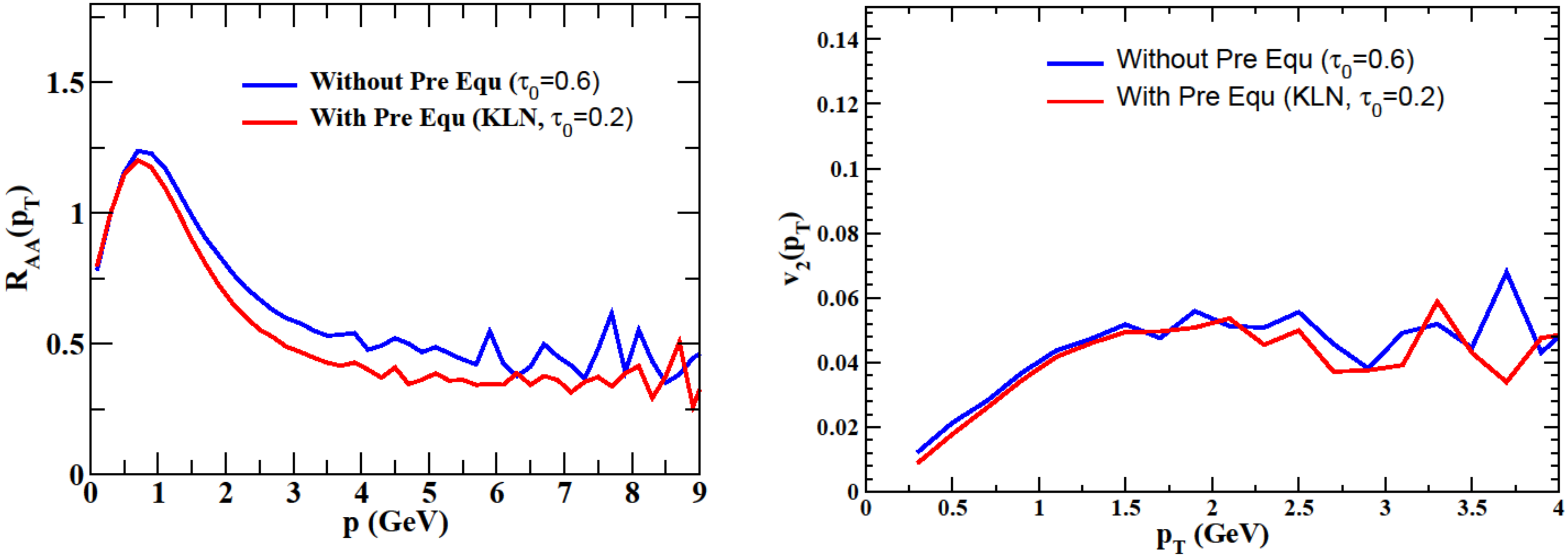


Das, Ruggieri, Mazumder, Greco, Alam
JPG 42 (2015)095108

It will be interesting to study the role of Pre-equilibrium on R_{AA} and v_2.

RAA and v2 @ RHIC

(Au+Au@200AGeV, b=8 fm)

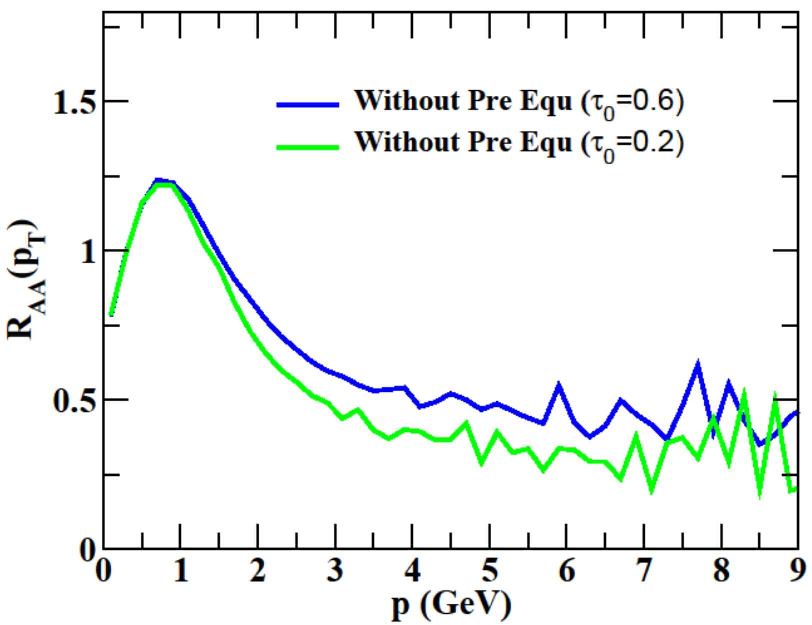


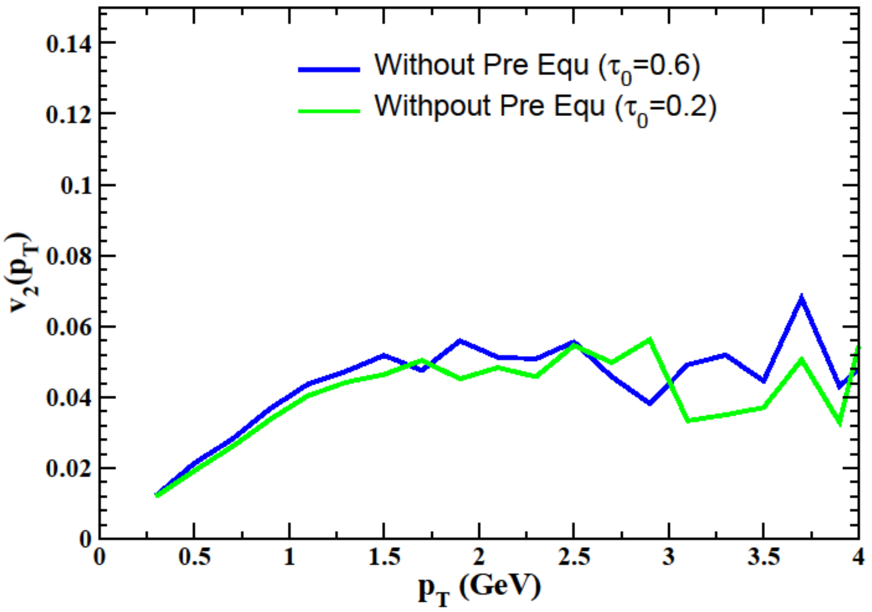
Under Preparation

Pre-equilibrium phase affect the RAA significantly.
Impact on v2 is nominal.

RAA and v2 @ RHIC

(Au+Au@200AGeV, b=8 fm)





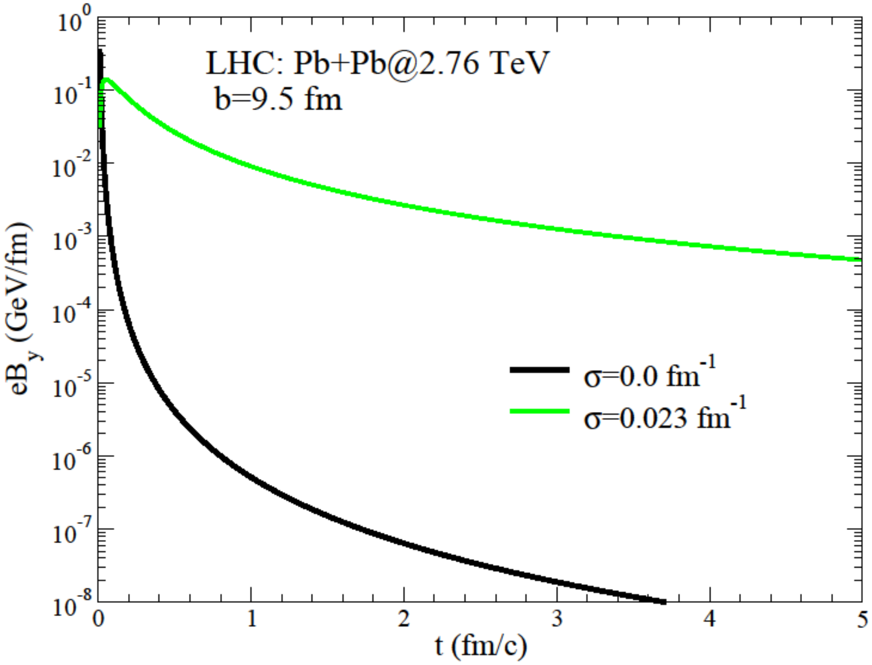
One can mock the impact of the pre-equilibrium phase with early locally thermalized QGP !

Impact of EM field on heavy quark dynamics at LHC

$$dp_j = -\Gamma p_j dt + \sqrt{dt} C_{jk}(t, p + \xi dp)\rho_k + F_{ext} dt$$

$$F_{ext} = q(E^{'} + v \times B^{'})$$

$$E' = \gamma (E + v \times B) - (\gamma - 1)(E \cdot \hat{v})\hat{v}$$
$$B' = \gamma (B - v \times E) - (\gamma - 1)(B \cdot \hat{v})\hat{v}$$

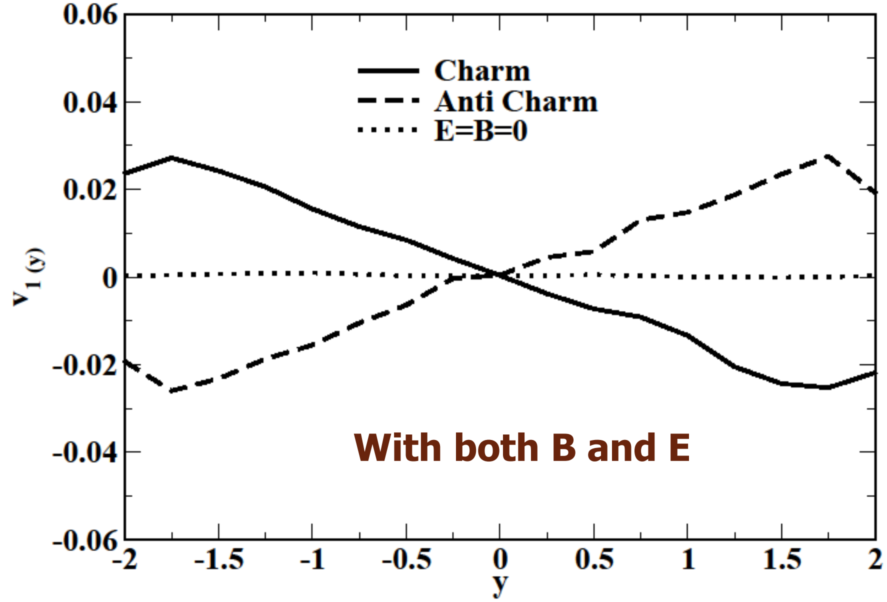


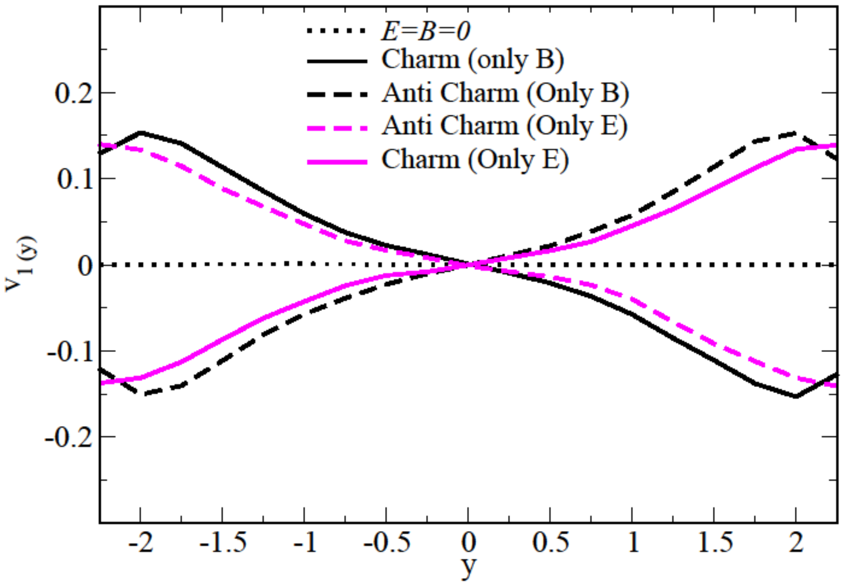
We consider both E and B.
Bx=Bz=0
And Ey=Ez=0

$$v_1 = < \frac{p_x}{p_T} >$$

Gusoy, Kharzeev and Rajagopal
PRC 89, 054905 (2014)

Heavy quark v1@LHC





The simulation is done starting from tau_0=0.2 fm.

The sign of v1 due to B is decided by $v \times B$

E act opposite to B.

Das, Plumari, Chartarjee, Scardina, Greco, Alam
Under preparation

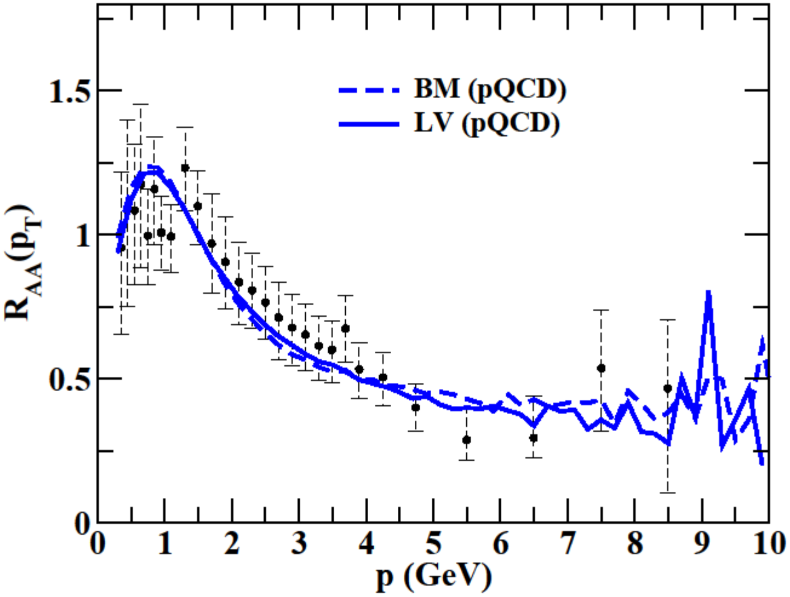
At RHIC the v1 could be around 1-2 % on which we are working currently.

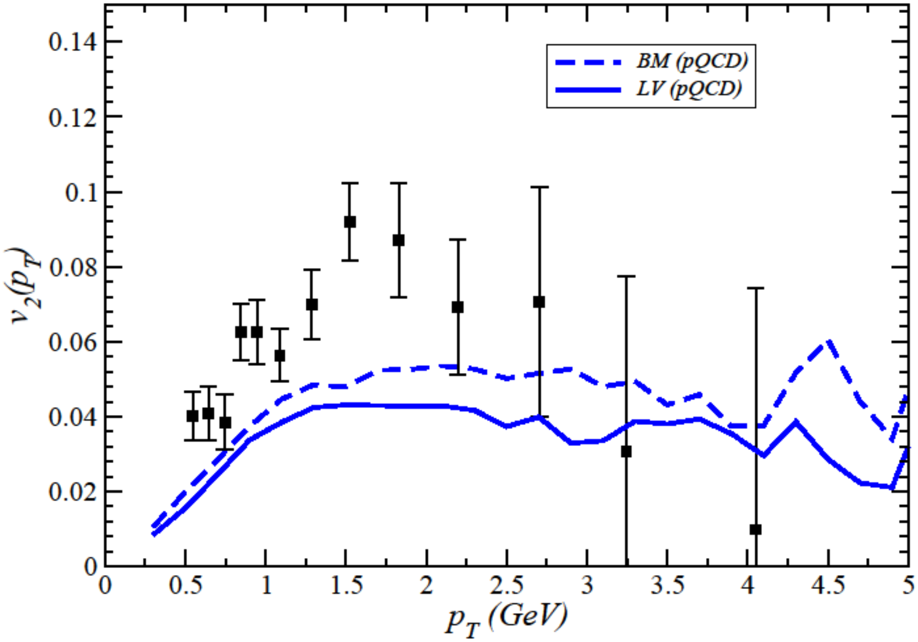
Summary & Outlook ……

- R_{AA}- v_2 of charm quarks seems to indicate:
- Drag about constant in T or weak T dependence to describe both RAA and v2 simultaneously.
- Simultaneous study of RAA and v2 can put constrain on various energy loss models.
- Boltzmann dynamics more efficient for v_2 even at fixed $R_{AA.}$
- Hadronization by coalescence of heavy quarks as well as the role of hadronic medium modify R_{AA} vs v_2 relation toward a bette agreement with the data.
- Pre-equibrium phase have significant impact on RAA but one can mock it with early locally thermalized QGP phase .
- Heavy quark have a finite v1 due to the presence of strong EM field created at heavy ion collision which can be measurable.
- Implementation of all these effects including radiation within a single framework (within Boltzmann equation) is going on……..

Thank You . . . .

R_{AA} and v2 at RHIC at mD=gT

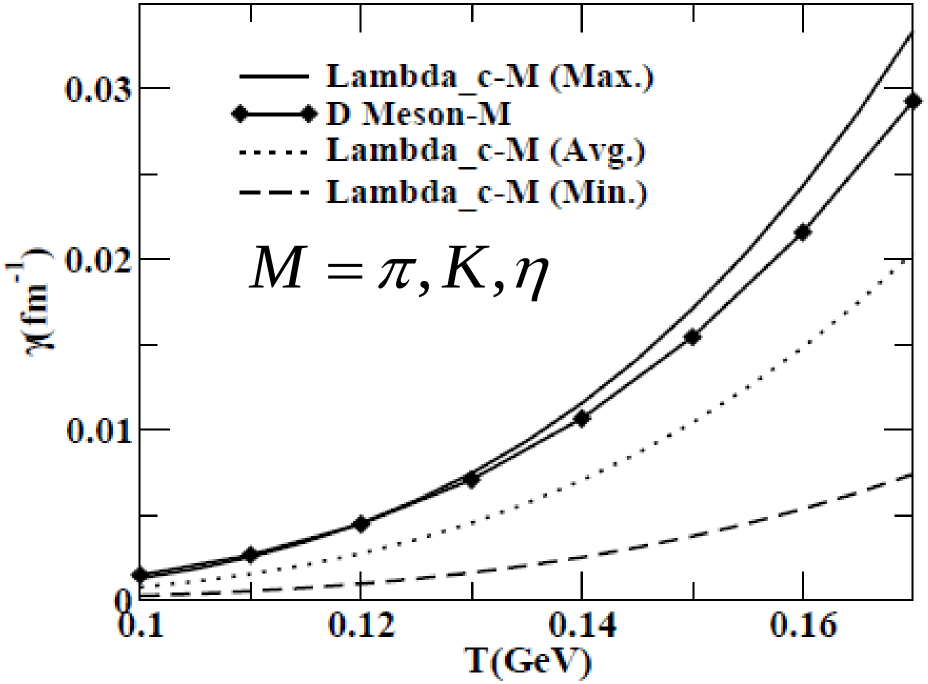


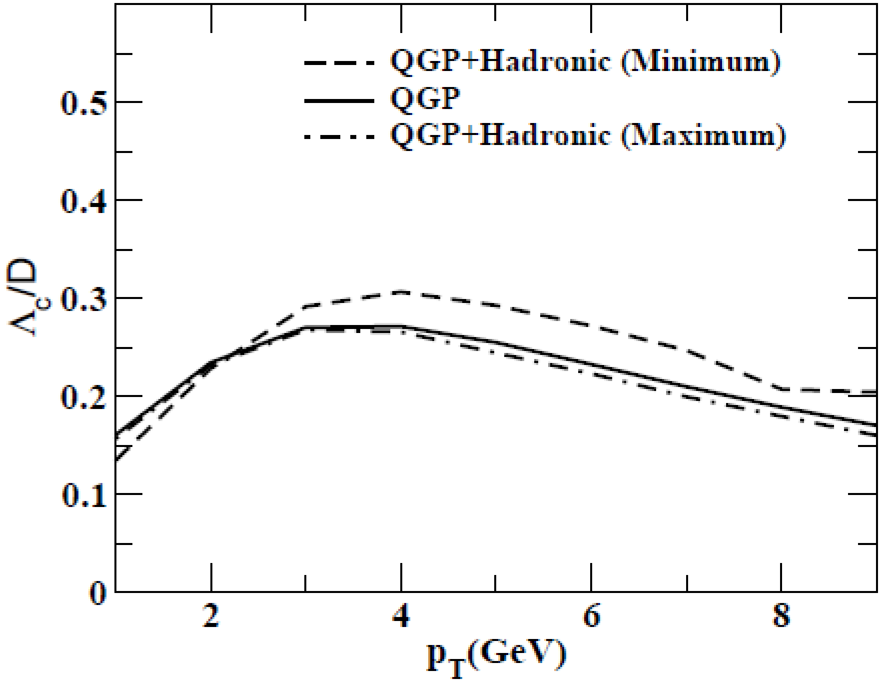


Das, Scardina, Plumari and Greco
PRC,90,044901(2014)

At fixed RAA Boltzmann approach generate larger v2 .
(depending on mD and M/T)

Heavy Baryon to Meson Ratio





Ghosh, Das, Greco, Sarkar and Alam
PRD,90, 054018 (2014)

Z. Liu, S. Zhu, PRD 86, 034009 (2012);
NPA 914,494 (2013

Lee, Ohnishi, Yasui, Yoo, Ko
PRL,222310,100(2008)

J/Psi in Hadronic Phase

$$J + V \to \eta_c \to J/\Psi + V$$

$$\eta_c + V \to J/\psi \to \eta_c + V$$

$$V = \rho, \omega, \phi$$

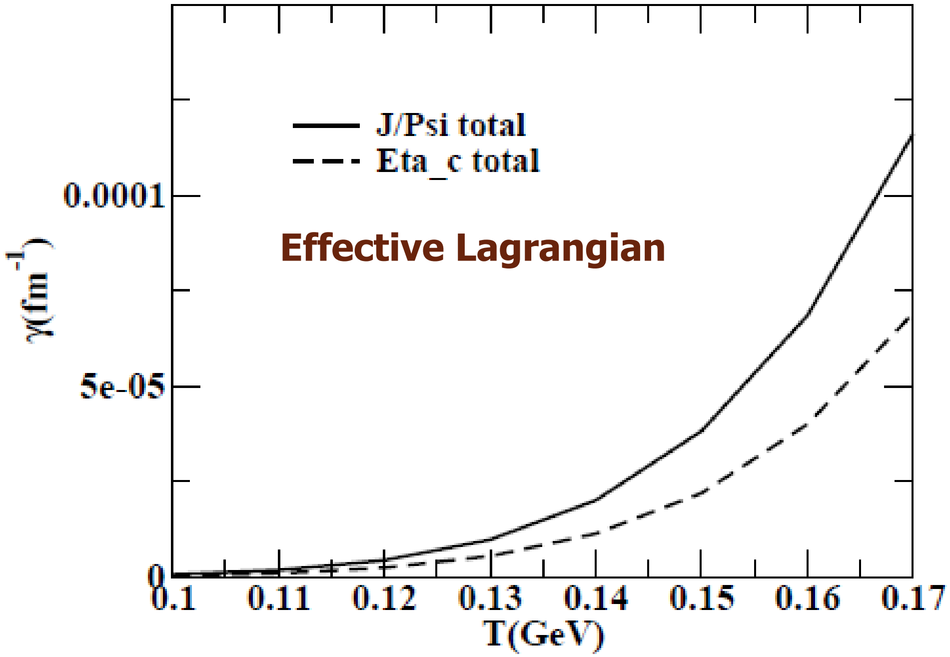
Haglin ,Gale, PRC 63, 065201(2001).

$$J/\psi\pi \to J/\psi\pi \qquad \eta_c\pi \to \eta_c\pi$$

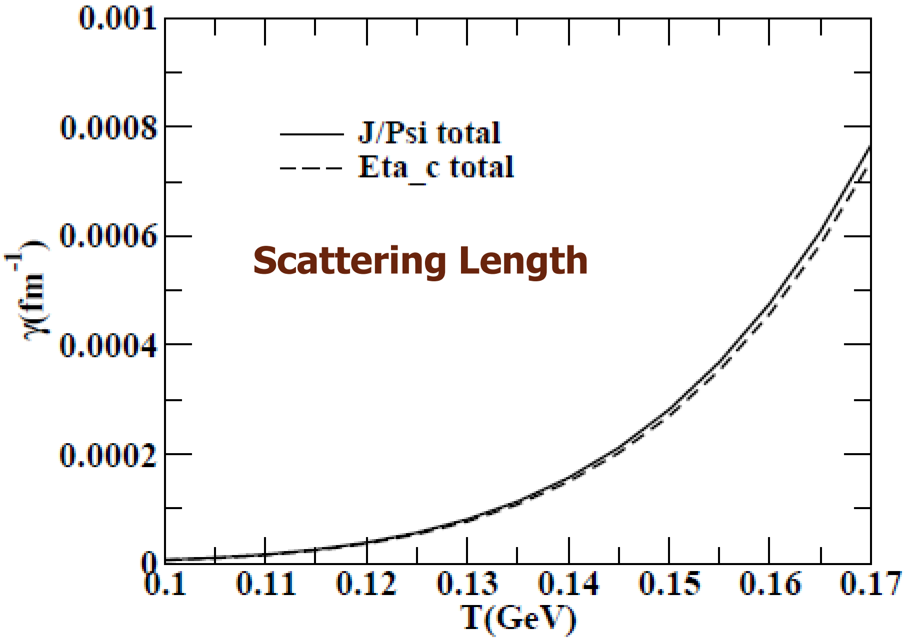
$$J/\psi\rho \to J/\psi\rho \qquad \eta_c\rho \to \eta_c\rho$$

$$J/\psi N \to J/\psi N \qquad \eta_c N \to \eta_c N$$

Yokokawa, Sasaki, Hatsuda and Hayashigaki
PRD 74, 034504 (2006)



Effective Lagrangian



Scattering Length

Mitra, Ghosh, Das, Sarkar and Alam
arXiv:1409.4652

Boltzmann Kinetic equation

$$\left(\frac{\partial}{\partial t}+\frac{P}{E}\frac{\partial}{\partial x}+\mathrm{F}.\frac{\partial}{\partial p}\right)f\left(x,\,p,\,t\right)=\left(\frac{\partial f}{\partial t}\right)_{col}$$

- The plasma is uniform ,i.e., the distribution function is independent of x.
- In the absence of any external force, F=0

$$\boxed{R(p,t)=\left(\frac{\partial f}{\Gamma\partial t}\right)_{col}=\int d^3k\left[\omega(p+k,k)f(p+k)-\omega(p,k)f(p)\right]}$$

$$\omega(p,k)=g\int\frac{d^3q}{(2\pi)^3}f'(q)v_{q,p}\sigma_{p,q\to p-k,q+k}$$ → is rate of collisions which change the momentum of the charmed quark from p to p-k

$$\omega(p+k,k)f(p+k)\approx\omega(p,k)f(p)+k.\frac{\partial}{\partial p}(\omega f)+\frac{1}{2}k_ik_j\frac{\partial^2}{\partial p_i\partial p_j}(\omega f)$$

$$\boxed{\frac{\partial \mathbf{f}}{\partial \mathbf{t}}=\frac{\partial}{\partial \mathbf{p_i}}\left[\mathbf{A_i}(\mathbf{p})\mathbf{f}+\frac{\partial}{\partial \mathbf{p_j}}\left[\mathbf{B_{ij}}(\mathbf{p})\mathbf{f}\right]\right]}$$

Fokker-Planck equation

B. Svetitsky PRD 37(1987)2484

where we have defined the kernels

, $\mathbf{A_i}=\int \mathbf{d^3k\,\omega(p,k)k_i}$ → **Drag Coefficient**

$\mathbf{B_{ij}}=\int \mathbf{d^3k\,\omega(p,k)k_ik_j}$ → **Diffusion Coefficient**

H. v. Hees and R. Rapp
arXiv:0903.1096

Fokker planck equation can be solved
Stocastically by Langevin eqation

$$dx_j=\frac{p_j}{E}dt$$

$$dp_j=-\Gamma p_jdt+\sqrt{dt}\,C_{jk}(t,\,p+\xi dp)\rho_k$$

Γ **is the drag force and** C_{ij} **is the stochastic force.**

I) LPM effect : Suppression of bremsstrahlung and pair production.

Formation length ($l_f = \frac{\hbar}{q_\perp}$) : The distance over which interaction is spread out

1) It is the distance required for the final state particles to separate enough that they act as separate particles.

2) It is also the distance over which the amplitude from several interactions can add coherently to the total cross section.

As q⊥ increase → ℓ_f reduce → Radiation drops proportional

S. Klein, Rev. Mod. Phys **71** (1999)1501

(II) Dead cone Effect : Suppression of radiation due to mass

$$\boxed{\frac{1}{\sigma}\frac{d^2\sigma}{dz\,d\theta^2} \sim C_F \frac{\alpha_s}{\pi}\frac{1}{z}\frac{\theta^2}{\left(\theta^2+4\gamma\right)^2}}$$

$$\textit{where } z = 2 - x_1 - x_2 \quad \textit{and} \quad \gamma = \frac{m^2}{s}$$

Where $x_1 = 2E_q/\sqrt{s}$ and $x_2 = 2E_{\bar{q}}/\sqrt{s}$ $\longrightarrow$ the energy fraction of the final state quark and anti-quark.

Radiation from heavy quarks suppress in the cone from θ =0 (minima) to θ=2 √γ (maxima)

Radiative Energy Loss

$$
\begin{aligned}
A_i^{2\to 2} &= \frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3 2E_q}\int\frac{d^3q'}{(2\pi)^3 2E_{q'}}\int\frac{d^3p'}{(2\pi)^3 2E_{p'}} \\
&\times \frac{1}{\gamma}\sum |M|^2_{2\to 2}(2\pi)^4\delta^4(p+q-p'-q')\hat{f}(\mathbf{q})(1\pm\hat{f}(\mathbf{q}'))(p-p')_i
\end{aligned}
$$

$$
\begin{aligned}
A_i^{2\to 3} &= \frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3 2E_q}\int\frac{d^3q'}{(2\pi)^3 2E_{q'}}\int\frac{d^3p'}{(2\pi)^3 2E_{p'}} \\
&\times \int\frac{d^3k_5}{(2\pi)^3 2E_5}\frac{1}{\gamma}\sum |M|^2_{2\to 3}(2\pi)^4\delta^4(p+q-p'-q'-k_5) \\
&\times \hat{f}(E_q)(1\pm\hat{f}(E_{q'}))(1+\hat{f}(E_5))\theta(\tau-\tau_F)\theta(E_p-E_5)(p-p')_i
\end{aligned}
$$

$$
\begin{aligned}
A_i^{2\to 3} &= A_i^{2\to 2}\times\int\frac{d^3k_5}{(2\pi)^3 2E_5}12g_s^2\frac{1}{k_\perp^2} \\
&\times \left(1+\frac{M^2}{s}e^{2y}\right)^{-2}[1+\hat{f}(E_5)]\theta(\tau-\tau_F)\theta(E_p-E_5)
\end{aligned}
$$

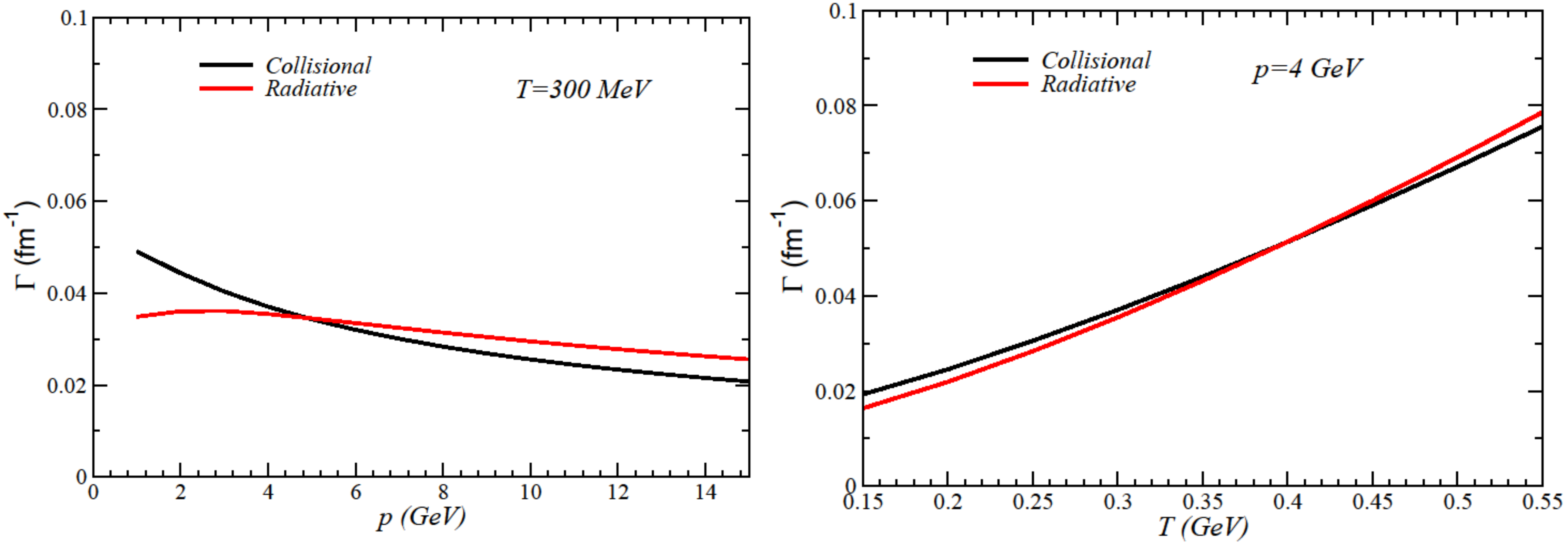
Mazumder, Bhattacharyya, Alam
PRD,89 (2014) 014002

$$
|M|^2_{2\to 3} = |M|^2_{2\to 2}\times 12g_s^2\frac{1}{k_\perp^2}\left(1+\frac{M^2}{s}e^{2y}\right)^{-2}
$$

Abir, Greiner, Martinez, Mustafa, Uphoff
Phys. Rev. D 85, 054012 (2012)

$$
\boxed{\Gamma_{eff} = \Gamma_{Coll} + \Gamma_{Rad}} \qquad \boxed{D_{eff} = D_{Coll} + D_{Rad}}
$$

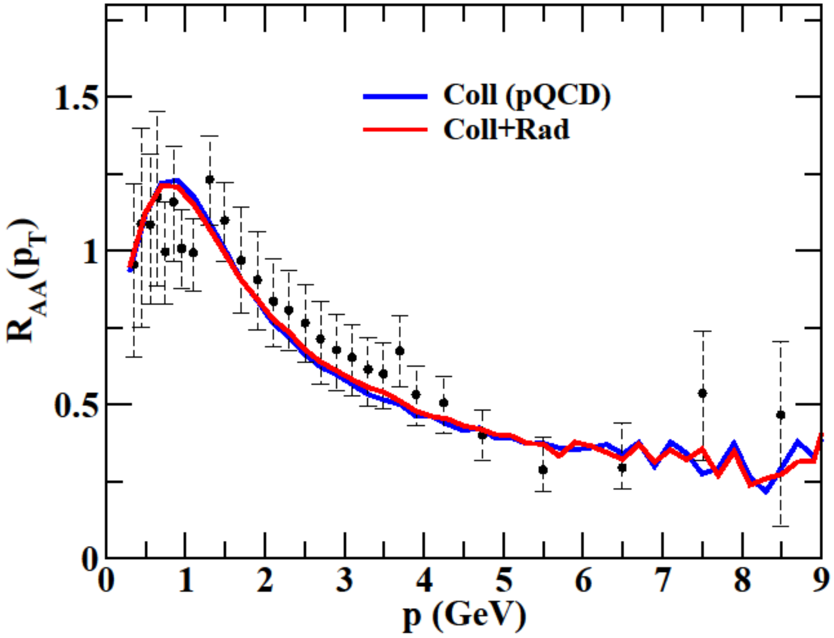
Radiative vs Collisional

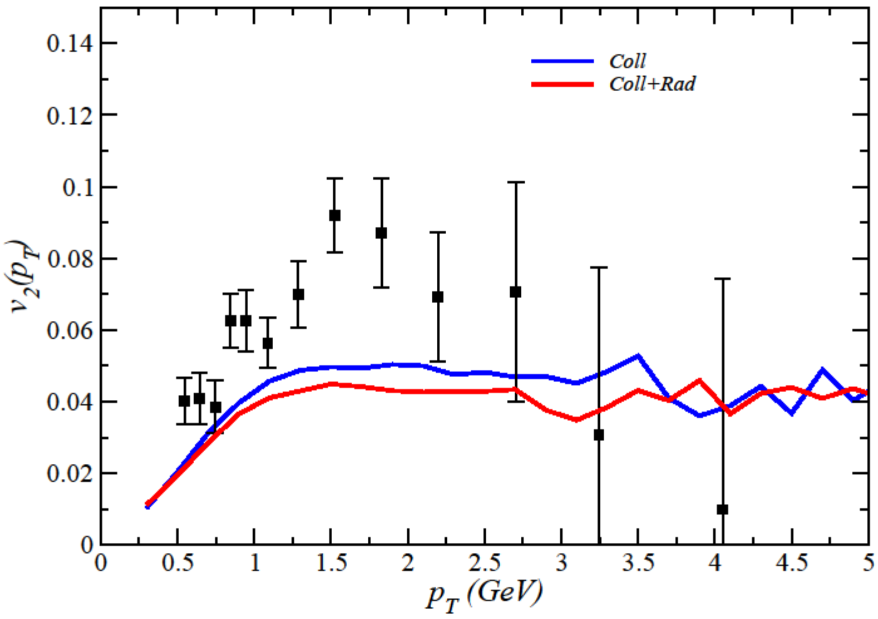


At High momentum radiative loss dominate over collisional loss

Das, Alam , Mohanty
PRC, 82,014908,2010

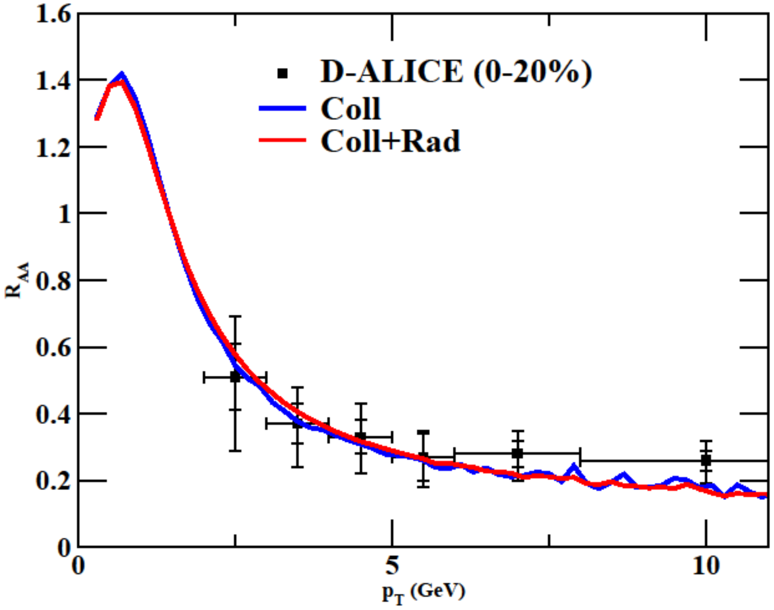
RAA and v2 @ RHIC (Collisional vs Collisonal+Radiative)

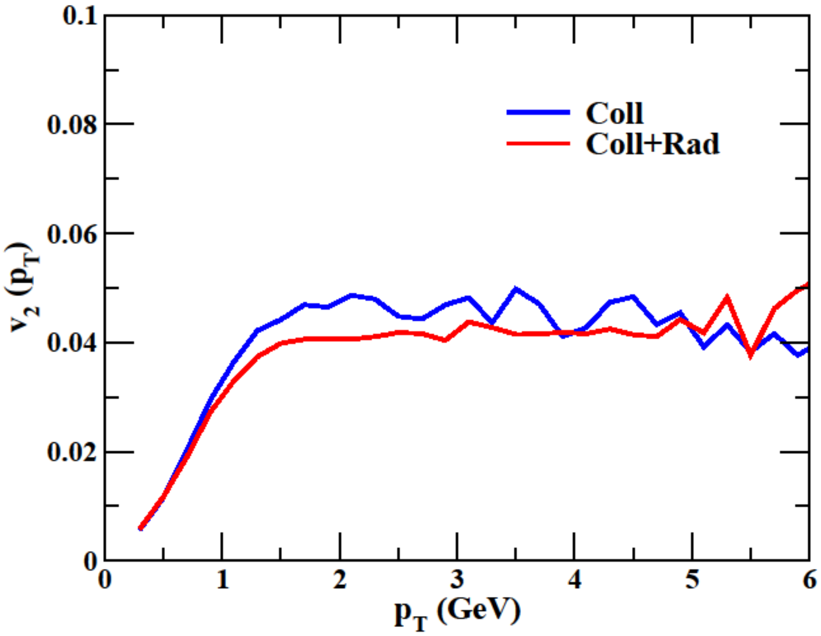




Das, Scadina, Plumari, Greco, Alam
Under prepairtion

RAA and v2 @ ALICE (Collisional vs Collisonal+Radiative)





Uphoff, Fochler, Xu, Greine
arXiv:1408.2964

Nahrgang, Aichelin, Bass ,Gossiaux, Werne
arXiv:1409.1464

Das, Scadina, Plumari, Greco, Alam
Under prepairtion