Heavy Quark Dynamics in QCD matter

Santosh Kumar Das

In collaboration with: Vincenzo Greco
Francesco Scadina
Salvatore Plumari
OUTLINE OF MY TALK

- Introduction

- Impact of $T$ dependence drag on heavy quarks observables
  1) Nuclear suppression factor
  2) Elliptic flow

- Heavy quark momentum evolution: Langevin vs Boltzmann

- Impact of pre-equilibrium phase on heavy quark observables.

- Impact of the electromagnetic filed on heavy quark dynamics
  (sizable heavy quark $v_1$)

- Summary and outlook
Heavy Quark & QGP

At very high density and temperature hadrons melt to a new phase of matter called Quark Gluon Plasma (QGP).

Produced by pQCD process (out of Equil.)

They go through all the QGP life time

No thermal production

\[ M_{c,b} \gg \Lambda_{QCD} \]

\[ \tau_{c,b} \gg \tau_{QGP} \]

\[ M_{c,b} \gg T_0 \]

SPS to LHC

\[ \sqrt{s} = 17.3\text{GeV} \text{ to } 2.76\text{TeV} \quad \sim 100 \text{ times} \]

\[ T_i = 200 \text{ MeV} \text{ to } 600 \text{ MeV} \quad \sim 3 \text{ times} \]
Boltzmann Kinetic equation

\[
\left( \frac{\partial}{\partial t} + \frac{p}{E} \frac{\partial}{\partial x} + F \cdot \frac{\partial}{\partial p} \right) f(x, p, t) = \left( \frac{\partial f}{\partial t} \right)_{col}
\]

\[
R(p, t) = \left( \frac{\partial f}{\partial t} \right)_{col} = \int d^3k [\omega(p + k, k)f(p + k) - \omega(p, k)f(p)]
\]

\[
\omega(p, k) = g \int \frac{d^3q}{(2\pi)^3} f'(q)v_{q,p} \sigma_{p,q \to p-k,q+k}
\]

is rate of collisions which change the momentum of the charmed quark from p to p-k

\[
\omega(p + k, k)f(p + k) \approx \omega(p, k)f(p) + k \cdot \frac{\partial}{\partial p}(\omega f) + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}(\omega f)
\]

\[
\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i} \left[ A_i(p)f + \frac{\partial}{\partial p_j} [B_{ij}(p)f] \right]
\]

where we have defined the kernels

\[ A_i = \int d^3k \omega(p, k)k_i \rightarrow \text{Drag Coefficient} \]

\[ B_{ij} = \int d^3k \omega(p, k)k_i k_j \rightarrow \text{Diffusion Coefficient} \]

- The plasma is uniform, i.e., the distribution function is independent of x.
- In the absence of any external force, F=0.

B. Svetitsky PRD 37(1987)2484
Langevin Equation

\[ dx_j = \frac{p_j}{E} \, dt \]
\[ dp_j = -\Gamma p_j \, dt + \sqrt{dt} \, C_{jk}(t, p + \xi dp) \rho_k \]

where \( \Gamma \) is the deterministic friction (drag) force
\( C_{ij} \) is stochastic force in terms of independent Gaussian-normal distributed random variable

\[ \rho = (\rho_x, \rho_y, \rho_z) \quad , \quad P(\rho) = \left( \frac{1}{2\pi} \right)^3 \exp\left( -\frac{\rho^2}{2} \right) \]

With \( <\rho_i(t)\rho_k(t')> = \delta(t-t')\delta_{jk} \)
\( \xi = 0 \) the pre-point Ito interpretation of the momentum argument of the covariance matrix.

H. v. Hees and R. Rapp
arXiv:0903.1096
Heavy flavor at RHIC

At RHIC energy heavy flavor suppression is similar to light flavor.

Simultaneous description of RAA and v2 is a tough challenge for all the models.
Heavy Flavors at LHC

JHEP 1209 (2012) 112

Pb-Pb, $\sqrt{s_{NN}} = 2.76$ TeV
Centrality 0-20%

- WHDG rad+coll
- POWLANG
- Aichelin et al, Coll+LPM rad
- BAMPS
- TAMU elastic
- UrQMD
Charm dynamics with upscaled pQCD cross section

Fokker-Plank for charm interaction in a hydro bulk

\[ D \propto \int d^3k \left| M_{g(q)c \rightarrow g(q)c} (k, p) \right|^2 k^2 \]

It’s not just a matter of pumping up pQCD elastic cross section: too low \( R_{AA} \) or too low \( v_2 \)

Moore & Teaney, PRC71 (2005)

Data-2004

Data
Time evolution of Heavy quarks observables

Das, Scardina, Plumari, Greco
arXiv:1509:06307

RAA and dN/dphi_ccbar developed during the early stage of the evolution $T_i$

$V_2$ developed during the later stage of the evolution $T_c$

$T$ dependence of the interaction i.e. the transport coefficients are the essential ingredient for the simultaneous description of HQ observables.
T- dependence of the Drag Coefficient

Drag Coefficient

\[ \gamma(T) = \frac{4\pi}{11\ln(2\pi T\Lambda)} \]

\[ m_D^2 = 4\pi\alpha_{QCD}(T)T^2 \]

\[ \gamma_{AdS/CFT} = k \frac{T^2}{M} \]

\[ g_{QPM}(T) = \left(\frac{48\pi^2}{(11N_c - 2N_f)\ln \left(\frac{T}{T_c} - \frac{T_s}{T_c}\right)}\right)^{\frac{1}{2}} \]

\[ m_q^2 = \frac{1}{6} \left(N_c + \frac{1}{2}N_f\right) g^2 T^2 \]

\[ m_g^2 = \frac{N_c^2 - 1}{8N_c} g^2 T^2 \]

\[ \alpha_{QPM}(T), \quad m_{q,g} = 0 \]

we mean simply the coupling of the QPM, but with a bulk of massless q and g.

**pQCD (Cambridge)**

\[ \alpha_{pQCD} = \frac{4\pi}{11\ln(2\pi T\Lambda)} \]

\[ m_D^2 = 4\pi\alpha_{pQCD}(T)T^2 \]

**AdS/CFT**

\[ \gamma_{AdS/CFT} = k \frac{T^2}{M} \]

Quasi-Particle-Model (fit to IQCD & P)

\[ g_{QPM}(T) = \left(\frac{48\pi^2}{(11N_c - 2N_f)\ln \left(\frac{T}{T_c} - \frac{T_s}{T_c}\right)}\right)^{\frac{1}{2}} \]

\[ m_q^2 = \frac{1}{6} \left(N_c + \frac{1}{2}N_f\right) g^2 T^2 \]

\[ m_g^2 = \frac{N_c^2 - 1}{8N_c} g^2 T^2 \]

\[ \alpha_{QPM}(T), \quad m_{q,g} = 0 \]

we mean simply the coupling of the QPM, but with a bulk of massless q and g.

**References**

Gubser, PRD, 74, 126005 (2006)


Das and Davody, PRC, 89, 054912 (2014)


Plumari at al. PRD, 84, 094004 (2011)
Light flavor sector:

Liao, Shuryak, PRL102 (2009) 202302
Scardina, Toro, Greco, PRC, 82 (2010) 054901
Xu, Liao, Gyulassy, arXiv:1411.3637

Das, Scardina, Plumari, Greco
PLB 747 (2016) 260-264
RAA and v2 @ ALICE

Das, Scardina, Plumari, Greco
PLB 747 (2015) 260-264

Eskola, Paukkunen, Salgado
JHEP,0807, 102 (2008)

S. Cao, G.-Y. Qin, and S. A. Bass
PRC 88(2013)044907
Heavy quark momentum evolution: Langevin vs Boltzmann

\[
\omega(p + k, k) f(p + k) \approx \omega(p, k) f(p) + k \cdot \frac{\partial}{\partial p} (\omega f) + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j} (\omega f)
\]

Boltzmann Equation

Fokker Planck

It will be interesting to study both the equation in an identical environment to ensure the validity of this assumption at different momentum transfer and their subsequent effects on RAA and v2.

Langevin dynamics:

\[
dx_j = \frac{p_j}{E} \, dt
\]

\[
dp_j = -\Gamma p_j \, dt + \sqrt{dt} C_{jk} (t, p + \xi dp) \rho_k
\]

\(\Gamma\) is the deterministic friction (drag) force

\(C_{ij}\) is stochastic force in terms of independent Gaussian-normal distributed random variable.

H. v. Hees and R. Rapp
arXiv:0903.1096
Transport theory

\[ p^\mu \partial_\mu f(x,p) = C_{22} \]

We consider two body collisions

\[ C_{22} = \frac{1}{2E_1} \int \frac{d^3p_2}{(2\pi)^3} \frac{d^3p'_1}{(2\pi)^3} \frac{1}{\nu} \int \frac{d^3p'_2}{(2\pi)^3} \frac{d^3p'_2}{(2\pi)^3} f_1 f_2 |M_{1'2' \rightarrow 12}|^2 (2\pi)^4 \delta^4(p_1 + p'_2 - p_1 - p_2) \]

\[ \Delta t \rightarrow 0 \]
\[ \Delta^3x \rightarrow 0 \]

Exact solution

Collision integral is solved with a local stochastic sampling

[ Z. Xhu, et al. PRC71(04)]
Greco et al PLB670, 325 (08)]
Momentum evolution starting from a $\delta$ (Charm) in a Box

In case of Langevin the distributions are Gaussian as expected by construction.

In case of Boltzmann the charm quarks does not follow the Brownian motion.

Das, Scardina, Plumari and Greco
PRC, 90, 044901 (2014)
Evolution: Boltzmann vs Langevin (Charm)

Momentum evolution starting from a $\delta$ (Charm) in a Box

\[
\frac{dN}{d^3p_{\text{initial}}} = \delta(p - 2\text{GeV})
\]

Langevin

Boltzmann

In case of Langevin the distributions are Gaussian as expected by construction

In case of Boltzmann the charm quarks follow the Brownian motion: At Low Momentum.
Momentum evolution starting from a $\delta$ (Bottom)

Langevin

In a Box

Boltzmann

For bottom quarks it works better.
At fixed RAA Boltzmann approach generate larger v2.
(depending on mD and M/T)

With isotropic cross section one can describe both RAA and V2 simultaneously within the Boltzmann approach!
Summary on the build-up of $v_2$ at fixed $R_{AA}$

$R_{AA}$ and $V_2$ are correlated but still one can have $R_{AA}$ about the same while $V_2$ can change up to a factor 2-3 

$\gamma(T)$ + Boltzmann dynamics + hadronization + hadronic phase

Das, Scardina, Plumari, Greco
arXiv:1509:06307
Impact of Pre-equilibrium Phase

It will be interesting to study the role of Pre-equilibrium on $R_{AA}$ and $v_2$.

Drescher, Nara PRC 75, 034905 (2007)
Hirano, Nara Y PRC 79, 064904 (2009)
Schenke, Tribedy, Venugopalan PRC 89, 024901 (2013).
Ruggieri, Scardina, Plumari, Greco PRC 89, 054914 (2014)

Das, Ruggieri, Mazumder, Greco, Alam JPG 42 (2015)095108
Pre-equilibrium phase affect the RAA significantly. Impact on v2 is nominal.
RAA and $v_2$ @ RHIC

(Au+Au@200AGeV, $b=8$ fm)

One can mock the impact of the pre-equilibrium phase with early locally thermalized QGP!
Impact of EM field on heavy quark dynamics at LHC

\[ dp_j = -\Gamma p_j dt + \sqrt{dt} C_{jk}(t, p + \xi dp) \rho_k + F_{ext} dt \]

\[ F_{ext} = q(E' + \nu \times B') \]

\[ E' = \gamma (E + \nu \times B) - (\gamma - 1) (E \cdot \hat{v}) \hat{v} \]

\[ B' = \gamma (B - \nu \times E) - (\gamma - 1) (B \cdot \hat{v}) \hat{v} \]

We consider both E and B.
Bx=Bz=0
And Ey=Ez=0

\[ \nu_1 = \frac{p_x}{p_T} \]

Gusoy, Kharzeev and Rajagopal
PRC 89, 054905 (2014)
The simulation is done starting from $\tau_0=0.2$ fm. The sign of $v_1$ due to $B$ is decided by $v \times B$. 

At RHIC the $v_1$ could be around 1-2% on which we are working currently.

Das, Plumari, Chartarjee, Scardina, Greco, Alam 
Under preparation
Summary & Outlook ......

- $R_{AA} - v_2$ of charm quarks seems to indicate:
  - Drag about constant in $T$ or weak $T$ dependence to describe both $R_{AA}$ and $v_2$ simultaneously.
  - Simultaneous study of $R_{AA}$ and $v_2$ can put constrain on various energy loss models.
  - Boltzmann dynamics more efficient for $v_2$ even at fixed $R_{AA}$.
  - Hadronization by coalescence of heavy quarks as well as the role of hadronic medium modify $R_{AA}$ vs $v_2$ relation toward a better agreement with the data.
  - Pre-equilibrium phase have significant impact on $R_{AA}$ but one can mock it with early locally thermalized QGP phase.
  - Heavy quark have a finite $v_1$ due to the presence of strong EM field created at heavy ion collision which can be measurable.

- Implementation of all these effects including radiation within a single framework (within Boltzmann equation) is going on........
Thank You
$R_{AA}$ and $v_2$ at RHIC at $m_D = g T$

At fixed $R_{AA}$ Boltzmann approach generate larger $v_2$.
(depending on $m_D$ and $M/T$)

Das, Scardina, Plumari and Greco
PRC, 90, 044901 (2014)
Heavy Baryon to Meson Ratio

\[ M = \pi, K, \eta \]

Z. Liu, S. Zhu, PRD 86, 034009 (2012); NPA 914,494 (2013)

Lee, Ohnishi, Yasui, Yoo, Ko PRL,222310,100(2008)

Ghosh, Das, Greco, Sarkar and Alam PRD,90, 054018 (2014)
**J/Psi in Hadronic Phase**

\[ J + V \rightarrow \eta_c \rightarrow J/\Psi + V \]

\[ \eta_c + V \rightarrow J/\psi \rightarrow \eta_c + V \]

\[ V = \rho, \omega, \phi \]

Haglin, Gale, PRC 63, 065201(2001).

\[ J/\psi \pi \rightarrow J/\psi \pi \quad \eta_c \pi \rightarrow \eta_c \pi \]

\[ J/\psi \rho \rightarrow J/\psi \rho \quad \eta_c \rho \rightarrow \eta_c \rho \]

\[ J/\psi N \rightarrow J/\psi N \quad \eta_c N \rightarrow \eta_c N \]

Yokokawa, Sasaki, Hatsuda and Hayashigaki

PRD 74, 034504 (2006)

---

**Effective Lagrangian**

**Scattering Length**

Mitra, Ghosh, Das, Sarkar and Alam

arXiv:1409.4652
Boltzmann Kinetic equation

\[
\left( \frac{\partial}{\partial t} + \frac{p}{E} \frac{\partial}{\partial x} + \textbf{F}. \frac{\partial}{\partial p} \right) f(x, p, t) = \left( \frac{\partial f}{\partial t} \right)_{\text{col}}
\]

\[ R(p, t) = \left( \frac{\partial f}{\partial t} \right)_{\text{col}} = \int d^3k \left[ \omega(p + k, k) f(p + k) - \omega(p, k) f(p) \right] \]

\[
\omega(p, k) = g \int \frac{d^3q}{(2\pi)^3} f'(q) v_{q,p} \sigma_{p,q \rightarrow p-k,q+k}
\]

is rate of collisions which change the momentum of the charmed quark from \( p \) to \( p-k \)

\[
\omega(p, k) f(p) \approx \omega(p, k) f(p) + k \frac{\partial}{\partial p} (\omega f) + \frac{1}{2} k i k_j \frac{\partial^2}{\partial p_i \partial p_j} (\omega f)
\]

\[
\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i} \left[ A_i(p) f + \frac{\partial}{\partial p_j} \left[ B_{ij}(p) f \right] \right]
\]

where we have defined the kernels,

\[ A_i = \int d^3k \omega(p, k) k_i \rightarrow \text{Drag Coefficient} \]

\[ B_{ij} = \int d^3k \omega(p, k) k_i k_j \rightarrow \text{Diffusion Coefficient} \]

Fokker-Planck equation

B. Svetitsky PRD 37(1987)2484

H. v. Hees and R. Rapp
arXiv:0903.1096

Fokker planck equation can be solved Stocastically by Langevin eqation

\[
dx_j = \frac{p_j}{E} \, dt
\]

\[
dp_j = -\Gamma p_j \, dt + \sqrt{dt} C_{jk}(t, p + \xi dp) \rho_k
\]

\( \Gamma \) is the drag force and \( C_{ij} \) is the stochastic force.
I) LPM effect: Suppression of bremsstrahlung and pair production.

Formation length \( l_f = \frac{\hbar}{q_{\perp}} \): The distance over which interaction is spread out

1) It is the distance required for the final state particles to separate enough that they act as separate particles.

2) It is also the distance over which the amplitude from several interactions can add coherently to the total cross section.

As \( q_{\perp} \) increase \( \rightarrow l_f \) reduce \( \rightarrow \) Radiation drops proportional

S. Klein, Rev. Mod. Phys 71 (1999)1501

(II) Dead cone Effect: Suppression of radiation due to mass

\[
\frac{1}{\sigma} \frac{d^2 \sigma}{dz d\theta^2} \sim C_F \frac{\alpha_s}{\pi} \frac{1}{z} \frac{\theta^2}{(\theta^2 + 4\gamma)^2}
\]

where \( z = 2 - x_1 - x_2 \) and \( \gamma = \frac{m^2}{s} \)

Where \( x_1 = 2E_q/\sqrt{s} \) and \( x_2 = 2E_{\bar{q}}/\sqrt{s} \) \( \rightarrow \) the energy fraction of the final state quark and anti-quark.

Radiation from heavy quarks suppress in the cone from \( \theta = 0 \) (minima) to \( \theta = 2\sqrt{\gamma} \) (maxima)
Radiative Energy Loss

\[ A_{i}^{2\rightarrow 2} = \frac{1}{2E_{p}} \int \frac{d^{3}q}{(2\pi)^{3}2E_{q}} \int \frac{d^{3}q'}{(2\pi)^{3}2E_{q'}} \int \frac{d^{3}p'}{(2\pi)^{3}2E_{p'}} \times \frac{1}{\gamma} \sum |M|_{2\rightarrow 2}^{2}(2\pi)^{4}\delta^{4}(p + q - p' - q') \hat{f}(q)(1 \pm \hat{f}(q'))(p - p')_{i} \]

\[ A_{i}^{2\rightarrow 3} = \frac{1}{2E_{p}} \int \frac{d^{3}q}{(2\pi)^{3}2E_{q}} \int \frac{d^{3}q'}{(2\pi)^{3}2E_{q'}} \int \frac{d^{3}p'}{(2\pi)^{3}2E_{p'}} \times \int \frac{d^{3}k_{5}}{(2\pi)^{3}2E_{5}} \frac{1}{\gamma} \sum |M|_{2\rightarrow 3}^{2}(2\pi)^{4}\delta^{4}(p + q - p' - q' - k_{5}) \times \hat{f}(E_{q})(1 \pm \hat{f}(E_{q'}))(1 + \hat{f}(E_{5}))\theta(\tau - \tau_{F})\theta(E_{p} - E_{5})(p - p')_{i} \]

\[ A_{i}^{2\rightarrow 3} = A_{i}^{2\rightarrow 2} \times \int \frac{d^{3}k_{5}}{(2\pi)^{3}2E_{5}} 12g_{s}^{2} \frac{1}{k_{\perp}^{2}} \times \left(1 + \frac{M^{2}}{s}e^{2y}\right)^{-2} \left[1 + \hat{f}(E_{5})\right]\theta(\tau - \tau_{F})\theta(E_{p} - E_{5}) \]

\[ |M|_{2\rightarrow 3}^{2} = |M|_{2\rightarrow 2}^{2} \times 12g_{s}^{2} \frac{1}{k_{\perp}^{2}} \left(1 + \frac{M^{2}}{s}e^{2y}\right)^{-2} \]

\[ \Gamma_{\text{eff}} = \Gamma_{\text{Coll}} + \Gamma_{\text{Rad}} \]

\[ D_{\text{eff}} = D_{\text{Coll}} + D_{\text{Rad}} \]
At High momentum, radiative loss dominate over collisional loss.

Das, Alam, Mohanty
PRC, 82, 014908, 2010
RAA and v2 @ RHIC (Collisional vs Collisonal+Radiative)

Das, Scadina, Plumari, Greco, Alam
Under prepairtion
RAA and $v_2$ @ ALICE (Collisional vs Collisonal+Radiative)

**Graphs:**
- **Left:** $D$-ALICE (0-20%)
  - D-ALICE (0-20%)
  - Coll
  - Coll+Rad

- **Right:** $v_2(p_T)$
  - Coll
  - Coll+Rad

**References:**
- Uphoff, Fochler, Xu, Greine
  - arXiv:1408.2964
- Nahrgang, Aichelin, Bass, Gossiaux, Werne
  - arXiv:1409.1464
- Das, Scadina, Plumari, Greco, Alam
  - Under preparation