Vorticity and Λ polarization in event-by-event (3+1)D viscous hydrodynamics

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RH.Fang, LG.Pang, Q.Wang & XN.Wang arXiv :1604.04036

LG.Pang, H.Petersen, Q.Wang & XN.Wang arXiv:1605.04024

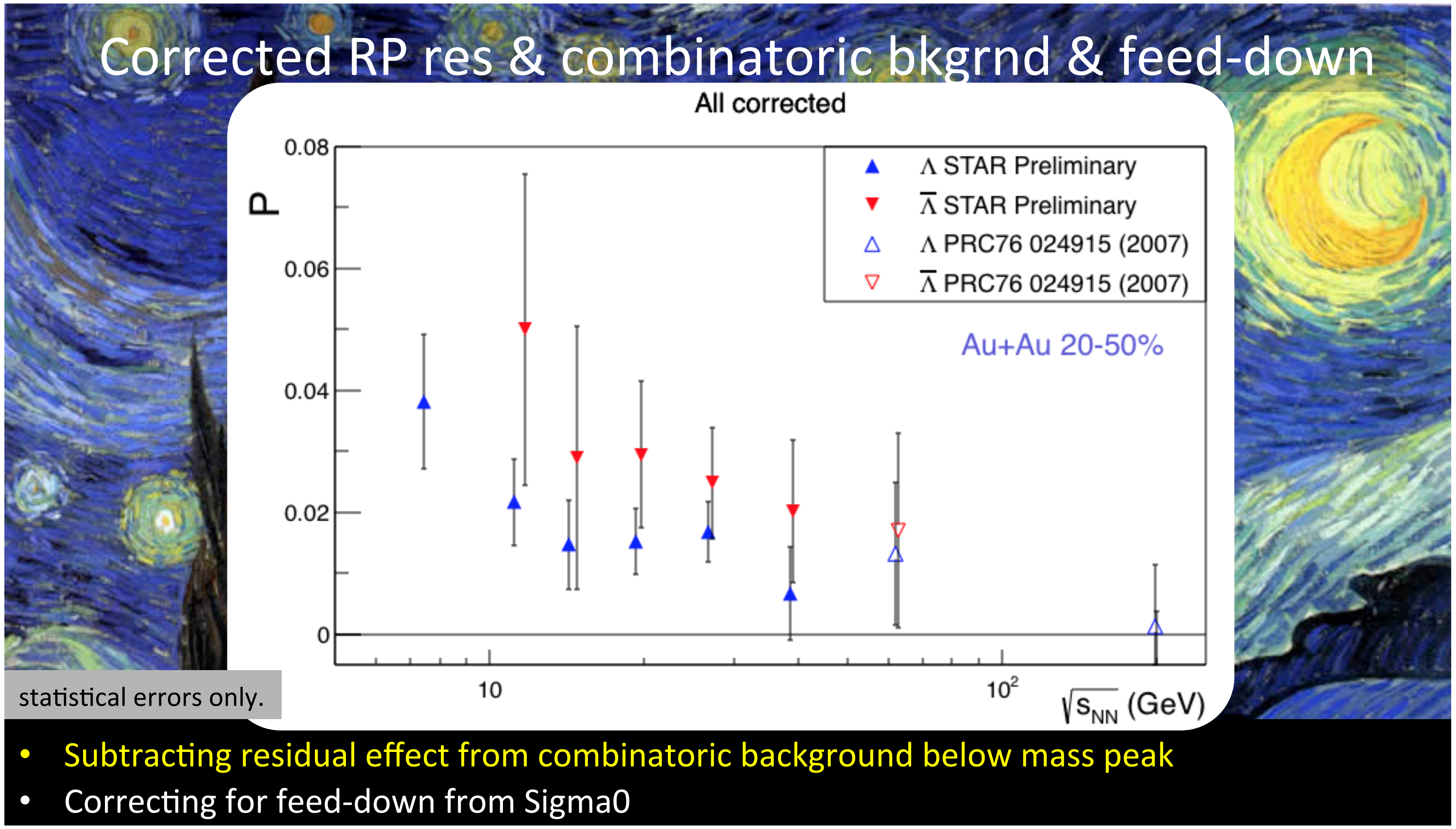
FIAS Frankfurt Institute for Advanced Studies

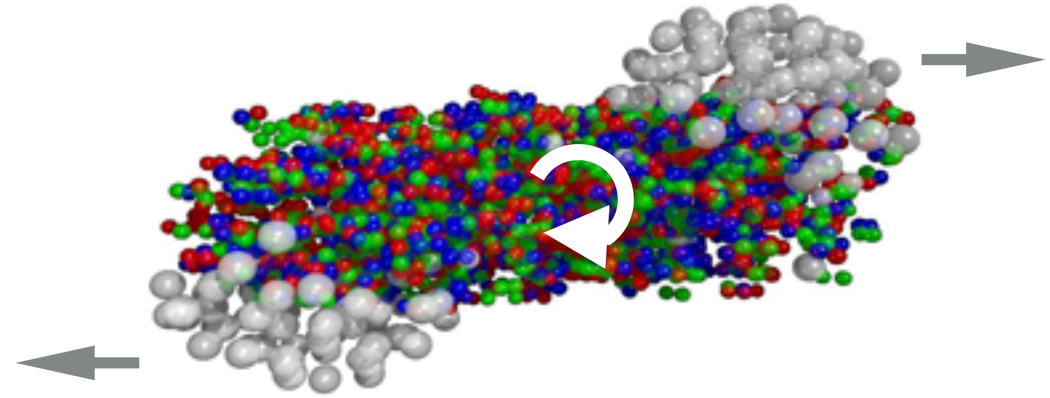
HELMHOLTZ | ASSOCIATION

HIC for FAIR Helmholtz International Center

Polarization in A+A

Mike Lisa, UCLA vorticity Workshop Feb 2016





$$\Lambda(uds) \to \pi^- + p$$

$$\bar{\Lambda} > \Lambda > 0$$

- Global Polarization: more fermions have spin direction parallel or anti-parallel to the direction of global orbital angular momentum

- How to build the bridge between angular momentum and spin polarization?

Z. T. Liang, X. N. Wang, PRL. 94 (2005) 102301, F. B., F. Piccinini, Ann. Phys. 323 (2008) 2452; F. B., F. Piccinini, J. Rizzo, PRC 77 (2008) 024906

Spin polarization in Equilibrium

Dirac Eq. $$[\gamma^\mu(i\partial_\mu + e_q A_\mu) - m]\,\psi(x) = 0$$

Spin: vorticity coupling Magnetic coupling

$$\delta E_s = \frac{\hbar}{2}\mathbf{n}\cdot\omega + e_q\hbar\frac{\mathbf{n}\cdot\mathbf{B}}{E_p}$$

$$\Pi = \frac{1}{2}\int\frac{d^3p}{(2\pi)^3}\left[f(E_p - \delta E_s) - f(E_p + \delta E_s)\right]$$

$$\approx -\int\frac{d^3p}{(2\pi)^3}\delta E_s\frac{\partial f(E_p)}{\partial E_p}$$

gradient expansion

fermion susceptibility

Becattini & Ferroni, EJPC 52 (2007) 597, Betz, Gyulassy & Torrieri, PRC 76 (2007) 044901, Becattini, Piccinini & Rizzo, PRC 77 (2008) 024906, Beccatini, Csernai & Wang, PRC 87 (2013) 034905, Xie, Glastad & Csernai, PRC 92 (2015) 064901, Deng & Huang, arXiv 1603.06117

Quantum Kinetic Theory

S.Pu, JH.Gao, ZT.Liang, Q.Wang & XNW, PRL 109(2012) 232301

Wigner function:

RH.Fang, LG.Pang, Q.Wang & XN.Wang arXiv :1604.04036

$$W_{\alpha\beta}(x,p) = \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y} \langle \bar{\psi}_\beta(x+\frac{1}{2}y)\psi_\alpha(x-\frac{1}{2}y)\rangle$$

Quantum Kinetic Equation for Wigner function:

$$\left[\gamma_\mu \left(p^\mu + \frac{i}{2}\hbar\partial_x^\mu\right) - m\right] W(x,p) = 0.$$

Polarization from spin-vorticity coupling:

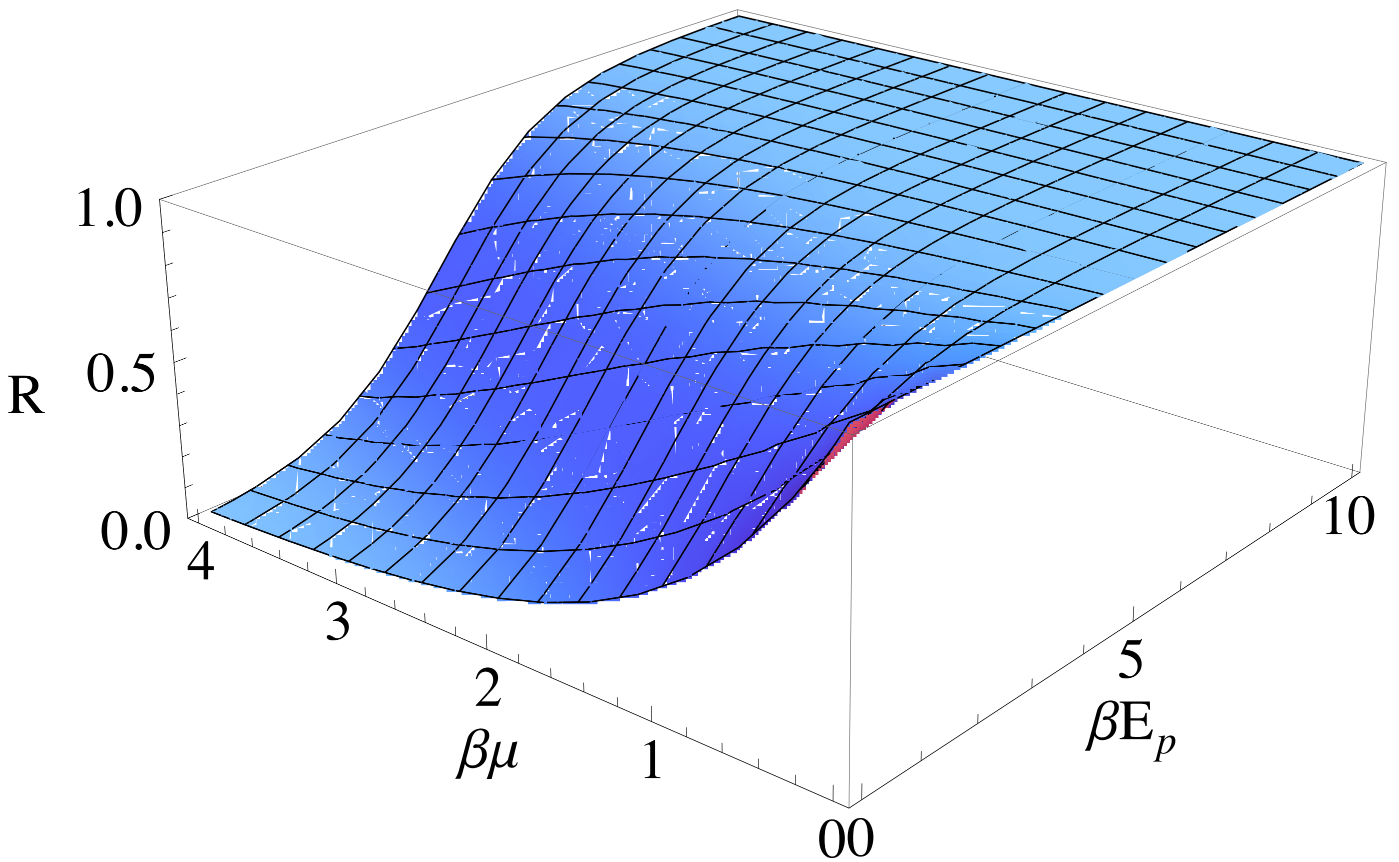
$$\Pi^\alpha(x) = \frac{1}{2}\hbar\omega^\alpha \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{e^{\beta(E_p-\mu)}}{[e^{\beta(E_p-\mu)}+1]^2} + \frac{e^{\beta(E_p+\mu)}}{[e^{\beta(E_p+\mu)}+1]^2} \right\}$$

vorticity fermion anti-fermion

Polarization fermion/anti-fermion

$$R = \frac{[\Pi/\rho]_{\text{fermion}}}{[\Pi/\rho]_{\text{anti-fermion}}}$$

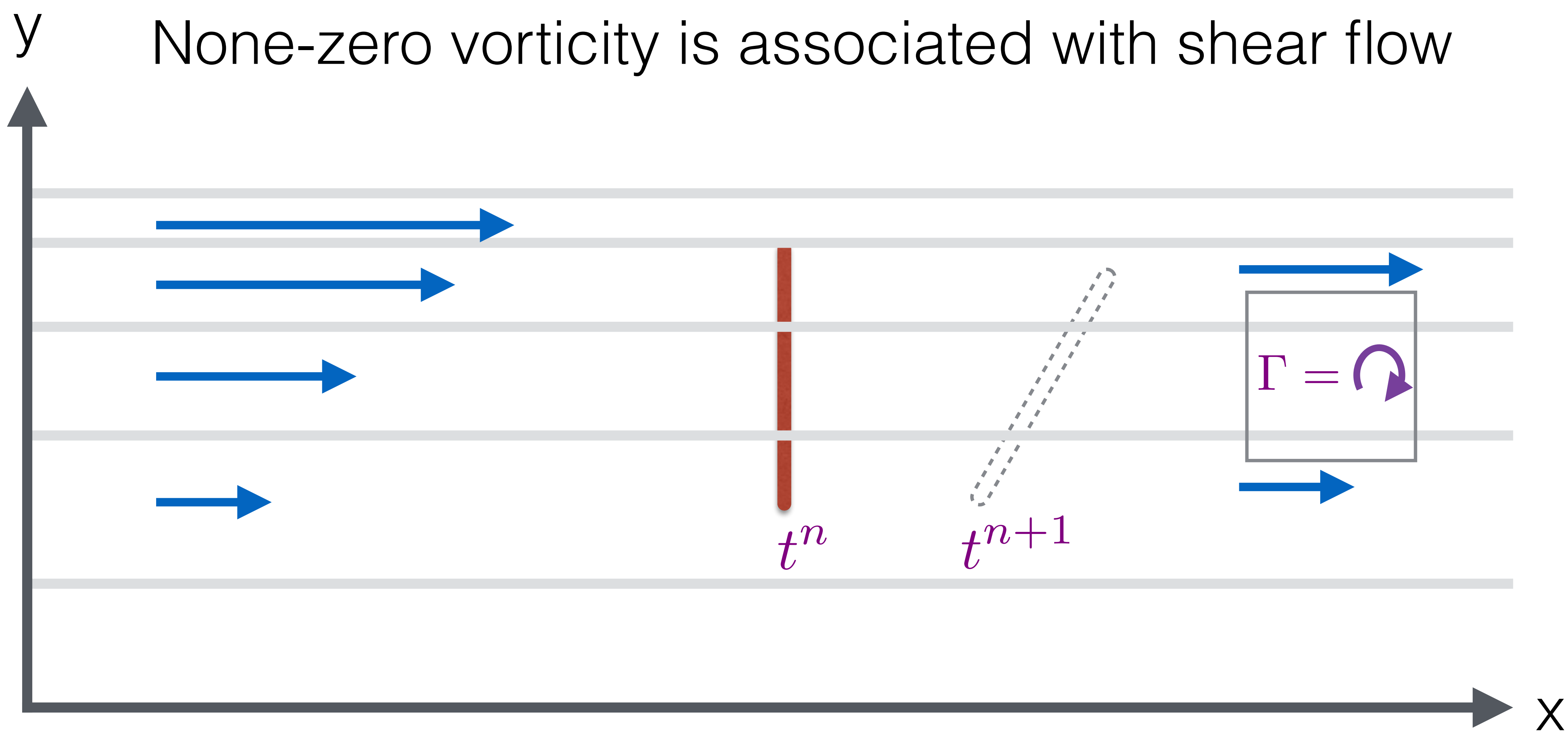
$$\Pi \approx -\int \frac{d^3p}{(2\pi)^3} \delta E_s \frac{\partial f(E_p)}{\partial E_p}$$



- Because of Pauli blocking, anti-fermions are easier to be polarized than fermions.
- The effect is strong if we consider the constituent quark model— the polarization of Lambda equals to the polarization of strange quark.

RH.Fang, LG.Pang, Q.Wang & XN.Wang arXiv :1604.04036

Vorticity and shear flow

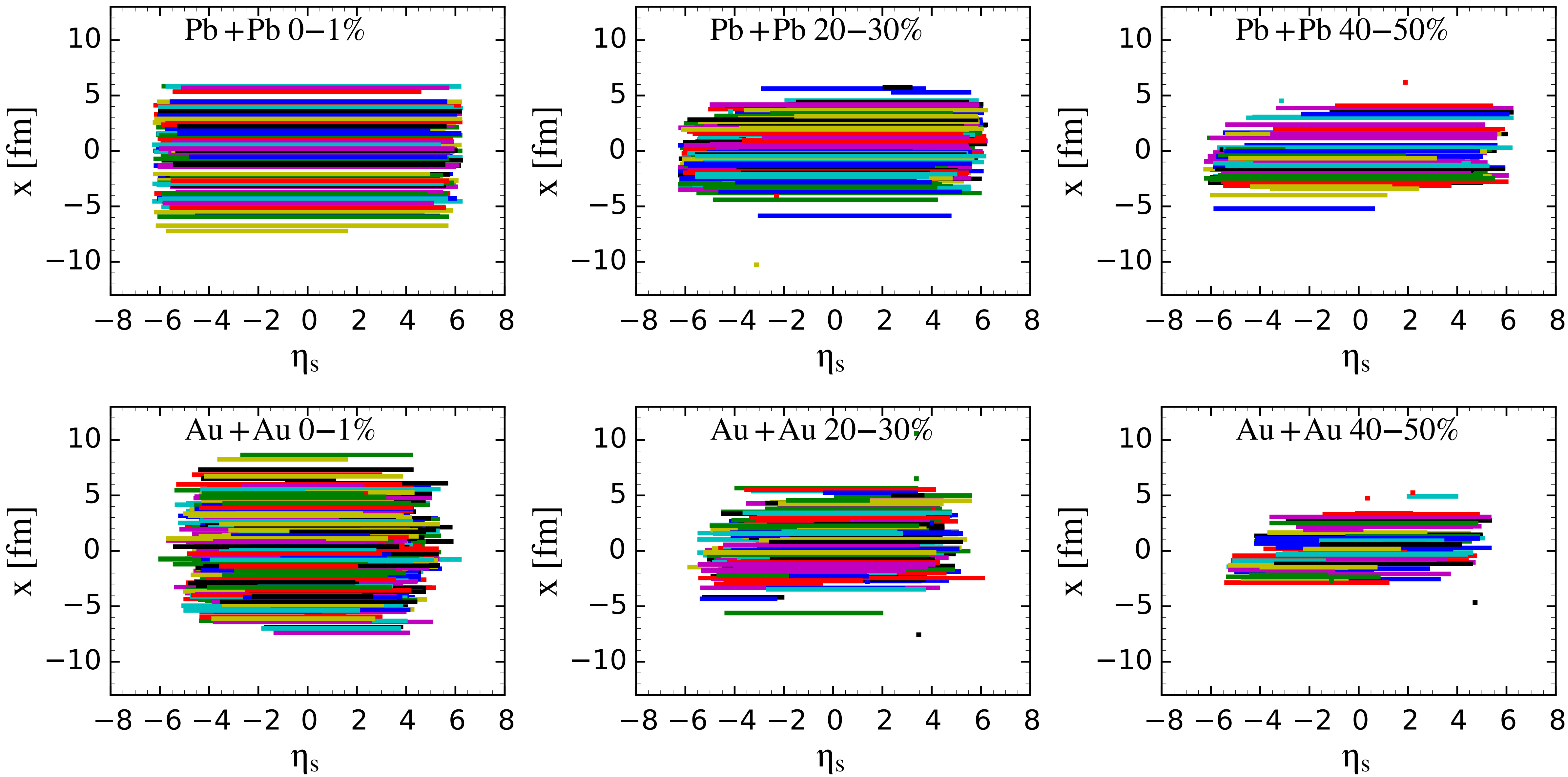


vorticity: $\omega_z = \partial_x v_y - \partial_y v_x$

circulation: $\Gamma = \oint \vec{v} \cdot d\vec{r} = \iint \nabla \times \vec{v} \cdot d\vec{A} = \iint \vec{\omega} \cdot d\vec{A}$

Shear flow in QGP

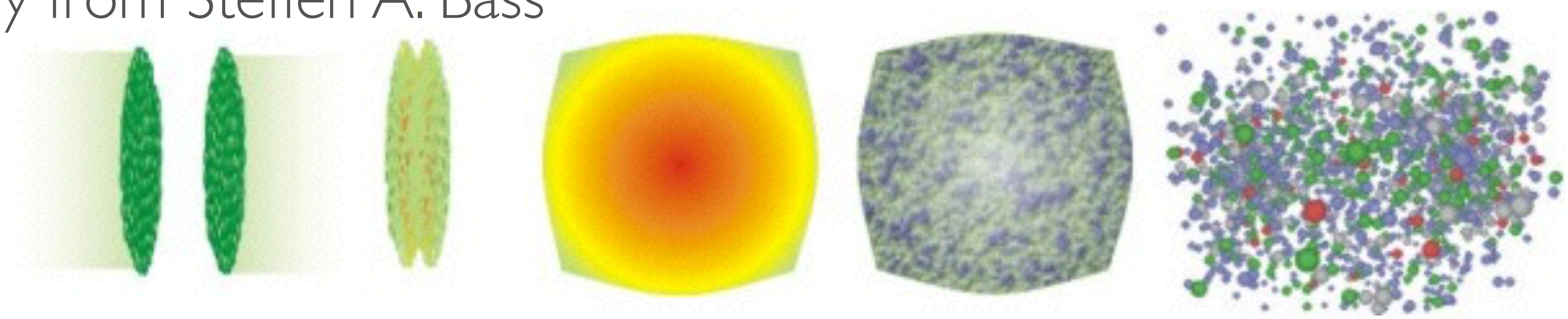
The initial condition for (3+1)D hydrodynamics is given by HIJING/AMPT model. Where the length of soft strings is sensitive to beam energy and centrality.



L.G.Pang, H.Petersen, G.Y.Qin, V.Roy, X.N.Wang, Eur.Phys.J. A52 (2016) no.4, 97

Method: (3+1)D viscous hydro

Fig originally from Steffen A. Bass



$$\nabla_\mu T^{\mu\nu} = 0 \tag{1}$$

$$\Delta^{\mu\nu\alpha\beta} u^\lambda \nabla_\lambda \pi_{\alpha\beta} = -\frac{\pi^{\mu\nu} - \pi^{\mu\nu}_{\rm NS}}{\tau_\pi} - \frac{4}{3}\pi^{\mu\nu}\nabla_\lambda u^\lambda \tag{2}$$

where

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu} + \pi^{\mu\nu} \tag{3}$$

$$\Delta^{\mu\nu\alpha\beta} = \frac{1}{2}(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\nu\alpha}\Delta^{\mu\beta}) - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta} \tag{4}$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu,\ g^{\mu\nu} = diag(1, -1, -1, -\tau^{-2}) \tag{5}$$

ε and P are the energy density and pressure, u^μ is the fluid velocity vector. ∇_μ is the covariant derivative.

- Constraints: $P = P(\varepsilon)$, $u_\mu u^\mu = 1$, $u_\mu \pi^{\mu\nu} = 0$, $\pi^\mu_\mu = 0$.

- CLVisc—A (3+1)D viscous hydrodynamics parallelized on GPU using OpenCL. LG.Pang, Y.Hatta, XN.Wang & BW.Xiao Phys.Rev. D91 (2015) no.7, 074027
- High performance computing cluster: GSI Green Cube, GPUs AMD FirePro s9150

Is there local vorticity in QGP?

Curl free — Divergence free (vorticity)

$$\mathbf{F} = -\nabla\mathbf{\Phi} + \nabla\times\mathbf{A}$$

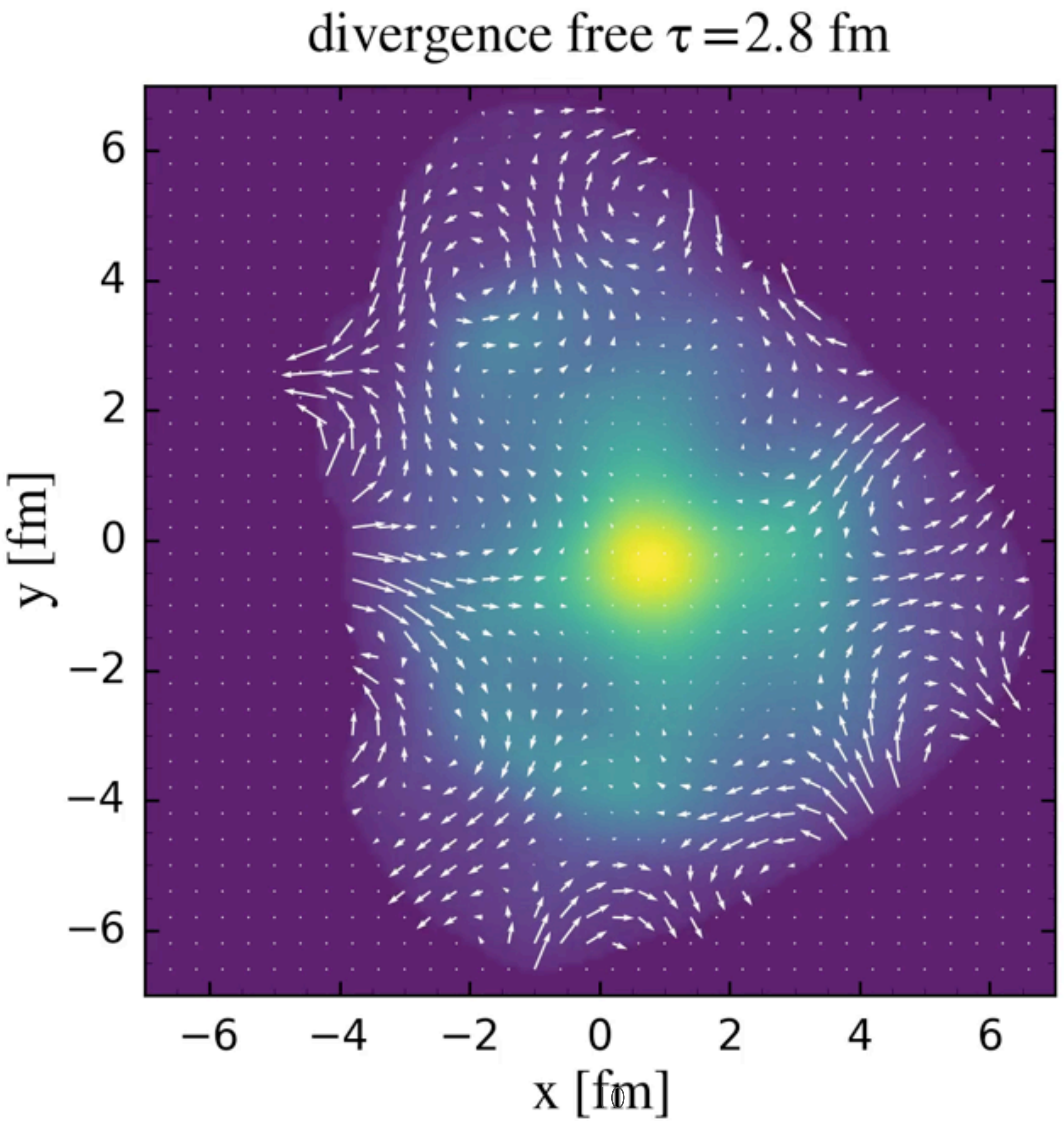
scalar potential $$\mathbf{\Phi}(\mathbf{r}) = \frac{1}{4\pi}\int\frac{\nabla'\cdot\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}dV'$$

vector potential $$\mathbf{A}(\mathbf{r}) = \frac{1}{4\pi}\int\frac{\nabla'\times\mathbf{F}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}dV'$$

Helmholtz-Hodge decomposition University of Vermont.

Let $\mathbf{F}$ be a smooth and rapidly decaying vector fields in three dimension, it can be decomposed into a curl-free component and a divergence-free component:

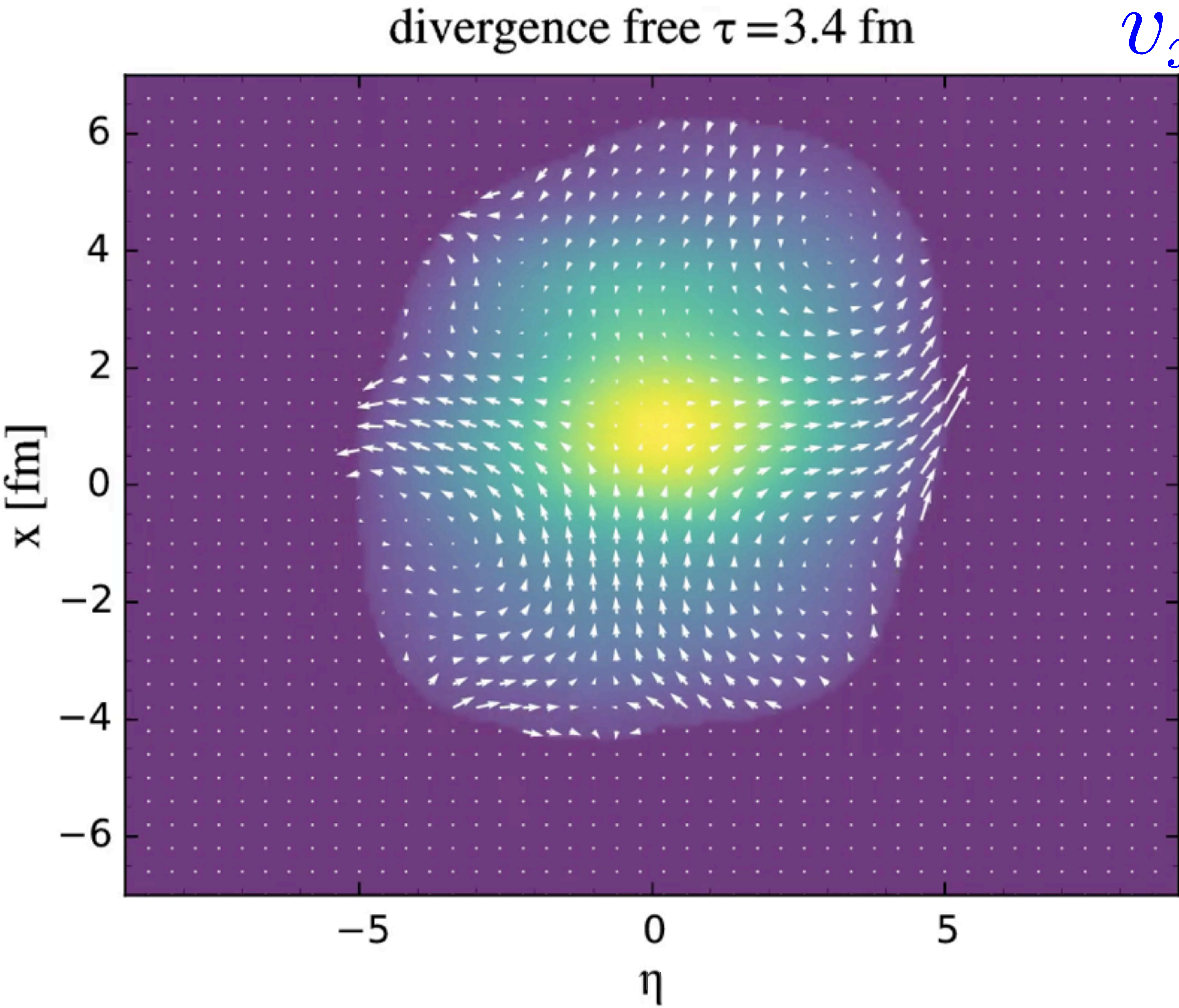
Vorticity in QGP (trans. plane)



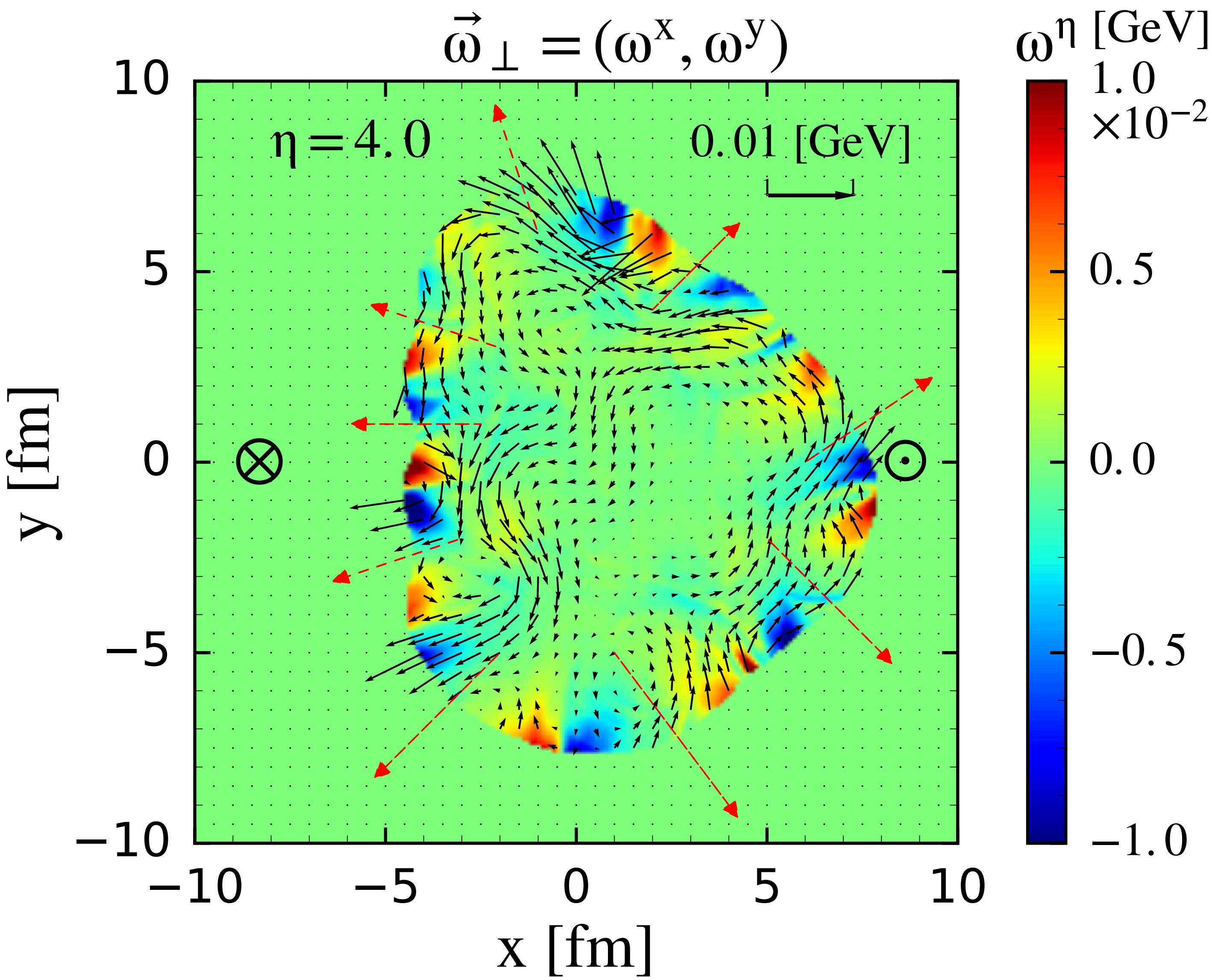
v_x, v_y

Vorticity in QGP (reaction plane)

$v_x \, , \, v_\eta$

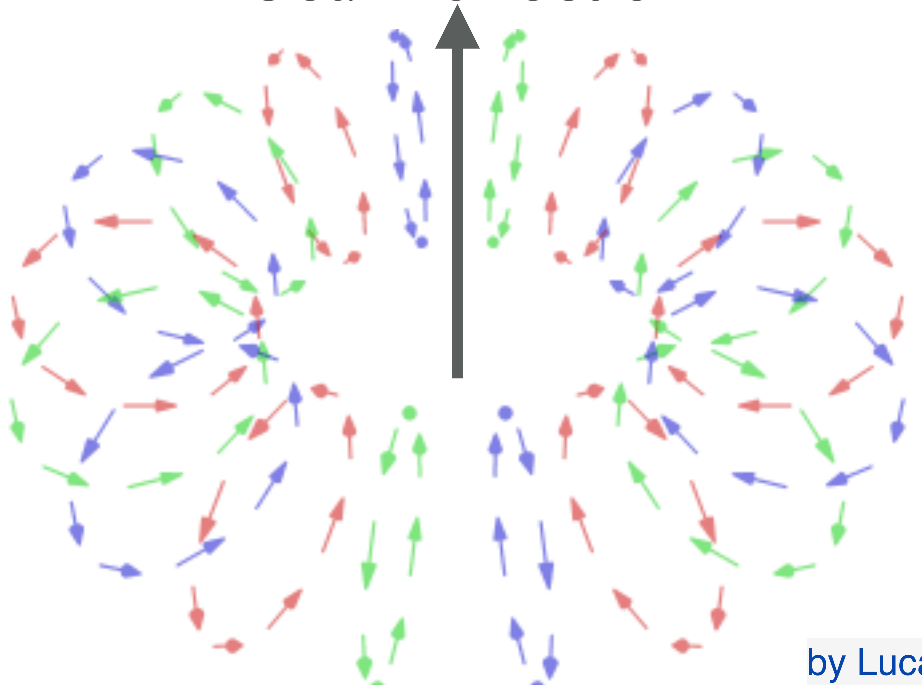


Vorticity in QGP: rich structure



- Vortex pair in 2D
- Vortex ring in 3D = Toroidal (smoke ring) vortical fluid

- Azimuthal angle dependence.
- Rapidity dependence
  LG.Pang, H.Petersen, Q.Wang & XNW arXiv:1605.04024



by Lucas V. Barbosa from WiKi Pedia

- Vortex ring appears in both the transverse and longitudinal direction
- It is thus interesting to study the spin correlation

Polarization on the freeze-out hyper surface

Becattini, F. et al. Annals Phys. 338 (2013) 32-49 arXiv:1303.3431

RH.Fang, LG.Pang, Q.Wang & XN.Wang arXiv :1604.04036

$$P^\mu \equiv \frac{d\Pi^\mu(p)/d^3p}{dN/d^3p} = \frac{\hbar}{4m} \frac{\int d\Sigma_\alpha p^\alpha \Omega^{\mu\nu} p_\nu n_f (1-n_f)}{\int d\Sigma_\alpha p^\alpha n_f}$$

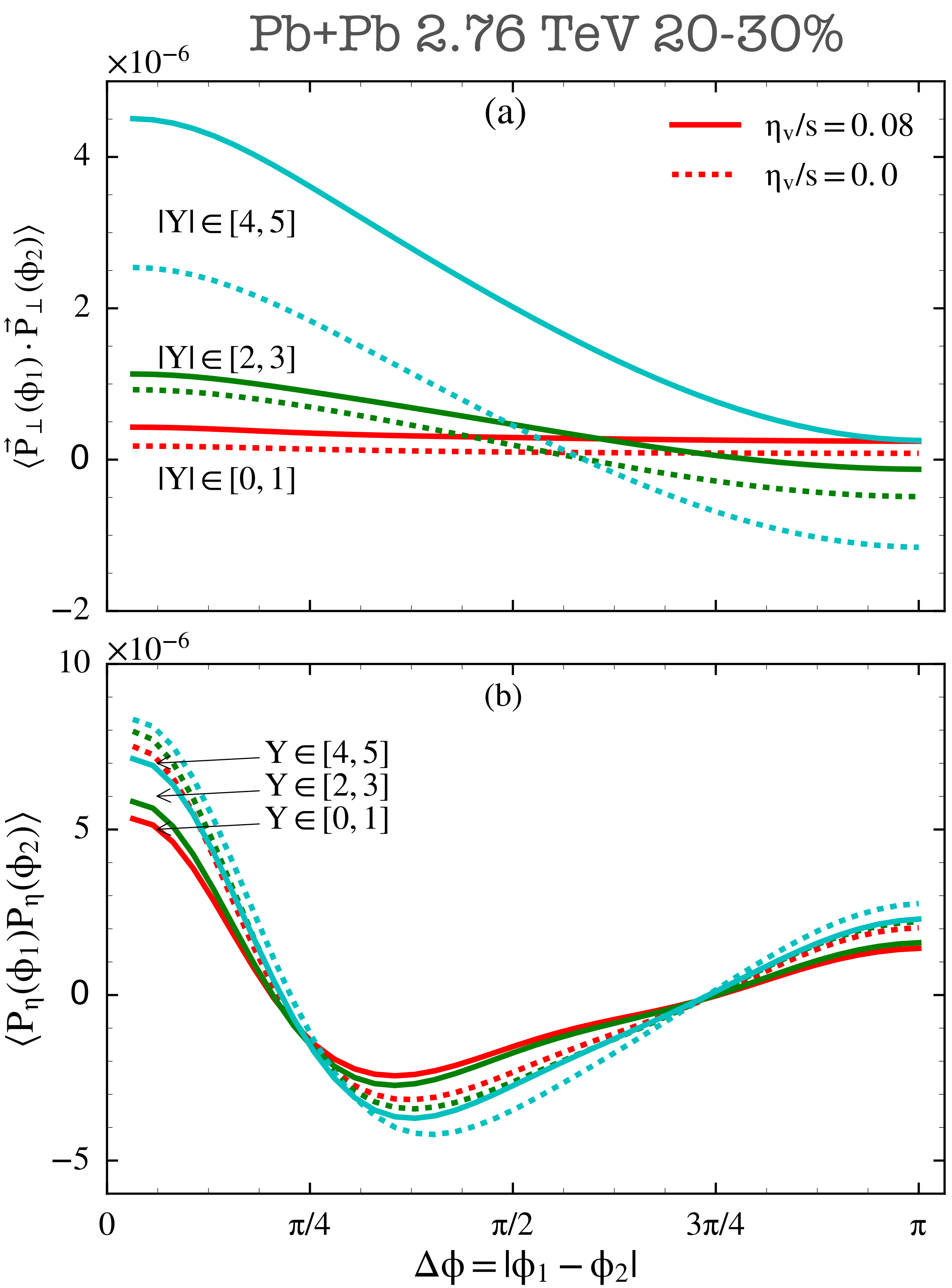
where $\Omega^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\partial_\rho(u_\sigma/T)$ is the thermal vorticity

thermal vorticity = convective + acceleration + conduction

- convective: spatial gradients of fluid velocity
- acceleration: temporal gradients of fluid velocity
- conduction: spatial and temporal gradients of temperature

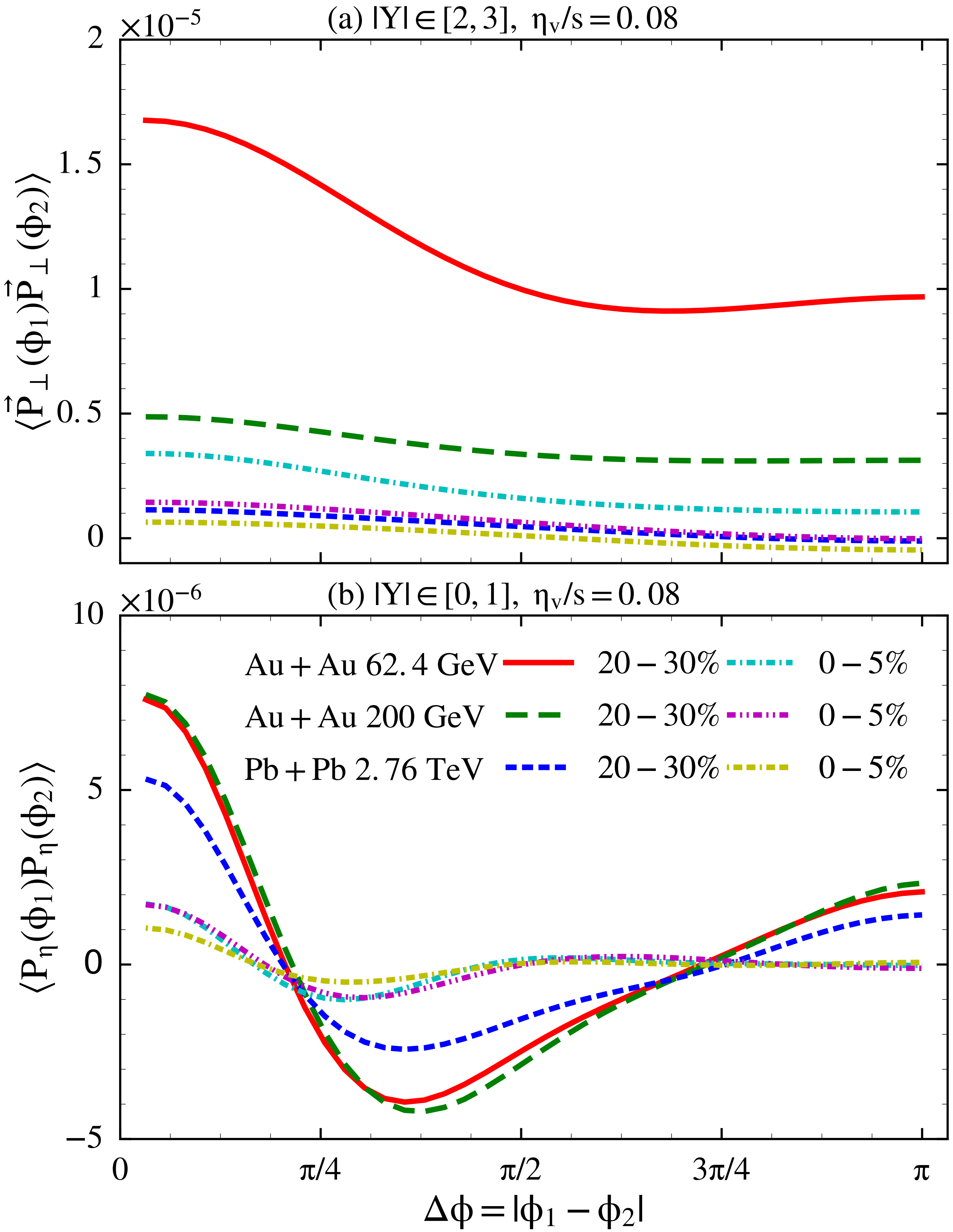
F.Becattini, L.P.Csernai, D.J.Wang, PRC88 (2013), 034905 Xie, Glastad, & Csernai PRC92 (2015) 064901, F.Becattini, et el. EPJC 2015, 406, LG.Pang, H.Petersen, Q.Wang & XN.Wang arXiv: 1605.04024, I. Karpenko, CPOD 2016, Poland and SQM

Lambda spin correlation



- (a) $\cos(\Delta\phi)$ azimuthal distribution due to local polarization (vortex ring).
- Shifts means global polarization caused by global orbital angular momentum
- Shear viscosity increases global polarization
- (b) Longitudinal spin is captured by spin correlation

Lambda spin correlation



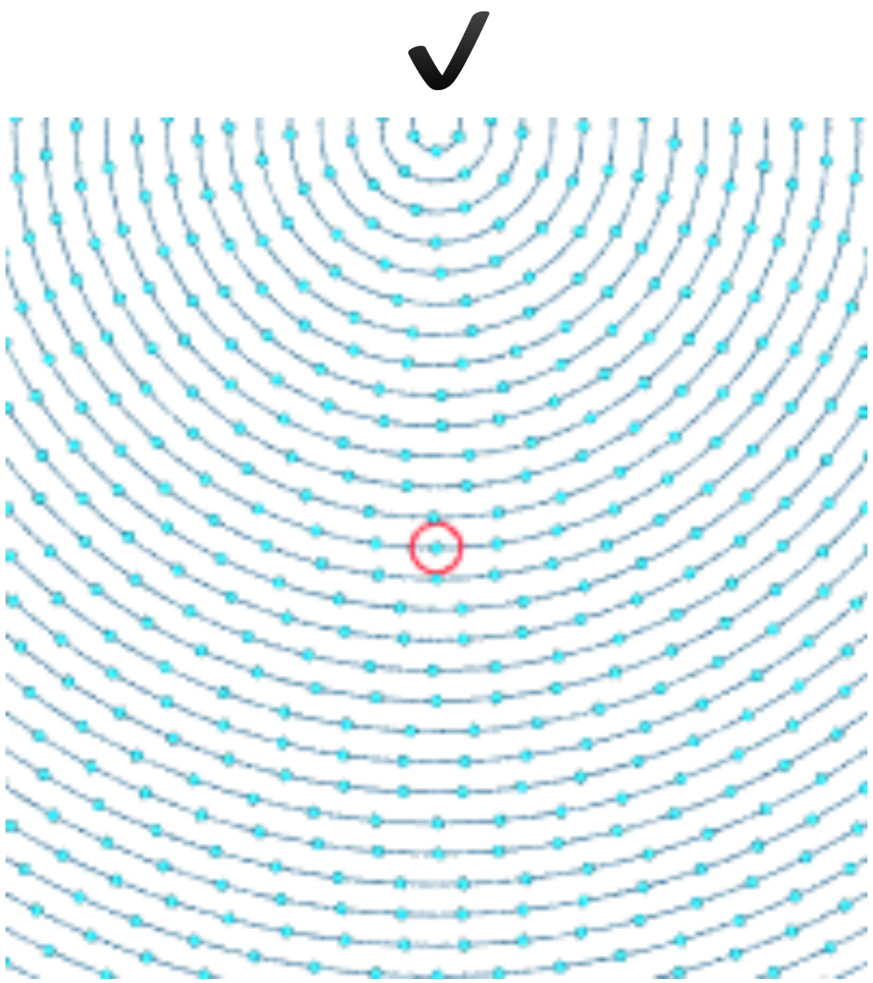
- (a) Polarization are stronger at lower beam energies and peripheral collisions.
- (b) The longitudinal spin correlation is coupled to the transverse collision geometry
- The beam energy dependence for longitudinal spin is weak.

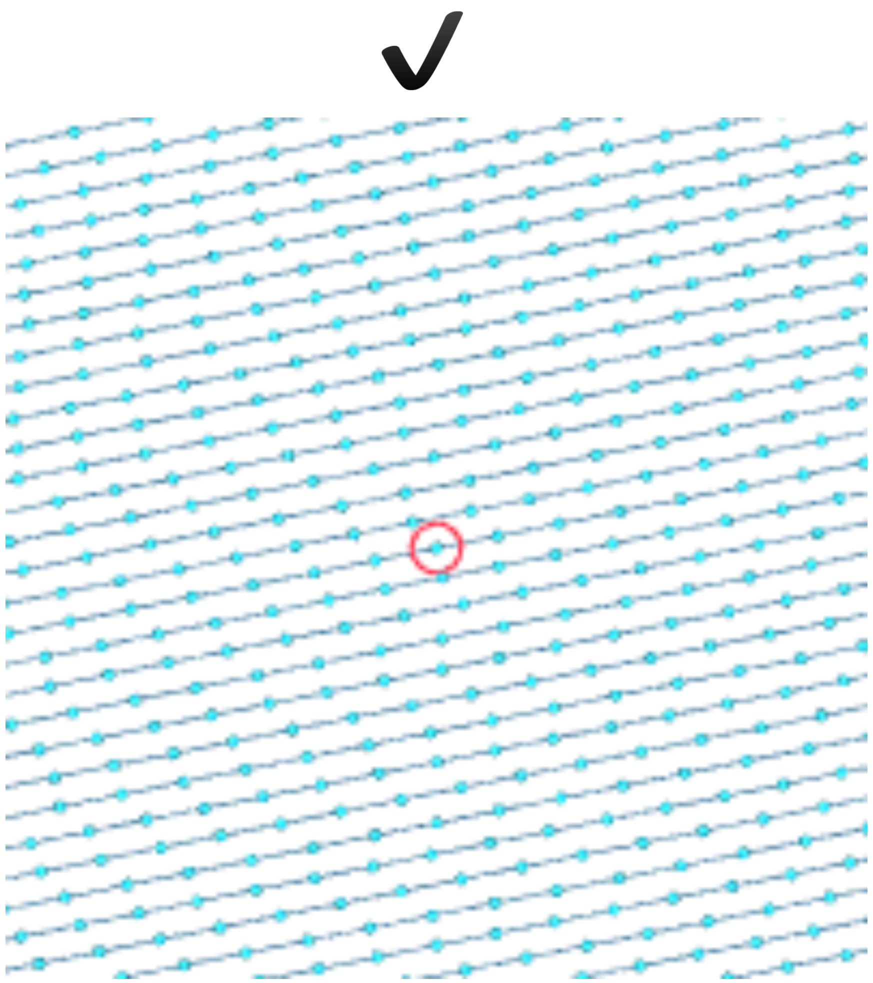
Summary

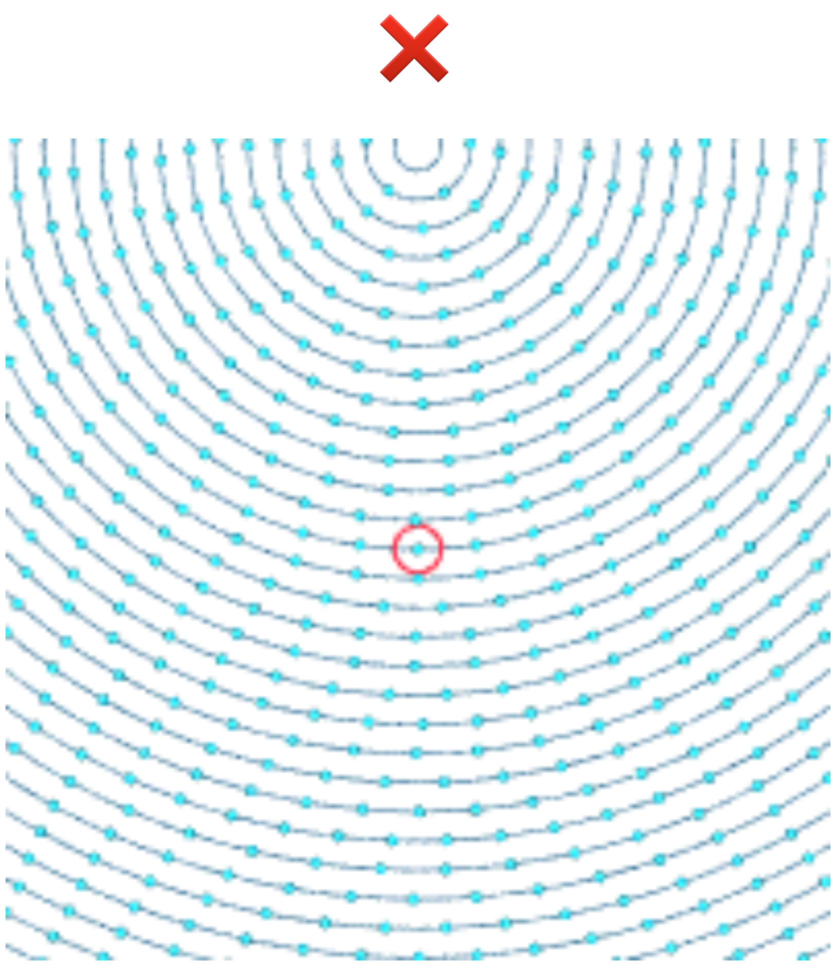
- Vortical fluid is visible by bare eyes with Helmholtz-Hodge decomposition.
- Fermions are locally polarized due to spin-vorticity coupling.
- Fermions are globally polarized because of global orbital angular momentum.
- Shear viscosity increases global polarization.
- Global polarization has strong beam energy, collision geometry, rapidity and shear viscosity dependence.
- Spin-vortex ring coupling can be studied in spin-spin correlation.

“Backups”

Vorticity in fluid dynamics







Rigid-body-like vortex $v \propto r$	Parallel flow with shear	Irrotational vortex $v \propto 1/r$

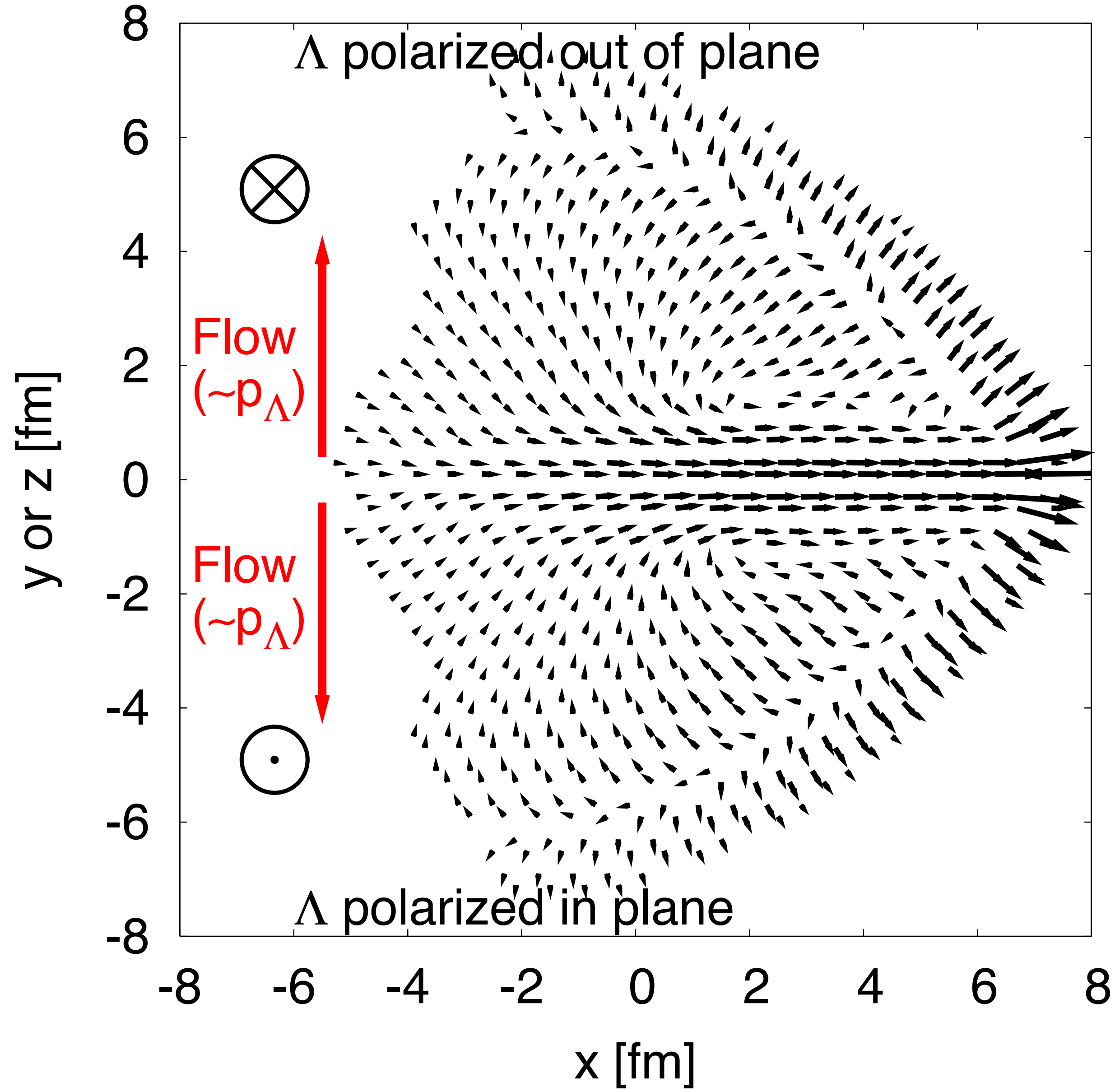
$$\vec{\omega} \equiv \nabla \times \vec{u}$$

From wikipedia by Jorge Stolfi

- Vorticity is pseudo-vector fields defined by the curl of fluid velocity vector.

Vortex ring by fast jet

Betz, Gyulassy, & G. Torrieri PRC76:044901,2007



Fast "jet" traversing the system