1. Introduction
2. Signatures of flux tube fragmentation:
   (i) an ordered, correlated chain of hadrons in every flux-tube fragmentation event
   (ii) two-hadron angular correlations in the charge sector and strangeness sector
   (iii) comparison with experiment
3. The boundary separating flux tube fragmentation and hard scattering in the space of \((p_T, \sqrt{s_{pp}})\).
4. Conclusions
This talk on flux tube fragmentation is based on


Related articles on hard scattering and flux tube fragmentation:


Introduction (I)

(i) pp reaction mechanisms provide insights in AA collision.

(ii) In a pp collision, flux-tube fragmentation is expected to dominate at low $p_T$ and low $\sqrt{s_{pp}}$

(iii) Hard scattering is expected to dominate at high $p_T$ and high $\sqrt{s_{pp}}$

The two processes cross over at the boundary $p_{Tb}(\sqrt{s_{pp}})$

(iii) Other processes, resonance production, recombinations, …
The signature for hard scattering is well known. It is provided by two-hadron angular correlations.

But, what is the signature for flux tube fragmentation?
(i) The *signature* for flux tube fragmentation is not known.

(ii) The *boundary* $p_{Tb}(\sqrt{s_{pp}})$ between flux tube fragmentation and hard-scattering is not known.

(iii) The *number of flux tubes in a pp collision* is not known.

(Is it one flux tube? two flux tubes? or more flux tubes?)

Therefore, we need the signature of flux tube fragmentation.
Two signatures of flux tube fragmentations
(I) A many-hadron signature of flux tube fragmentation

A flux tube fragmentation produces a chain of hadrons ordered in rapidity, and all $q\bar{q}$ pair production obeys local conservation laws.

Therefore, when hadrons are ordered in rapidity, the flavors of all constituents of the hadrons are correlated along the chain, and neighboring hadrons are likely to be azimuthally back-to-back correlated, on an event-by event basis.

The many-hadron signature requires the identification of particles detected in the events. This predicted signature is yet to be observed.

(II) Two-hadron angular correlation signature of flux tube fragmentation


How are these results obtained?
Pairs of $q$ and $\bar{q}$ are produced: $p_1, p_2, p_3, p_4$ ($p_T^i, p_{zi}, \text{charge, flavor}$).

Antiquark $p_1$ combines with quark $p_2$ to form meson $P_{12}$. Antiquark $p_3$ combines with quark $p_4$ to form meson $P_{34}$.

We would like to calculate two-hadron transverse momentum correlations between mesons $P_{12}$ and $P_{34}$, longitudinal momentum correlations between mesons $P_{12}$ and $P_{34}$, charge correlations between mesons $P_{12}$ and $P_{34}$, and flavor correlations between mesons $P_{12}$ and $P_{34}$.
Transverse correlations between mesons $P_{12}$ and $P_{34}$

Transverse momentum distribution for non-adjacent hadrons or uncorrelated mesons

$$
\frac{dN_T^N}{d\vec{\rho}_{Ti}d\vec{\rho}_{Ti}d\vec{\rho}_{Ti}d\vec{\rho}_{Ti}} = \left( \prod_{i=1,2,3,4} \frac{d\vec{q}_i \exp \left( -\frac{\vec{q}_i^2}{2\sigma_q^2} \right)}{(\sqrt{2\pi\sigma_q})^2} \right) \left( \prod_{i=1}^4 \frac{d\vec{p}_{Ti} \exp \left( -\frac{\vec{p}_{Ti}^2}{2\sigma_u^2} \right)}{(\sqrt{2\pi\sigma_u})^2} \delta(\vec{p}_{Ti} - \vec{p}_{Ti}' - \vec{q}_i) \right)
$$

Transverse momentum distribution for adjacent hadrons with correlations

$$
\frac{dN_T^A}{d\vec{\rho}_{Ti}d\vec{\rho}_{Ti}d\vec{\rho}_{Ti}d\vec{\rho}_{Ti}} = \left( \prod_{i=1,2,4} \frac{d\vec{q}_i \exp \left( -\frac{\vec{q}_i^2}{2\sigma_q^2} \right)}{(\sqrt{2\pi\sigma_q})^2} \right) \left( \prod_{i=1}^4 \frac{d\vec{p}_{Ti} \exp \left( -\frac{\vec{p}_{Ti}^2}{2\sigma_u^2} \right)}{(\sqrt{2\pi\sigma_u})^2} \delta(\vec{p}_{Ti} - \vec{p}_{Ti}' - \vec{q}_i) \right)
$$

$$[\vec{q}_3 = -\vec{q}_2]$$
Transverse correlations between mesons $P_{12}$ and $P_{34}$

From the variables $\vec{p}_{T1}, \vec{p}_{T2}, \vec{p}_{T3}, \vec{p}_{T4}$, we construct

$$\vec{P}_{T12} = \vec{p}_{T1} + \vec{p}_{T1}, \quad \vec{p}_{T12} = \vec{p}_{T1} - \vec{p}_{T1}$$

$$\vec{P}_{T34} = \vec{p}_{T3} + \vec{p}_{T4}, \quad \vec{p}_{T34} = \vec{p}_{T3} - \vec{p}_{T4}$$

We integrate over $\vec{p}_{T12}$ and $\vec{p}_{T34}$.

We label

$$\vec{P}_{T12} = (|\vec{P}_{T12}|, \varphi_{12})$$

$$\vec{P}_{T34} = (|\vec{P}_{T34}|, \varphi_{34})$$

We further integrate over all variables except

$$\Delta \varphi = \varphi_{12} - \varphi_{34}$$

we get from

$$\frac{dN_T^N}{d\vec{p}_{Ti}d\vec{p}_{Ti}d\vec{p}_{Ti}d\vec{p}_{Ti}}$$

and from

$$\frac{dN_T^A}{d\vec{p}_{Ti}d\vec{p}_{Ti}d\vec{p}_{Ti}d\vec{p}_{Ti}}$$

to

$$\frac{dN_T^N}{d\Delta \varphi}$$

$$\frac{dN_T^A}{d\Delta \varphi}$$
Results of two-hadron transverse correlation $\frac{dN_T^A}{d\Delta\phi}$ and $\frac{dN_T^N}{d\Delta\phi}$ in flux tube fragmentation

A=adjacent hadrons
N=non-adjacent hadrons
Longitudinal Momentum Distributions of Mesons $P_{12}$ and $P_{34}$

From $p_{z1}, p_{z2}, p_{z3}, p_{z4}$, we construct

$$P_{z12} = p_{z1} + p_{z1}, \quad p_{z12} = p_{z1} - p_{z2}$$
$$P_{z34} = p_{z3} + p_{z4}, \quad p_{z34} = p_{z3} - p_{z4}$$

The relative degrees of freedom, $p_{z12}$ and $p_{z34}$, have to do with the binding of the individual mesons through a relative longitudinal interaction. They are implicitly included when we consider bound mesons $P_{12}$ and $P_{34}$.

The remaining degrees of freedom, $P_{z12}$ and $P_{z34}$ can be represented by $y_{12}$ and $y_{34}$.

What is the hadron longitudinal momentum distribution at NA61/SHINE energies of $\sqrt{s_{pp}} \sim 20$ GeV?

$$\frac{dN_{\pi}}{dy_{12}} = \frac{N_{\pi 0} \exp\left(-\frac{y_{12}^2}{2\sigma_y^2}\right)}{(\sqrt{2\pi}\sigma_y)^2}$$
Two-hadron longitudinal correlations between $P_{12}$ and $P_{34}$

Longitudinal momentum distribution for a non-adjacent or an uncorrelated hadron pairs

$$\frac{dN_{Yy}^{uncor}}{dy_{12} dy_{34}} = \frac{N_{\pi 0}^2 \exp\left(-\frac{y_{12}^2 + y_{34}^2}{2\sigma_y^2}\right)}{(\sqrt{2\pi}\sigma_y)^2}$$

$$\frac{dN_{Yy}^{uncor}}{d\Delta y \ d\Sigma_y} = \frac{N_{\pi 0}^2 \exp\left(-\frac{4\sigma_y^2}{(\Delta y)^2 + \Sigma_y^2}\right)}{(\sqrt{2\pi}\sigma_y)^2}, \quad \Delta y = y_{12} - y_{34}, \quad \Sigma_y = y_{12} + y_{34}$$

Upon integration over $\Sigma_y$, we get for non-adjacent or uncorrelated pair $\Delta y$ distribution

$$\frac{dN_{Yy}^{mixed \ event}}{d\Delta y} = \frac{dN_{Yy}^{uncor}}{d\Delta y} \propto \frac{N_{\pi 0}^2 \exp\left(-\frac{4\sigma_y^2}{(\Delta y)^2}\right)}{(\sqrt{2\pi}\sigma_y)^2}$$
Two-hadron longitudinal correlations between Mesons $P_{12}$ and $P_{34}$

Longitudinal momentum distribution for two adjacent hadrons

\[
\frac{dN_y^A}{dy_{12} dy_{34}} = \frac{N_{\pi 0}^2 \exp\left(-\frac{y_{12}^2}{2\sigma_y^2} - \frac{y_{34}^2}{2\sigma_y^2}\right)}{(\sqrt{2\pi}\sigma_y)^2} \frac{1}{1 + \exp\left(\frac{|y_{12} - y_{34}| - w}{a}\right)},
\]

\[
\frac{dN_y^A}{d\Delta y d\Sigma_y} = \frac{N_{\pi 0}^2 \exp\left(-\frac{(\Delta y)^2 + \Sigma_y^2}{4\sigma_y^2}\right)}{(\sqrt{2\pi}\sigma_y)^2} \frac{1}{1 + \exp\left(\frac{|\Delta y| - w}{a}\right)},
\]

$\Delta y = y_{12} - y_{34}$

Upon integration over $\Sigma_y$, we get for adjacent hadron pair

\[
\frac{dN_y^A}{d\Delta y} \propto \frac{N_{\pi 0}^2 \exp\left(-\frac{(\Delta y)^2}{4\sigma_y^2}\right)}{(\sqrt{2\pi}\sigma_y)^2} \frac{1}{1 + \exp\left(\frac{|\Delta y| - w}{a}\right)}
\]

\[
\frac{dN_y^A}{d\Delta y} / \frac{dN_y^{\text{mixed event}}}{d\Delta y} = \frac{1}{1 + \exp\left(\frac{|\Delta y| - w}{a}\right)}
\]
Two-hadron longitudinal correlations between $P_{12}$ and $P_{34}$

Longitudinal momentum distribution for two non-adjacent hadrons

$$
\frac{dN^N_y}{dy_{12} dy_{34}} = \frac{N^2\pi_0 \exp\left(-\frac{y_{12}^2}{2\sigma_y^2} - \frac{y_{34}^2}{2\sigma_y^2}\right)}{(\sqrt{2\pi}\sigma_y)^2} \frac{1}{1 + \exp\left(\frac{w - |y_{12} - y_{34}|}{a}\right)}, \quad W = \frac{1}{dN/dy}
$$

$$
\frac{dN^N_y}{d\Delta y d\Sigma y} = \frac{N^2\pi_0 \exp\left(-\frac{(\Delta y)^2 + \Sigma_y^2}{4\sigma_y^2}\right)}{(\sqrt{2\pi}\sigma_y)^2} \frac{1}{1 + \exp\left(\frac{w - |\Delta y|}{a}\right)}, \quad \Delta y = y_{12} - y_{34}
$$

Upon integration over $\Sigma y$, we get for adjacent hadron pair

$$
\frac{dN^N_y}{d\Delta y} \propto \frac{N^2\pi_0 \exp\left(-\frac{(\Delta y)^2}{4\sigma_y^2}\right)}{(\sqrt{2\pi}\sigma_y)^2} \frac{1}{1 + \exp\left(\frac{w - |\Delta y|}{a}\right)}
$$

$$
\frac{dN^N_y}{d\Delta y} / \frac{dN^N_y}{d\Delta y}_{\text{mixed event}} = \frac{1}{1 + \exp\left(\frac{w - |\Delta y|}{a}\right)}
$$
Charge and flavor correlations between mesons $P_{12}$ and $P_{34}$

In non-adjacent case, $q\bar{q}$ pairs are produced at $p_1, p_2, p_3, p_4$.

We enumerate all possible flavor SU(3) $q\bar{q}$ pairs. Each $s\bar{s}$ pair production is associated with a reduced probability $f_s$.

We shall keep contributions only up to the order $f_s$.

$$p_1 = (u, d, s), \quad p_2 = (\bar{u}, \bar{d}, \bar{s}), \quad p_3 = (u, d, s), \quad p_4 = (\bar{u}, \bar{d}, \bar{s}),$$

In adjacent case, the $q\bar{q}$ pair at $p_2$, and $p_3$ are correlated:

$$p_1 = (u, d, s), \quad (p_2, p_3) = (u, \bar{u}), (d, \bar{d}), (s, \bar{s}), \quad p_4 = (\bar{u}, \bar{d}, \bar{s}),$$
Charge and flavor correlations between mesons $P_{12}$ and $P_{34}$

We enumerate all possible flavor SU(3) quark-antiquark pairs.

We get the charges and flavors quantum numbers of $P_{12}$ and $P_{34}$. This enumeration gives the probability

$P^A(\nu)$ for configuration $\nu$ in adjacent $P_{12}$ and $P_{34}$

$P^N(\nu)$ for configuration $\nu$ in non-adjacent $P_{12}$ and $P_{34}$.

Two meson distribution function $\frac{dN^{AN}(\nu)}{d\Delta \phi d\Delta y}$ is

$$\frac{dN(\nu)}{d\Delta \phi d\Delta y} = \frac{dN^A(\nu)}{d\Delta \phi d\Delta y} \times P^A(\nu) + \frac{dN^N(\nu)}{d\Delta \phi d\Delta y} \times P^N(\nu)$$

Correlation function for two flux tubes

$$C(\Delta \phi, \Delta y) = \frac{dN_{sub}(\nu)}{d\Delta \phi d\Delta y \times \frac{dN^{A}(\nu)}{d\Delta \phi d\Delta y} \times P^A(\nu) + \frac{dN^N(\nu)}{d\Delta \phi d\Delta y} \times P^N(\nu) + P^N(\nu)) \frac{1}{8\pi^2}$$

$$\left( P^A(\nu) + P^N(\nu) \right) \frac{1}{8\pi^2} + P^N(\nu) \right) \frac{1}{8\pi^2}$$
We obtain two-hadron angular correlations as signatures for flux tube fragmentation.

Remarks

- Resonance production and subsequent resonance decay also lead to signals similar to the flux tube fragmentation.

- They are estimate to be about 10-20% at SPS and RHIC energies in central rapidity region, but higher at the projectile fragmentation and target fragmentation regions.
The boundary between flux tube fragmentation and hard-scattering at $\sqrt{s_{pp}}=200$ GeV is $p_T^{b} \sim 0.5$ GeV/c.
STAR data at $\sqrt{s_{pp}}=200$ GeV for $p_T < 0.5$ GeV/c

unlike charge hadrons

like charge hadrons
1. For $p_T > 0.1$ GeV/c, the production mechanism is a combination of flux tube fragmentation and hard scattering.

2. For $p_T > 1$ GeV, production mechanism is predominantly hard scattering.
Flux tube fragmentation
unlike charges

Comparison of theoretical
results with NA61/SHINE
data

NA61/SHINE pp data for two unlike charges

$\sqrt{s_{pp}}=6.3 \text{ GeV}$

$\sqrt{s_{pp}}=7.6 \text{ GeV}$

$\sqrt{s_{pp}}=8.7 \text{ GeV}$

$\sqrt{s_{pp}}=12.3 \text{ GeV}$

$\sqrt{s_{pp}}=17.2 \text{ GeV}$
As the collision energy increases, the cross-over boundary $p_{Tb}$ moves to lower values of $p_T$. 

Schematic illustration:
Conclusions

• The occurrence of a chain of ordered and correlated hadrons provides a signature of flux tube fragmentation. This signature is yet to be observed.

• Two-hadron angular correlations in the charge and strangeness sectors provide other signatures of flux tube fragmentation.

• Two-hadron correlation data reveal that flux tube fragmentation dominates the low $p_T$, low $\sqrt{s_{pp}}$ region whereas hard scattering dominates the high $p_T$, high $\sqrt{s_{pp}}$ region.

• The cross over boundary between flux tube fragmentation and hard scattering moves to a lower $p_T$ as collision energy increases. At LHC energies, hard scattering dominates over a very extensive region of $p_T$. 