Multi-strange hadrons and the precision extraction of QGP properties in the RHIC-BES domain

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Strangeness as a probe of QCD matter

- Multistrange hadrons Ω and ϕ are expected to have small hadronic cross sections
  $\Rightarrow$ Potentially good constraints for chemical freezeout region

Modeling approach:

Model parameters (input): $\vec{x} = (x_1, ..., x_n)$

$(\tau_0,\ R_{\text{trans}},\ R_{\text{long}},\ \eta/s,\ \epsilon_C)$

$\Downarrow$

Model output $\vec{y} = (y_1, ..., y_m) \Leftrightarrow$ Experimental values $\vec{y}^{\text{exp}}$

$(N_{\text{ch}},\ \langle p_T \rangle,\ v_2, ...)$

Goal: Find the "true" values of the input parameters, for which $\vec{x}^* \Rightarrow \vec{y}^{\text{exp}}$.
What is the level of uncertainty associated with the proposed values?

Transport + hydrodynamics hybrid model

Karpenko, Huovinen, Petersen, Bleicher, Phys.Rev.C, 91, 064901 (2015)

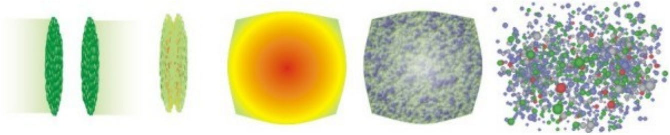


Image source: S. A. Bass

- Initial State from UrQMD[1] hadron+strings cascade
- Start the hydrodynamical evolution when nuclei have passed through each other: $\tau_0 \geq \frac{2R_{\text{nucleus}}}{\sqrt{\gamma_{CM}^2 - 1}}$
- Particle properties (energy, baryon number) to densities: 3D Gaussians with "smearing" parameters R_{trans}, R_{long} ($\equiv \sqrt{2}\sigma$)
- 3+1D viscous hydrodynamics[2] with viscosity parameter η/s
- Transition from hydro back to transport ("particlization") when energy density $\epsilon \leq \epsilon_C$

[1] S. A. Bass *et al.*, Prog. Part. Nucl. Phys. **41**, 255 (1998), M. Bleicher *et al.*, J. Phys. G **25**, 1859 (1999).

[2] Iu. Karpenko *et al.*, Comput.Phys.Commun. 185, 3016 (2014).

Bayes' theorem

Given a set $X = \{\vec{x}_k\}_{k=1}^{N}$ of points in parameter space and a corresponding set $Y = \{\vec{y}_k\}_{k=1}^{N}$ of points in observable space,

$$P(\vec{x}^*|X, Y, \vec{y}^{\text{exp}}) \propto P(X, Y, \vec{y}^{\text{exp}}|\vec{x}^*) P(\vec{x}^*)$$

- $P(\vec{x}^*|X, Y, \vec{y}^{\text{exp}})$ is the *posterior* probability distribution of $\vec{x}^*$ for given $(X, Y, \vec{y}^{\text{exp}})$
- $P(\vec{x}^*)$ is the *prior* probability distribution (simplest case: ranges of parameter values)
- $P(X, Y, \vec{y}^{\text{exp}}|\vec{x}^*)$ is the *likelihood* of $(X, Y, \vec{y}^{\text{exp}})$ for given $\vec{x}^*$ (to be determined with statistical analysis)

  $$P(X, Y, \vec{y}^{\text{exp}}|\vec{x}^*) = \exp\left(-\tfrac{1}{2}(\vec{y}^* - \vec{y}^{\text{exp}})^T \Sigma^{-1} (\vec{y}^* - \vec{y}^{\text{exp}})\right),$$

  where Σ is the covariance matrix. In this study $\Sigma = \text{diag}(\sigma \vec{y}^{\text{exp}})^2$, with $\sigma = 0.05$ as a global estimate of relative uncertainty, and $\vec{y}^*$ is model output at $\vec{x}^*$

Analysis procedure

Model parameters (input): τ_0, R_{trans}, R_{long}, η/s, ϵ_C
Uniform priors: input parameter space = 5-D hypercube

⇓

Hybrid model ⇒ Gaussian process; statistical emulator trained with model data, necessary for quick evaluation of the likelihood

⇓

Experimental data ⇒ Random walk (MCMC) in the input parameter space, guided by the likelihood

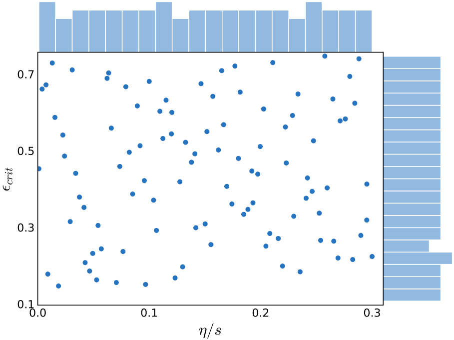
⇓

Posterior distribution of input parameters (best-fit values + uncertainty)

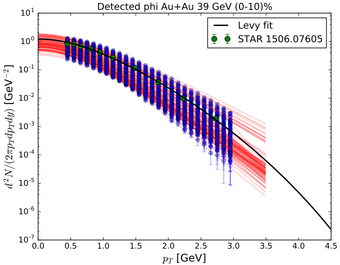
Investigated parameter ranges

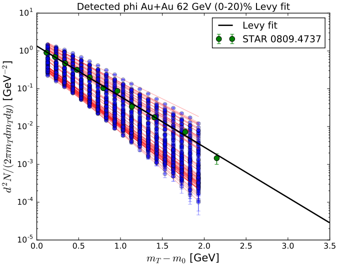
To provide the Gaussian process with a proper representation of the hybrid model, training points are sampled evenly over whole parameter space using Latin hypercube method

- Shear viscosity over entropy density η/s: 0.001 - 0.4
- Transport-to-hydro transition time τ_0: 0.4 - 3.1 fm
- Transverse Gaussian smearing of particles R_{trans}: 0.2 - 2.2 fm
- Longitudinal Gaussian smearing of particles R_{long}: 0.2 - 2.2 fm
- Hydro-to-transport transition energy density ϵ_C: 0.15 - 0.75 GeV/fm^3



ϕ Levy fits to dN/dp_T, dN/dm_T at $\sqrt{s_{NN}} = 39, 62.4$ GeV

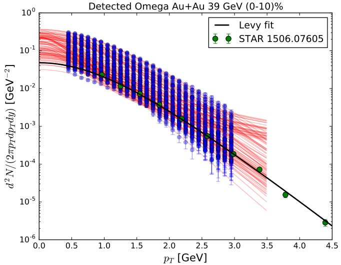




- "Detected ϕ": ϕ which has both its decay products surviving to the end of UrQMD simulation
- Levy fit: $Y \frac{(n-1)(n-2)}{2\pi nT(nT+m_0(n-2))} \left(1 + \frac{\sqrt{p_T^2+m_0^2}-m_0}{nT}\right)^{-n}$ PRC 71, 064902

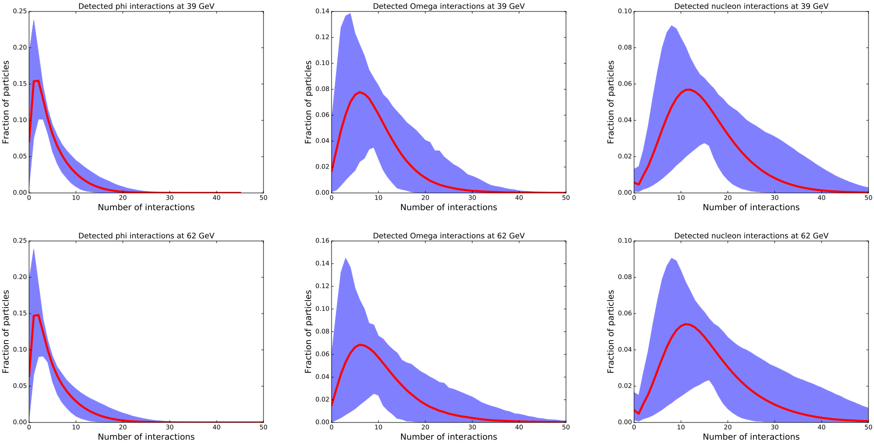
  Fit parameters: Y = yield, T = slope, n = exponent
- ϕ yield extracted from Levy fits = ϕ count from simulation output $\Rightarrow$ apples-to-apples comparison with data
- Underestimation of detected ϕ with most parameter combinations

Ω Levy fits to dN/dp_T at $\sqrt{s_{NN}} = 39, 62.4$ GeV



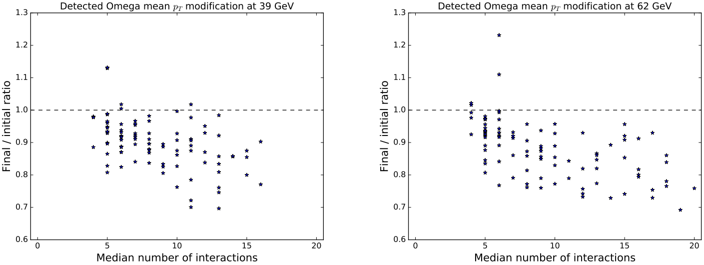
- Ω considered "stable" in UrQMD $\Rightarrow$ "detected" = "total"
- Fit quality suffers from low statistics at higher p_T
- Yield is typically overestimated; however, more points around the experimental value compared to ϕ

Number of interactions



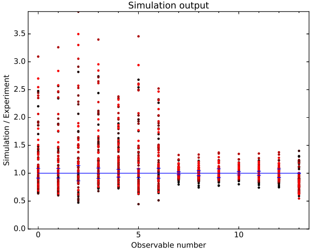
- ϕ typically has a few interactions during its lifetime
- Most Ω experience ≈ 10 interactions, but notable portion > 20! Effect on p_T spectrum?

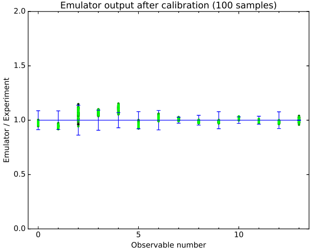
Effect of interactions on Ω mean p_T

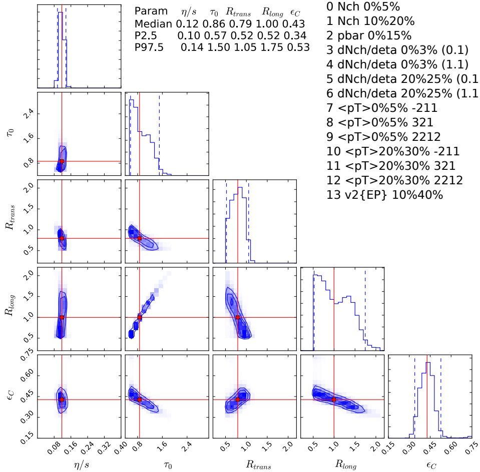


- Suppression up to 30% detected for some training points
- Large variations even with the same median value of interactions
- Check p_T distribution at hypersurface vs. final distribution once the best-fit parameter values are determined

62.4 GeV analysis without Ω





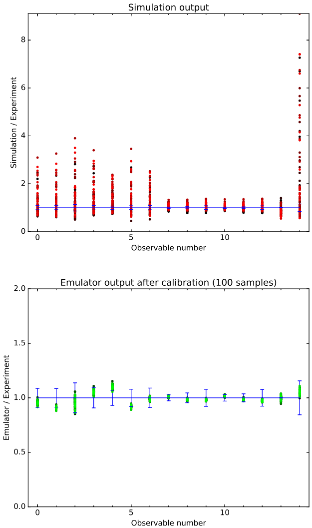


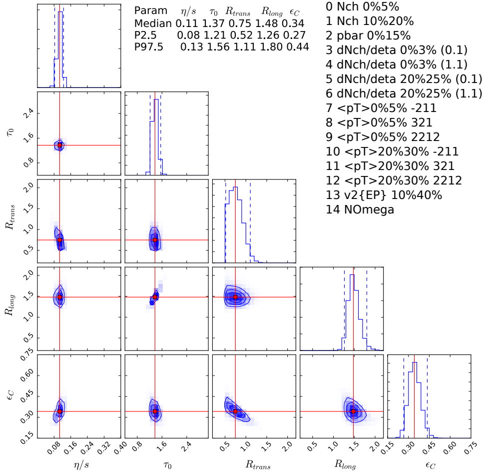
Param	η/s	τ_0	R_{trans}	R_{long}	ϵ_C
Median	0.12	0.86	0.79	1.00	0.43
P2.5	0.10	0.57	0.52	0.52	0.34
P97.5	0.14	1.50	1.05	1.75	0.53

0 Nch 0%5%
1 Nch 10%20%
2 pbar 0%15%
3 dNch/deta 0%3% (0.1)
4 dNch/deta 0%3% (1.1)
5 dNch/deta 20%25% (0.1)
6 dNch/deta 20%25% (1.1)
7 <pT>0%5% -211
8 <pT>0%5% 321
9 <pT>0%5% 2212
10 <pT>20%30% -211
11 <pT>20%30% 321
12 <pT>20%30% 2212
13 v2{EP} 10%40%

Results preliminary

62.4 GeV analysis with Ω



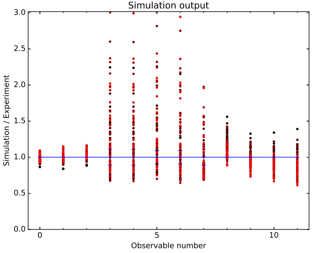


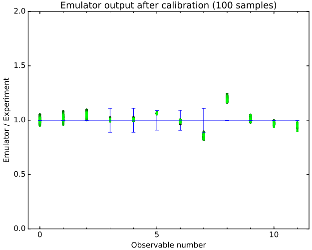
Param	η/s	τ_0	R_{trans}	R_{long}	ϵ_C
Median	0.11	1.37	0.75	1.48	0.34
P2.5	0.08	1.21	0.52	1.26	0.27
P97.5	0.13	1.56	1.11	1.80	0.44

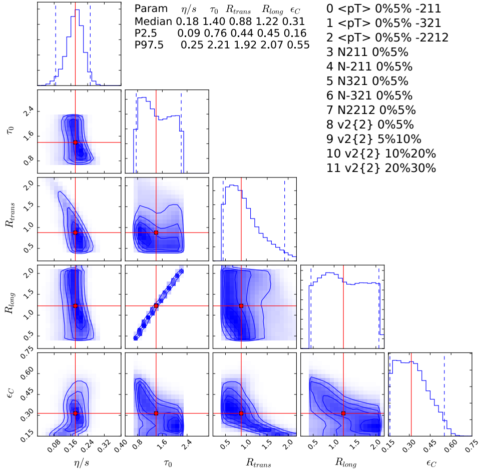
0 Nch 0%5%
1 Nch 10%20%
2 pbar 0%15%
3 dNch/deta 0%3% (0.1)
4 dNch/deta 0%3% (1.1)
5 dNch/deta 20%25% (0.1)
6 dNch/deta 20%25% (1.1)
7 <pT>0%5% -211
8 <pT>0%5% 321
9 <pT>0%5% 2212
10 <pT>20%30% -211
11 <pT>20%30% 321
12 <pT>20%30% 2212
13 v2{EP} 10%40%
14 NOmega

Results preliminary

39 GeV analysis without Ω

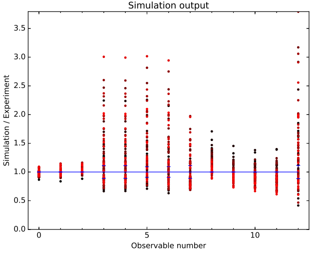


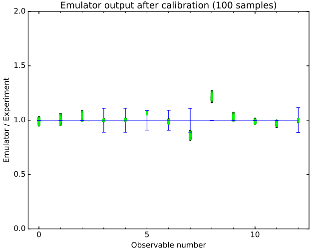


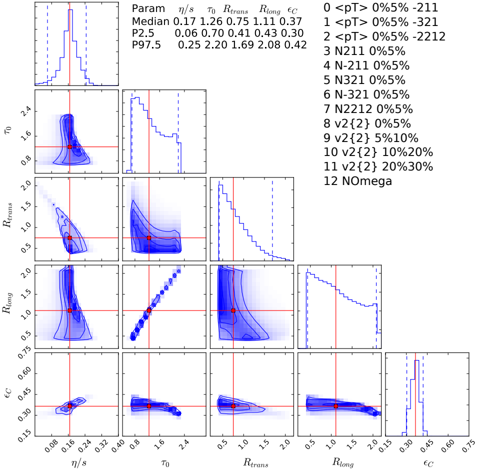


Results preliminary

39 GeV analysis with Ω

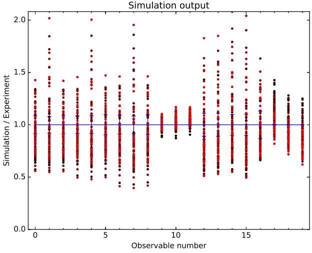


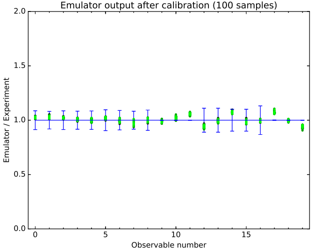


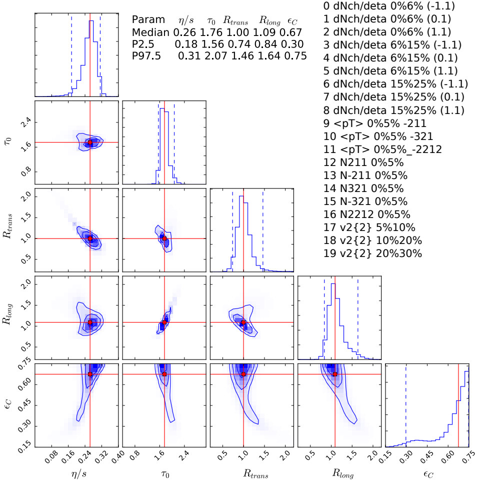


Results preliminary

19.6 GeV analysis without Ω

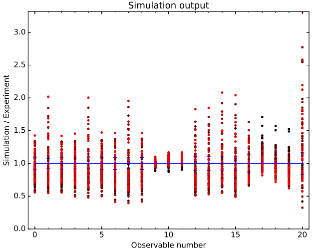


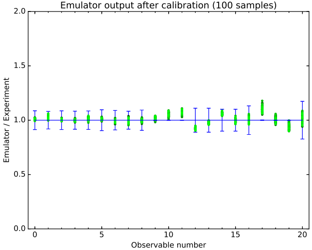


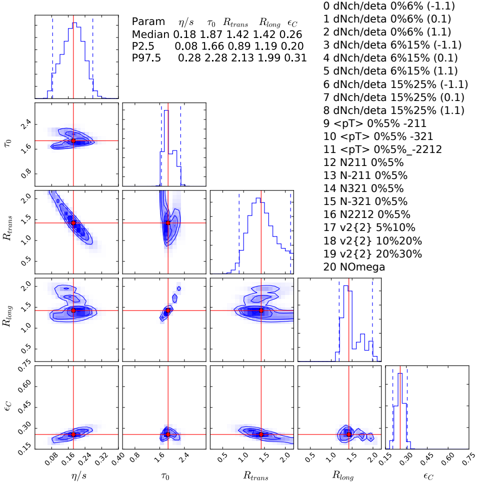


Results preliminary

19.6 GeV analysis with Ω







Results preliminary

Summary and outlook

- Gaussian processes allow the emulation of complex models, making it possible to investigate multidimensional parameter spaces within reasonable computational effort
- First results from the statistical analysis of a transport+hydro hybrid model suggest that multi-strange hadrons provide very important constraints on model parameters at lower collision energies
- Ω has less interactions than nucleons, but still enough to potentially distort the signal from hypersurface. Need to run simulation with posterior median values and compare p_T distributions before and after UrQMD afterburner
- ϕ needs to be also included in statistical analysis
  - ϕs detected based on UrQMD analysis seem to underestimate the measured value
  - Quantify the uncertainty by allowing the "true number" of ϕ vary between the "detected" and "total" values of UrQMD

Extra slides

Gaussian process

http://dan.iel.fm/george

Assumption: Set Y_a of values of observable y_a, corresponding to set X of points in parameter space, has a multivariate normal distribution:

$$Y_a \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

where $\boldsymbol{\mu} = \mu(X) = \{\mu(x_1), ..., \mu(x_N)\}$ is the mean (often set to 0 by centering the data) and

$$\boldsymbol{\Sigma} = \sigma(X, X) = \begin{pmatrix} \sigma(\vec{x}_1, \vec{x}_1) & \cdots & \sigma(\vec{x}_1, \vec{x}_N) \\ \vdots & \ddots & \vdots \\ \sigma(\vec{x}_N, \vec{x}_1) & \cdots & \sigma(\vec{x}_N, \vec{x}_N) \end{pmatrix}$$

is the covariance matrix with covariance function $\sigma(\vec{x}, \vec{x}')$ (model-dependent choice; constant, linear, exponential, periodic, ...).

Drawing samples from a Gaussian process

- Define a vector of N points, $\vec{x} = (x_1, ..., x_N)$, on which to evaluate the GP
- Compute covariance matrix $\Sigma_{ij} = \sigma(x_i, x_j)$
- Compute Cholesky decomposition $\Sigma = SS^T$
- The vector $\vec{y} = \mu(\vec{x}) + S\vec{u}$, $u_i \sim N(0, 1)$, defines a GP sample

The joint distribution of observations $Y_o = (y(x_{o1}), ..., y(x_{ok}))$ and predictions $Y_p = (y(x_{p1}), ..., y(x_{pn}))$ is

$$\begin{pmatrix} Y_p \\ Y_o \end{pmatrix} = \mathcal{N}\left(\begin{pmatrix} \mu_p \\ \mu_o \end{pmatrix}, \begin{pmatrix} \Sigma_{pp} & \Sigma_{po} \\ \Sigma_{op} & \Sigma_{oo} \end{pmatrix}\right)$$

resulting to posterior mean (prediction based on known values)

$$\bar{\mu}(X_p) = \mu_p + \Sigma_{po}\Sigma_{po}^{-1}(Y_o - \mu_o)$$

and posterior variance $\bar{\Sigma} = \Sigma_{pp} - \Sigma_{po}\Sigma_{oo}^{-1}\Sigma_{op}$.

Gaussian process

Choice: Squared-exponential covariance function with a noise term

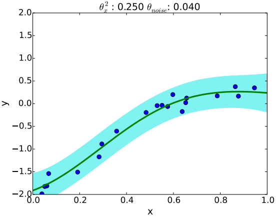
$$\sigma(\vec{x},\vec{x}') = \theta_0 \exp\left(-\sum_{i=1}^{n} \frac{(x_i - x_i')^2}{2\theta_i^2}\right) + \theta_{\text{noise}}\delta_{\vec{x}\vec{x}'}$$

The *hyperparameters* $\vec{\theta} = (\theta_0, \theta_1, ...\theta_n, \theta_{\text{noise}})$ are not known a priori and must be estimated from the given data

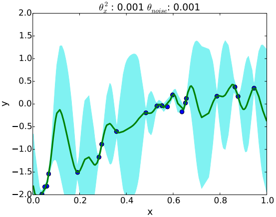
$\Rightarrow$ emulator training: Maximise the marginal likelihood (aka "evidence")

$$\log P(Y|X,\vec{\theta}) = \underbrace{-\frac{1}{2}Y^T\mathbf{\Sigma}^{-1}(X,\vec{\theta})Y}_{\text{data fit}} \underbrace{-\frac{1}{2}\log|\mathbf{\Sigma}(X,\vec{\theta})|}_{\text{complexity penalty}} \underbrace{-\frac{N}{2}\log(2\pi)}_{\text{normalization}}$$

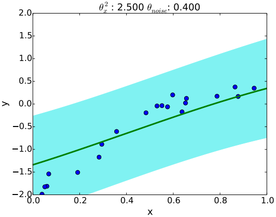
1-D example



Target GP

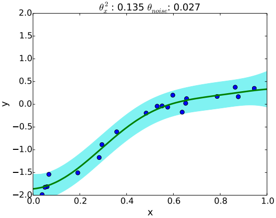


Overfit



Underfit

$\Downarrow$



After training

Markov Chain Monte Carlo

"emcee": D. Foreman-Mackey *et al.*, Publ. Astron. Soc. Pacific **125**, 306 (2013), arXiv:1202.3665

The posterior distribution is sampled with Markov Chain Monte Carlo (MCMC) method

- Random walk in parameter space, where each step is accepted or rejected based on a relative likelihood (calculated in terms of principal components)
- Converges to posterior distribution as number of steps $N \to \infty$
- Acceptance fraction a_f of steps measures the quality of random walk
  - $a_f \sim 0 \Rightarrow$ walker "stuck"
  - $a_f \sim 1 \Rightarrow$ purely random walk
  - aim for 0.2-0.5
- Autocorrelation time = Number of steps between independent samples
  "Burn-in" takes a few autocorrelations,
  gathering enough samples $\sim \mathcal{O}(10)$ autocorrelations