Discovering the QCD critical point with net-proton fluctuations

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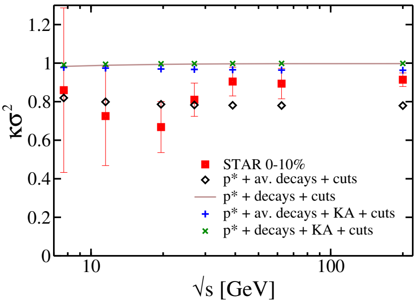
Prelude

- event-by-event fluctuations of the net-proton number expected to signal a critical point (CP) in the QCD phase diagram
- associated with fluctuations in the chiral order parameter σ
- higher-order fluctuations depend stronger on correlation length
- final stage processes could wash out critical fluctuation signals:



- isospin redistribution
- resonance decays
- ...

→ Can critical fluctuation signals survive resonance decays?

Y. Hatta and M.A. Stephanov, PRL 91 (2003) 102003; M.A. Stephanov, PRD 65 (2002) 096008; *ibid.* PRL 102 (2009) 032301; M. Kitazawa and M. Asakawa, PRC 86 (2012) 024904; M. Nahrgang et al., Eur. Phys. J. C 75 (2015) 573; L. Adamczyk et al. [STAR Collaboration], PRL 112 (2014) 032302

Critical fluctuations based on Ising-model EoS

$\rightarrow$ need to know σ-field fluctuations

under universality hypothesis:
order parameter magnetization

$$M(r,H) = M_0 R^\beta \theta$$

with parametric representation

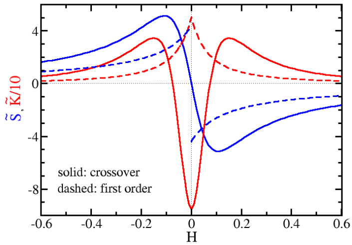
$$r = R(1-\theta^2)\,,\; H = H_0 R^{\beta\delta}\tilde{h}(\theta)$$

σ-field cumulants:

$$\langle(\delta\sigma)^n\rangle_c = \left(\frac{T}{V}\right)^{n-1}\left(\frac{\partial^{n-1}M}{\partial H^{n-1}}\right)_r$$

$\rightarrow$ valid in scaling regime

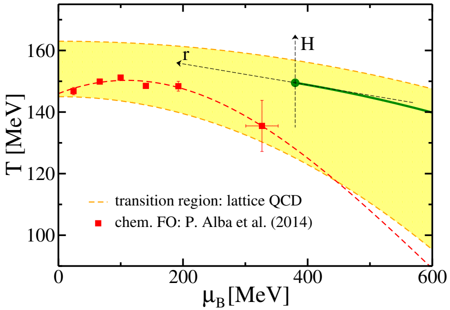
- CP located at $(r,H)=(0,0)$
- for $r>0$ crossover regime
- for $r<0$ first-order phase transition at $H=0$



$\tilde{S} \sim \langle(\delta\sigma)^3\rangle/\langle(\delta\sigma)^2\rangle$, $\tilde{K} \sim \langle(\delta\sigma)^4\rangle_c/\langle(\delta\sigma)^2\rangle$

P. Schofield, PRL 22 (1969) 606; R. Guida and J. Zinn-Justin, NPB 489 (1997) 626

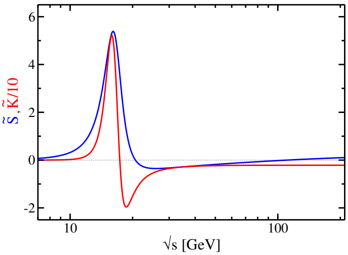
Mapping to QCD thermodynamics



What is unknown?

- location of CP in (μ_B, T)
- size of critical region
- mapping $r(\mu_B, T)$ and $H(\mu_B, T)$

→ one preferred direction for r in CP: along first-order transition line



What do we know?

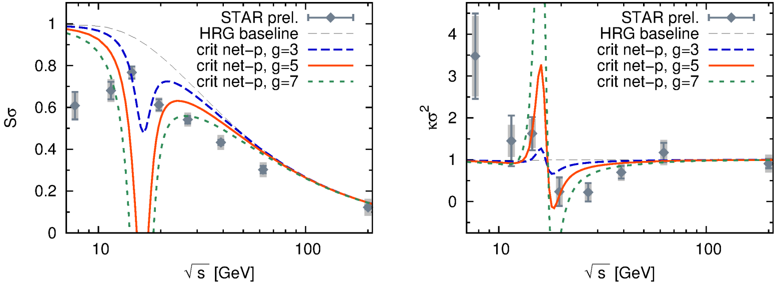
- $T_{c,0} = (154 \pm 9)$ MeV
- chiral crossover curvature $\kappa_c = 0.007 \dots 0.02$
- chemical freeze-out parameters

Y. Aoki et al., JHEP 06 (2009) 088; A. Bazavov et al., PRD 85 (2012) 054503; O. Kaczmarek et al., PRD 83 (2011) 014504; G. Endrodi et al., JHEP 04 (2011) 001; C. Bonati et al., PRD 92 (2015) 054503; R. Bellwied et al., PLB 751 (2015) 559; P. Cea et al., PRD 93 (2016) 014507; P. Alba et al., PLB 738 (2014) 305; A. Bazavov et al., PRD 93 (2016) 014512; J.-W. Chen et al., 1603.05198

Coupling to observable fluctuations

- effective interaction of particles with σ-field, e.g. $g\bar{p}\sigma p$
- additional critical fluctuation contributions:

$$\delta f_{\text{crit}} = -\frac{g}{T} f^0 (1 \pm f^0) \frac{m}{E} \delta\sigma$$



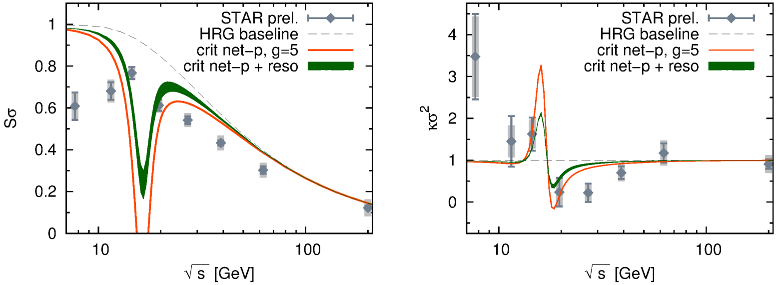
→ critical fluctuations imprinted in net-proton fluctuations

→ magnitude of signal depends on coupling g

M.A. Stephanov et al., PRL 81 (1998) 4816; *ibid.* PRD 60 (1999) 114028; C. Athanasiou et al., PRD 82 (2010) 074008; J. Thäder et al. [STAR Collaboration], 1601.00951

Influence of resonance decays

- resonance decay is a probabilistic process $\rightarrow$ significant contribution to fluctuations
- two limiting cases: no coupling to σ-field vs. chiral model inspired coupling via $g_R = \frac{g}{3}\frac{m_R}{m_p}(3 - |S_R|)$



$\rightarrow$ resonance decays reduce $S\sigma$ by $\sim$ 40% and $\kappa\sigma^2$ by $\sim$ 50%

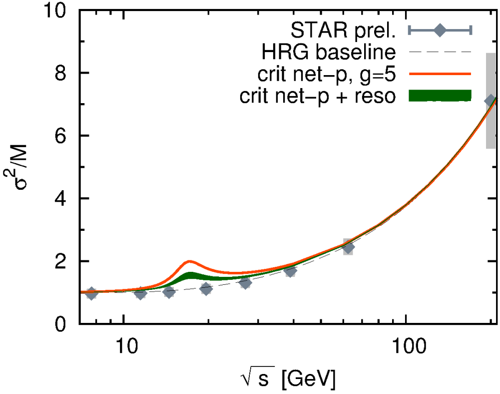
but signal survives!

D. Zschiesche et al., PRC 63 (2001) 025211; V. Dexheimer and S. Schramm, Astroph. J. 683 (2008) 943; J. Thäder et al. [STAR Collaboration], 1601.00951

What about σ^2 / M ?

Absolutely no deviation from the HRG baseline seen in the data.

⇒ provides additional important constraints!



J. Thäder et al. [STAR Collaboration], 1601.00951

What remains to be done

- too early for a quantitative comparison with data:

  input so far: equilibrium σ-field fluctuations
  neglects dynamical and non-equilibrium effects
  $\rightarrow$ critical slowing down, memory effects

$\Rightarrow$ see talk by C. Herold (NχFD)

- at $\mu_B \neq 0$, σ mixes with the net-baryon density n_B (and e and $\vec{m}$):

  within Ginzburg-Landau formalism $\rightarrow$ the diffusive mode n_B becomes the true critical (slow) mode near the CP in the long-time dynamics

$\Rightarrow$ calls for a fluid dynamical treatment including fluctuations!

B. Berdnikov and K. Rajagopal, PRD 61 (2000) 105017; H. Fuji, M. Ohtani PRD70 (2004); M. Stephanov, D. Son PRD70 (2004); M. Nahrgang et al., PRC 84 (2011) 024912; S. Mukherjee et al., PRC 92 (2015) 034912; C. Herold et al., PRC 93 (2016) 021902

Fluid dynamical fluctuations

Conventional fluid dynamics propagates thermal averages of the energy density, pressure, velocities, charge densities, etc.

However, ...

- ... already in equilibrium there are thermal fluctuations!
- ... the fast processes, which lead to local equilibration also lead to noise!

Conventional ideal fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq}$$
$$N^{\mu} = N^{\mu}_{\rm eq}$$

Y. Minami, T. Kunihiro, PTP122 (2010); P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); J. Kapusta, B. Müller, M. Stephanov PRC85 (2012); J. Kapusta, J. Torres-Rincon PRC86 (2012); C. Chafin, T. Schäfer, PRA87 (2013); P. Romatschke, R. Young, PRA87 (2013); P. Kovtun, G. Moore, P. Romatschke, JHEP1407 (2014); J. Kapusta, C. Young, PRC90 (2014); C. Young, J. Kapusta, C. Gale, S. Jeon, B. Schenke, PRC91 (2015); K. Murase, T. Hirano, 1304.3243; *ibid.* 1601.02260

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Conventional viscous fluid dynamics:

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Fluctuating viscous fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{eq}} + \Delta T^{\mu\nu}_{\mathrm{visc}} + \Xi^{\mu\nu}$$

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- In second-order fluid dynamics there are relaxation equations for $\Xi^{\mu\nu}, \ldots$:

$$u^{\gamma}\partial_{\gamma}\Xi^{\langle\mu\nu\rangle} = -\frac{\Xi^{\mu\nu} - \xi^{\mu\nu}}{\tau_{\pi}}$$

with (white) noise correlators in linear response theory

$$\langle \xi^{\mu\nu}(x)\xi^{\alpha\beta}(x')\rangle = 2T\left[\eta(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}) + (\zeta - 2/3\eta)\Delta^{\mu\nu}\Delta^{\alpha\beta}\right]\delta^4(x - x')$$

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Fluid dynamical fluctuations - nonlinearities

- correlation functions in linearized fluid dynamics describe noninteracting modes
- if nonlinearities are included $\rightarrow$ interaction of modes
  $\rightarrow$ modification of correlations
  $\rightarrow$ contributions to transport coefficients, ...
- symmetrized correlator:

$$G_S^{xyxy}(\omega, \mathbf{0}) = \int \mathrm{d}^3x \mathrm{d}t\, e^{i(\omega t - \mathbf{k}\cdot\mathbf{x})} \left\langle \frac{1}{2}\{T^{xy}(t,\mathbf{x}), T^{xy}(0,\mathbf{0})\} \right\rangle$$

- for the shear-shear contribution $\Rightarrow$

$$G_{R,\mathrm{shear-shear}}^{xyxy}(\omega, \mathbf{0}) = -\frac{7T}{90\pi^2}\Lambda^3 - i\omega\frac{7T}{60\pi^2}\frac{\Lambda}{\gamma_\eta} + (i+1)\omega^{3/2}\frac{7T}{90\pi^2}\frac{1}{\gamma_\eta^{3/2}}$$

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cutoff-dependent fluctuation contribution to pressure

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cutoff-dependent fluctuation contribution to pressure

cutoff-dependent correction to η

P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); C. Chafin, T. Schäfer, PRA87 (2013); P. Romatschke, R. Young, PRA87 (2013)

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cutoff-dependent fluctuation contribution to pressure

cutoff-dependent correction to η

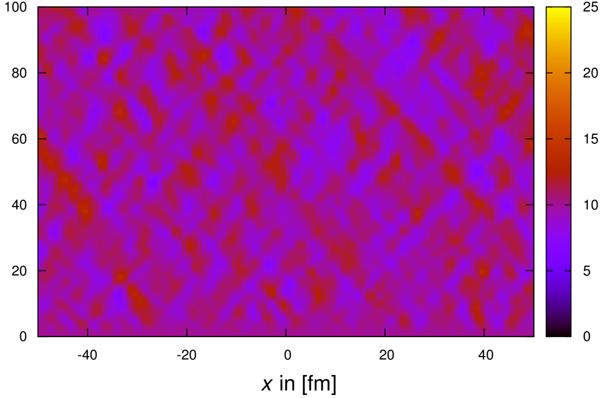
frequency-dependent contribution to η and τ_π

P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); C. Chafin, T. Schäfer, PRA87 (2013); P. Romatschke, R. Young, PRA87 (2013)

Fluid dynamical fluctuations - 1+1d

- Static "box" with periodic boundary conditions in relativistic $1+1$d fluid dynamics
- Initialized at $e_0 = 10$ GeV/fm^3 (without fluctuations nothing would happen)

consider time evolution in [fm] of e in [GeV/fm^3] for $\Delta x = 1.0$ fm



M. Nahrgang, MB, Y. Karpenko, S. Bass, T. Schäfer, work in progress

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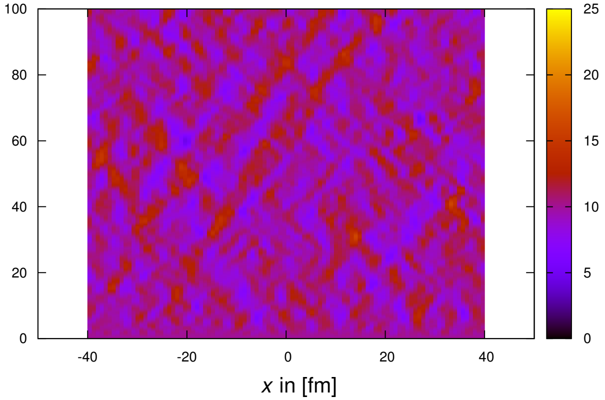
consider time evolution in [fm] of e in [GeV/fm^3] for $\Delta x = 0.8$ fm



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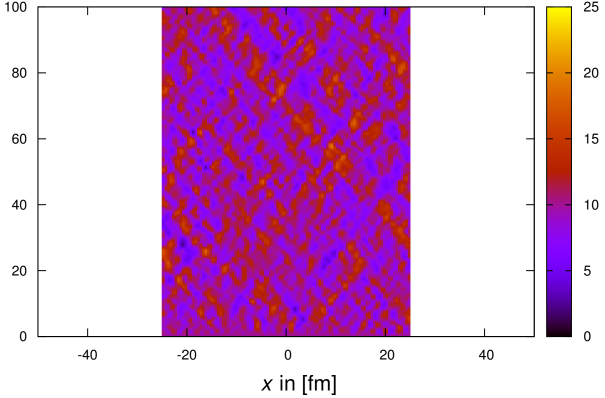
consider time evolution in [fm] of e in [GeV/fm^3] for $\Delta x = 0.5$ fm



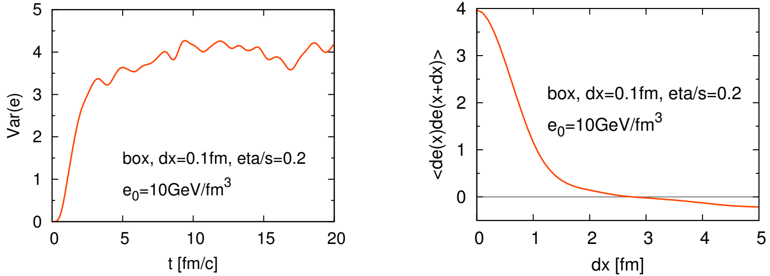
M. Nahrgang, MB, Y. Karpenko, S. Bass, T. Schäfer, work in progress

Fluid dynamical fluctuations - 3+1d

- Static box with periodic boundary conditions in relativistic $3 + 1$d fluid dynamics

$$\partial_\mu T^{\mu\nu} = \partial_\mu \left(T^{\mu\nu}_{\mathrm{eq}} + \Delta T^{\mu\nu}_{\mathrm{visc}} + \Xi^{\mu\nu} \right) = 0$$

time evolution of the variance $\langle (\Delta e)^2 \rangle$: $\langle \delta e(x) \delta e(x + dx) \rangle$ correlation function:

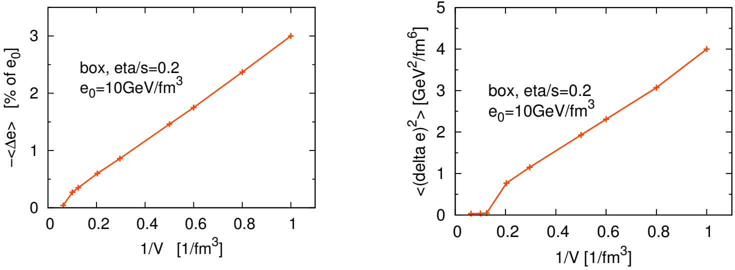


- The variance $\langle (\Delta e)^2 \rangle$ saturates after $\sim$ 5 fm.
- Fluctuations are large in the computational cell of fluid dynamics $\Rightarrow$ noise correlated over $\sim$ 1 fm^3 – reproduced.

M. Nahrgang, MB, Y. Karpenko, S. Bass, T. Schäfer, work in progress

Fluid dynamical fluctuations - 3+1d nonlinearities

- Important check: equilibrium expectations for fluctuations and nonlinear effects!



- Proportionality to $1/V$ reproduced for the correction to the average and the variance of energy density in the local rest frame.
- Implementing fluid dynamical fluctuations is important,

  but requires a sustained and systematic effort!

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Conclusions

- qualitative features of the critical point can be studied with phenomenological models:
  - → critical fluctuation signals in net-proton fluctuations are reduced by resonance decays but survive
- for a realistic dynamical treatment need to apply fluctuating dissipative fluid dynamics:
  - → in 1+1d ”box”: evolution of fluctuations clearly visible, volume dependence tested successfully
  - → in 3+1d box: expectations for modifications due to nonlinear effects verified, correlations reproduced

⇒ next: study more realistic expansion scenarios!