An Application of Functional Renormalization Group Method for Superdense Nuclear Matter

Gergely Gábor Barnaföldi, Péter Pósfay, Antal Jakovác

References:
arXiv:1604.01717 [hep-th], *Eur. Phys.* J. C (2015) **75**: 2, PoS(EPS-HEP2015)369

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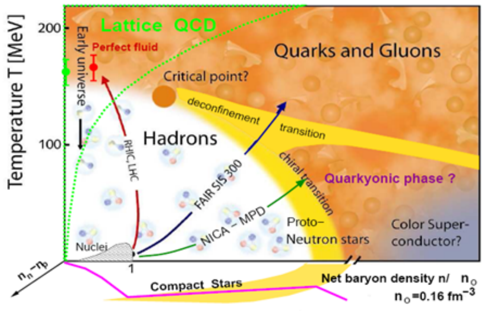
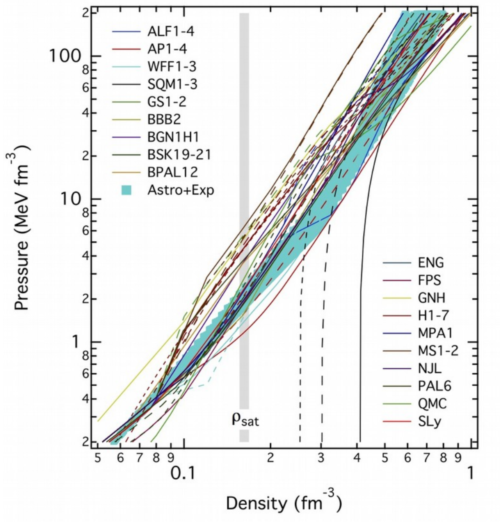
Outline

- Motivation
- Introduction to FRG method
- Anzatz for the effective action:
  - Fermi gas model at finite temperature with a Yukawa coupling
- Solving Wetterich equation for finite chemical potential
  - Local polynomial approximation
  - Wetterich equation at zero temperature
  - Solution techniques
- Results and comparison of the FRG results to other models

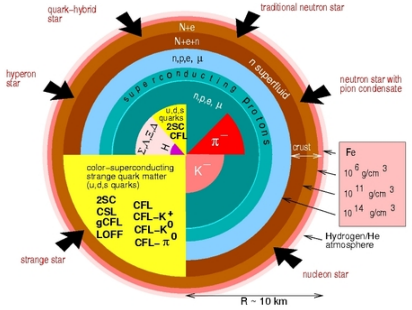
Motivation

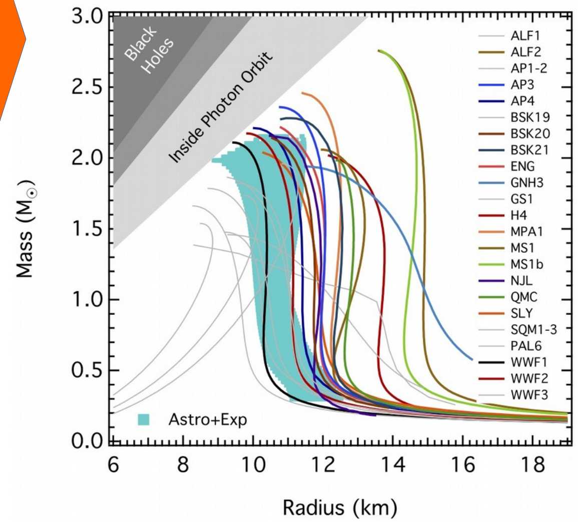
EoS from exp & theory



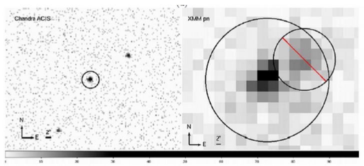


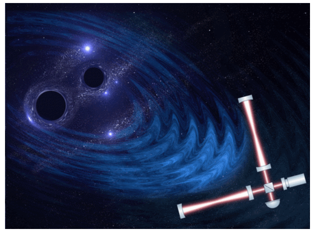
Application in compact stars

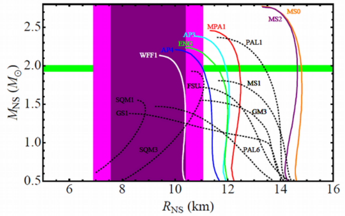


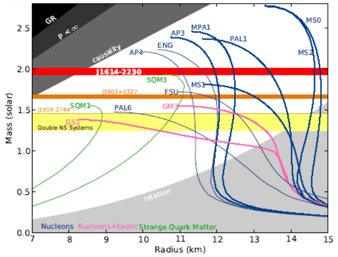


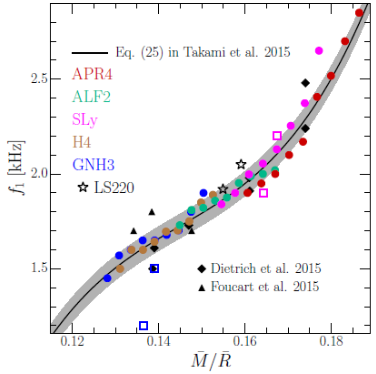
Constraints by astropysical observations





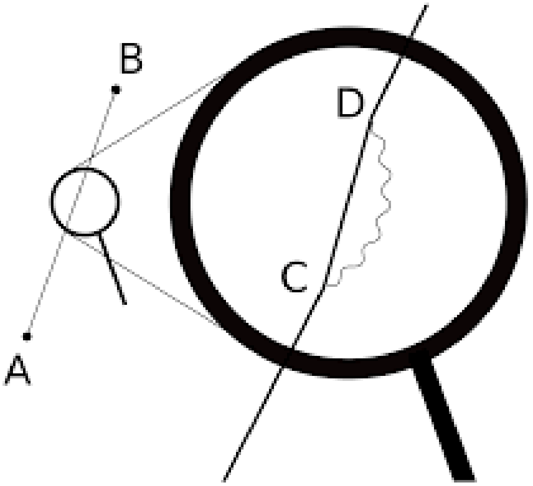


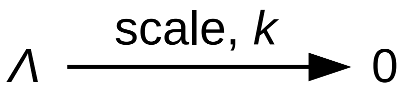


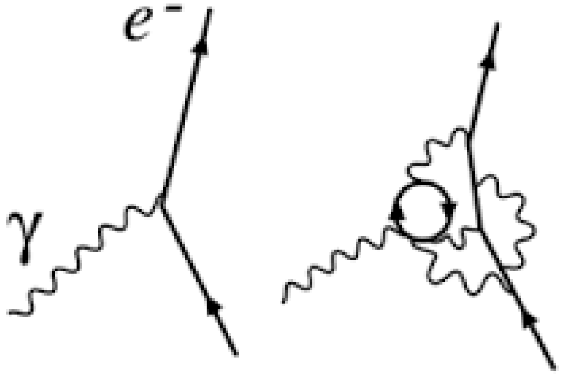


Motivation for FRG

- It is hard to get effective action for an interacting field theory: e.g.: EoS for superdense cold matter ($T \rightarrow 0$ and finite μ)
- Taking into account quantum fluctuations using a scale, k
  - Classical action, $S=\Gamma_{k\rightarrow\Lambda}$ in the UV limit, $k \rightarrow \Lambda$
  - Quantum action, $\Gamma=\Gamma_{k\rightarrow 0}$ in the IR limit, $k \rightarrow 0$
- FRG Method
  - Smooth transition from macroscopic to microscopic
  - RG method for QFT
  - Non-perturbative description
  - Not depends on coupling
  - BUT: Technically it is NOT simple







Functional Renormalization Group (FRG)

- FRG is a general non-perturbative method to determine the effective action of a system.
- **Scale dependent effective action (k scale parameter)**

$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \, STr \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

Wetterich equation

k=Λ
Classical action

Integration

k=0
Quantum fluctuations included

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Wetterich equation

- **Ansatz** for the integration,
  - not need to be perturbative
  - scale-dependent coupling

$$\Gamma_k = \sum_{l=1}^{l=N} \frac{g_l(k)}{l!} \hat{O}_l$$

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Wetterich equation

- **Regulator**
  - Determines the modes present on scale, k
  - Physics is regulator independent

Ansatz: Interacting Fermi-gas model

Ansatz for the effective action:

$$\Gamma_k[\varphi,\psi] = \int d^4x \left[\bar{\psi}\,(i\partial\!\!\!/ - g\varphi)\,\psi + \frac{1}{2}(\partial_\mu\varphi)^2 - U_k(\varphi)\right]$$

Fermions : *m=0*, **Yukawa-coupling** generates mass

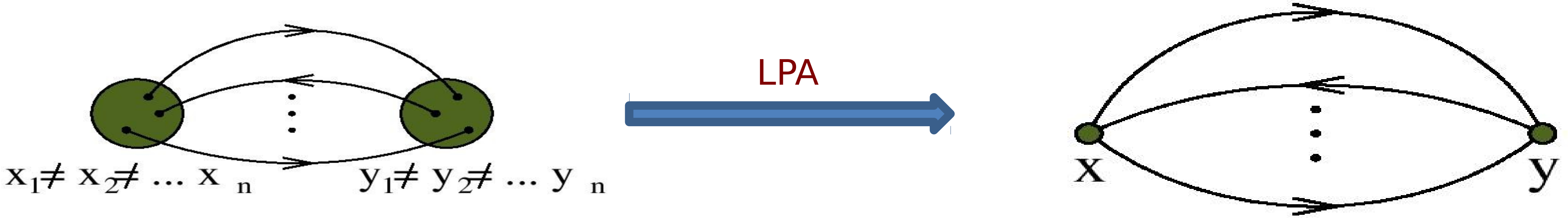
Bosons: the **potential** contains self interaction terms

We study the scale dependence of the potential only!!

Local Potential Approximation (LPA)

What does the ansatz exactly mean ?

LPA is based on the assumption that the contribution of these two diagrams are close. *(momentum dependence of the vertices is suppressed)*



This implies the following ansatz for the effective action:

$$\Gamma_k\left[\psi\right] = \int d^4x \left[\frac{1}{2}\psi_i K_{k,ij}\psi_j + U_k\left(\psi\right)\right]$$

Interacting Fermi-gas at finite temperature

Ansatz for the effective action:

$$\Gamma_k[\varphi, \psi] = \int d^4x \left[\bar{\psi} (i\partial\!\!\!/ - g\varphi) \psi + \frac{1}{2} (\partial_\mu \varphi)^2 - U_k(\varphi) \right]$$

Wetterich -equation

$$\partial_k U_k = \frac{k^4}{12\pi^2} \left[\frac{1 + 2n_B(\omega_B)}{\omega_B} + 4 \frac{-1 + n_F(\omega_F - \mu) + n_F(\omega_F + \mu)}{\omega_F} \right]$$

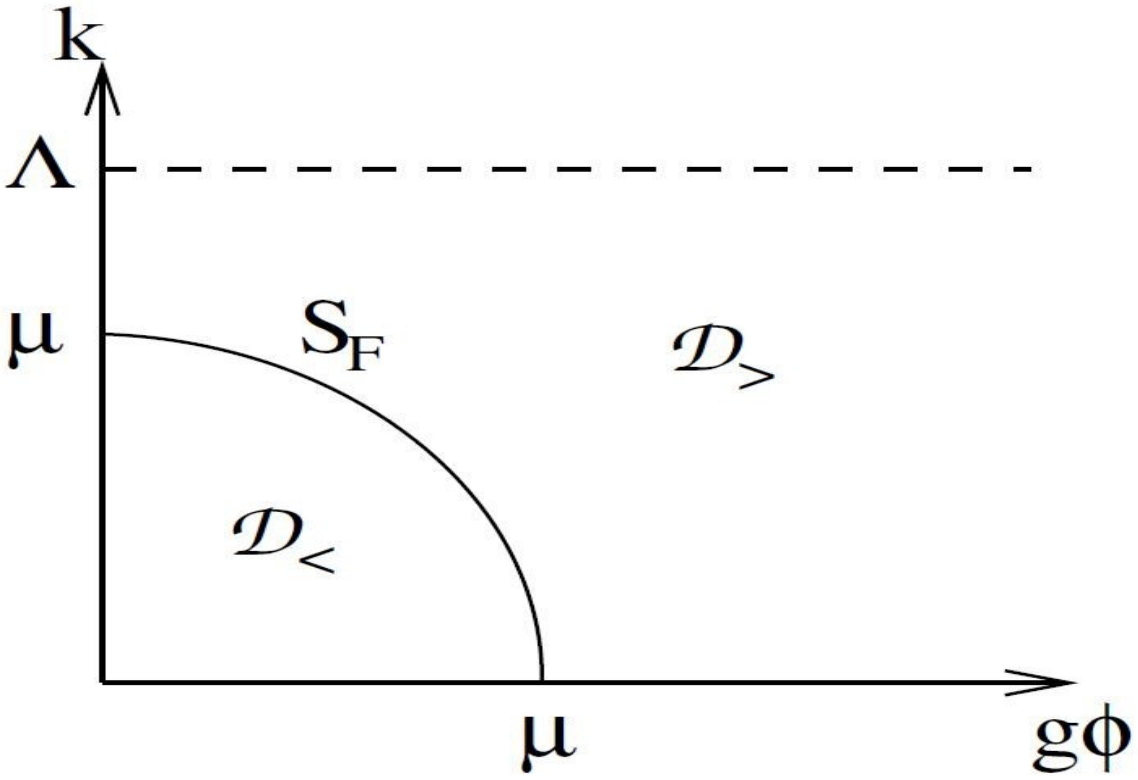
Bosonic part **Fermionic part**

$$U_\Lambda(\varphi) = \frac{m_0^2}{2}\varphi^2 + \frac{\lambda_0}{24}\varphi^4 \qquad \omega_F^2 = k^2 + g^2\varphi^2 \qquad \omega_B^2 = k^2 + \partial_\varphi^2 U \qquad n_{B/F}(\omega) = \frac{1}{1 \mp e^{-\beta\omega}}$$

Interacting Fermi-gas at zero temperature

T=0 , μ≠0 ⟹ $n_F(\omega) \to \Theta(-\omega)$

We have two equations for the two values of the step function each valid on different domain



Interacting Fermi-gas at zero temperature

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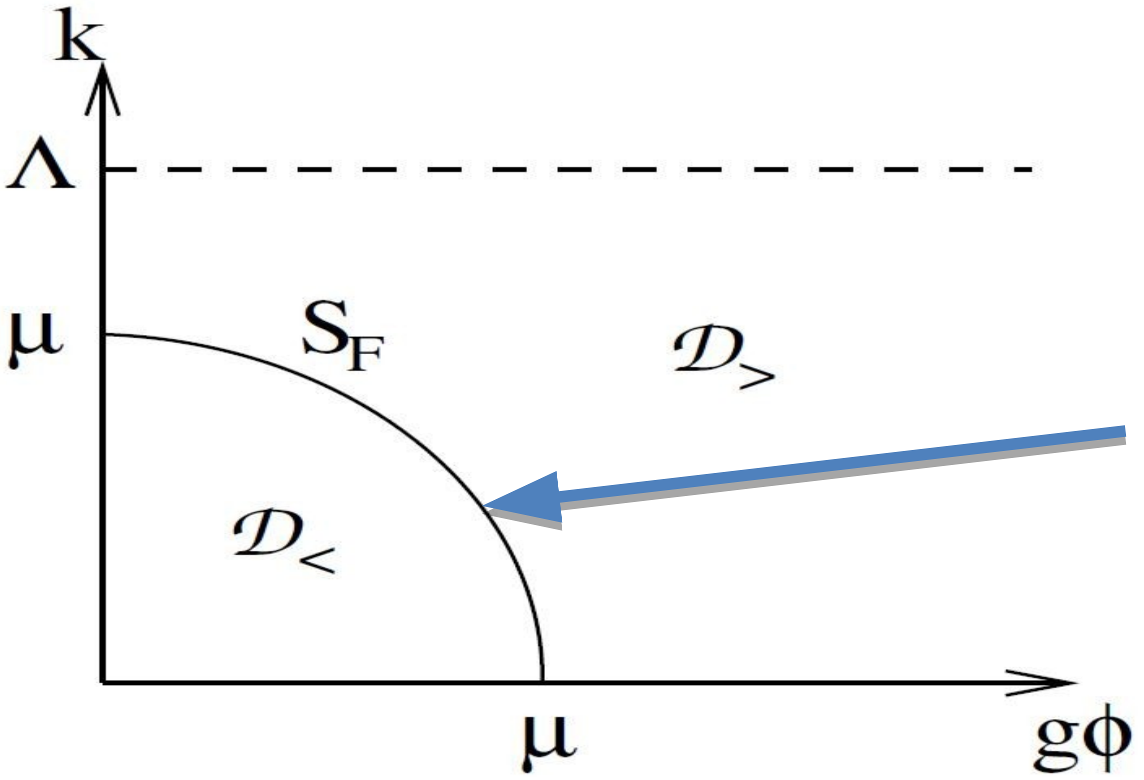


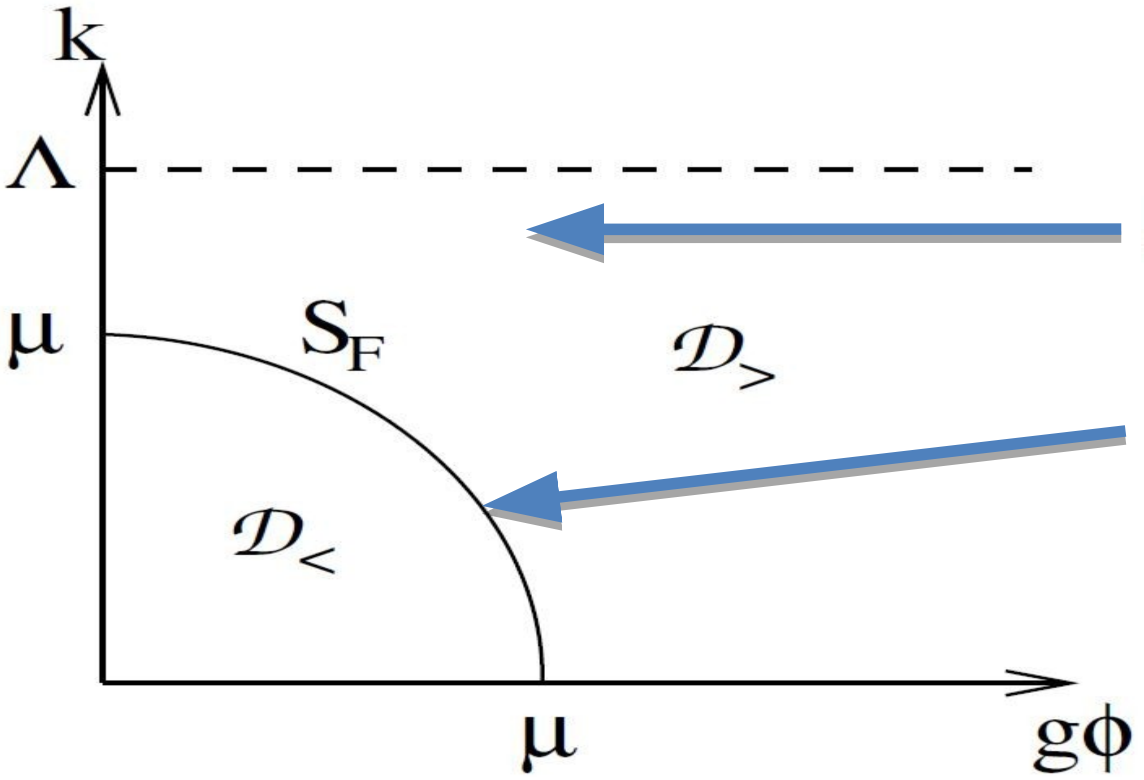
$$k_F = \sqrt{\mu^2 - g^2\varphi^2};$$

Fermi-surface

Interacting Fermi-gas at zero temperature

T=0 , μ≠0 ⟹ $n_F(\omega) \to \Theta(-\omega)$

We have two equations for the two values of the step function each valid on different domain



$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\frac{1}{\omega_B} - \frac{4}{\omega_F}\right]$$

$$k_F = \sqrt{\mu^2 - g^2\varphi^2};$$

Fermi-surface

Interacting Fermi-gas at zero temperature

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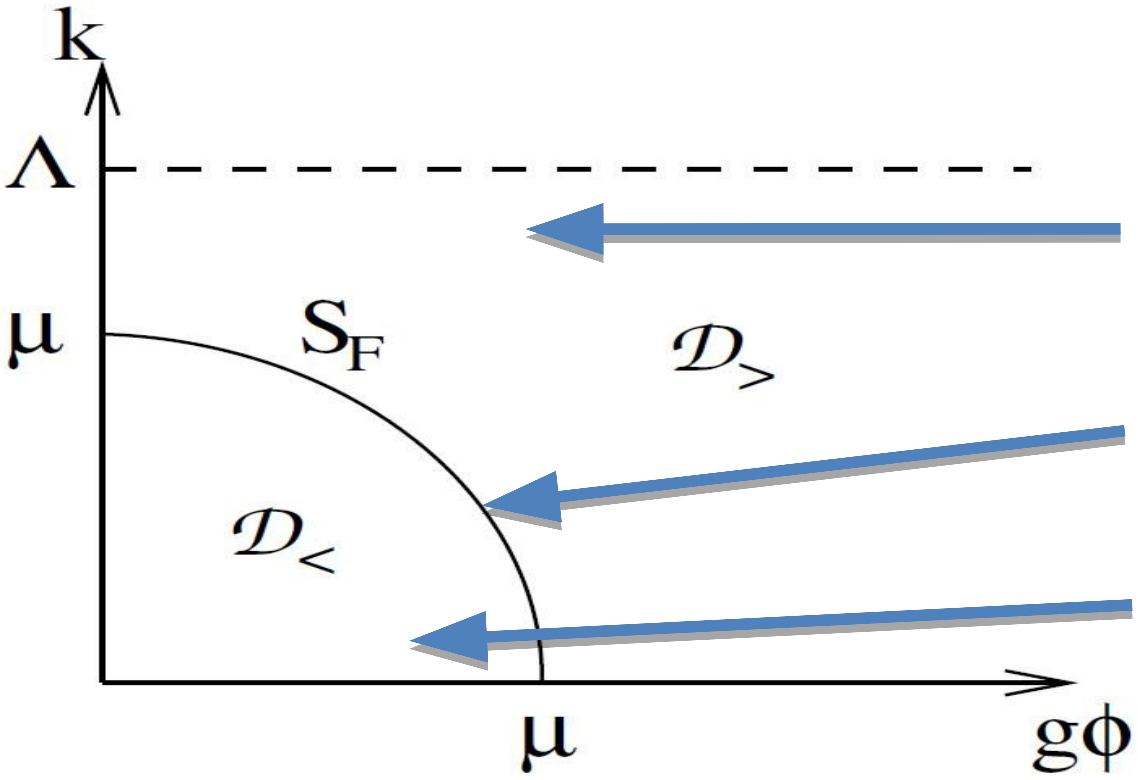


$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\frac{1}{\omega_B} - \frac{4}{\omega_F}\right]$$

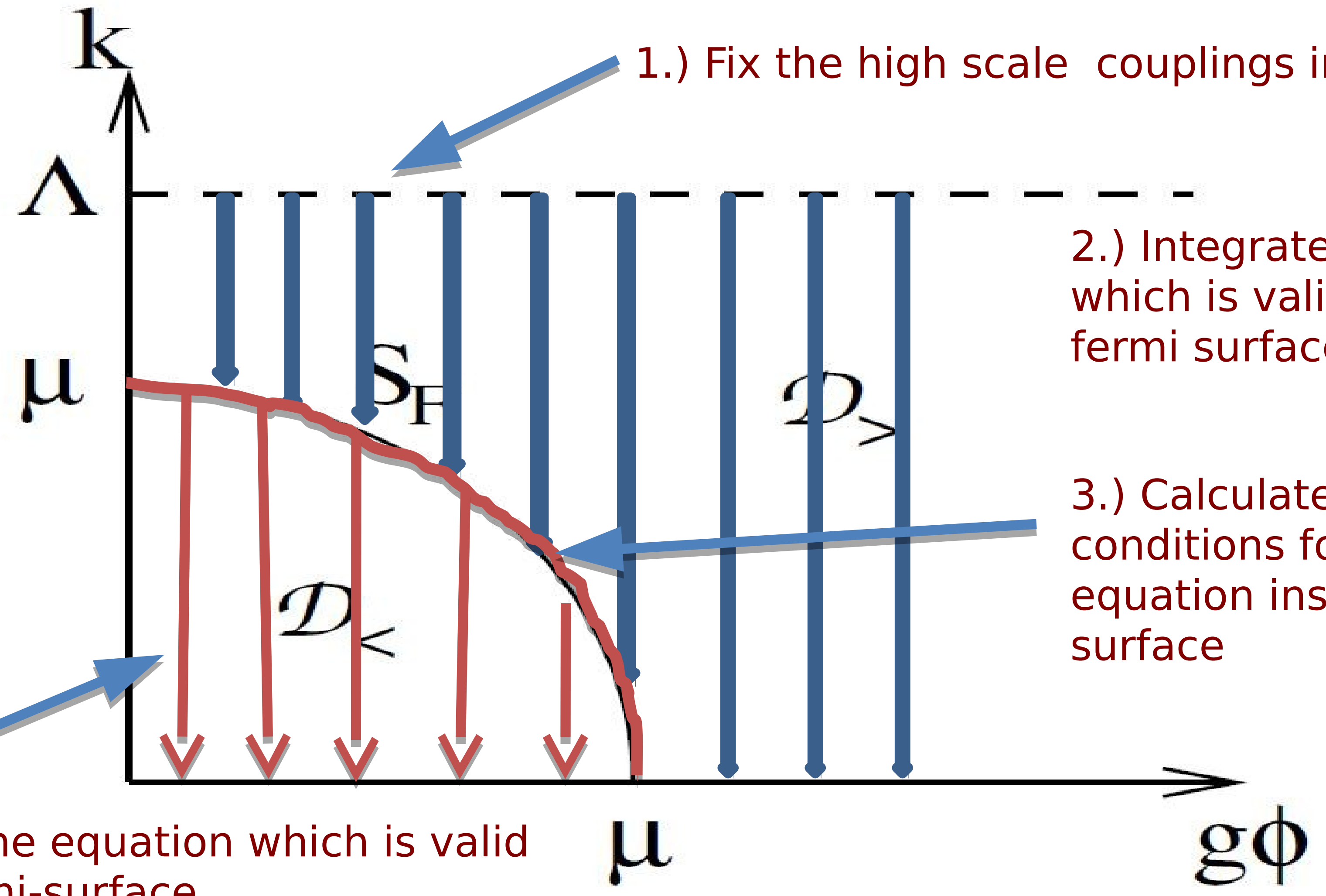
$$k_F = \sqrt{\mu^2 - g^2\varphi^2},$$

Fermi-surface

$$\partial_k U_k = \frac{k^4}{12\pi^2}\frac{1}{\omega_B}$$

Fermionic vacuum fluctuations and thermodynamic fluctuations cancel

Integration of the Wetterich-equaiton



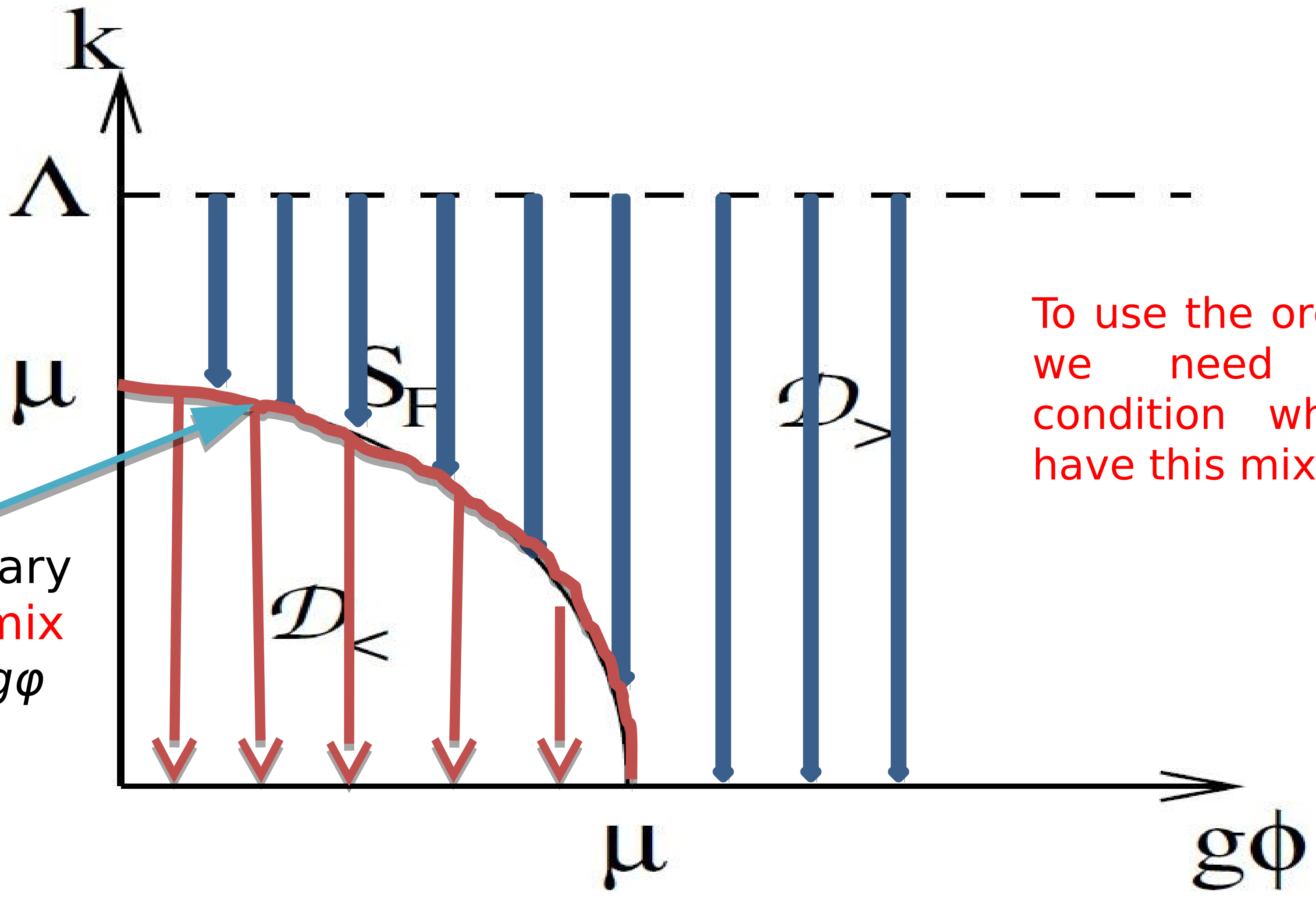
1.) Fix the high scale couplings in the theory

2.) Integrate the equation which is valid outside of the fermi surface

3.) Calculate the initial conditions for the other equation inside the fermi surface

4.) Integrate the equation which is valid below the Fermi-surface

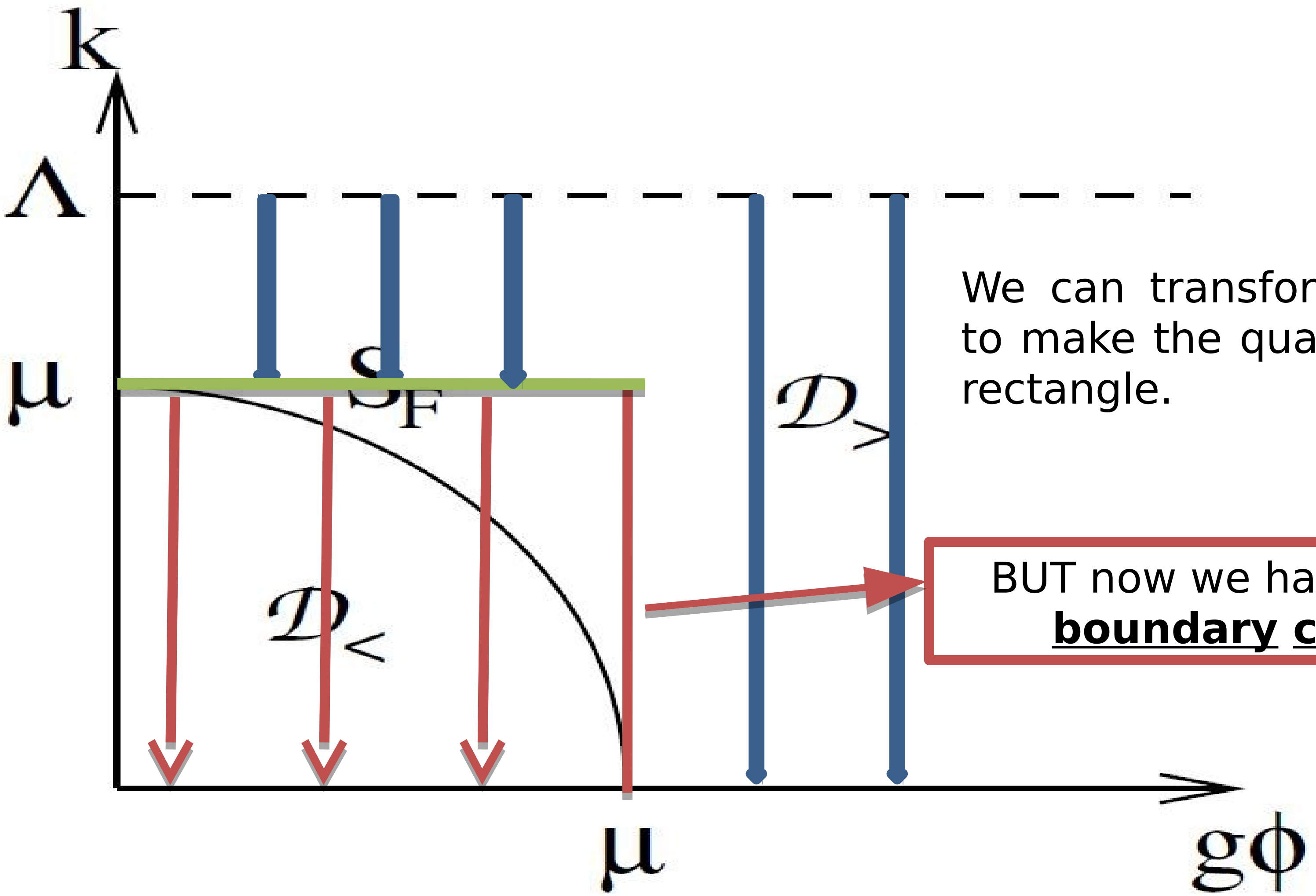
BUT…



The boundary condition mix the *k* and *gφ*

To use the orginal method we need an initial condition which do not have this mixing

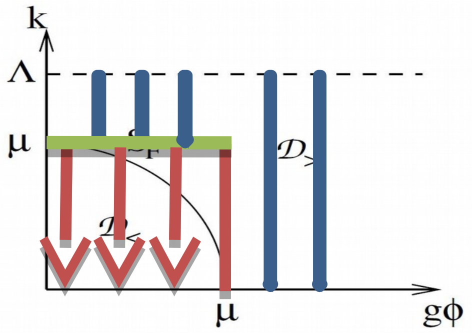
Solution: Need to transform the variables



We can transform the variables to make the quarter circle into a rectangle.

BUT now we have a well defined **boundary condition** too!

Solution: Circle → Rectangle transformation



- Coordinate transformation is required with: $(k, \varphi) \mapsto (x, y)$
  - mapping the Fermi-surface to rectangle
  - Keep the symmetries of the diff. eq.
  - Circle-rectangle transformation: $x = \varphi_F(k), \quad y = \dfrac{\varphi}{x}$

- Transformation of the potential: $\tilde{U}(x, y) = V_0(x) + \tilde{u}(x, y)$

  with boundary condition at the Fermi-surface, V_0

- Transformed Wetterich-eq: $x\partial_x \tilde{u} = -xV_0' + y\partial_y \tilde{u} - \dfrac{g^2 (kx)^3}{12\pi^2} \dfrac{1}{\sqrt{(kx)^2 + \partial_y^2 \tilde{u}}}.$

- and the new boundary conditions: $\tilde{u}(x = 0, y) = \tilde{u}(x, y = \pm 1) = 0.$

Solution of transformed Wetterich by an orthogonal system

- Solution is expanded in an orthogonal basis to accommodate the strict boundary condition in the transformed area

$$\tilde{u}(x,y) = \sum_{n=0}^{\infty} c_n(x) h_n(y) \quad h_n(1) = 0 \quad \int_0^1 dy\, h_n(y) h_m(y) = \delta_{nm}$$

- The square root in the Wetterich-equation is also expanded:

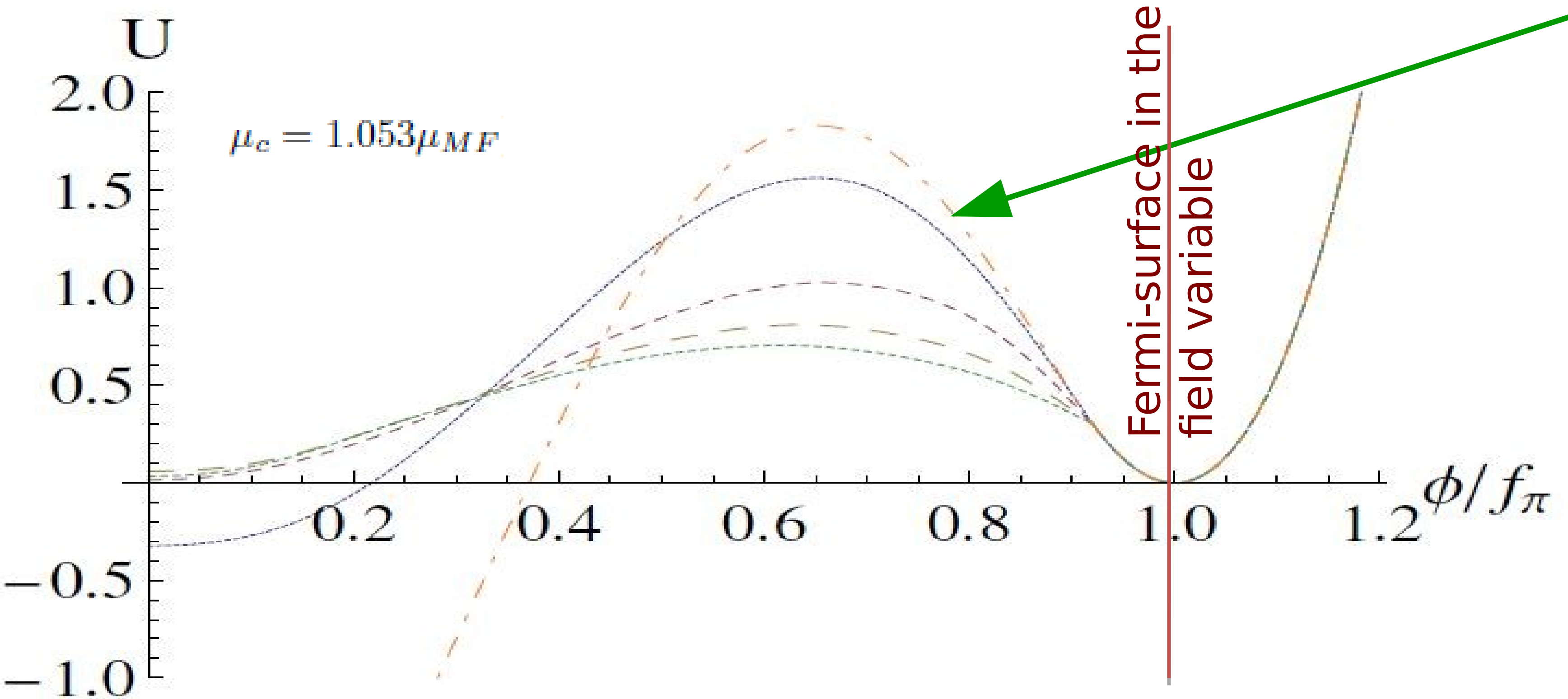
$$x c_n'(x) = \int_0^1 dy\, h_n(y) \left[-x V_0' + y \partial_y \tilde{u} - \frac{g^2 (kx)^3}{12\pi^2} \sum_{p=0}^{\infty} \binom{-1/2}{p} \frac{(\partial_y^2 \tilde{u} - M^2)^p}{\omega^{2p+1}} \right]$$

Where: $\omega^2 = (kx)^2 + M^2$

Expanded square root

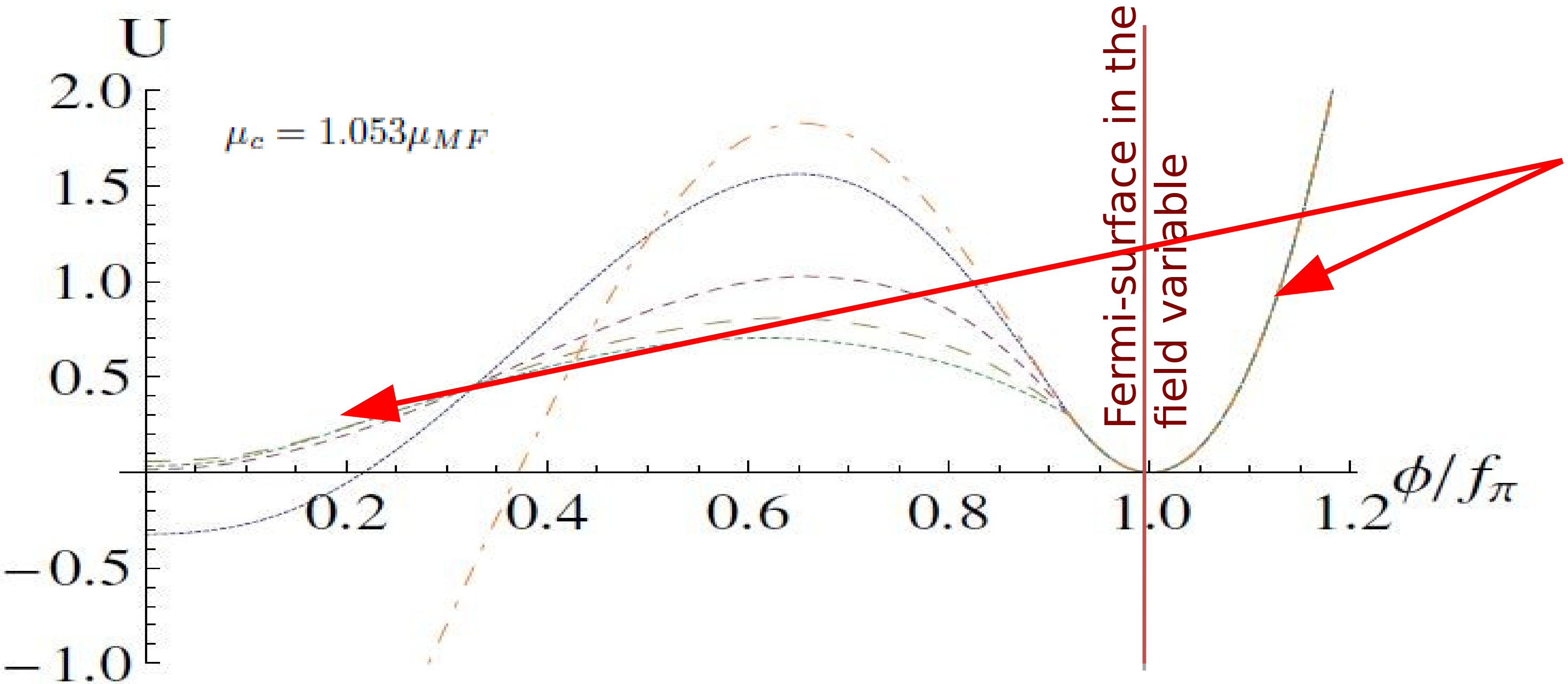
We use harmonic base: $h_n(y) = \sqrt{2} \cos q_n y, \quad q_n = (2n+1)\frac{\pi}{2}$

Result: The Effective Potential & Comparison



Potential in one-loop approximation

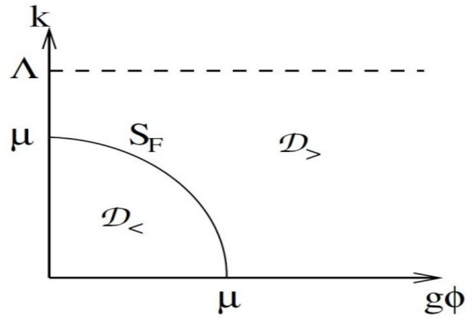
Result: The Effective Potential & Comparison



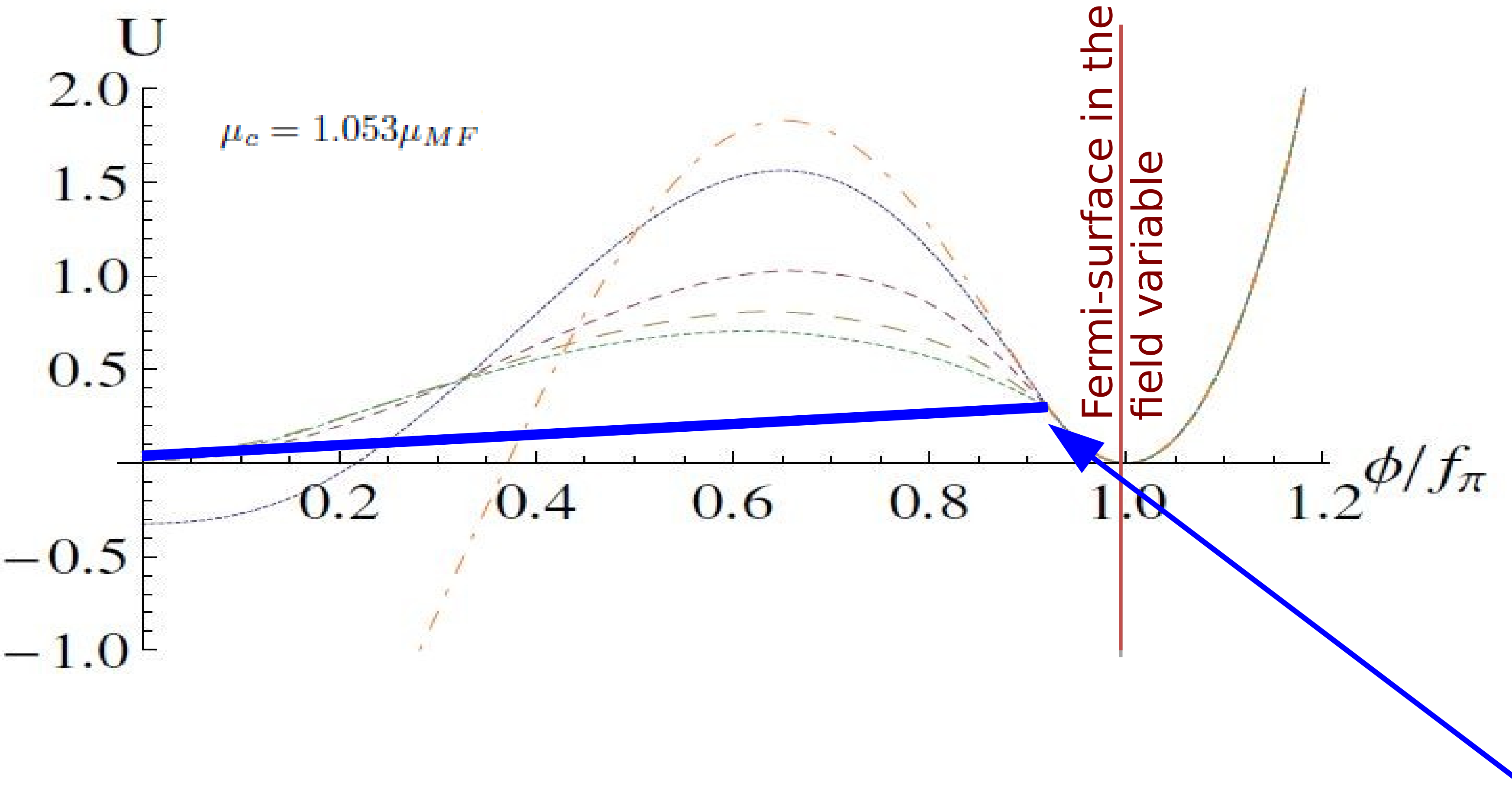
Potential in one-loop approximation

Higher orders of the Taylor-expansion for the square root converge fast where the potential is **convex**
→ **coarse grained action**

Solution changes only below Fermi-surface, since switch to another equation



Result: The Effective Potential & Comparison

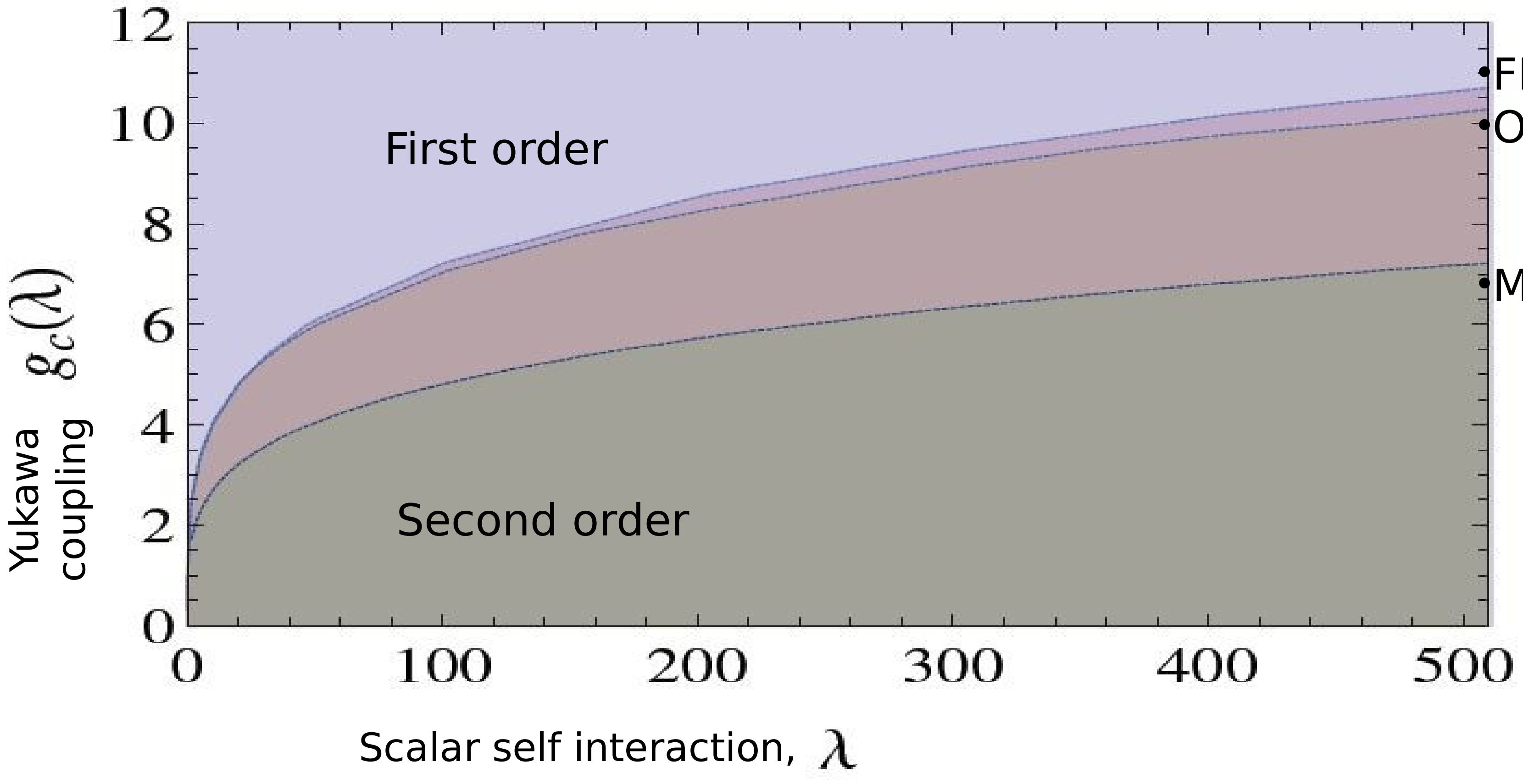


Potential in one-loop approximation

Higher orders of the Taylor-expansion for the square root converge fast where the potential is **convex**
→ **coarse grained action**

In the **concave** part of the potential solution is slowly converges to a straight line, because the free energy (effective potential) must be convex from thermodynamical reasons
→ **Maxwell construction**

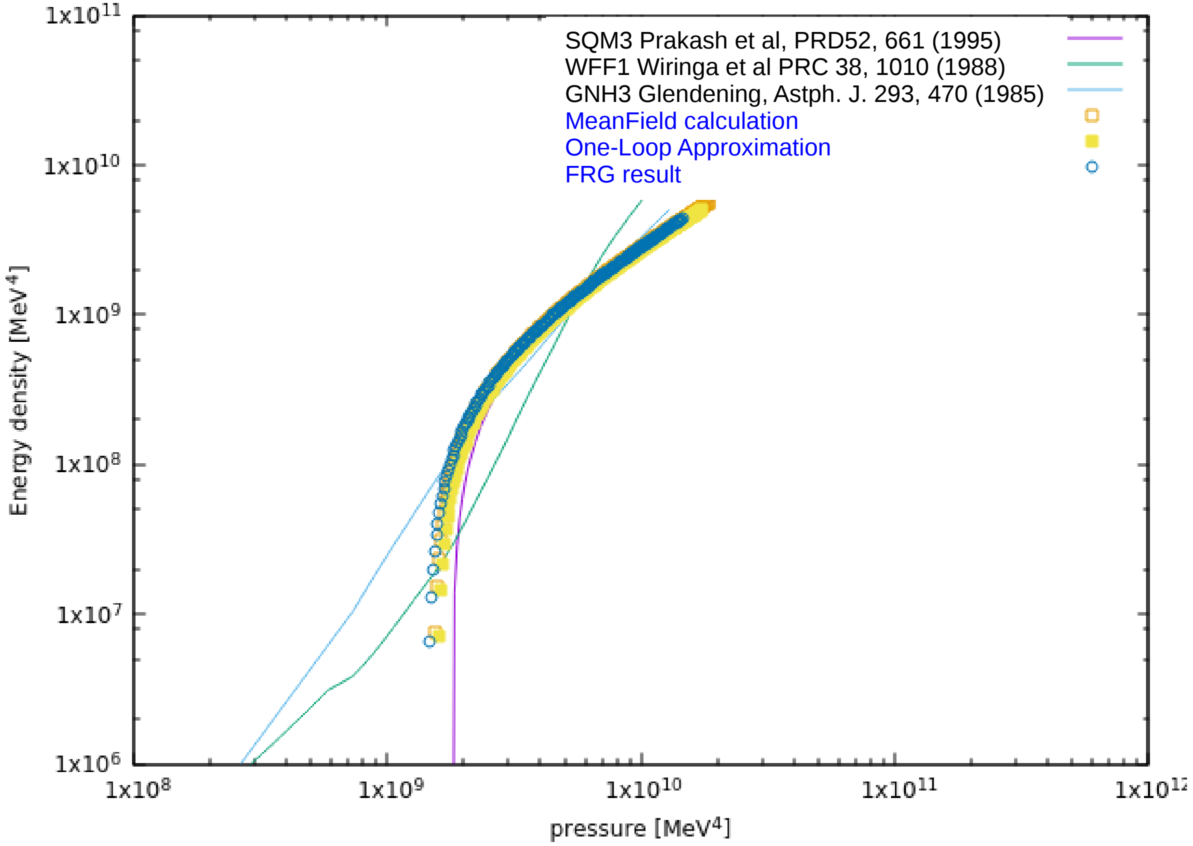
Result: Phase structure of interacting Fermi gas model

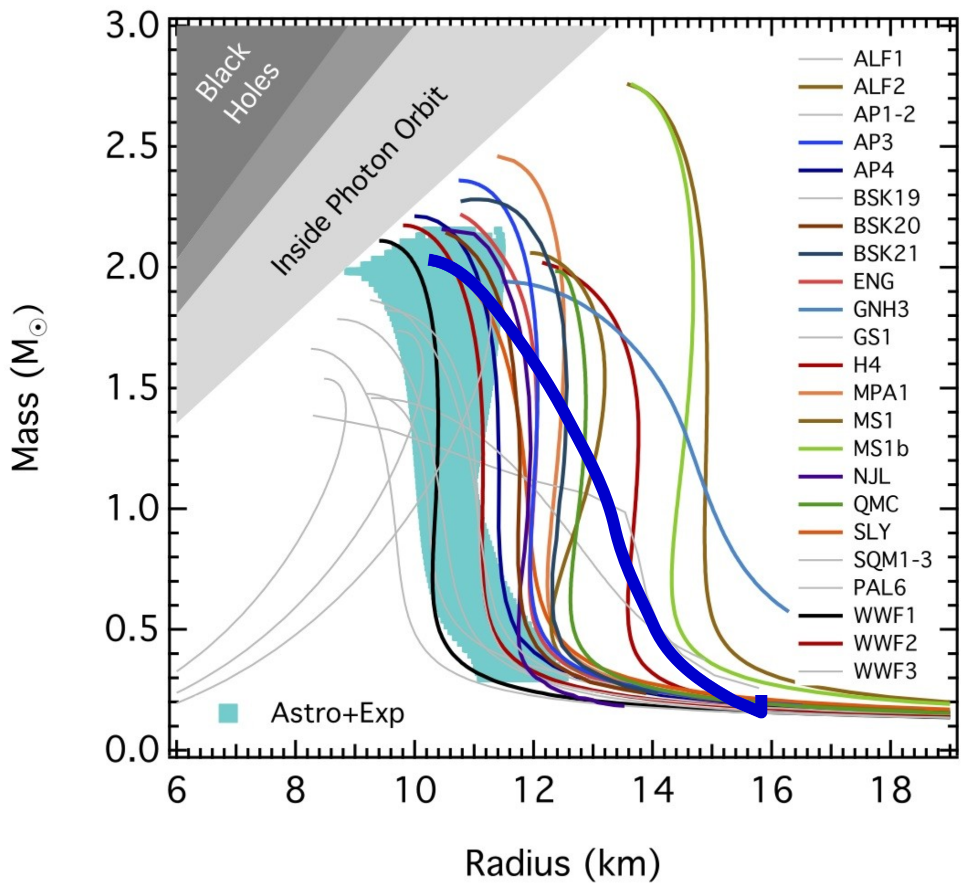


- FRG method
- One loop approximation
- Mean field approximation

Result: Test in a Compact Star

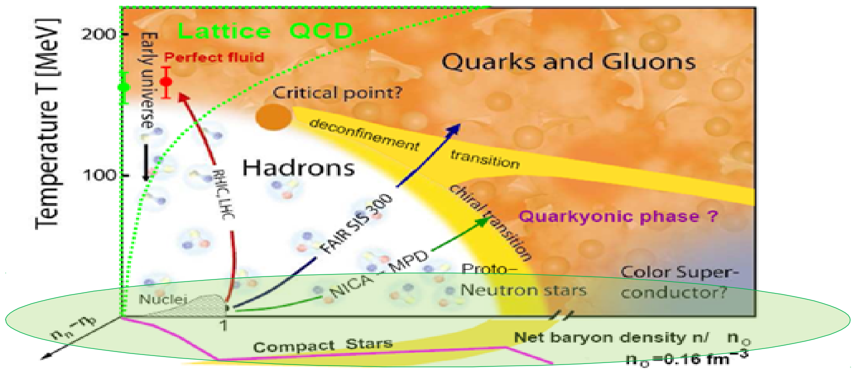
- Compare FRG EoS to SQM3, GNH3, WFF1 → TOV result on M(R) diagram



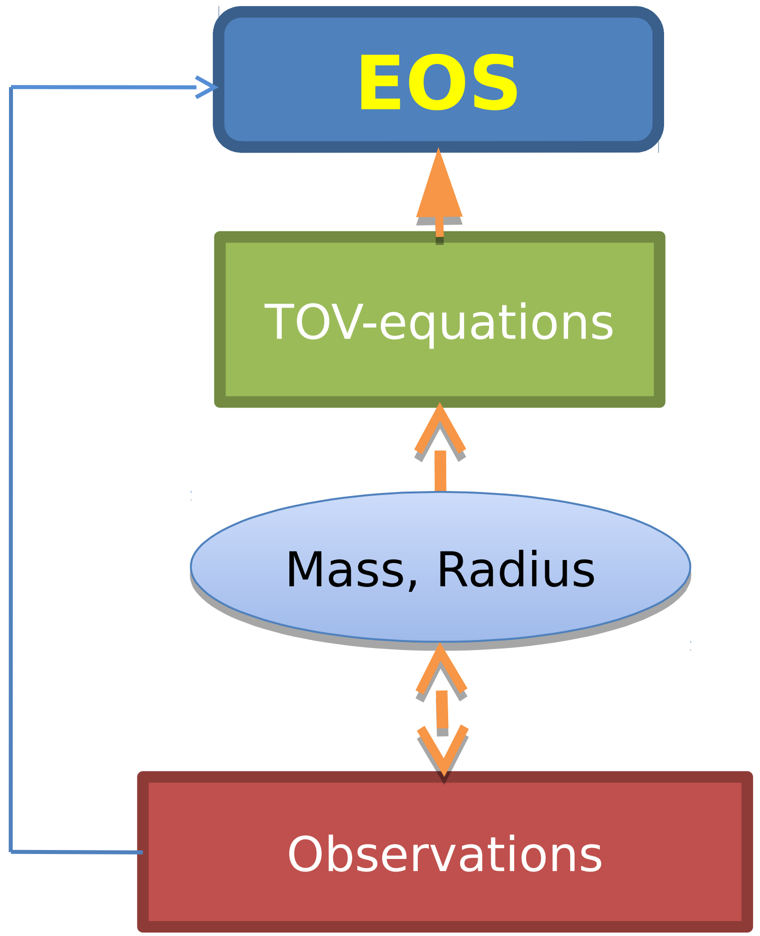


Summary

- FRG method were used to obtain the effective potential for
  - interacting Fermi gas with a simple Yukawa coupling
  - Concave part of the potential converges slowly to a line → Maxwell construction
  - Convex part of the potential → Coarse Grained action
  - Chiral phase transition is reproduced
- Based on FRG method, now we can have a technique to make:
  - An effective model for the hardly accessible part of the phase diagram at:
    - Low temperature
    - High-density
    - Finite chemical potential
- Stay tuned: these all we need for Compact Star EoS



Motivation for Using FRG



What are the effects of quantum fluctuations on the Equation of State (EOS) ?

What is the difference between the same parameters in mean field and quantum flucuations included ?

- Compressibility (important for neutron star mass!)
- Binding energy
- Surface tension of nuclear matter

FRG is a general method to take quantum fluctuations into account.