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Vorticity in heavy-ion collisions

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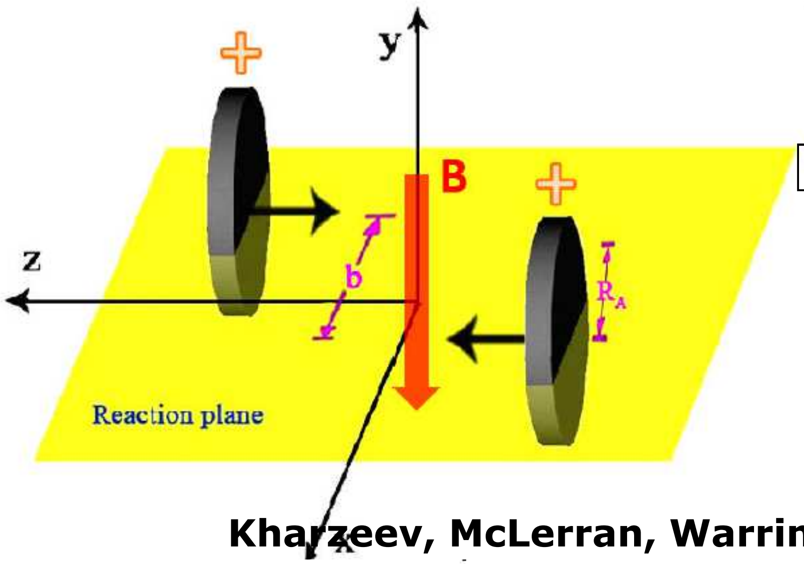
Outline

- Introduction
- Vorticity in heavy-ion collisions
- Event-by-event fluctuation of vorticity orientation
- Summary

Introduction

Introduction

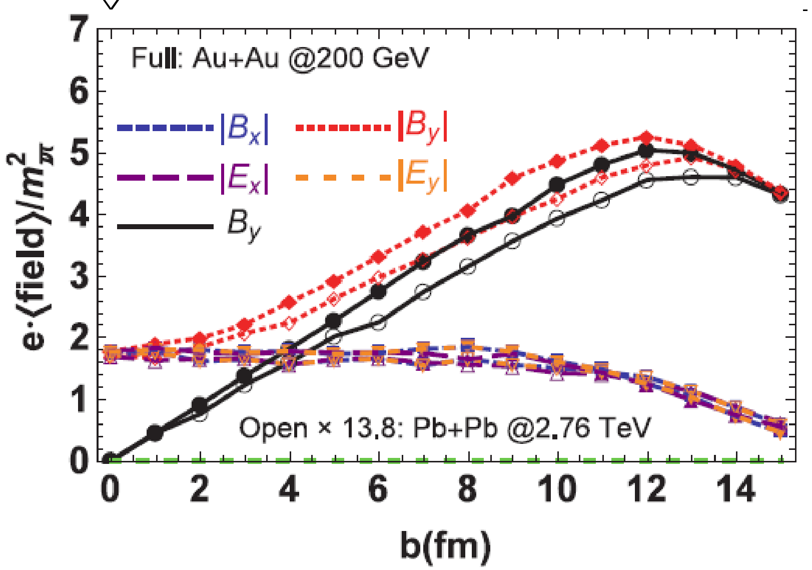
Consider a non-central heavy-ion collision



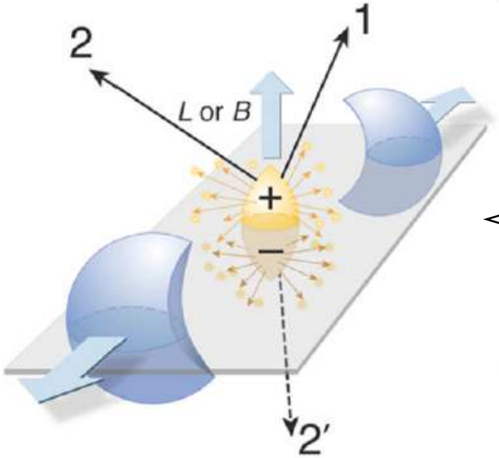
generate magnetic field

Actually, B field is very strong

Kharzeev, McLerran, Warringa 2007



Chiral magnetic effect

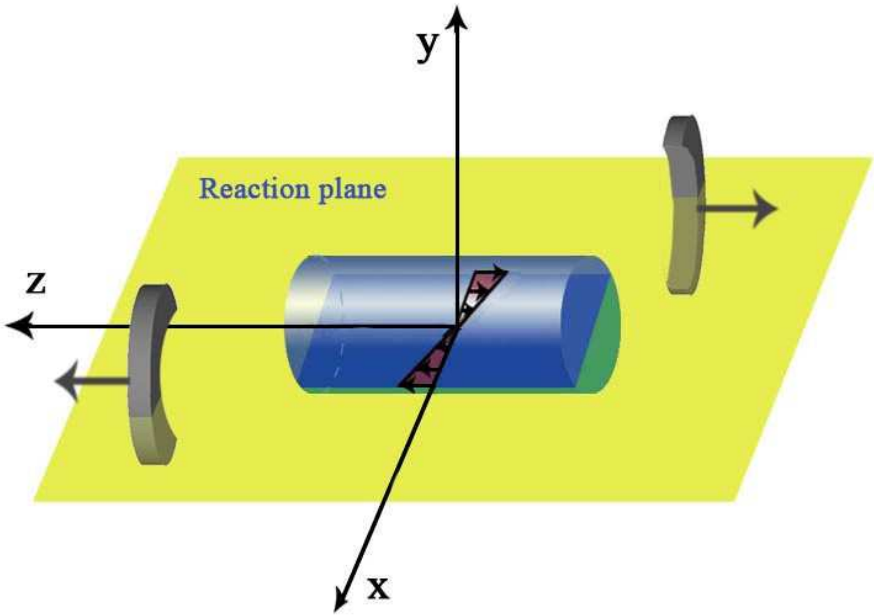


$$\mathrm{J}_V = \frac{N_c e}{2\pi^2}\mu_A \mathrm{B}$$

Skokov et al 2009, 2012, Deng and XGH 2012, 2015, … …

Introduction

Consider a non-central heavy-ion collision



Finite angular moment (AM)

⇩

Manifested as flow shear*

⇩

Finite vorticity(local rotation)

$$\vec{\omega} = \vec{\nabla} \times \vec{v}$$

$$J_0 \sim Ab\sqrt{s}/2 \quad \Longrightarrow \quad J \sim \int d^3x I(x)\omega(x)$$

$I(x) \sim [x^2 - (x \cdot \hat{\omega})^2]\varepsilon(x)$ is the moment of inertia density

J_0 is about 10^6 for RHIC Au+Au @ 200 GeV,
system volume is ~ fm^3, very large AM density

***For low energy collision, the system after collision may be globally rotating**

Introduction

Such vorticity can bring interesting phenomena

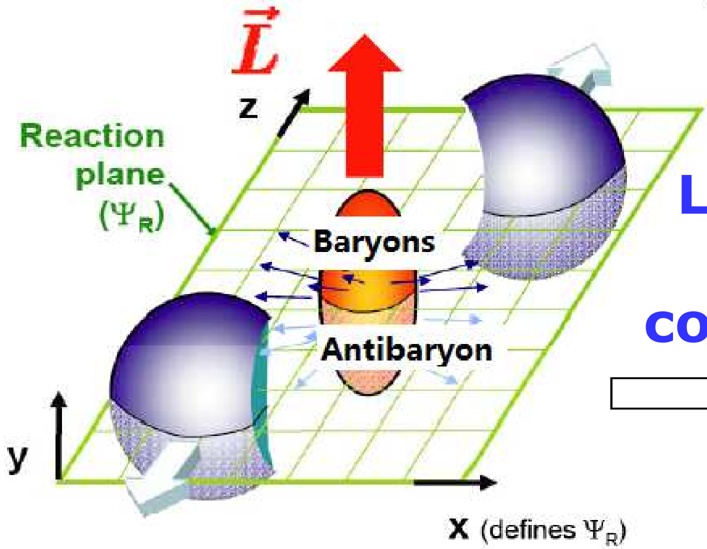
Chiral vortical effect: vorticity + chiral anomaly

(Kharzeev, Zhitnitsky 2007, Erdmenger etal 2009, Son, Surowka 2009, Banerjee etal 2011, Landsteiner etal 2011)

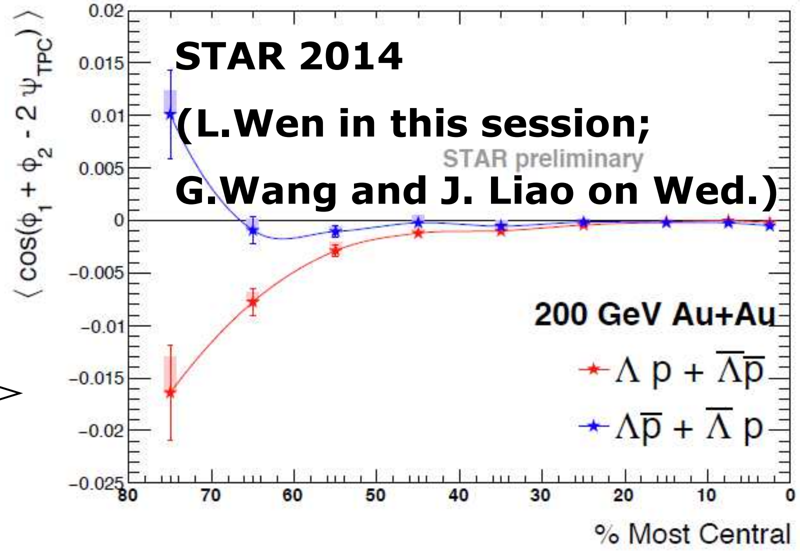
$$\vec{j} = \chi \vec{\omega}, \qquad \chi = N_c \mu \mu_5 / (2\pi^2)$$

$$\vec{j}_5 = \chi_5 \vec{\omega}, \qquad \chi_5 = N_c [T^2/12 + (\mu^2 + \mu_5^2)/(4\pi^2)]$$

Phenomenology: baryon-antibaryon separation w.r.t reaction plane



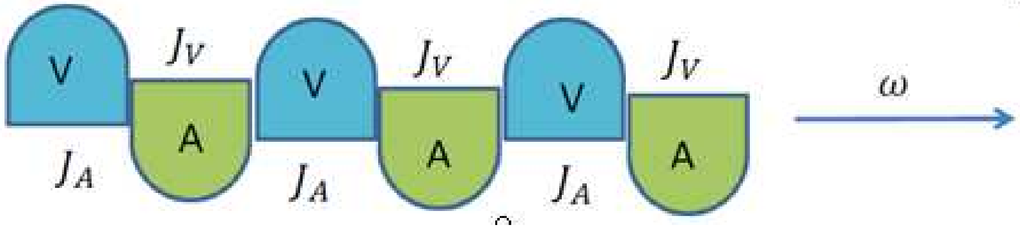
Lambda-proton correlation



Introduction

Such vorticity can bring interesting phenomena

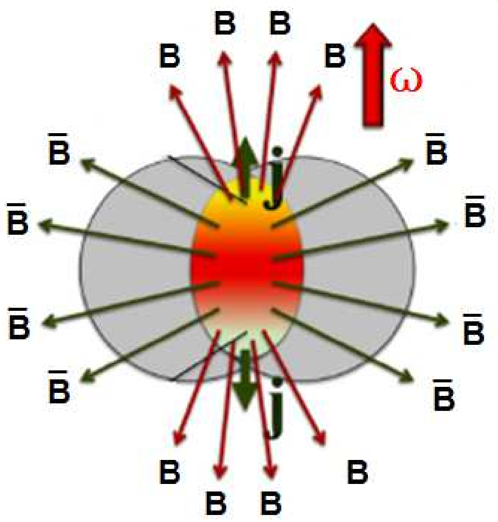
Chiral vortical wave: collective modes from CVE
(Jiang, Liao, XGH 2015)

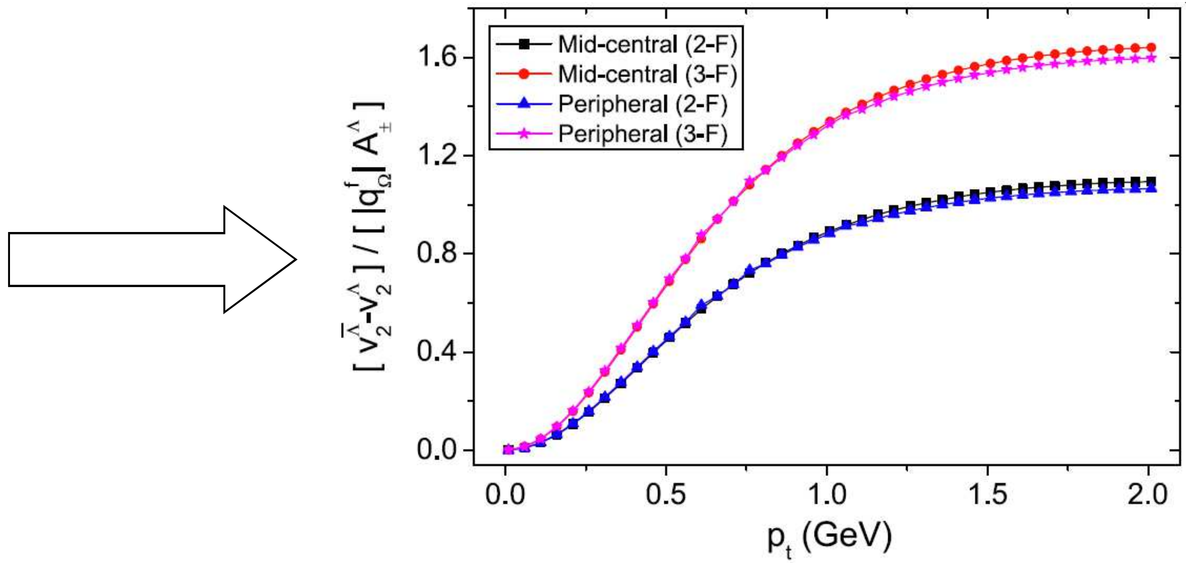


$$\partial_t n_{L,R} = \pm \frac{2\omega\alpha^2}{\pi^2} n_0 \partial_x (n_{L,R})$$

CVW velocity

Phenomenology: v2 splitting between baryons and antibaryons

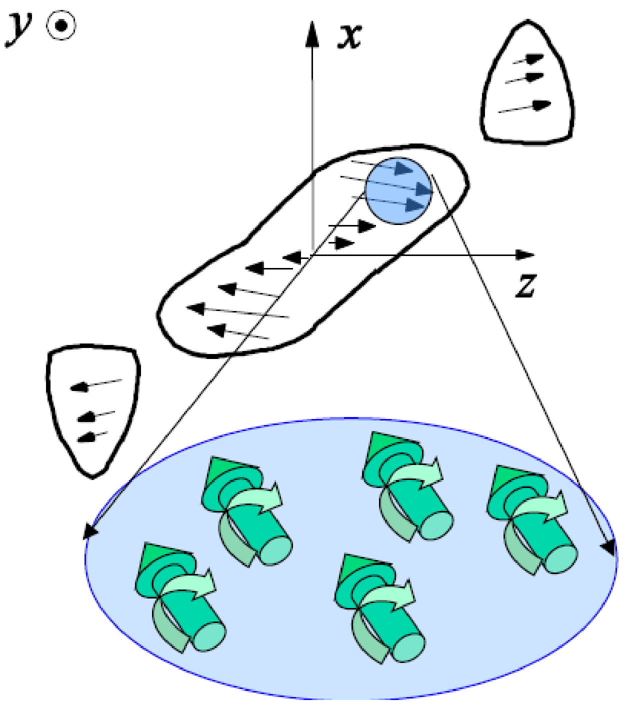




Introduction

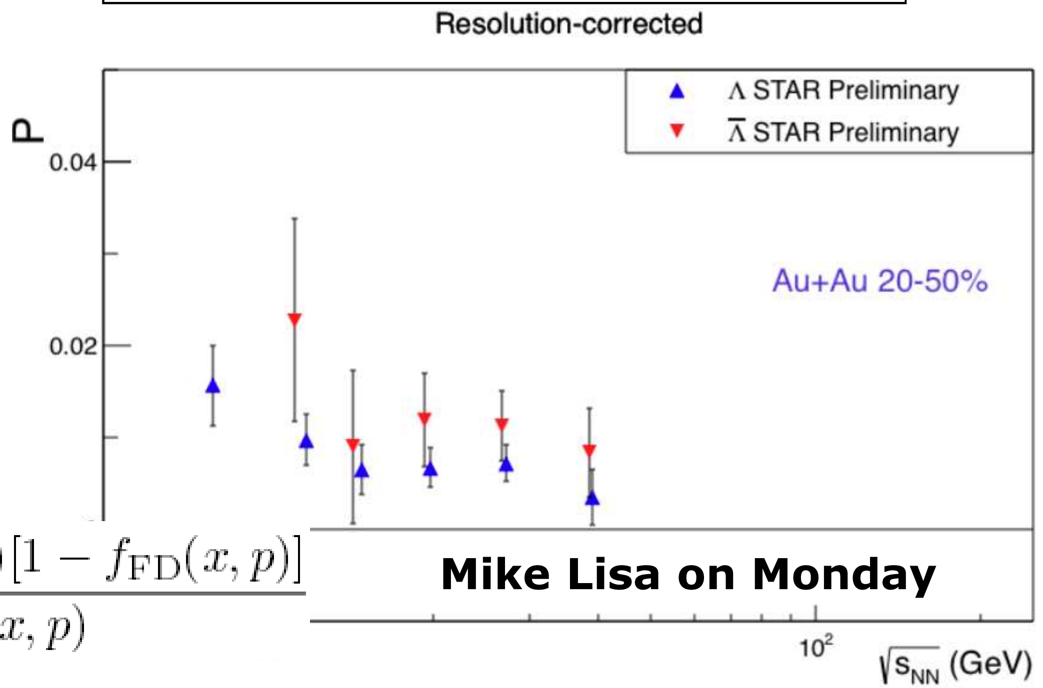
Such vorticity can bring interesting phenomena

Global spin polarization of quarks due to spin-vorticity coupling **(Liang and Wang 2005; Becattini etal 2013; Wang etal 2016)**
(Becattini on Monday Pang and Karpenko in this session)



Hadrons can inherit the spin polarization of quarks

E.g., Lambda polarization



$$\mathscr{P}^{\alpha}(p) = \frac{\hbar}{4m} \frac{\int d\Sigma_\lambda p^\lambda \tilde{\Omega}^{\alpha\sigma} p_\sigma f_{\rm FD}(x,p)[1 - f_{\rm FD}(x,p)]}{\int d\Sigma_\lambda p^\lambda f_{\rm FD}(x,p)}$$

Mike Lisa on Monday

Vorticity in heavy-ion collisions

Based on: W.T.Deng and XGH, arXiv: 1603.06117

Other works (sorry for having no enough time to show all these results):

Becattini etal, 2008, 2010, 2015
Csernai etal, 2011-2015
Huang,Huovinen,Wang, 2011
Sorin etal, 2013
Jiang, Lin, Liao, 2016
Pang, Petersen, Wang, Wang, 2016

(See talks by Becattini, Lisa, Pang, Karpenko, Liao)

Vorticity in HICs

As vorticity play a key role in CVE and Lambda polarization, we now study vorticity itself in detail

Event-by-event generation of vorticity in HICs by using HIJING model

Definition of velocity field

$$v_1^a(x) = \frac{1}{\sum_i \Phi(x,x_i)} \sum_i \frac{p_i^a}{p_i^0} \Phi(x,x_i) = \frac{J^a}{J^0}$$ **~ Particle flow velocity**

$$v_2^a(x) = \frac{\sum_i p_i^a \Phi(x,x_i)}{\sum_i [p_i^0 + (p_i^a)^2/p_i^0] \Phi(x,x_i)} = \frac{T^{0a}}{T^{00}+T^{aa}}$$ **~ Energy flow velocity**

Smearing function Phi

$$\Phi_G(x,x_i) = \frac{K}{\tau_0 \sqrt{2\pi\sigma_\eta^2} 2\pi\sigma_r^2} \exp\left[-\frac{(x-x_i)^2+(y-y_i)^2}{2\sigma_r^2} - \frac{(\eta-\eta_i)^2}{2\sigma_\eta^2}\right]$$

Parameters are so chosen that with hydro, it is consistent with data of hadron spectra and v2 **(Pang, Wang, Wang 2012)**

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Definition of vorticity field (for each definition of v)

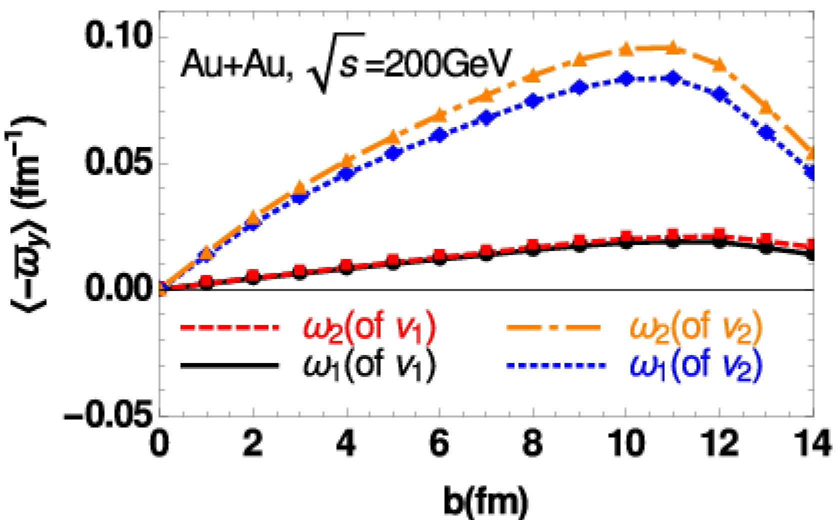
$$\omega_1 = \nabla \times v,$$ **~ nonrelativistic definition**

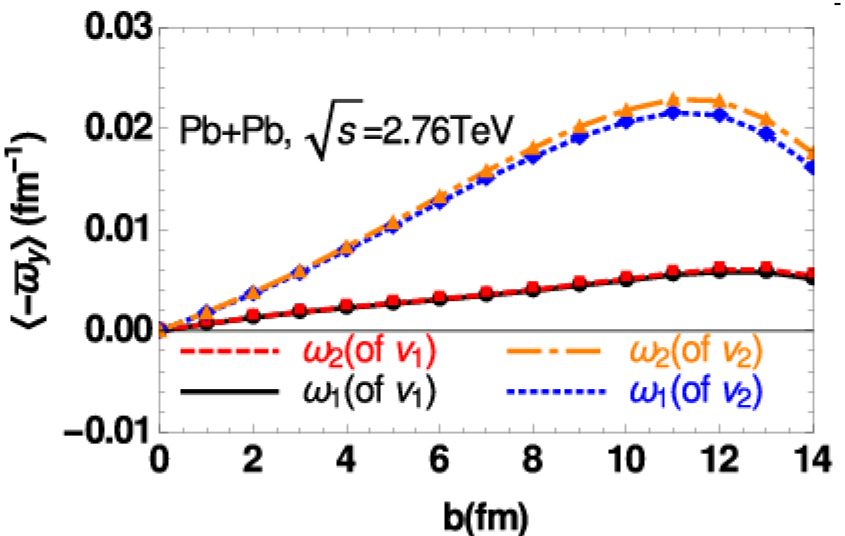
$$\omega_2 = \gamma^2 \nabla \times v.$$ **~ relativistic definition with Lorentz correction**

≈ spatial component of $\omega^\mu = \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$

Vorticity in HICs

Impact parameter dependence of vorticity

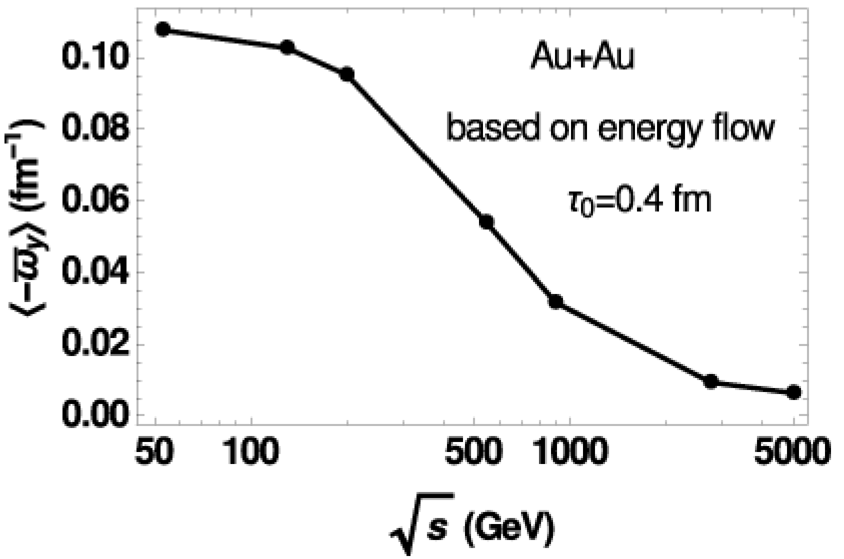


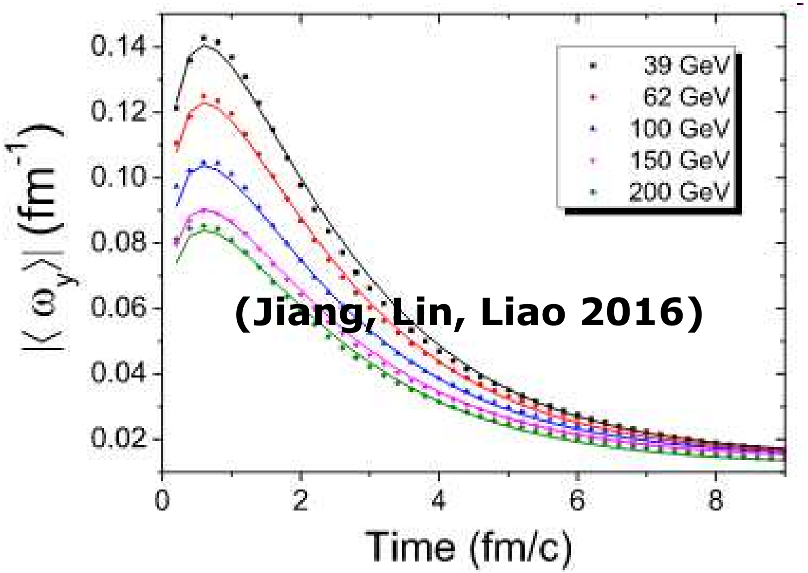


- **Showed is the vorticity at zero rapidity at initial time averaged over the reaction zone and averaged over 10^5 events.**
- **Vorticity of energy flow at RHIC at b=10 fm is 10^22 Hz. (Fastest man-made rotation via laser light ~ 10^7 Hz (Arita etal Nat.Comm. 2013))**
- **RHIC: Take T~300 MeV, T*vorticity~10^4 MeV^2 comparable to magnetic field eB. But at LHC, initial vortical effect is smaller than eB**
- **At b<2R_A, increase with b; then drops. Anglular momentum of the overlapping region has a similar behavior.**

Vorticity in HICs

Collision energy dependence





•**Consistent with the Lambda polarization result of STAR**

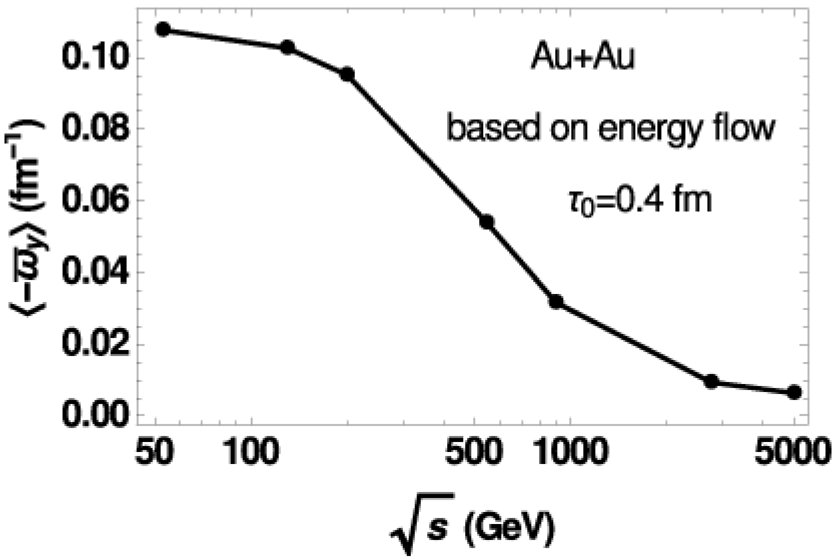
•**Total angular momentum increases with energy, but vorticity at zero rapidity decreases with energy. Reason: with energy grows, moment of inertia increases quickly; more AM carried by finite rapidity particles**

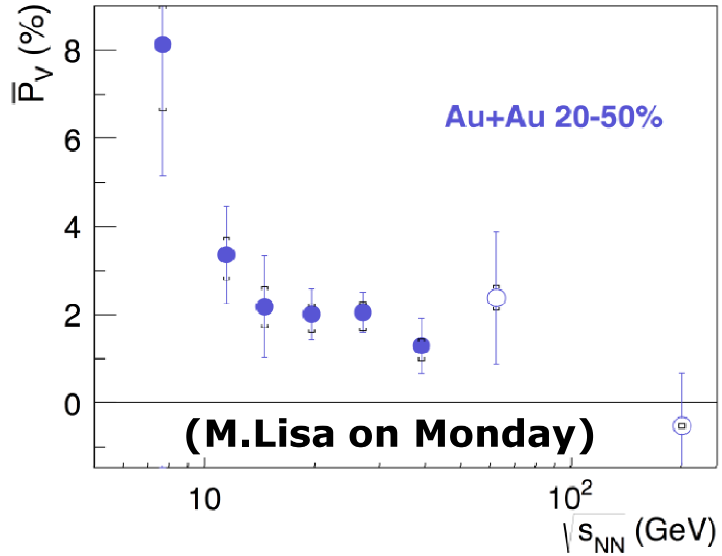
•**Higher collision energy, closer to Bjorken, thus smaller vorticity**

•**Indicates stronger chiral vorticty effect at lower energy (in addition to the fact that mu is also larger for lower energy)**

Vorticity in HICs

Collision energy dependence





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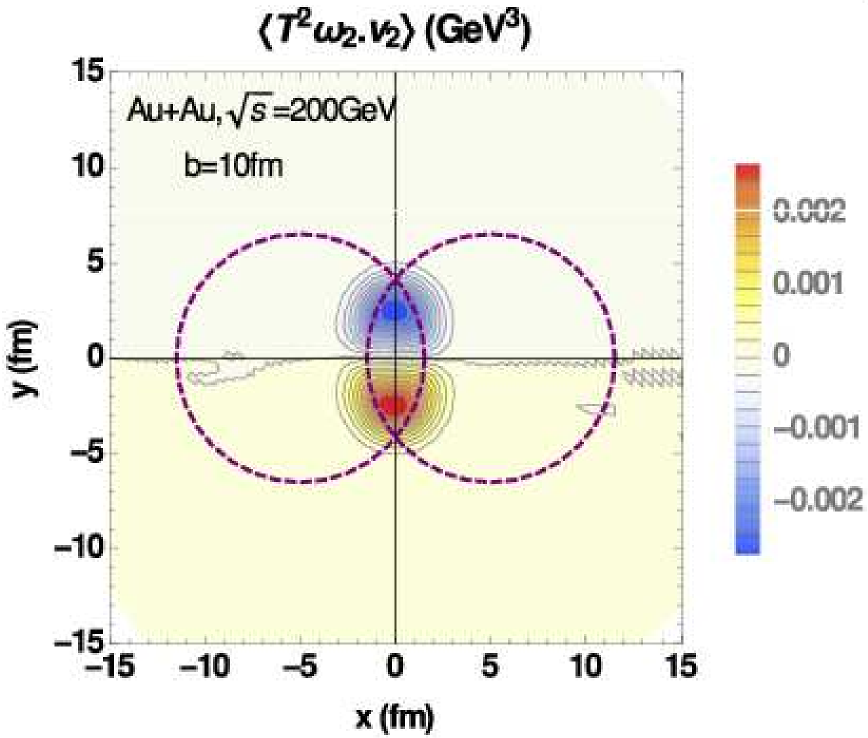
Vorticity in HICs

Flow Helicity distribution



- **Helicity separation w.r.t the reaction plane**

- **Without anomaly, under ideal relativistic hydro. Eq.:**

$$\frac{d}{dt}\int d^3\mathbf{x}T^2\vec{v}\cdot\vec{\omega}_2 = 0$$

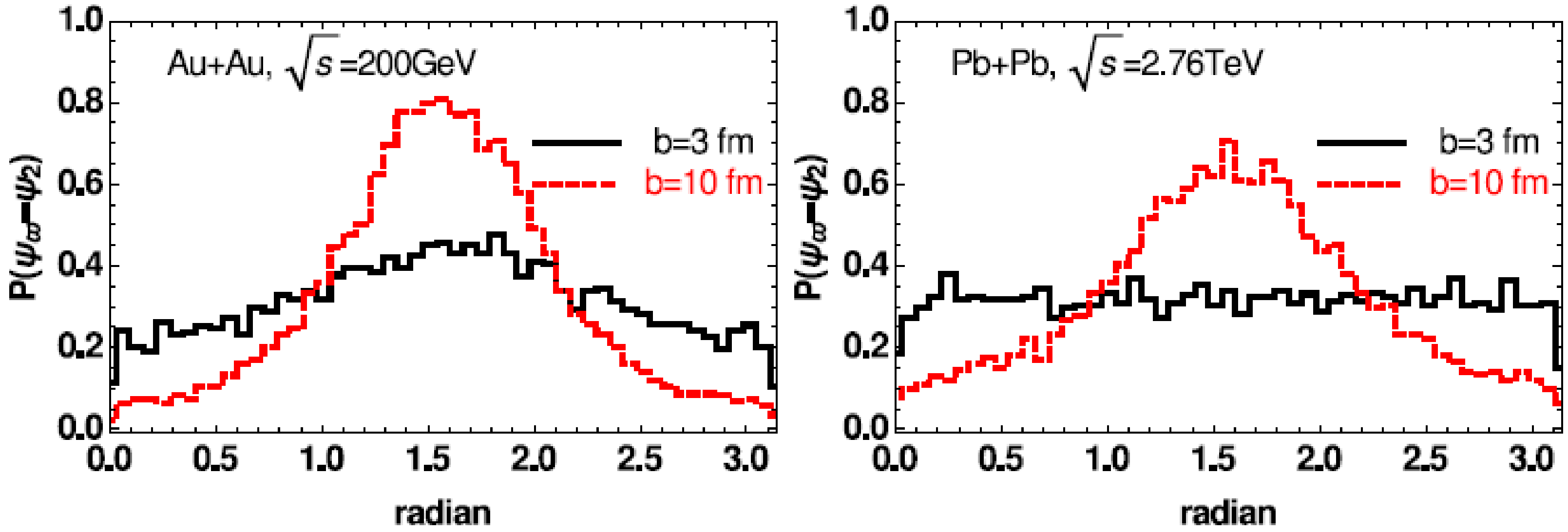
- **In anomalous hydro:**

$$\frac{d}{dt}\int d^3\mathbf{x}T^2\vec{v}\cdot\vec{\omega}_2 = \frac{12}{N_c}\frac{d}{dt}\int d^3\mathbf{x}n_5$$

- **A mechanism to generate fermion chirality and mu_5**
- **Should enter anomalous hydro**

Vorticity in HICs

Event-by-event fluctuation of vorticity orientation



- **Shown is histogram of azimuthal angle of vorticity relative to participant plane (PP)**
- **Clear event-by-event fluctuation in vorticity orientation**
- **For small b, fluctuation so strong that correlation with PP is lost**
- **Large b, Gaussian around pi/2**

Vorticity in HICs

Such fluctuation can strongly influence vorticity driven effects, e.g., chiral vortical effect.

Consider the experimental measured correlation:

$$\gamma_{\alpha\beta} = \langle \cos(\phi_\alpha + \phi_\beta - 2\psi_2) \rangle$$

CVE induced two particle distribution:

$$f_{\alpha\beta}^{\mathrm{CVE}} \propto \boldsymbol{\omega}^2 \cos(\phi_\alpha - \psi_\omega) \cos(\phi_\beta - \psi_\omega)$$

$$\Rightarrow \quad \gamma_{\alpha\beta} \propto \langle \boldsymbol{\omega}^2 \cos[2(\psi_\omega - \psi_2)] \rangle$$

If no fluctuation:

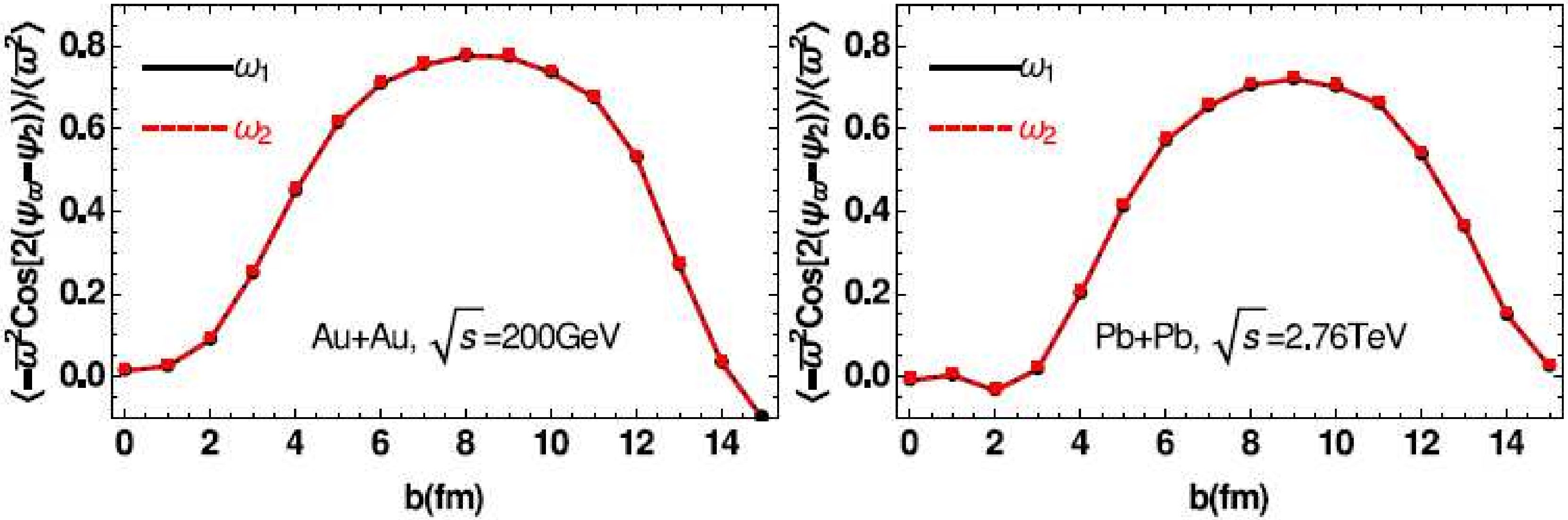
$$\gamma_{\alpha\beta} \propto \langle \boldsymbol{\omega}^2 \rangle$$

Thus this correlation quantifies azimuthal fluctuation:

$$R_2 = \frac{1}{\langle \bar{\omega}^2 \rangle} \langle \bar{\omega}^2 \cos[2(\psi_\omega - \psi_2)] \rangle$$

Vorticity in HICs

Azimuthal correlation between vorticity and participants



•Vorticity of particle flow and energy flow show same correlation

•At very central and very peripheral, small correlation. Reason: either vorticity or participant angle fluctuates strongly.

•Very little dependence on collision energy, as it is geometry dominated.

•Strongest correlation at b~8-9 fm. Suppression factor ~ 0.8.

Vorticity in HICs

Time evolution (qualitative argument)

(Nonrelativistic) Vorticity equation:

$$\frac{\partial \omega}{\partial t} = \nabla \times (v \times \omega) + \nu \nabla^2 \omega$$

with kinematic shear viscosity:

$$\nu = \eta/(\varepsilon + P) = T^{-1}(\eta/s)$$

- **Reynolds number:** $\mathrm{Re} = UL/\nu$
- **If Re<<1 with initial profile** $\omega(0, x) = \omega_0 e^{-x_\perp^2/\sigma_r^2}$

$$\frac{\partial \omega}{\partial t} = \nu \nabla^2 \omega \Longrightarrow \omega(t, x) = \omega_0 \frac{\sigma_r^2}{\sigma_r^2 + 4\nu t} \exp\left(-\frac{x_\perp^2}{\sigma_r^2 + 4\nu t}\right)$$

- **Decay slowly for t < sigma^2/4v**

Vorticity in HICs

Time evolution (qualitative argument)

- If Re>>1 with initial profile

$$\frac{\partial \omega}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{\omega})$$

- Vortex line is frozen in the fluid. Vorticity will decay due to QGP expansion.
- Suppose longitudinal Bjorken expansion and transverse Gaussian initial entropy distribution caused transverse expansion

$$s(\boldsymbol{x}_\perp) = s_0 \exp\left(-\frac{x^2}{2a_x^2} - \frac{y^2}{2a_y^2}\right)$$

- Vorticity decays:

$$\omega_y(t, \boldsymbol{x}) = \frac{t_0}{t} \exp\left[-\frac{c_s^2}{2a_x^2}(t^2 - t_0^2)\right] \omega_y(t_0, \boldsymbol{x}_0)$$

- For t< 7 fm, inversely proportional to t

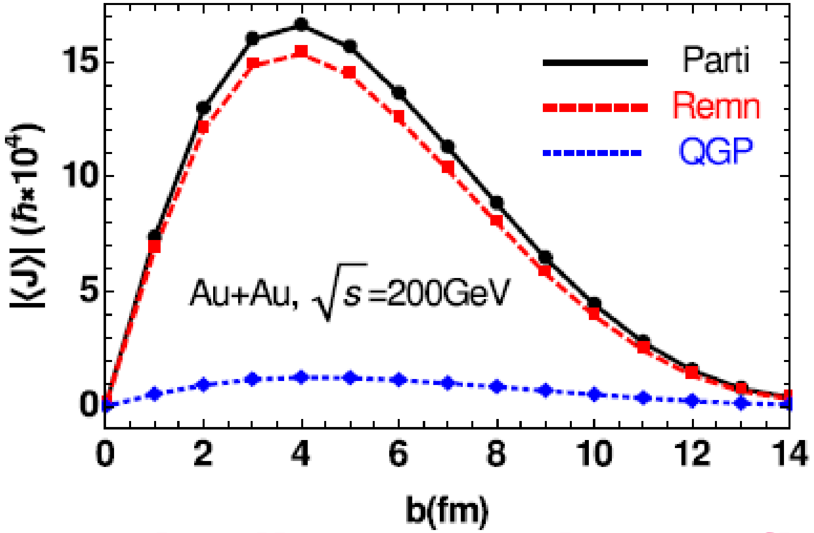
Summary

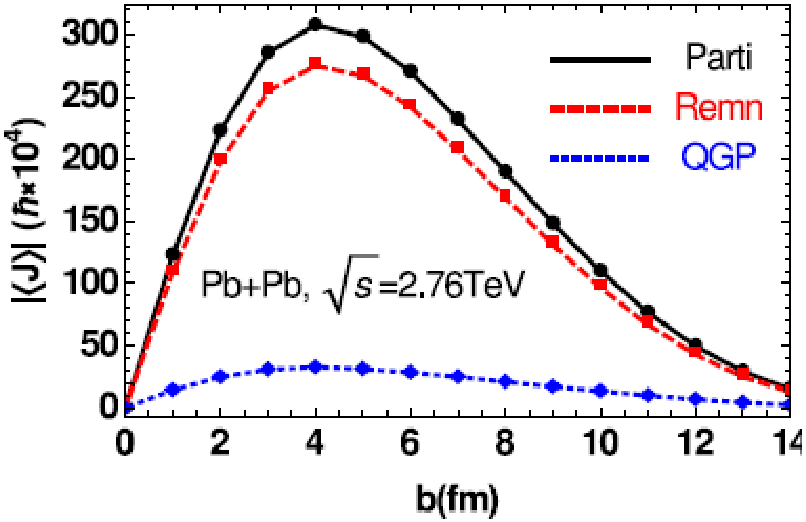
- Noncentral heavy-ion collisions generate flow shear and vorticity
- The vorticity increases with centrality (for b<2R) but decreases with collision energy
- The vorticity orientation suffers from strong event-by-event fluctuation
- Flow helicity separate w.r.t the reaction plane
- (Velocity profile, vorticity spatial distribution, rapidity distribution, etc, in 1603.06117)

Thank you very much for your attention

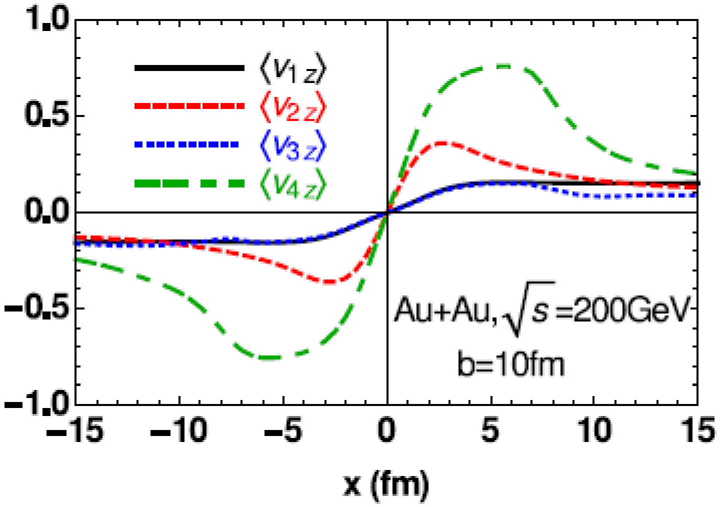
Vorticity in HICs

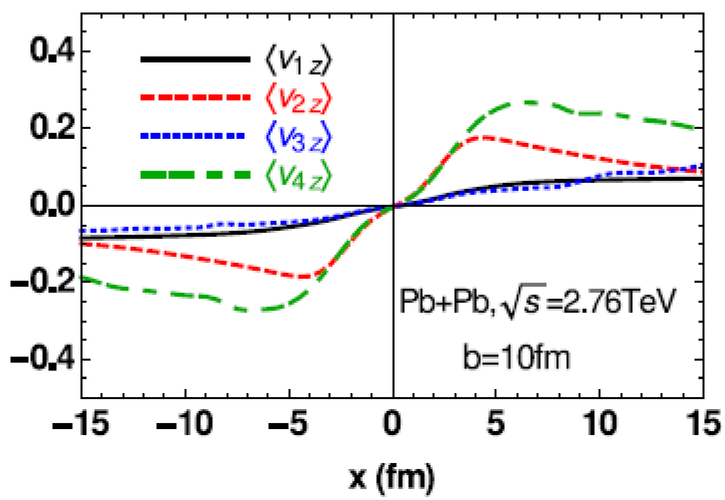
Anglular momentum of the overlapping region





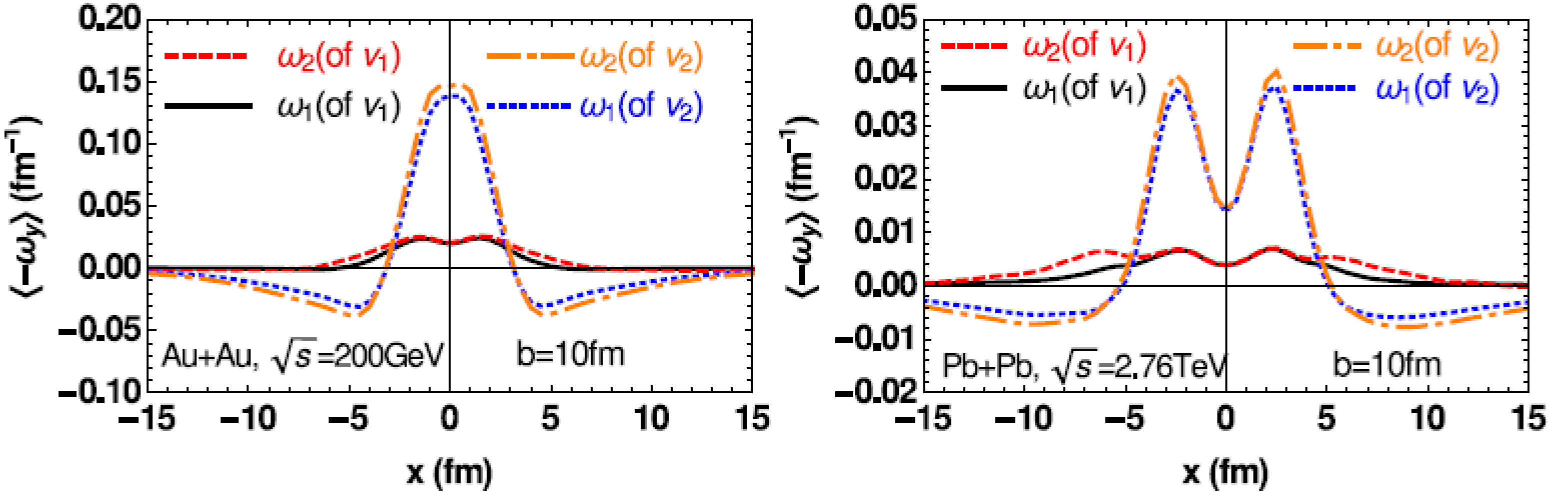
Longitudinal velocity profile





Backup

Spacial distribution



Rapidity dependence