




Analysis of hadron yield data within HRG model with multi-component eigenvolume corrections

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In collaboration with Horst Stoecker
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Strangeness in Quark Matter 2016

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FIAS Frankfurt Institute
for Advanced Studies 

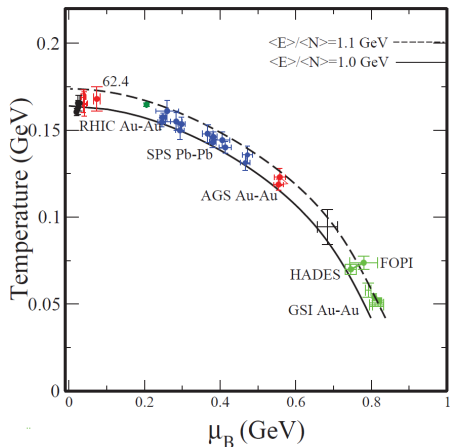
GOETHE
UNIVERSITÄT
FRANKFURT AM MAIN 

HGS-HiRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

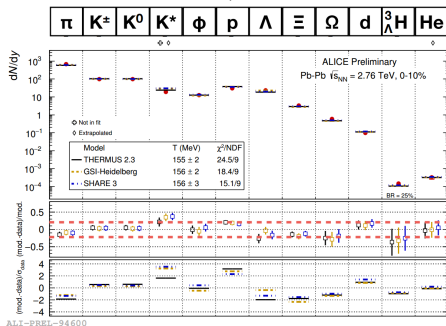
Chemical freeze-out curve

Freeze-out parameters from χ^2 fits within HRG model

Cleymans et al. PRC (2006); Andronic et al. NPA (2006); Becattini et al. PRC (2006).



ALICE Pb+Pb $\sqrt{s_{NN}} = 2.76 \text{ TeV}$



ALICE Collab., SQM2015, QM2015

Chemical freeze-out in HIC mapped on QCD phase diagram but ...

- How **robust** are the conclusions based on **ideal gas**?
- Is there really a **sharp** freeze-out with well-defined temperature?

Interacting hadron gas

- In realistic hadron gas there are attractive and **repulsive** interactions
- Attraction already included by resonances
- Model repulsive interactions by **eigenvolume** correction
- **Van der Waals** procedure: $V \rightarrow V - vN$

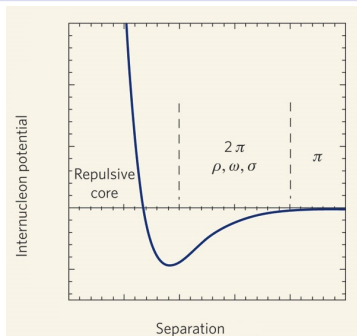
$$P = \frac{NT}{V - vN}$$

In GCE: transcendental equation for pressure¹

$$P(T, \mu) = P^{\text{id}}(T, \mu - vP), \quad n(T, \mu) = n^{\text{id}}(T, \mu^*) / (1 + v n^{\text{id}}(T, \mu^*))$$

In multi-component system $V \rightarrow V - \sum_i v_i N_i$ (“**Diagonal**” EV model)²

$$P(T, \mu) = \sum_i P_i^{\text{id}}(T, \mu_i - v_i P), \quad n_i(T, \mu) = n_i^{\text{id}}(T, \mu_i^*) / (1 + \sum_j v_j n_j^{\text{id}}(T, \mu_j^*))$$



¹D.H. Rischke, M.I. Gorenstein, H. Stoecker, W. Greiner, Z.Phys. C 51, 485 (1991)

²G.D. Yen, M.I. Gorenstein, W. Greiner, S.N. Yang, Phys. Rev. C 56, 2210 (1997)

EigenVolumes: Scenario 0

How to choose eigenVolumes for different hadrons?

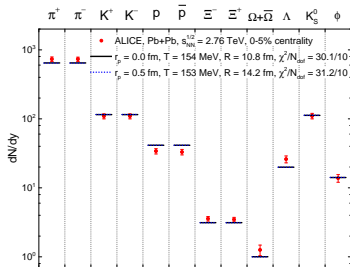
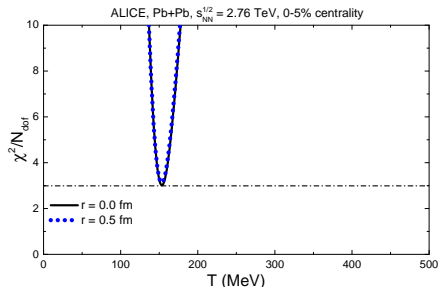
Not many constraints \Rightarrow consider different scenarios

Scenario 0: Constant eigenVolume for all hadrons ($v_i \equiv v$)

In this case in Boltzmann approximation

$$n_i(T, \mu) = \frac{n_i^{\text{id}}(T, \mu_i) e^{-vP/T}}{1 + \sum_j v n_j^{\text{id}}(T, \mu_j) e^{-vP/T}}$$

$$\text{and} \quad \frac{n_i(T, \mu)}{n_j(T, \mu)} = \frac{n_i^{\text{id}}(T, \mu)}{n_j^{\text{id}}(T, \mu)}$$

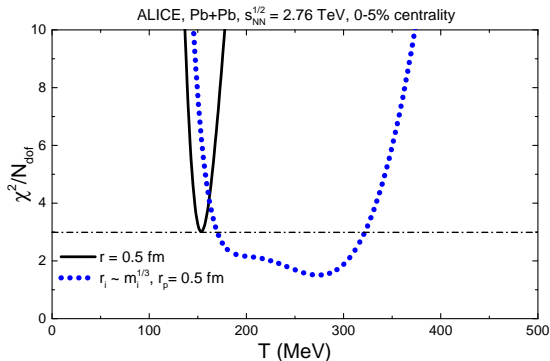


- Simplest and most **commonly used** parametrization
- EigenVolume effects essentially **cancel out** in yield ratios
- **No change** in T or μ_B compared to point-particle case

Eigen volumes: Scenario 1

Scenario 1: Mass-proportional eigen volumes ($v_i = m_i/\varepsilon_0$ or $r_i \sim m_i^{1/3}$)³

- Bag model inspired
- Obtained originally for heavy Hagedorn states
- Results in stronger suppression of heavier hadrons

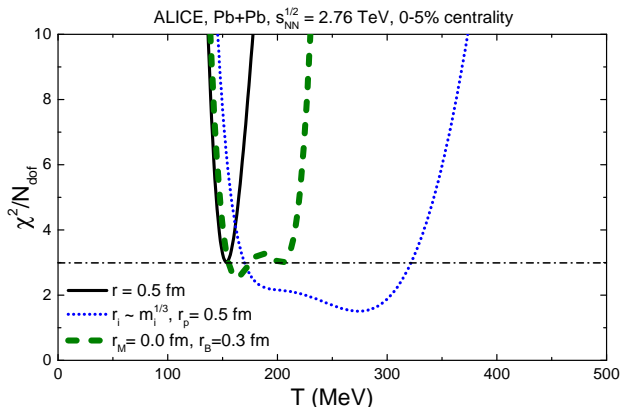


Drastic changes in ALICE χ^2 profile, also high sensitivity on ε_0
For $r_p = 0.5$ fm global minimum at $T \simeq 270$ MeV

³Hagedorn, Rafelski, Phys. Lett. B (1980); Kapusta, Olive, Nucl. Phys. A (1983)

Scenario 2: Two-component model: different volumes for mesons and baryons

We consider particular case $r_M = 0$ and $r_B = 0.3$ fm, has been compared to lattice successfully⁴

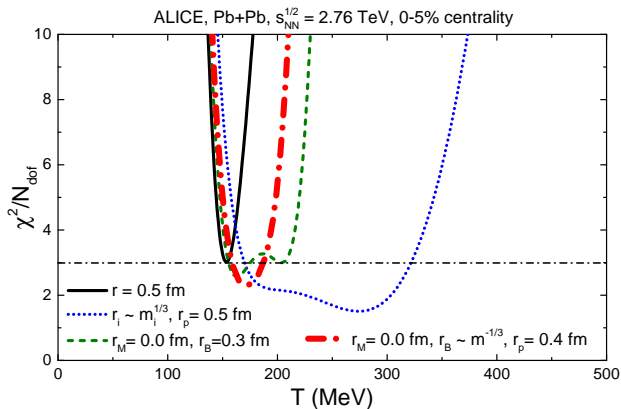


Wide irregular minimum in $T = 155 - 210$ MeV range

⁴A. Andronic, P. Braun-Munzinger, J. Stachel, M. Winn, Phys. Lett. B 718, 80 (2012).

Scenario 3: Point-like mesons and **reverse bag** model for baryons $v_B \sim 1/m$

Strange baryons have generally smaller volumes than non-strange ones



Result: $T_{\text{ch}} = 175 \pm 20$ MeV

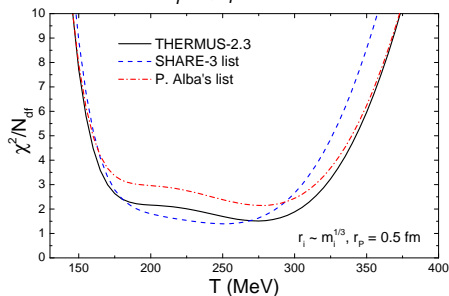
Many other options possible...

Dependence on particle list and decay branching ratios

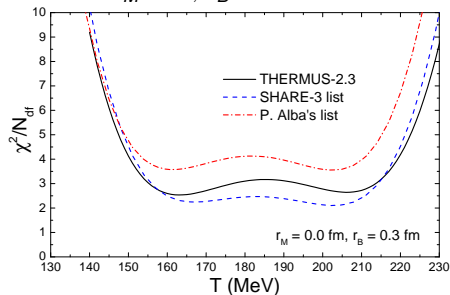
Thermal fits may be sensitive to input particle list

Cross-check: **THERMUS-2.3** and **SHARE-3.0** (publicly available),
and list from P. Alba which includes many **unconfirmed states**

$$r_i \sim m_i^{1/3}, r_p = 0.5 \text{ fm}$$



$$r_M = 0, r_B = 0.3 \text{ fm}$$



χ^2 profile shape is rather insensitive to details of particle list

Picture can be more complicated if light nuclei are considered

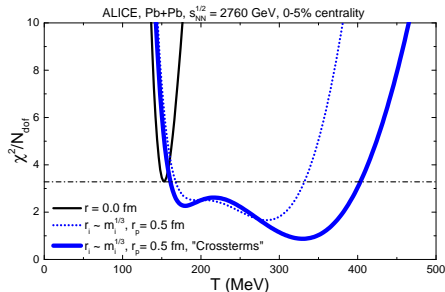
Crossterms eigenvolume model

The “**Diagonal**” EV model we used is not perfectly consistent with **virial expansion** for multi-component system of hard spheres

$$P(T, \{n_i\}) = T \sum_i n_i + \sum_{ij} b_{ij} n_i n_j + \dots \quad \text{with} \quad b_{ij} = \frac{2\pi}{3} (r_i + r_j)^3$$

On the other hand, the “**Crossterms**” eigenvolume model is⁵

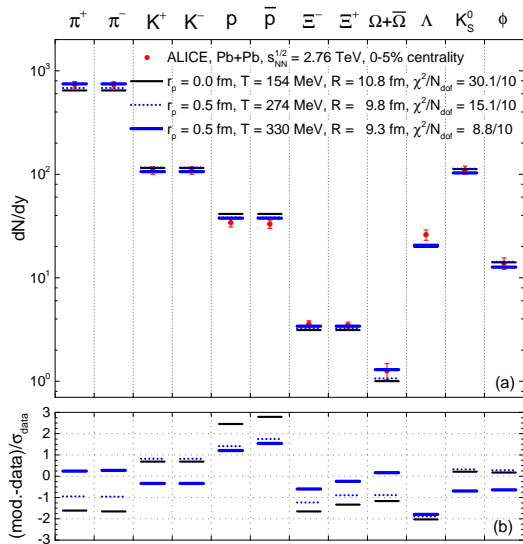
$$P(T, \{n_i\}) = T \sum_i \frac{n_i}{1 - \sum_j v_j n_j}, \quad \Rightarrow \quad P(T, \{n_i\}) = T \sum_i \frac{n_i}{1 - \sum_j \tilde{b}_{ji} n_j},$$



- Scenario 1: $r_i \sim m_i^{1/3}$
- “Crossterms” give even stronger effect
- $\chi^2/N_{\text{df}} : 30/10 \rightarrow 15/10 \rightarrow 9/10$
- $T_{\text{ch}} : 155 \rightarrow 270 \rightarrow 320$ MeV

⁵M.I. Gorenstein, A.P. Kostyuk, Ya.D. Krivenko, J. Phys. G 25, L75 (1999)

ALICE yields within bag-like eigenvolume parametrization



So what about other experiments at lower collision energies?

Finite net-baryon density \Rightarrow additional fit parameter μ_B

Fits to NA49 Pb+Pb 4π data at $\sqrt{s_{NN}} = 6.3, 7.6, 8.8, 12.3,$ and 17.3 GeV,
and STAR Au+Au dN/dy data at $\sqrt{s_{NN}} = 200$ GeV

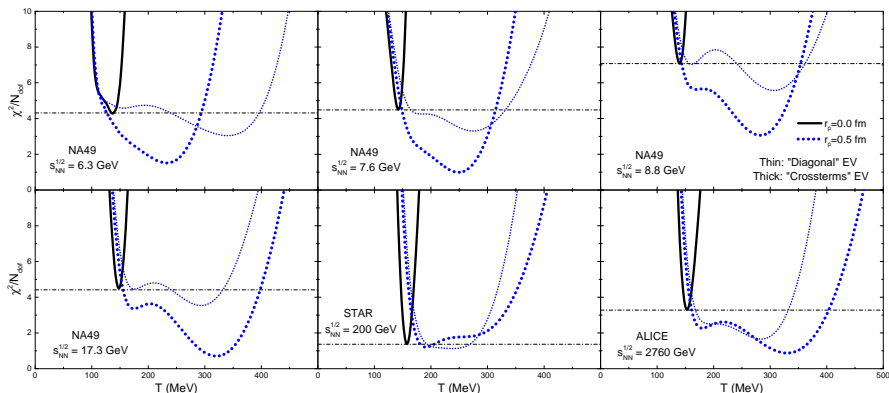
χ^2 profile at lower energies

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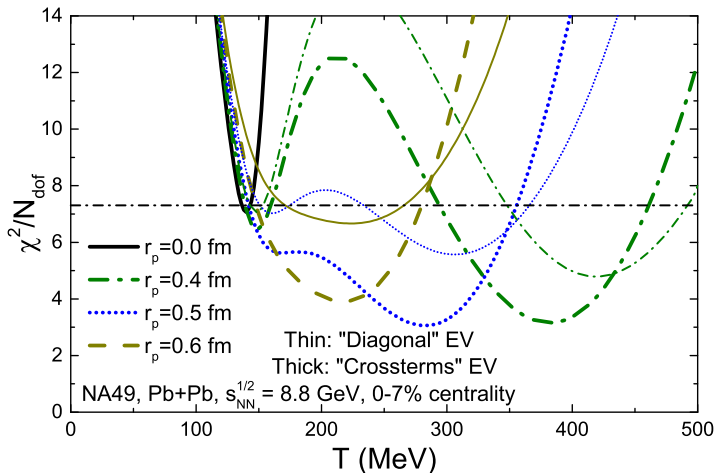
“Crossterms” model with $r_i = r_p (m_i/m_p)^{1/3}$ and $r_p = 0.5$ fm



All the same effect, improved χ^2 , huge sensitivity

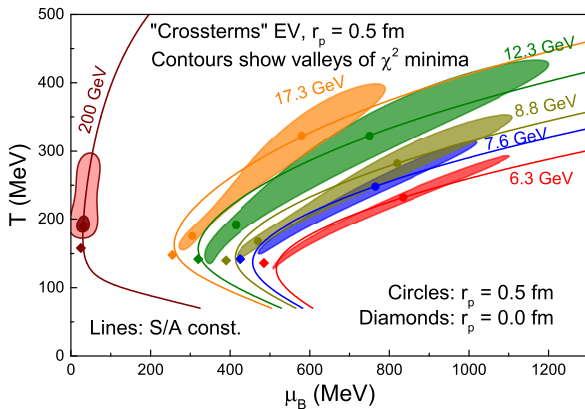
χ^2 profile at lower energies

“Crossterms” model with $r_i = r_p (m_i/m_p)^{1/3}$ and $r_p = 0.4, 0.5, 0.6$ fm



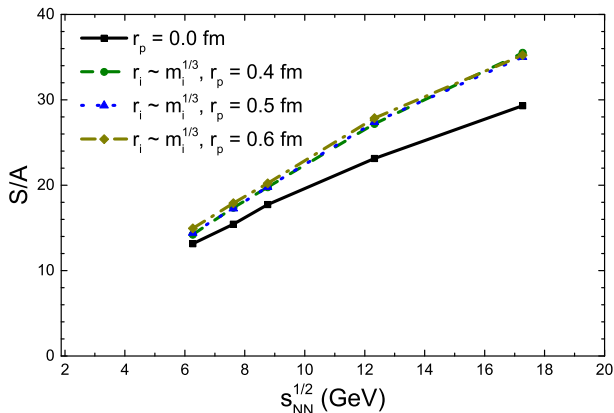
Huge sensitivity on r_p

The $T-\mu_B$ dependence gives a more complete picture



- Conclusions based on point-particle HRG are not robust
- T and μ_B are clearly correlated
- Entropy per baryon S/B approx. constant along valleys of χ^2 minima
- Compatible with **isentropic** expansion and **continuous** freeze-out?

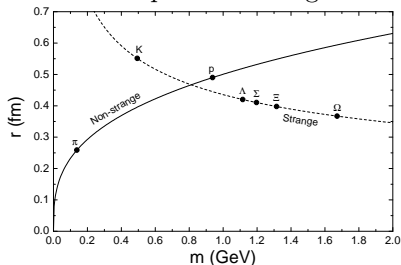
Entropy per baryon excitation function



- S/A at global minima are same for different finite values of r_p
- Entropy per baryon is a **robust** observable
- On the other hand E/N is NOT constant, 1.2-1.5 GeV in EV-HRG
- Interpretation of results prone to controversy, needs further studies

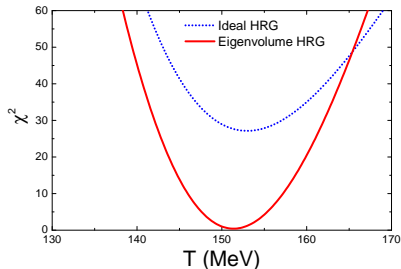
Flavor-dependent eigenvolumes

Next step: different eigenvolumes for strange and non-strange hadrons



- χ^2/N_{dof} : 27.1/8 \rightarrow 0.42/6
- Remarkably small χ^2
- No dramatic change in T
- Same for all other centralities

- **Smaller** volumes for **strange** particles
- Non-strange: $v_i \sim m_i$
- Strange: $v_i \sim 1/m_i$ (reverse!)
- Normalization from best fit to ALICE data



P. Alba, V. Vovchenko, M.I. Gorenstein, H. Stoecker, arXiv:1606.06542

- Thermal fits are **extremely sensitive** to eigenvolume interactions
- Chemical freeze-out parameter values from **ideal HRG** are **not unique**
- **Entropy per baryon** is a robust observable, E/N is not
- Mass-proportional eigenvolumes improve agreement with data and lead to generally wider and irregular χ^2 minima. Obtained results hint on **isentropic expansion** and continuous chemical freeze-out
- **Flavor-dependent** eigenvolumes lead to essential improvement with data
- Proper **restrictions** on eigenvolumes are really needed!

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Thanks for your attention!

Backup slides

Some details about implementation

- Own implementation of eigenvolume HRG written in C++ is used
- Solves eigenvolume models, also many other features
- Auxiliary tool: GUI written within Qt framework
- Where possible cross-checked with other codes

Thermal model calculations

Database file: D:/THERMUS23/Database.dat Load database...

Thermal model Fit to experiment Energy dependence Contour plots Event generator

Name	PDG ID	Mass	Stable?	Baryon?	Neutral?	Charge	Strangeness	Charm	Prim. density	Prim. multiplicity	Tot.
1 pi0	111	0,13498	*		*				0,0221724	92,8755	228
2 pi+	211	0,13957	*			+1			0,0200586	84,0212	196
3 pi-	-211	0,13957	*			-1			0,023665	99,1277	222
4 K+	321	0,49368	*			+1	+1		0,00701418	29,3809	40,4
5 K-	-321	0,49368	*			-1	-1		0,00201722	8,44971	13,5
6 K0S	310	0,49767	*				(hidden)	0	0	0	27,1
7 K0L	130	0,49767	*				(hidden)	0	0	0	27,1
8 K0	311	0,49767	*				+1	0,00739406	30,9721	41,7	
9 K0bar	-311	0,49767	*								
10 eta	221	0,5473	*								
11 rho+	213	0,7711	*								
12 rho-	-213	0,7711	*								
13 rho0	113	0,7711	*								
14 omega(782)	223	0,78257	*								
15 f0(600)	9000221	0,8	*								
16 Kstar(892)+	323	0,89166	*								
17 Kstar(892)-	-323	0,89166	*								
18 Kstar(892)0	313	0,8961	*								
19 Kstar(892)0bar	-313	0,8961	*								
20 p	2212	0,93827	*	*							
21 pbar	-2212	0,93827	*		*						
22 n	2112	0,93956	*	*							
23 nbar	-2112	0,93956	*		*						
24 eta(958)	331	0,95778	*				0,66667 (hidden)	0,000203562	0,852678	0,93	
25 f0(980)	9010221	0,98	*					0,000184701	0,773676	0,78	

Show only stable particles Show calculation results...

Model:

Ideal HRG EV-HRG CE-HRG S-CE-HRG C-CE-HRG

Statistics:

Boltzmann Quantum

Hadron radius (fm): 0,30

Parameters:

T (MeV): 125,00 γ_5 : 1,00 R (fm): 10,000

μ_1 (MeV): 450,00 μ_2 (MeV): 86,81 μ_3 (MeV): -9,39

B: 2 S: 0 Q: 2

strio: 0,400

ite resonance width Renormalize branching ratios

kellization:

openMP

ulate Write to file...

mark:

aryon density = 0,080138 fm⁻³

aryon number = 332,256

lectric charge = 134,902

trangeness = 2,97647e-09

harm = 0

= 0,4

= 1,65152e-11

= nan

Total scaled variance = 1

Calculation time = 81 ms

pi-

Information:

Name = pi-

PDG ID = -211

Mass = 139,57 MeV

Type = Meson

Stable? = Yes

Neutral? = No

Spin degeneracy = 1

Electric charge = -1

Strangeness = 0

Charm = 0

Strangeness content = 0

Charm content = 0

Hard core radius = 0,5 fm

Particle production:

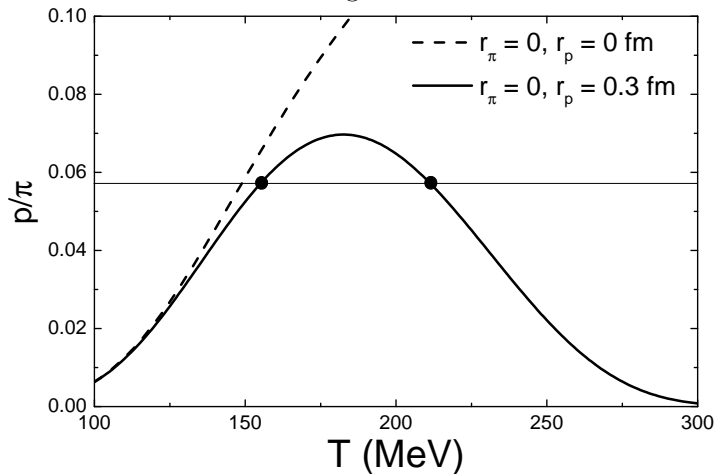
Primordial density = 0,023665 /fm³

Primordial multiplicity = 99,1277

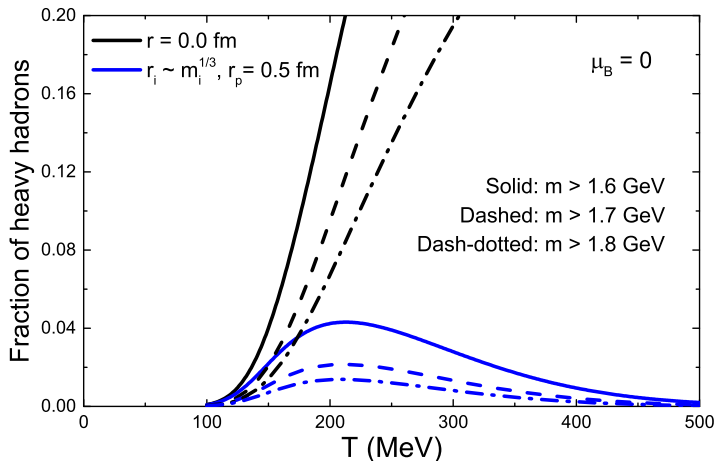
Total multiplicity = 222,491

Source	Multiplicity	Fraction (%)
Direct thermal	99,1277	44,5535
Decays from thermal Delta(1232)-	21,8199	9,80705
Decays from thermal rho-	11,3422	5,09779
Decays from thermal rho0	10,5203	4,7284
Decays from thermal omega(782)	7,31558	3,28803
Decays from thermal Delta(1232)0	6,79205	3,05272
Decays from thermal Kstar(892)0	5,27301	2,36998
Decays from thermal Delta(1600)-	5,23363	2,35229

Origin of two local minima: non-monotonic dependence of individual ratios due to eigenvolumes

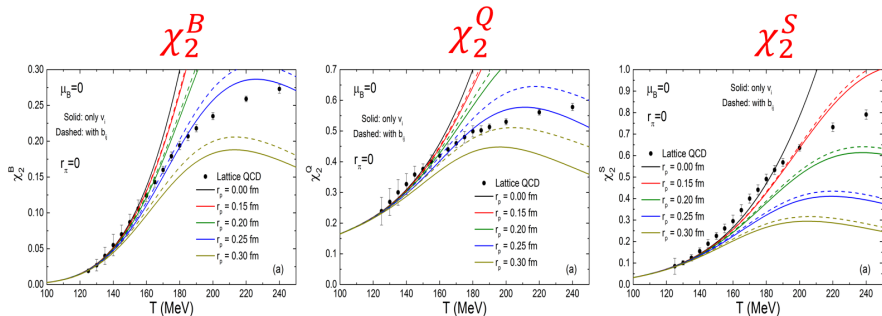


Heavy baryons contribution



Hagedorn divergences are tamed within eigenvolume model
Limiting temperature may be artefact of using point-particle gas

Susceptibilities in eigenvolume HRG



Strangeness susceptibility behave differently from baryon and electric charge
Hint at flavor dependence of eigenvolumes?